# Production Function of Skilled and Unskilled Labour

# in a Model of a Non-Growing Russian Economy

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## Abstract

This paper builds a production function of skilled and unskilled labour for the economy that does not grow. The paper uses the constant elasticity of substitution production function (CES) of skilled and unskilled labour on the micro level as an important building stone. The paper obtains a production function on the macro level using the generalised Houthakker-Johansen model. The paper uses the macro production function as a core for description of production sectors in the model of Russian economy. Using Russian statistical data on output variables, this paper discussed the identification of new medium-run employment and wage equations of the model. The identification is based on the accuracy of fitting measured by the Theil's index and on the accuracy of direction measured by the correlation index. The paper shows that an employment problem of the skilled workers in Russia follows from the problem of absence of proper investments.

*JEL classification:* E22, E24, O33

*Keywords:* production function, skilled and unskilled labour, transition economy

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The paper is published in the Intrenational Labour Market Conference Proceedings, October 11-12, 1999. PP.560-575.

## 1. Introduction

Soviet industry in 1970s - 1990s was inefficient, but the Russian population before 1990s had a very high level of education. That was Russian main asset and Russian chance to grow, assimilating modern technologies. But command system of economic relations did not use the chance for technological modernisation. On the contrary, market economies have the advantage of providing a withdrawal for relatively inefficient organisations: enterprises that are relatively inefficient cannot pay their bills, and disappear.

Market economies had modernised their production funds after impact of the energy's crisis in 1970s. It gave the opportunity to proceed theirs economic growth in 1980s - 1990s. The market mechanism provides enough quick permanent modernisation: the production funds with ineffective technologies are unmade (firms are bankrupted); the workers are restudied and are removed to the new production capacities with modern technologies.

The command system of economic regulation instead modernisation used the special system of price formation. The prices of industrial sectors were exceeded, and the plants used unlimited credit for theirs mutual accountings. As a result they used production capacities with old ages without any modernisation.

When one uses old production capacities without replacement and creates new ones with decreasing rate, the range of productivity becomes wide. In Olenev and Pospelov (1989) it is shown that the less the range of productivity is the more sustained economic system is. Therefore, under ageing of the production plants and equipment the economic system tends to disintegration. To show this it was necessary to use macro production function obtained by aggregation of micro level descriptions, exactly, the

Houtekker-Johansen model of production function for a sector described by distribution of capacities over technologies [see Johansen (1972).]

Shananin (1997a, 1997b) has investigated the generalised Houthekker-Johansen model for the case of positive homogeneous production functions on the micro level. The generalised model uses more than one production factor. Shananin has proved the theorems on the existence of the solution, necessary and sufficient conditions of optimality for the corresponding problem of the optimal resource distribution. The production functions aggregated from micro description have properties postulated in neo-classical economic theory.

Each technology is characterised by norm of unskilled labour expenses per unit of output, labour intensity $x \in R_1^+$. At each moment of time we have the distribution of capacities for sector number $i$ over the labour intensities [or over technologies]: $m_i(x)$, where $i = 1,\dots,N$, if sectors are $N$ in number.

Let an index $j$ specifies a kind of labour: $j = 1$ corresponds to skilled labour, and $j = 2$ - to unskilled one. Let a value $l_{ij}(x)$ defines a flow of labour resources number $j$ selected per unit of the sector $i$ capacity appropriated to technology $x$. So, at each moment of time we have distribution of skilled and unskilled labour resources over the technologies: $l_{ij}(x)m_i(x)$, where $j = 1,2$ and $i = 1,\dots,N$.

Utilisation rate of production capacity that appropriates to technology $x$ is equal

$$v_i(x) = min\ \{1,\ F_0[l_{i1}(x),\ l_{i2}(x)]/x\}, \tag{1.1}$$

where $F_0(*)$ is a positive homogeneous production function on the micro level. In our case, the production function converts skilled and unskilled labour units into effective labour units (effective labour intensity). The total output of sector $i$ is equal

$$Y_i = \int_0^\infty \min\{1,\ F_0[l_{i1}(x),\ l_{i2}(x)]/x\}\ m_i(x)dx\ \ (i = 1,\dots,N). \tag{1.2}$$

The employment in sector $i$ is defined by the available flows of labour $L^S_{ij}$:

$$L_{ij}, = \int_0^\infty \; l_{ij}(x)m_i(x)dx \le L^S_{ij}, \; (i = 1, \ldots, N; \; j = 1, 2). \qquad (1.3)$$

The problem of optimal resource's distribution for each sector $i$ is setting as[2]:

$$\int_0^\infty \min\{1, F_0[l_1(x), l_2(x)] \,/\, x\} \; m(x)dx \to \max_{l_1, l_2}, \qquad (1.4)$$

$$L_1 = \int_0^\infty \; l_1(x)m(x)dx \le L^S_1, \qquad (1.5)$$

$$L_2 = \int_0^\infty \; l_2(x)m(x)dx \le L^S_2, \qquad (1.6)$$

$$l_2(x) \ge 0, \;\; l_2(x) \ge 0. \qquad (1.7)$$

Let's consider a profit, which one may obtain from one unit of the production capacity with technology x

$$\pi(p, \boldsymbol{w}, x) = \sup_{\mathbf{l} \ge 0} \big(p \, min(1, F_0[\mathbf{l}]/x) - \boldsymbol{wl}\big), \qquad (1.8)$$

where $p$ is the selling price[3], $\boldsymbol{w} = (w_1, w_2)$ - the labour prices or wages, $\boldsymbol{l} = (l_1, l_2)$ - the used labour. Then, the following theorem is hold.

