DO AGGREGATE MEASURES OF MISMATCH MEASURE MISMATCH?
A TIME SERIES ANALYSIS OF EXISTING CONCEPTS

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Abstract:

This paper discusses the performance of popular mismatch indexes proposed by Lilien (1982), Jackman and Roper (1987), Jackman, Layard and Savouri (1991), and Lambert (1988). Results in the literature show that, with the exception of Lambert's index, the measures of mismatch reveal decreasing or at least stable mismatch for European labour markets. This contradicts micro evidence which suggests declining mobility. Some time series analysis tackles this puzzle, and shows that indices consisting of aggregate time series may lead to false conclusions. Measures of mismatch fail when time series shift upward: Without changing the *relative structure* between individual groups (regions, skills, occupation), this paper shows that trending time series imply a decreasing mismatch for all but one index. The only exception is Lambert's *rho*. Here stochastic trends lead to a spurious *increase* of mismatch.

Keywords: Mismatch, labour mobility, time series analysis

JEL classification: J60, C22, C43

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1. Introduction

Many researchers explained the persistent and high level of unemployment in Europe as a result of "mismatch". They argue that these economies do not have the flexibility to match their unemployed with available employment opportunities. It is widely believed that recent episodes of economic history including the two OPEC oil shocks have caused adjustment processes including significant shifts in employment across industrial sectors, skills and regions. Moreover, during the late seventies and eighties the introduction of new technologies suggested an increasing pace of job reallocation, leading to an increasing mismatch among different skill groups, and - since new technologies are often concentrated in a few regions - across different geographical areas.

Surprisingly, existing empirical measures of mismatch indicate little, if any increase in mismatch during the 1980s. As presented in Jackman, Layard and Savouri (1991), Layard, Nickell and Jackman (1991) and country papers edited by Padoa-Schioppa (1991), recent international evidence even suggests the opposite: Before 1975, in many cases measured mismatch was higher than afterwards (see Abraham (1991) for a survey of the international evidence). In contrast, the country papers in Drèze and Bean (1990) show that measures of mismatch based on rationing models indicate a steady increase in mismatch after the first OPEC oil price shock.

Stochastic trends and/or global shifts in unemployment are natural suspects to explain this findings. The purpose of this paper is to see whether the performance of the most prominent measures of mismatch, namely those presented by Jackman and Roper (1987), Jackman, Layard and Savouri (1991), Lilien (1982) and Lambert (1988), might be determined by spurious factors arising from "trending" time series.

The results reveal that conventional measures of mismatch depend on the nature of the underlying time series of employment, unemployment and vacancies. The puzzling existence of contradictory results can be explained by the nonstationarity of the underlying time series.

The paper is organized as follows. Section 2 surveys some empirical evidence of measured mismatch and contrasts micro and macro results. An overview of mismatch indicators is given in Section 3. The time series analysis of the indexes of mismatch follows in Section 4. Section 5 offers a few concluding remarks.

2. Some puzzling evidence.

Measuring mismatch is a popular topic in empirical economics and in economic policy advice. However, as pointed out by Abraham (1991), the existing studies do not reveal a clear pattern of mismatch after 1975. Abraham expresses a reluctance to accept the evidence concerning skill mismatch: She concludes "*... that given all of the problems that stand in the way of constructing a believable skill mismatch indicator I am unwilling, in spite of the lack of positive evidence, to conclude that skill mismatch has in fact not worsened*" (Abraham, 1991, p. 478). With respect to regional mismatch, in her opinion the empirical evidence is less ambigous and thus she concludes that increasing geographical mismatch does not seem to be a general problem.

With regard to Gemany, one may also believe that regional mismatch has increased in the 1980s. Evidence of German time series does not show a clear trend, as it is given by estimated measures of mismatch in Jackman, Layard and Savouri (1991), Franz and König (1986) and Franz (1991). Franz (1991), for instance, using 141 regional labour market districts and the concept proposed Jackman and Roper (1987) (see below), detects an upward shift between 1976 and 1979 but no clear-cut positive or negative trend thereafter. Franz concludes that " *... regional mismatch does not seem to be able to contribute much to the outward shift of the Beveridge curve ...*".

If mismatch has to do with insufficient mobility, as most people believe (see, for instance, Barro, 1988; see also the survey in Section 3), then Table 1.a provides some micro evidence consistent with the view of a *worsened* matching process which is hindered by decreasing mobility. Comparing the time periods 1955-1970 and 1971-1985, a study of the German employment institute (IAB, Institut für Arbeits- und Berufsforschung) reveals a significant decline in regional labour mobility: For the group of all workers the share of people moving for professional reasons declined from 17.2% to 10.4% (male workers, for women the source only contains the results from a survey in 1985). Mobility increases with qualification. Distinguishing between employed and unemployed people, we observe that the largest drop (in terms of the share 1985/1970) occured for the lowest skill category of employed workers.