***Shananin's Theorem*[4]*:*** A distribution of resources $\boldsymbol{l}(x) = (l_1(x), l_2(x))$ is the optimal solution of the problem (1.4)-(1.7) if [and subject to the condition $\boldsymbol{L} > 0$ - if and only if] there is the simultaneously existence of the numbers $p \ge 0$, $\boldsymbol{w} = (w_1, w_2) \ge 0$, that satisfy the following conditions:

1) firstly,

$$w_j \Big( \int_0^\infty \; l_j(x)m(x)dx - L_j \Big) = 0 \;\; (j = 1, 2) \qquad (1.9)$$

2) secondly, for each $x \ge 0$ such that $p < q_0(w_1 x, w_2 x)$ we have $\boldsymbol{l}(x) = 0$

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[2] Index $i$ is omitted for simplicity of writing.

[3] This description involves also outlays on products of other sectors. For example, in Leontief's model we have, if $a_{ij}$ - outlay's norm of product $i$ in sector $j$, then presentation of price $p$ for sector $i$ is: $p = p_i - \Sigma_j \, p_j a_{ji}$.

[4] The proof of the Theorem in more common form you may find in Shananin (1997a).

3) thirdly, for each $x \geq 0$ such that $p > q_0(w_1x, w_2x)$ we have

$$F_0[\boldsymbol{l}(x)] = x \quad \text{and} \quad p - \boldsymbol{wl}(x) = \pi(p, \boldsymbol{w}, x). \tag{1.10}$$

where function $q_0(\boldsymbol{q})$ is defined as

$$q_0(\boldsymbol{q}) = \inf_{\mathbf{l} \geq 0, F_0(\mathbf{l})>0} \{(\boldsymbol{ql})/F_0(\boldsymbol{l})\}, \ \boldsymbol{q} \in R_+^{\,2}. \tag{1.11}$$

The function $F(\boldsymbol{L})$, that matched the vector $\boldsymbol{L} \geq 0$ and the optimal value of the functional (1.4) in the problem (1.4)-(1.7) calls “production function on the macro level.”

According to the methods of the system analysis of evolving economy [Petrov, Pospelov, and Shananin (PPSh, 1996)] the extraction of a system’s fragment for study requires the special attention and should come true on the basis of system’s model as a whole. Then it becomes clear, what is lost at such extraction and whether it is possible to lose it. Thus, it is necessary to test the description of production function in the whole model of economy.

Stiglitz (1999) argue that at least part of failures of the reforms was an excessive reliance on textbook models of economics. PPSh argue that it is necessary to built models described real situation of reforming economy. As such model in this paper it is used a modified PPSh model of Russian economy of transition period.

The remainder of the paper is organised as follows. Section 2 offers a core of the model of production function on the macro level. Section 3 offers a sketch of description of production capacities' dynamics in non-growth and growth economies. Section 4 presents production capacities' dynamics in non-growth economy. Section 5 builds production function for non-growth Russian economy. Section 6 shows some results of using the obtained production function in the modified PPSh model of Russian economy and presents some results of model's identification. Section 7 presents conclusions. Annex presents some graphical results of identification.

**2. Production function on macro level**

Let's denote a ratio of skilled to unskilled labour by $\lambda = l_1/l_2$, and name it "skilled ability". Marginal rate of unskilled labour substitution by skilled labour for any homogeneous function depends only of the skilled ability $\lambda$. For instance, for homogeneous first-degree function $F_0(l_1, l_2) = l_2 f(\lambda)$ we have $S_1 = - dl_1/dl_2 = f(\lambda)/f'(\lambda) - \lambda$. As such function, let's use the next CES-function

$$F_0[l_1, l_2] = \alpha l_2[1 + \beta\lambda^{-\rho}]^{-1/\rho}, \tag{2.1}$$

where $\alpha > 0, \beta > 0, \rho > -1$.

Let's suppose that the unskilled labour is always used in accordance with its norm. So that $l_2(x) = x$. Let's suppose that the parameters of production function on each technology x are the same. Then, for used production capacity with technology $x$ we may find the value of the skilled ability $\lambda(x)$according condition (1.10) and form (2.1) of the production function:

$$\lambda = (\beta / (\alpha^{\rho} - 1))^{1/\rho}, \tag{2.2}$$

and the value of skilled labour $l_1(x) = x\lambda$. So that, if the parameters of production function $\alpha, \beta,$and $\rho$ don't depend of technology $x$, then we have by (2.2) that the skilled ability $\lambda$ also don't depend of technology x.