A possible explanation of reduced worker mobility is increasing home ownership, including low-skilled workers. Bover, Muellbauer and Murphy (1990) suggest the problem that high house prices might lead to a "mobility trap", i.e. a reduction of mobility due to local and financial commitments. Official statistics, summarized in Table 1.b, are consistent with this hypothesis:

While the number of households in rented houses, flats etc. remained more or less stable, the number of owner occupied dwellings and owner-occupied houses increased steadily. The ratio between both was 55.6% in 1972 and it had increased to 70.6% in 1985.

German time series evidence of occupational mismatch (Franz, 1991, p. 119, using 327 professions and the measure by Jackman and Roper, 1987) shows a more or less constant mismatch between 1976 and 1982 and a sharp drop during the following years. Again, micro evidence suggests the opposite. The share of people never changing occupation rose from 62.8% in 1979 to 72.8% in 1986. Recent anecdotal evidence of structural changes due to new technologies, oil price shocks and other disturbing influences leads many economists to thinking that the coincidence of such adjustment processes and the increasing reluctance to change occupations resulted in an *increase* of mismatch (Abraham, 1991, for instance).

Table 1.: Some evidence on labour mobility

a) Regional mobility: Share of employed workers who changed their home for professional reasons during the years 1955-1970 and 1971-1985, in %

	Men			Women
Group	1970	1985	Ratio 85/70	1985
All workers	17.2	10.4	0.6	7.7
Unskilled workers ("Hilfs-,angelernte Arbeiter)				
- unemployed	17	14	0.82	11
- employed	12	5	0.42	3
Skilled workers ("Fach-,Vorarbeiter, etc.")				
- unemployed	13	11	0.85	8*
- employed	11	5	0.45	3
Low and medium ranked administrative and managerial employees ("einfache und mittlere Angestellte")				
- unemployed	29	13	0.45	10
- employed	19	10	0.53	8
High ranked administrative and managerial employees ("gehobene und leitende Angestellte")				
- unemployed	43	35*	0.81	18*
- employed	33	20	0.61	15

Table 1.a is based on two surveys performed by the German "Institut für Arbeitsmarkt- und Berufsforschung" (IAB) in 1970 and 1985, containing 60973 men in 1970, and 8177 men and 5304 women in 1985

* Shares are calculated using less than 100 observations

Source: Institut für Arbeitsmarkt- und Berufsforschung der Bundesanstalt für Arbeit (1988), p.130-131

b) Living in owner-occupied houses and owner-occupied dwellings

Year	Owner-occupied residence	Others (rented house, flat, etc.)	Ratio "Owning/lodging"
1972	7.5	13.5	55.6
1982	9.3	13.9	66.9
1985	9.6	13.6	70.6

Source: Statistical Yearbook (Statistisches Jahrbuch), various issues

c) Skill mobility: Share of the German labour force that changed its occupation - in %.[1)]

	1979	1985/86
Never changed occupation	62.8	72.8
One change of occupation	24.8	18.9
More than one change	12.4	8.3

1) The share is calculated using the response to the question: "After finishing your school or your professional education did your professional activity change once or more than once to such a degree that it could be refered to as a change of occupation ("Berufswechsel")?

Source: Zentralarchiv für empirische Sozialforschung - ZA Studie 1243 (1979, p.325), 29769 observations, and ZA Studie 1790 (1985/1986, p.77), 26361 observations

A time series example and international cross section data published in Padoa-Schioppa (1991) highlight the puzzling relationships between the level of unemployment and the level of mismatch. Among the measures of mismatch presented in this book, the longest time series available is the one by Bentolila and Dolado (1991, p. 191, 1962-1989). Furthermore, Jackman, Layard and Savouri (1991) present a cross section consisting of measures of mismatch by occupation for 11 nations in 1987.

Figure 1 displays the striking negative correlation between the time path of Spanish unemployment and the index of regional mismatch (according to the definition of Jackman, Layard and Savouri, 1991). After Franco’s death and the start of new political institutions in 1975, the negative correlation was almost perfect. In terms of Pearson’s correlation coefficient, the correlation coefficient for the whole period 1962 - 1986 is -0.84 (t-value = 7.4).

[Figure 1 about here]

Jackman, Layard and Savouri (1991, Table 2.3, see also Layard, Nickell, Jackman, 1991, p.288) calculate a mismatch indicator using the variance of relative unemployment rates and apply it to various countries using data for 1987. At a first glance, one again gets the impression that mismatch is more or less the inverse of unemployment. The calculation of Spearman’s rank-correlation confirms that conjecture: The coefficient is -0.47; taking a more robust trimming approach that deletes outliers (here: maximum and minimum of both unemployment and mismatch) leads to a highly significant negative correlation of -0.73.

A further example is provided by Lilien's index of interindustrial dispersion calculated by Flanagan (1987, see also Franz, 1991). This index takes the following values: 2.64 in 1960-64, 3.21 in 1965-69, 3.21 in 1970-74, 2.29 in 1975-79 and 1.85 in 1980-83. During these periods, average unemployment rates were (in percent) 0.8, 1.1, 1.2, 4.4 and 6.7, respectively.

In contrast, the country papers in Drèze and Bean (1990) provide estimates of mismatch based on a macroeconomic rationing model (Sneessens and Drèze, 1986, Lambert, 1988). For all European countries and for the U.S., the index of mismatch increases linearly over time and reaches its maximum at the end of the observation period (1986). Bentolila and Dolado (1991) also present some estimated indexes of mismatch based on this concept they and confirm this relationship: Structural unemployment (SURE, see below) increased steadily and reaches the peak at about 10% in 1985.

So far the reason for that is unknown. It is at least surprising that rising unemployment and both falling and increasing indexes of mismatch do exist simultaneously.

3. Aggregate measures of mismatch: An overview.

Several measures of mismatch exist in the literature. The most common ones are discussed in the contributions edited by Padoa-Schioppa (1991). They originated in the papers of Jackman and Roper (1987), Jackman, Layard and Savouri (1991), Lilien (1982) and Lambert (1988).

Jackman and Roper (1987) start their framework by formalizing a general definition provided by Turvey (1977), who defines structural unemployment as existing where "*there is a mismatch betweeen vacant jobs and unemployed workers such that if the latter were available with different skills and/or in different places the level of unemployment would fall"*. To make this measure operational, Jackman and Roper (1987) specify a job hiring function (a matching function in a more recent terminology) H :

$$H_i = H(U_i, V_i), \tag{1}$$

where U_i and V_i are the number of unemployed workers and vacancies in category *i* (sector, skill, region, ...) and H_i is the number of job hires per unit time period.

Following Turvey (1977), Jackman and Roper (1987) define structural unemployment as that sectoral allocation of the existing stock of unemployment which, given the sectoral allocation of vacancies, maximizes aggregate hires subject to $\Sigma_i U_i =$ constant and V_i given.

The solution to this allocation problem is that the ratio of unemployment to vacancies is identical across sectors. This implies $u_1 = v_1, u_2 = v_{2,} \ldots, u_K = v_K$ where $u_i = U_i / \Sigma_i U_i$ and $v_i = V_i / \Sigma_i V_i$. A natural way to define mismatch is thus

$$M_1 = \frac{1}{2}\Sigma_i \mid u_i - v_i \mid, 0 \le M_1 \le 1. \tag{2}$$

Using the matching function (1) in form of a Cobb-Douglas specification, Jackman and Roper (1987) derive a second indicator given by

$$M_2 = 1 - \Sigma_i (u_i v_i)^{1/2}, 0 \le M_2 \le 1. \tag{3}$$

Figure 2 displays some estimates for occupational mismatch in Germany, based on the indexes M_1 and M_2. Both indexes are based on 40 professional classifications of the years 1951 until 1992. M_1 and M_2 have some remarkable time patterns if one compares both time series with unemployment. Until the recession in 1975, mismatch seemed to be procyclical and parallel to unemployment. After 1975, however, mismatch and unemployment have been running in opposite directions.

[Figure 2 about here]

Both M_1 and M_2 require job vacancy data, at a disaggregate level if possible. This is a prohibitive obstacle for many countries, where such data are not available (for instance, the U.S. even lacks *aggregate* vacancy data; therefore, American case studies use the "help wanted" index as an approximation for U.S. job vacancies (see Abraham, 1987)). The advantage of the third index,

$$M_3 = \frac{1}{2} Var\left(\frac{U_i / N_i}{\Sigma U_i / \Sigma N i} \right), \tag{4}$$

proposed by Jackman, Layard and Savouri (1991), is that it is exclusively based on unemployment rates (N_i = number of workers). The authors derive their index from a monopolistic competition framework related to the NAIRU ("non-accelerating inflation rate of unemployment", see Layard and Bean, 1989, for a description of this concept).

A standard index of industrial mismatch is Lilien's (1982) sigma, which is based on employment growth. This index measures the relative dispersion of growth rates among industries:

$$M_4(t) = \left(\Sigma_i \frac{L_{it}}{L_t} (\Delta \log L_{it} - \Delta \log L_t)^2 \right)^{1/2}, \tag{5}$$

where $L_t = \Sigma L_{it}$ is aggregate employment.

Lilien's index is easy to compute because it does not require data on unemployment by industry. The most severe criticism has been raised by Abraham and Katz (1986), who show that Lilien's sigma is not invariant with respect to aggregate demand fluctuations and, as a result, the pure "frictional" component of unemployment cannot be disentangled. Despite this problem, Lilien's sigma is one of the most popular indices of structural change in empirical work (see Brunello, 1991, Franz, 1991, and Morisette and Salvas-Bronsard, 1993, for recent applications).