If $\nu$ is a best technology in the sector, $\nu(t) = inf\{x: m(t, x) > 0\}$, and $M(t)$ is a total production capacity of the sector:

$$M(t) = \int_{\nu}^{\infty} m(t, x)dx, \tag{2.3}$$

then, in accordance with (1.4)-(1.7), we have an implicit form of production function on macro level

$$Y(t) = M(t)\, f(t, z), \tag{2.4}$$

where $z = z_1/\lambda = z_2$, $z_j = L_j(t)/M(t)$, ($j$ =1,2), and production function $f(t, z)$ is defined by the following system of equations:

$$f(t, z) = \int_{\nu}^{\xi(t,z)} h(t, x)dx, \tag{2.5}$$

$$z = \int_{\nu}^{\xi(t,z)} xh(t, x)dx \le min(z^S{}_1/\lambda,\ z^S{}_2), \tag{2.6}$$

where denoted $z^S{}_j = L^S{}_j(t)/M(t)$, ($j$ =1,2). A structure of the production capacities distribution on technologies, $h(t, x)$, is defined as $h(t, x) = m(t, x)/M(t)$. So that, we have production function if production capacities is filled out by all kinds of labour in order with increase of labour intensity x.

Profit on the unit of production capacity at the moment $t$, $\pi(t, x)$, in accordance with (1.8) is defined as

$$\pi(t, x) = p(t) - (w_1(t)\lambda + w_2(t))x. \tag{2.7}$$

Then, the value $\xi(t, z)$ from (2.5)-(2.6) is defined as minimum of three values:

$$\xi(t, z) = min\ [\xi_\pi(t),\ \xi_1(t, z^S{}_1),\ \xi_2(t, z^S{}_2)], \tag{2.8}$$

where $\xi_\pi(t) = p(t) / (w_1(t)\lambda + w_2(t))$ [$\pi(t, \xi_\pi) = 0$], and the values $\xi_1(t, z^S{}_1)$, $\xi_2(t, z^S{}_2)$ are defined by conditions $z = z^S{}_1\lambda$, $z = z^S{}_2$, respectively [see (2.6)]. It is clear, that parameters must be such that we have the existence of three values, $\xi_\pi$, $\xi_1$, and $\xi_2$. Suppose, that it is so. Then, we have three following cases:

$$\xi = \xi_1 \le min\ (\xi_\pi,\ \xi_2), \tag{2.9}$$

$$\xi = \xi_2 \le min\ (\xi_\pi,\ \xi_1), \tag{2.10}$$

$$\xi = \xi_\pi \le min\ (\xi_1,\ \xi_2). \tag{2.11}$$

The case (2.9) describes situation when we have a lack of skilled labour, the case (2.10) - a lack of unskilled labour, and the case (2.11) - a lack of demand on this production with prices $p$, $w_1$, $w_2$ or, that is the same, surplus of outlays, which one is described by high relative wages $w_1/p$, and $w_2/p$. The last case describes also a surplus of skilled and unskilled labour.

So that, the production function (2.5)-(2.6) may be used as a check-up of situation when we have surplus of skilled labour in country's massive military-industrial sector and lack of skilled labour in new private sector. The check-up may be done on the basis of model identification on real statistic data, if the model rightly describes main processes in the economy.

Now, in accordance with new denotations, we may define the unemployment in the sector by

$$U = M(t)[(z^S{}_1 - z_1) + (z^S{}_2 - z_2)]. \tag{2.12}$$

**3. Description of sector's production capacity dynamics**

The production capacity dynamics of a production cell may be described by the process of depreciation as in Olenev, Petrov and Pospelov (1986), by process of learning-by-doing alike in Lucas (1993), and some combination of the processes. We will be consider that all this cases are grounded on the following hypothesis:

$1^0$. The employment on the working production cell is fixed at the moment of entry into the market till the moment of pull out of the market.

The hypothesis $1^0$ means that the number of skilled and unskilled labour positions on the cell is constant:

$$x(t, t_0)m(t, t_0) = \nu(t_0)\, I(t_0), \quad \lambda x(t, t_0)m(t, t_0) = \lambda \nu(t_0)\, I(t_0), \tag{3.1}$$

where $\nu(t_0)$ is the best technology at $t_0$, $\nu(t_0) = x(t_0, t_0)$ , and $I(t_0)$ is an initial value of the production capacity $m(t, t_0)$.

The process of depreciation is grounded on hypothesis $1^0$ and on the next one:

$2^0$. The output on the working production cell is dropped down with the constant rate of depreciation as a result of stochastic process of breakage[5].

The hypothesis $2^0$ means that production capacity $m(t, t_0)$ of the cell, created at $t_0$ is described by the following equation:

$$m(t, t_0) = I(t_0) \exp(-\mu(t - t_0)), \tag{3.2}$$

where $\mu$ is the rate of the production capacity's depreciation. Then (3.1) and (3.2) imply that the norm of unskilled labour intensity follows

$$x(t, t_0) = \nu(t_0) \exp(\mu(t - t_0)). \tag{3.3}$$

At each moment $t$ we have a density of production capacity over technology $x$, $m(t, x)$. In variables $t, x$ (3.2) rewrites as $m(t, x)\Delta x = I\Delta\nu_0 \exp(-\mu a)$, and (3.3) as $x = \nu_0 \exp(\mu a)$, where $a = t - t_0$ is the age of production capacity. Then $\Delta x = \Delta\nu_0 \exp(\mu a)$, and $m(t, x) = I \exp(-2\mu a)$. So, as the production capacity is ageing, it not only comes down on the technology (the labour intensity is increased, so that the productivity is decreased), but also widens over technologies [Figure 1].