The last index taken into consideration is Lambert's (1988) *rho*, that is derived in a rationing framework. Each micromarket consists of two components, labour demand, D_{it}, and labour supply, S_{it}. The observed, or transacted, value is the minimum of both latent components, $L_{it} = \min(D_{it}, S_{it})$. Assuming that D_{it} and S_{it} are log-normally distributed (see Smolny, 1993, for some empirical justification), Lambert (1988) shows that aggregate employment can be approximated by a CES function of aggregate components:

$$L_t \cong \left(D_t^{-\rho_t} + S_t^{-\rho_t} \right)^{(-1/\rho_t)}, \tag{6}$$

where $D_t = \Sigma_i D_{it}, S_t = \Sigma_i S_{it}$, and $L_t = \Sigma_i L_{it}$. The parameter ρ_t measures mismatch on the micromarkets, since it is inversely related to the dispersion of micro demands and supplies, more precisely to the difference of microeconomic disturbances (cf. Lambert, 1988, p. 124).

Empirical estimates of Lambert's *rho* are based on equation (6), which can be estimated by nonlinear least squares. To get time-variant indexes, ρ_t is formulated in terms of linear time trends and dummies (see the country papers in Drèze and Bean, 1990) or in terms of explanatory variables (see Entorf, König and Pohlmeier, 1992). An appealing characteristic of *rho* is its interpretation in terms of "structural unemployment at equity", SURE, which measures the amount of unemployment which would arise due to mismatch despite a hypothetical situation of aggregate equity D=S. Equation (6) reveals that such a situation leads to $L_t = 2^{-1/\rho_t} S_t$, and using the definition $UR = 1 - L/S$ gives

$$SURE_t = 1 - 2^{-1/\rho_t}. \tag{7}$$

4. Time series analysis.

Five indices of mismatch are presented in Section 3. The evidence in Figures 1 and 2 and other surprising results in Section 2 suggest that the interpretation of mismatch might be affected by trending employment and unemployment variables. The following analysis summarizes time series properties of M_1, M_2, M_3, M_4 and Lambert's *rho* in the presence of stochastic trends and, where possible, deterministic trends. Since no systematic variation of any particular sector is imposed, it is expected that mismatch indicators do not reveal any significant mismatch trend.

As can be seen from the following propositions, this expectation does not hold for aggregate mismatch indicators. Starting with M_1, the following properties hold:

Proposition 1:

(a) *If*

$$\frac{1}{K^+}\sum_{i\in I^+} U_i > \frac{1}{K^-}\sum_{i\in I^-} U_i$$

with

$$I^+ = \{i \mid u_i > v_i\}, I^- = \{i \mid u_i \le v_i\},$$

$K^+ =$ *number of elements in* I^+ *and* $K^- = K - K^+$, *then* M_1 *decreases in the presence of additive shifts in unemployment measured as* $U_i^* \equiv U_i + q$.

(b) *If sectoral unemployment* U_{it} *as well as sectoral vacancies* V_{it} *are random walks, then* $M_1(t)$ *converges weakly to a Cauchy distributed random variable as* $t \to \infty$.

(c) *If sectoral unemployment follows a random walk with drift* δ, *and if individual vacancies are generated according to a random walk with drift* θ, *then* $M_1(t)$ *tends in probability to zero for large t.*

Proof: See Appendix

Proposition 1.(a) implies that the concept M_1 very likely indicates mismatch if the economy is characterized by a coincidence of a high dispersion of unemployment and a relatively homogeneous set of vacancies. For the limiting case of completely homogeneous vacancies, the following Corollary applies.

Corollary:

Given identical vacancies $V_i = V$ for all i, but at least one i with $U_i \neq U_j, i \neq j$, a positive additive global shift in unemployment will decrease mismatch M_1.

Proof: See Appendix

In case of Jackman and Roper's (1987) alternative proposal, the presence of random walks with drift implies the same asymptotic property:

Proposition 2:

If unemployment and vacancies behave like random walks with drifts δ and θ, respectively, then M_2 converges in probability to zero for large t.

Proof: See Appendix

Jackman, Layard and Savouri's (1991) M_3 can be analysed in the presence of simple deterministic growth:

Proposition 3:

If unemployment U_i of individual groups i is globally shifting upwards by an amount q (such that the new individual level is $U_i + q$ for all i), then the measure of mismatch M_3 decreases.

Proof: See Appendix

A very strong result holds for Lilien's (1982) index. Simple random walks without any drift lead to decreasing mismatch measures:

Proposition 4:

If sectoral employment L_{it} follows a random walk (with or without drift), then Lilien's (1982) sigma (M_4) converges to zero for large t.

Proof: See Appendix

The final concept of mismatch is different from previous measures of mismatch. Lambert's (1988) *rho* is based on disequilibrium theory, its empirical evidence indicates growing mismatch after OPEC I, and Proposition 5 shows that mismatch *increases* if underlying time series reveal stochastic trends:

Proposition 5:

If the components $\ln D_{it}$ and $\ln S_{it}$ of the min-condition $\ln L_{it}$ behave like random walks (with or without drift), then Lambert's (1988) rho approaches zero for $t \to \infty$, i.e. the measure of structural underutilization "SURE" converges to 100% if $t \to \infty$.

Proof: See Appendix

So far, all proofs are based on additively growing time series, either in terms of deterministic growth or in terms of stochastic trends. It should be stressed that potential *multiplicative* increments do *not* affect presented measures of mismatch, as can be seen from the following remark.

Remark: *If the individual components of the indices $M_1 - M_5$, i.e. sectoral unemployment, vacancies or employment series, grow with a multiplicative factor, then measures of mismatch do not change.*

Proof: M_1 and M_2 are based on ratios $u_i = U_i/\Sigma U_i$ and $v_i = V_i/\Sigma V_i$. Thus, the ratios do not change if we replace U_i by $U_i(1+q)$ and V_i by $V_i(1+m)$ since the factors cancel out in the ratio. This also applies to M_3, which is based on the ratio $(U_i/N_i)/(\Sigma U_i/\Sigma N_i)$. Lilien's index uses logged differences (so that the constant factor disappears) and the ratio L_i/L, with the same effect as above. Lambert's *rho* represents the dispersion of microeconomic supply and demand disturbances. Mean values of $\ln D_t$ and $\ln S_t$ do not enter the variance in (7) and, hence, multiplicative changes cannot affect Lambert's *rho.*

5. Concluding remarks

Measuring mismatch is an important issue in policy advice. This paper discusses the performance of five popular mismatch indices in the presence of globally growing unemployment. Measures of mismatch reported in literature do indicate no worsening of mismatch (Padoa-Schioppa, 1991). On the other hand, estimates based on macroeconomic rationing models presented in Drèze and Bean (1990) reveal steadily growing mismatch in Europe. With respect to German data, the paper presents some micro evidence which suggests the same conclusion: Indicators of regional and occupational mobility reflect decreased mobility, most likely contributing to reported lack of skilled labour and high unemployment for wrongly skilled and unskilled workers.

The time series analysis of this paper reveals that conventional measures of mismatch fail when unemployment figures reveal additive upward shifts. The results show that with both deterministic and stochastic shifts, measures of mismatch are likely to decline without any changes in the *relative* structure of sectors, skills or other grouping criteria. In contrast, Lambert's *rho* indicates *increasing* mismatch in such situations, i.e. it is biased in the opposite direction.

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Appendix

Proof of Proposition 1:

(a) Let $U_i^* \equiv U_i + q$ for all i. Then M_1 becomes

$$M_1^*(q) = \Sigma\left|\frac{U_i^*}{\Sigma U_i^*} - \frac{V_i}{\Sigma V_i}\right| = \frac{\sum_{j\in I^+} U_j^*}{\sum_{i=1}^{K} U_i^*} - \frac{\sum_{j\in I^-} U_j^*}{\sum_{i=1}^{K} U_i^*} + \frac{\sum_{j\in I^-} V_j}{\sum_{i=1}^{K} V_i} - \frac{\sum_{j\in I^+} V_j}{\sum_{i=1}^{K} V_i}$$

We define $\Sigma^+ = \sum_{j\varepsilon I^+} U_j, \Sigma^- = \sum_{j\varepsilon I^-} U_j, S \equiv \Sigma_j U_j = \Sigma^+ + \Sigma^-$. The proposition follows if $\partial M_1^*/\partial q < 0$:

$$\frac{\partial M_1^*}{\partial q} = \frac{(K^+ - K^-)(S + Kq) - ((\Sigma^+ + K^+q) - (\Sigma^- + K^-q))K}{(S + Kq)^2}$$

Using $S = \Sigma^+ + \Sigma^-, K = K^+ + K^-$ and collecting terms leads to

$$\frac{\partial M_i^*}{\partial q} = \frac{2(K^+\Sigma^- - K^-\Sigma^+)}{(S + Kq)^2}.$$

Thus,

$$\frac{\partial M_1}{\partial q} < 0 \Leftrightarrow \frac{\Sigma^+}{K^+} > \frac{\Sigma^-}{K^-}.$$

(b) Let

$$U_{it} = U_{i.t-1} + \varepsilon_{it} = \sum_{\tau=1}^{t} \varepsilon_{i\tau} + U_{i0}, \qquad (A1)$$

$$V_{it} = V_{i.t-1} + \nu_{it} = \sum_{\tau=1}^{t} \nu_{i\tau} + V_{i0},$$

where ε_{it} and ν_{it} are generated according to ε_{it}~IID$(0,\sigma^2_{\varepsilon i})$, ν_{it}~IID$(0,\sigma^2_{\nu i})$ with U_{i0} and V_{i0} being non-zero starting values (thus, taking expectated values, unemployment and vacancies could be interpreted as deviation from natural rates U_{i0} and V_{i0}) . *IID* represents any well defined independently and identically distributed random variable.