Dynamic process, when production capacity on individual production lines is described only by process of depreciation, describes in particular the dynamic of production capacities in non-growth economy, that practically don't have net capital investment. Evidently, this case we had in Russian economy in 1992-1998.

[5] Shananin demonstrated that the stochastic process of equipment's breakage and repairing with slowly increasing of breakage's frequency conforms to capacity's decreasing with constant rate [see Danishevski, Fedorov, Shananin (1988)].

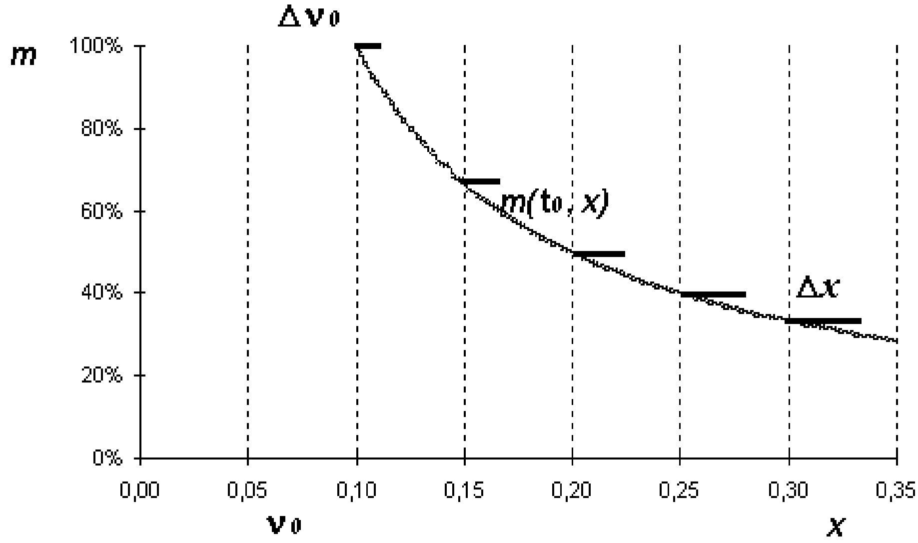


Figure 1

For description, what we will lose in process of transition without sufficient capital investments, let consider a process of learning-by-doing.

The process of learning-by-doing can be described as follows:

$$m(t, t_0) = J(t_0)\xi(t), \tag{3.4}$$

where $\xi(t)$ represents a degree of comparative cumulative experience in the production of the good initialised at $t_0$, $0 \le \xi(t) \le 1$, $J(t_0)$ is a maximal value of production capacity. The degree of cumulative experience is in turn defined by the differential equation:

$$d\xi(t)/dt = [1 - \xi(t)]\,\eta, \tag{3.5}$$

and the initial value $\xi(t_0)$, assumed to be less than or equal to one, of the experience variable on the date $t_0$ when production was begun. Here $\eta$ is a learning rate that depends on the level of skilled ability[6], $\eta = \eta(\lambda)$, $\eta'(\lambda) > 0$, and the value of this parameter is constant if skilled ability is fixed. In accordance with (3.5) cumulative experience increases as sooner as more is a variation between perfect and current level of the cumulative experience, $1 - \xi(t)$. The general solution to (3.5) is

$$\xi(t) = 1 - [1 - \xi(t_0)]\exp[-\eta(t - t_0)]. \tag{3.6}$$

Then (3.4) and (3.6) imply that production capacity follows

$$m(t, t_0) = J(t_0)\{1 - [1 - \xi(t_0)]\exp[-\eta(t - t_0)]\}. \tag{3.7}$$

Production capacity grows approaching it's maximal value, $J(t_0)$, with the increase of age, $t - t_0$, and the rate of this growth declines monotonously from $\eta[1-\xi(t_0)]/\xi(t_0)$ to zero[7]. Owing to hypothesis $1^0$ we have (3.1), where $I(t_0) = J(t_0)\xi(t_0)$ and $\nu(t_0)$ is the best technology for entry at $t_0$. Then (3.7) and (3.1) imply that the norm of unskilled labour intensity follows

$$x(t, t_0) = \nu(t_0)\xi(t_0)/\{1 - [1 - \xi(t_0)]\exp[-\eta(t - t_0)]\}. \tag{3.8}$$

Labour intensity decreases monotonously from $\nu(t_0)$ to $\nu(t_0)\xi(t_0)$, so that productivity, equalled to inverted labour intensity, grows from $1/\nu(t_0)$ to $1/[\nu(t_0)\xi(t_0)]$.

In variables $t, x$ (3.7) rewrites as $m(t, x)\Delta x = J[1 - (1 - \xi_0)\exp(-\eta a)]\Delta\nu_0$, and (3.8) as $x = \nu_0\xi_0/[1 - (1 - \xi_0)\exp(-\eta a)]$, where $a = t - t_0$ is the age of production capacity, and $\xi_0 = \xi(t_0)$. Then $\Delta x = \Delta\nu_0\, \xi_0/[1 - (1 - \xi_0)\exp(-\eta a)]$, and $m(t, x) = J[1 - (1-\xi_0)\exp(-\eta a)]^2$. So as the production capacity is ageing it not only comes up on the technology (the labour intensity is decreased, and the productivity is increased), but also compresses over technologies [Figure 2].

Lucas (1993) used the similar learning model, described the process of learning-by-doing, to explain episodes of very rapid income growth. Such growth was observed for some of the south-east countries in 1960-1990 (when effect of depreciation may be ignored). It is possible that the combination of depreciation's and learning-by-doing's processes describes some of the successful transition economies such as Poland.

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[6] The more educated workers there are, the more new ideas to improve productive efficiency there will be.

[7] Unlike learning model, presented in Lucas (1993), we don't have herein production growth without bound on individual product line.

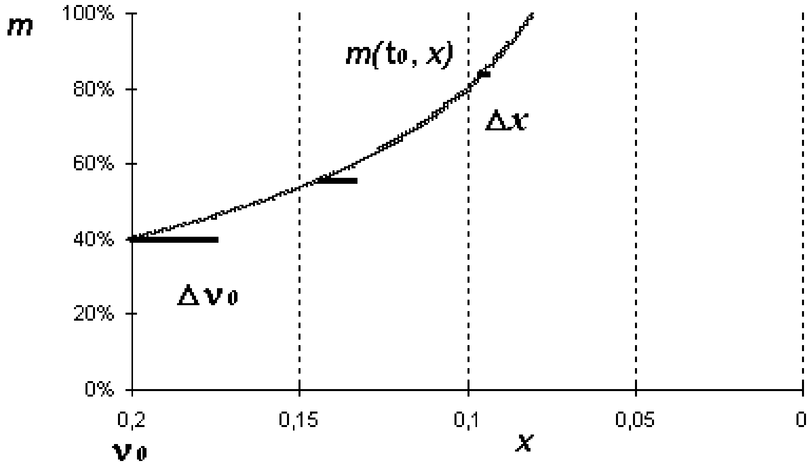


Figure 2

## 4. Dynamics of production capacities in non-growing economy

The dynamics of production capacities in non-growing Russian economy in 1996-1998 is described by the process of depreciation (3.1)-(3.3) and the following hypotheses

$3^0$ At moment *t* there are not technology with labour intensity less then a value $\nu > 0$, and all new capacities have this labour intensity $\nu$.

$4^0$ The new capacities are established continuously at the rate of $I(t)$.

If the age of production capacity with labour intensity *x* denotes $\theta(t, x)$, then according (3.3) we have

$$x = \nu exp[\mu\theta(t, x)]. \tag{4.1}$$

Let's consider the total capacity $\overline{M}(t, x)$ of production cells with the labour intensity less then *x*.

$$\overline{M}(t, x) = \int_{t-\theta(t,x)}^{t} I(\tau)exp[-m(t - \tau)]d\tau, \; (\overline{M} = 0, \text{ if } x < \nu). \tag{4.2}$$

Excluding $\theta$ from (4.1),(4.2) differentiated them by t and x when x > n, we obtain

$$\partial\overline{M}/\partial t = -\mu\,\overline{M}(t, x) + I(t) - \mu x\partial\overline{M}/\partial x. \tag{4.3}$$

The derivative $\partial \overline{M}/\partial x = m(t, x)$ is a density of the measure $\overline{M}$. Differentiating of (4.3) by $x$ gives

$$\partial m/\partial t = -2\mu m(t, x) - \mu x \partial m/\partial x \quad (x > \nu). \tag{4.4}$$

The boundary condition for (4.4) one may find from (4.1), (4.2).

$$m(t, \nu) = I(t)/(\mu\nu). \tag{4.5}$$

If total sum of labour positions in the sector is finite, then $xm(t, x) = x\partial \overline{M}/\partial x \to 0$ if $x \to \infty$, and from (4.3) one has the following equation for the sector's total capacity $M = \overline{M}(t, \infty)$:

$$dM/dt = I(t) - \mu M(t). \tag{4.6}$$

## 5. Production function for non-growing economy

Let us write down the production function (2.5),(2.6) as follows

$$M(t)f(t, z) = \int_{t-\theta(t,z)}^{t} I(\tau) \exp\{-\mu(t - \tau)\} d\tau, \tag{5.1}$$

$$M(t)z = \nu \int_{t-\theta(t,z)}^{t} I(\tau) d\tau, \tag{5.2}$$

where $\theta(t, z)$ is an age of the oldest production capacity. If denote the relative value of the new constructed production capacities by the agency of $\sigma(\tau) = I(\tau)/M(\tau)$, then in accordance with (4.6) we have $I(\tau) = \sigma(\tau)M(t)\exp\{-\int_{\tau}^{t} (\sigma(s) - \mu)ds\}$, and hence the production function (5.1),(5.2) overwrites as

$$f(t, z) = 1 - \exp\{\int_{t-\theta(t,z)}^{t} \sigma(\tau)\, d\tau\}, \tag{5.3}$$

$$z/\nu = 1 - \exp\{\mu\theta(t, z) - \int_{t-\theta(t,z)}^{t} \sigma(\tau) d\tau\} + \mu \int_{t-\theta(t,z)}^{t} \exp\{\mu(t - \tau) - \int_{\tau}^{t} \sigma(s)ds\} d\tau. \tag{5.4}$$

In order to obtain an analytical expression for the production function, it is necessary to make one or another assumption about the character of the change of function $\sigma(\tau)$, where $\tau$: $t - \theta(t, z) \le \tau \le t$. For example, if $\sigma(\tau) = \sigma = const$ and $\sigma > \mu > 0$ then

$$f(z) = 1 - [1 - (1 - \mu / \sigma)z / \nu]^{1/(1 - \mu/\sigma)}. \tag{5.5}$$

In this special case the loading of the total production capacity $f$ is entirely defined by the parameter of short-term control $z$. [see Olenev, Petrov, Pospelov (1986)].