Exploiting the same decomposition as in (a), we can write

$$M_1(t) = \frac{\sum_{j\varepsilon I^+} U_{jt} - \sum_{j\varepsilon I^-} U_{jt}}{\sum_{i=1}^{K} U_{it}} + \frac{\sum_{j\varepsilon I^-} V_{jt} - \sum_{j\varepsilon I^+} V_{jt}}{\sum_{i=1}^{K} V_{it}}. \qquad (A2)$$

As $t \to \infty$, asymptotic behaviour leads to the following expression for unemployment in category i U_{it} (see Banerjee et al., 1993 for details about the transformation of integrated time series to Wiener processes):

$$t^{-1/2} \sum_{\tau=1}^{t} \varepsilon_{i\tau} \Rightarrow A_{\varepsilon i} \equiv \sigma_{\varepsilon i} \int_0^1 dW(r), \qquad (A3)$$

where $\Rightarrow$ denotes weak convergence. In (A3), $A_{\varepsilon i}$ is a random variable that boils down to a simple normal representation according to the central limit theorem: $A_{\varepsilon i} \sim N(0, \sigma^2_{\varepsilon i})$. Analogously, the asymptotic behaviour of individual vacancy terms is $B_{\nu i} \sim N(0, \sigma^2_{\nu i})$. Dividing numerator and denominator by $t^{1/2}$ leads to the following asymptotic ratio of normally distributed random variables (note that because of dividing by $t^{1/2}$ starting values play no role in the limit):

$$M_1(t) \Rightarrow \frac{\sum_{j\varepsilon I^+} A_{jt} - \sum_{j\varepsilon I^-} A_{jt}}{\sum_{i=1}^{K} A_{it}} + \frac{\sum_{j\varepsilon I^-} B_{jt} - \sum_{j\varepsilon I^+} B_{jt}}{\sum_{i=1}^{K} B_{it}}.$$

Thus, the limiting distribution is distributed as Cauchy and does not have a mean. (Here Slutsky's Theorem does not help in calculating asymptotic means, since both numerator and denominator have zero expectation).

(c) Let

$$U_{it} = \delta_i + U_{t.t-1} + \varepsilon_{it} = t\delta_i + \sum_{\tau=1}^{t} \varepsilon_{i\tau} + U_{i0}$$

$$V_{it} = \theta_i + V_{t.t-1} + \nu_{it} = t\theta_i + \sum_{\tau=1}^{t} \nu_{i\tau} + V_{i0}$$

with ε_{it}, ν_{it}, U_{i0} and V_{i0} defined as above. δ_i and θ_i are individual drift parameters (the role and importance of drift terms in the analysis of random walks is demonstrated in Entorf, 1995).

We write M_1 as follows:

$$M_1(t) = \sum_{i=1}^{K} \left| \frac{U_{it} \sum_{j=1}^{K} V_{jt} - V_{it} \sum_{j=1}^{K} U_{jt}}{\left(\sum_{j=1}^{K} U_{jt}\right)\left(\sum_{j=1}^{K} V_{jt}\right)} \right|. \qquad (A4)$$

Looking at cross terms, for instance $U_{it}V_{jt}$, we see that the asymptotic behaviour is dominated by the product of the two deterministic trends $\delta_i\theta_j t^2$. All other terms grow with lower speed. See, for instance, the product $t\delta_i \Sigma v_{j\tau}$. According to (A3), this term grows with speed $t^{3/2}$.

Thus, dividing numerator and denominator by t^2 leads to the asymptotic result

$$M_1(t) \Rightarrow \sum_{i=1}^{K} \left| \frac{\delta_i \sum_{j=1}^{K} \theta_j - \theta_i \sum_{j=1}^{K} \delta_j}{\left(\sum_{j=1}^{K} \delta_j\right)\left(\sum_{j=1}^{K} \theta_j\right)} \right|.$$

Considering identical drifts $\delta_i = \delta$ and $\theta_i = \theta$ for all i yields the result of Proposition 1 (c).

Proof of Corollary:

Identical vacancies imply $v_i = 1/K$. Since

$$I^+ = \left\{ i \mid U_i / \left(\sum_{i=1}^{K} U_i \right) < \frac{1}{K} \Leftrightarrow U_i > \frac{\Sigma U_i}{K} \right\}$$

$$I^- = \left\{ i \mid U_i / \left(\sum_{i=1}^{K} U_i \right) \geq \frac{1}{K} \Leftrightarrow U_i \leq \frac{\Sigma U_i}{K} \right\},$$

averaging U_i of both sets I^+ and I^- leads to

$$\frac{1}{K^+} \sum_{i \varepsilon I^+} U_i > \frac{\Sigma_i U_i}{K} \geq \frac{1}{K^-} \sum_{i \varepsilon I^-} U_i$$

Proof of Proposition 2:

We consider

$$\Sigma_i (u_{it} v_{it})^{1/2} = \frac{\Sigma_i (U_{it} V_{it})^{1/2}}{(\Sigma_i U_{it})^{1/2} (\Sigma_i V_{it})^{1/2}}.$$

As before, we assume $U_{it} = \delta_i t + \Sigma \varepsilon_{i\tau} + U_{i0}, V_{it} = \theta_i t + \Sigma \nu_{i\tau} + V_{i0}$. Using (A3), we again make use of asymptotically dominating linear drift terms:

$$\Sigma_i (u_{it} v_{it})^{1/2} \Rightarrow \frac{\Sigma_i (\delta_i \theta_i)^{1/2}}{(\Sigma_i \delta_i)^{1/2} (\Sigma_i \theta_i)^{1/2}}.$$

This ratio converges to 1 in the case of common drift terms.