The average age of plants and equipment will remain unchanged. Indeed, in general case in the absence of the dismantling of old production capacities the average age of plants and equipment is defined as

$$A(t) = \frac{1}{M(t)} \int_{-\infty}^{t} (t-\tau) I(\tau) \exp\bigl(-\mu(t-\tau)\bigr) d\tau = \int_{0}^{\infty} \exp\left(-\int_{t-a}^{t} \sigma(s) ds\right) da. \tag{5.6}$$

Supposing here $\sigma(t) = \sigma = const$ we shall derive $A = 1/\sigma$.

In the case of present Russian economy the loading of total production capacity is defined in larger degree by the preceding evolving of production system, namely: by the past investment policy. It is shown in the increase the average age of plants and equipment: $A(1980) = 9.5$ years, $A(1995) = 14.1$ years[8]. To find the production function of Russian economy let us assume that function $\sigma(\tau)$ was changed as follows[9]

$$\sigma(\tau) = \begin{cases} \sigma_1, \tau \le t_1, \\ \sigma_1\left[1 - \beta_1\left(\tau - t_1\right)\right], & t_1 \le t < t_2, \\ \sigma_2\left[1 - \beta_2\left(\tau - t_2\right)\right], \tau \ge t_2, \end{cases} \tag{5.7}$$

[8] See Clifford Gaddy and Barry W Ickes (August 1998), "Why are Russian Enterprises Not Restructuring?" *Transition*. Vol.9, N. 4. P.1.

[9] It is very difficult to construct endogenous investment function for the model of transition economy, in which the volume of investment does not defined from a maximisation problem. This is the more so for the Russian economy in 1996-1998, in which the investment policy was essentially defined by the government decisions not private. All of this was enforced us to use exogenous investment function instead.

where $t_1 = 1980$, $t_2 = 1990$[10], $\sigma_1 > \sigma_2 > 0$, $\beta_1 > 0$, $\beta_2 > 0$. That is to say we assume that net investments made constant part of total capacity before 1980 ($\sigma_1 = 1 / A(1980) = 0.1053$ [$1$/year]), and after 1980 this value linear diminishes undergoing shock $(\sigma_2 - \sigma_1)$ in 1990. In accordance with our assumption the average age of plants and equipment up to 1980 has had constant value, $A(t) = A(t_1) = 1/\sigma_1 = 9.5$ years if $t \le t_1$, and then it is increased.

During the 7 years of transition not only new production capacities have been used to produce output, but also the capacities created before 1980 have been used. The main reason of this is that the only several new capacities was created after 1980. So, let us assume that the eldest age of capacities that used in production, $\theta(t, z)$, is more than age of production capacities created after 1980: $t - t_1 < \theta(t, z)$. Then (5.3), (5.4) and (5.7) give us the following expression of production function for $t > t_2$

$$f(t,z) = 1 - \Omega(t,\mathbf{s})^{-\mu/(\sigma_1-\mu)}\left[1 + \frac{\mu}{\sigma_1}\left(\Psi(t,\mu,\mathbf{s}) - 1\right) - \left(1 - \frac{\mu}{\sigma_1}\right)\frac{z}{\nu}\right]^{1/(1-\mu_1/\sigma_1)}, \qquad (5.8)$$

where $\mathbf{s}$ is vector of parameters that define function $\sigma(t)$, $\mathbf{s} = (t_1, \sigma_1, \beta_1, t_2, \sigma_2, \beta_2)$, and $\Omega(t, \mathbf{s})$, $\Psi(t, \mu, \mathbf{s})$ are defined as

$$\Omega(t,\mathbf{s}) = \exp\left\{(\sigma_1 - \sigma_2)(t - t_2) + \frac{\sigma_1\beta_1}{2}(t_2 - t_1)^2 + \frac{\sigma_2\beta_2}{2}(t - t_2)^2\right\}, \qquad (5.9)$$

$$\Psi(t,\mu,\mathbf{s}) = \exp\left\{\mu(t - t_1) - \sigma_1(t_2 - t_1) + \frac{\sigma_1\beta_1}{2}(t_2 - t_1)^2 - \sigma_2(t - t_2) + \frac{\sigma_2\beta_2}{2}(t - t_2)^2\right\} +$$

$$+ (\sigma_1 - \mu)\sqrt{\frac{\pi}{2\sigma_1\beta_1}}\left\{\Phi\left(\frac{\sigma_1 - \mu}{\sqrt{\sigma_1\beta_1}}\right) - \Phi\left(\frac{\sigma_1\left(1 - \beta_1(t_2 - t_1)\right) - \mu}{\sqrt{\sigma_1\beta_1}}\right)\right\} \times$$

$$\times \exp\left\{-(\sigma_1 - \mu)t + (\sigma_1 - \sigma_2)(t - t_2) + (\sigma_1\beta_1/2)t_2(t_2 - 2t_1) + (\sigma_2\beta_2/2)(t - t_2)^2\right\} +$$

[10] Note that estimation for the shock in investments as the end of 1990 differs from the official date for the

$$+\left(\sigma_1-\mu\right)\sqrt{\frac{\pi}{2\sigma_2\beta_2}}\left\{\Phi\left(\frac{\sigma_2-\mu}{\sqrt{\sigma_2\beta_2}}\right)-\Phi\left(\frac{\sigma_2\left(1-\beta_2\left(t-t_2\right)-\mu\right)}{\sqrt{\sigma_2\beta_2}}\right)\right\}\times$$

$$\times\exp\left\{-\left(\sigma_2-\mu\right)t+\left(\sigma_2\beta_2/2\right)t\left(t-2t_2\right)\right\}, \qquad (5.10)$$

where function $\Phi(x)$ is a table function, the probability integral,

$$\Phi(x)=\frac{2}{\sqrt{2\pi}}\int_0^x \exp\left\{-u^2/2\right\}du\,. \qquad (5.11)$$

Note, that equality $\sigma(t) = \sigma = const$, (or that is the same $\sigma_1 = \sigma_2 = \sigma$, $t_1 = t_2 = t$) in (5.8)-(5.10)) gives $\Omega(t, \boldsymbol{s}) = \Psi(t, \mu, \boldsymbol{s}) = 1$, and moves (5.8) in (5.5).

My estimations of the production function on the basis of statistical data, and under the assumptions $t_1 = 1980$ and $t_2 = 1990$, are: $\sigma_1 = 0.1053$ *[1/year]*, $\beta_1 = 0.026$ *[1/year]*, $\sigma_2 = 0.04$ *[1/year]*, $\beta_2 = 0.09$ *[1/year]*.

**6. Some results of numerical experiments on the model of Russian economy**

New model is constructed considering difference for the skilled level of workers. As a result the description of production sector was entirely changed in comparison with the original PPSh's model. The labour was not a limited resource in the original PPSh's model that described Russian economy for 1992-1995s. In the case of modified model the deficit of skilled or unskilled labour in a sector gives the reduction of the production capacity loading in the sector. It is defined in accordance with the production function obtained above. For description of the production block in the modified model let use the same structure as in the original model. In production we have three sectors. Sector 1 involves the economic sectors that produce current consumer goods, sector 2 - durable consumer and capital goods including military goods, and sector 3 - raw goods going for export and for production needs.

initiation of liberal reform as the beginning of 1992.

An hierarchy system of relationships of economic agents in our model doesn't change in comparison with the original PPSh's model. The framework involves the following economic agents: households, producers, importers, exporters, commercial banks, Government together with the Central Bank. The hierarchy structure is based on the Stackelberg game in which a dominant (or leader) agent moves first and a subordinate (or follower) agent moves second.

For instance, exporters within limits defined by the reaction of the Central bank of Russia dominate importers in the currency market. The exporters define such Rouble exchange rate that gives them a maximum of their net currency income. Importers dominate households in the consumption market. The importers define such consumer price index that gives them a maximum of their net currency income. It is assumed that the net currency income of exporters and importers don't work in Russian investment market and it is leaked abroad.

It is assumed that we don't have the equilibrium in the labour market. So that the processes on the labour market are described by the dynamic equations on the wages for skilled and unskilled labour in every sector, and on the assignment of the supply of all kinds of labour on sectors.

It is interesting to note that behaviour of total unemployment $U$ is followed from the kind of assignment equations for the part of skilled or unskilled labour supply belongs to a sector. In more exact terms, the part $\lambda_{ij}$ of total $j$-th labour supply $L_j^s$, which belongs to $i$-th sector, $L_{ij}^s = \lambda_{ij} L_j^s$, is defined by assignment equation. Preliminary numerical experiments showed that the total unemployment decreases when the assignment equations are described by the unemployment levels in sectors. And it increases when the wage levels describe them:

$$\frac{d\lambda_{ij}}{dt} = \frac{1}{\theta_j}\left(\frac{\sum_{k\neq i}\left(w_{ij} - w_{kj}\right)}{\sum_i w_{ij}}\right), 0 < \lambda_{ij} < 1,\ \Sigma_i \lambda_{ij} = 1\ (j = 1, 2). \qquad (6.1)$$

As illustrated in Figure 3, the last case agrees to statistical data. The axes are time and unemployment: there are plotted the months of 1996-2000s on the abscissa axis and the total unemployment $U$ on the ordinate axis.