Thus

$$M_2(t) = 1 - \Sigma (u_{it} v_{it})^{1/2} \Rightarrow 0$$

Proof of Proposition 3:

We write M_3 as $M_3 = var(K_i U_i / (\Sigma U_i))$ with $K_i = \Sigma N_i / N_i$; in order to simplify notation, we assume constant weights for all sectors. It follows

$$var\left(\frac{U_i}{\Sigma U_i} K \right) = \frac{K}{(\Sigma U_i)^2} \Sigma U_i^2 - 1. \qquad (A5)$$

Inserting $U_i + q$ for all i and neglecting the constant term "-1" (which is negligible with respect to intended partial derivatives) leads to

$$M_3^* = \frac{\Sigma(U_i + q)^2}{(\Sigma U_i + Kq)^2}$$

Since

$$\frac{\partial M_3^*}{\partial q} = \frac{2(\Sigma U_i + Kq)\,[(\Sigma U_i + Kq)^2 - K\Sigma(U_i + q)^2]}{(\Sigma U_i + Kq)^4)}$$

we obtain

$$\frac{\partial M_3^*}{\partial q} < 0$$

$$\Leftrightarrow \qquad (\Sigma U_i + Kq)^2 < K\Sigma(U_i + q)^2$$

$$\Leftrightarrow \qquad (\Sigma U_i)^2 < K\Sigma(U_i)^2$$

which holds if at least for one pair i,j (i≠j) $U_i \neq U_j$.

Proof of Proposition 4:

We can write

$$\Delta \ln L_{it} \approx \frac{L_{it} - L_{i.t-1}}{L_{it}} \qquad (A6)$$

and

$$\Delta \ln L_t \approx \frac{L_t - L_{t-1}}{L_t}.$$

When employment behaves like a random walk with drift, i.e. $L_{it} = \gamma_i t + \Sigma_\tau \nu_{i\tau}$, it follows from (A6) that

$$L_{it} - L_{i.t-1} = \gamma_i + \nu_{it}$$

and

$$L_t - L_{t-1} = \Sigma_i \gamma_i + \Sigma_i \nu_{it}.$$

According to the definition of Lilien's *sigma* (equation 9), and after taking squared values, we obtain

$$(M_4)^2 = \Sigma_i \left(\frac{\gamma_i + \nu_{it}}{L_{it}} - \frac{\Sigma_i(\gamma_i + \nu_{it})}{L_t} \right)^2 \frac{L_{it}}{L_t}$$

$$= \Sigma_i \frac{((\gamma_i + \nu_{it})L_t - L_{it}\Sigma_i(\gamma_i + \nu_{it}))^2}{L_{it}L_t^3}.$$

Following the same arguments concerning the speed of divergence of $\Sigma_\tau \nu_{i\tau}$ as above and inspecting highest linear trends in both numerator and denominator (which are found as t in L_t and L_{it} since $\Sigma_\tau \nu_{i\tau}$ diverges with speed $t^{1/2}$), the highest trend in the denominator is of order t^4 ($t{*}t^3$), whereas the numerator is only of order t^3. The dominating term $(L_{it}\Sigma\nu_{it})^2$ implies an asymptotic linear time trend $(tt^{1/2})^2$. Thus, for large samples Lilien's sigma goes to zero.

If the drift term γ_i is zero, then the dominating trend term is of order $t^{1/2}$. So the numerator grows with speed $t = (t^{1/2})\,(t^{1/2})$, whereas the denominator grows faster with speed $t^2 = (t^{1/2})\,(t^{3/2})$. Thus, irrespective of random walks contain drifts or not, Lilien's *sigma* converges to zero.

Proof of Proposition 5:

We define both components as random walks with drift:

$$\ln D_{it} = \gamma_d + \ln D_{i.t-1} + \varepsilon_{it}^d = \gamma_d t + \sum_{\tau=1}^{t} \varepsilon_{i\tau}^d$$

$$\ln S_{it} = \gamma_s + \ln S_{i.t-1} + \varepsilon_{it}^s = \gamma_s t + \sum_{\tau=1}^{t} \varepsilon_{i\tau}^s,$$

where both drifts are allowed to be zero. The stochastic components are sums of normally *N(0,1)*-distributed disturbances such that

$$E(\Sigma_\tau \varepsilon_\tau^d) = E(\Sigma_\tau \varepsilon_\tau^s) = 0,$$

$$E(\varepsilon_\tau^d \varepsilon_\tau^s) = r\sigma_d \sigma_s \forall \tau = 1, 2, \ldots, t, E(\varepsilon_\tau^d \varepsilon_j^s) = 0 \forall \tau \neq j,$$

$$E(\varepsilon_\tau^d \varepsilon_j^d) = 0 \text{for} \tau \neq j, E(\varepsilon_\tau^d \varepsilon_j^d) = \sigma_d^2 \text{for} \tau = j,$$

$$E(\varepsilon_\tau^s \varepsilon_j^s) = 0 \text{for} \tau \neq j, E(\varepsilon_\tau^s \varepsilon_j^s) = \sigma_s^2 \text{for} \tau = j.$$

We write the two processes in accordance with the assumptions of Lambert's theorem (see Lambert, 1988, Appendix A):

$$\ln D_{it} = d_t + u_{it},$$

$$\ln S_{it} = s_t + v_{it},$$

where we use the assumptions of Proposition 5

$$d_{it} = \gamma_d t, s_{it} = \gamma_s t,$$

$$u_{it} = \sum_{\tau=1}^{t} \varepsilon_{i\tau}^d, v_{it} = \sum_{\tau=1}^{t} \varepsilon_{i\tau}^s,$$

such that

$$\begin{pmatrix} u_{it} \\ v_{it} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \omega_{dt}^2 & \omega_{dst} \\ \omega_{sdt} & \omega_{st}^2 \end{pmatrix} \right),$$

where

$$\omega_{dt}^2 = var\left(\sum_{\tau=1}^{t} \varepsilon_\tau^d \right) = t\sigma_d^2, \omega_{st}^2 = var\left(\sum_{\tau=1}^{t} \varepsilon_\tau^s \right) = t\sigma_s^2,$$

$$\omega_{dst} = \omega_{sdt} = E\left(\sum_{\tau=1}^{t} \varepsilon_\tau^d \sum_{\tau=1}^{t} \varepsilon_\tau^s \right) = tr\sigma_d \sigma_s = r\omega_{dt}\omega_{st}.$$

The assumptions concerning the structure of underlying variances and covariances allows for the application to Lambert's (1988) theorem. According to the definition of Lambert's *rho* (Lambert, 1988, p. 124), it decreases with time t:

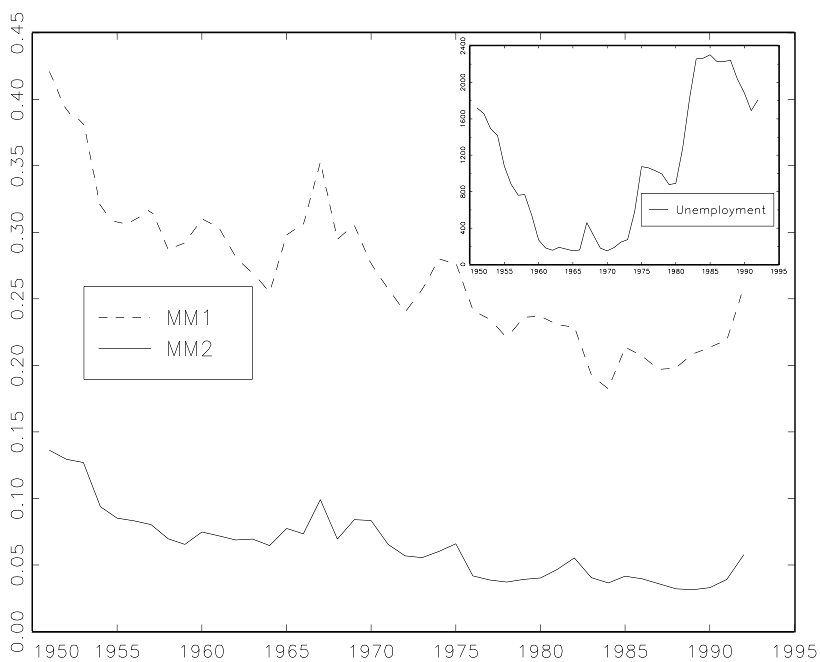
$$\rho_t = \frac{r}{var(u_t - v_t)} = \frac{r}{\omega_{dt}^2 + \omega_{st}^2 - 2r\omega_{st}\omega_{rt}}$$

$$= \frac{r}{t(\sigma_d^2 + \sigma_s^2 - 2r\sigma_d\sigma_s)}$$

Figure 1: Unemployment and mismatch in Spain

Note: Percentage points, *Source*: Bentolila and Dolado (1991),

Figure 2: Occupational mismatch in Germany, 1951-1992



Note: For the period 1951 to 1961 only 37 categories are available. During this period, four professions (metal-oriented) are summarized in a single group.
Sources: Leupoldt and Ermann (1973), Ermann (1984), Ermann (1988), Bundesanstalt für Arbeit: Amtliche Nachrichten der Bundesanstalt für Arbeit, various issues.