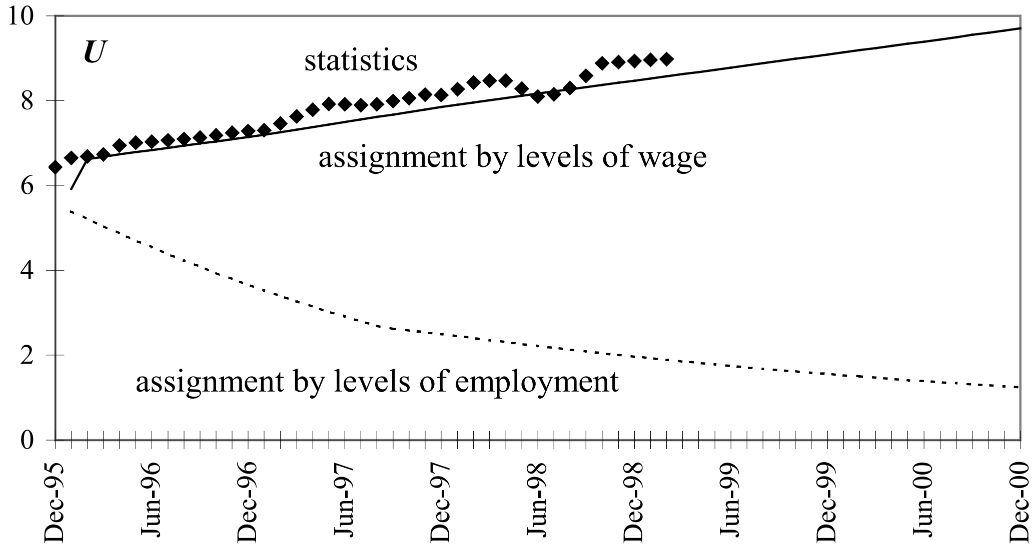


Figure 3. Total unemployment $U$ for different kinds of assignment equations [millions]

Figure 4 shows the dynamics of unemployment in Russia in 1996-2000s: skilled and unskilled labour unemployment in the sectors of economy, $U_{ij}$.

From this figure we notice that the unemployment $U_{21}$ of skilled workers in the second sector (which includes military-industrial complex) is increased. At the same time the unemployment of skilled workers in the first and third sector is absent, $U_{11} = U_{31} = 0$. The unemployment of unskilled workers behaves in reverse order: $U_{22} = 0$, $U_{12} > 0$, $U_{32} > 0$. In addition, unskilled workers' unemployment in the first sector $U_{12}$ is decreased, and unskilled workers' unemployment in the third sector $U_{32}$ is increased.

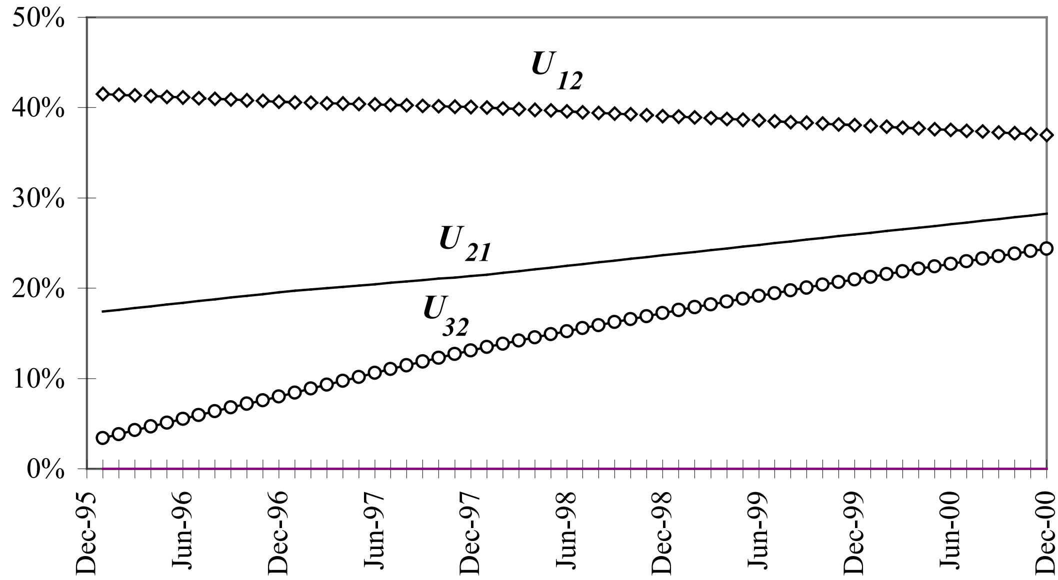

Figure 4. Skilled and unskilled labour unemployment in sectors 1-3 [%]

The results of identification are measured by Theil's inequality index and by the correlation coefficient.[11] They are exhibited in the Table 1. The part of "good" output variables with the Theil's inequality index less than 0.2 is equal 89% (the rate of agreement for computed and statistical data), the part of "good" output variables with the correlation coefficient more than 0.7 is equal 63% (the rate of the strength and direction of the linear relationship for computed and statistical data), and the part of "very good" output variables with the Theil's inequality index less than 0.2 and with the correlation coefficient more than 0.7 is equal 58%.

[11] The Theil's inequality index *TII* [Theil (1961)] is a measure of the degree to which one time series *C* differs from another *S*. It is greater then zero, and the more it is the more disagreement is. The correlation coefficient *CC* measures the strength and direction of the linear relationship between two variables and is bounded by -1 and 1.

*Table 1*. Model fitting accuracy measured by the Theil's inequality index (*TII*), and by the correlation coefficient (*CC*)

| **Output variable** | ***TII*** | ***CC*** |
|---|---|---|
| Expenditures of consolidated budgets | 0.186 | 0.231 |
| Revenues of consolidated budgets | 0.180 | 0.409 |
| Deficit of consolidated budgets | 0.351 | -0.209 |
| CPI | 0.086 | 0.989 |
| Exchange Rate | 0.058 | 0.997 |
| M0 | 0.078 | 0.866 |
| Personal income | 0.070 | 0.775 |
| Consumer expenditures | 0.114 | 0.903 |
| Real Personal Income | 0.103 | 0.544 |
| GDP | 0.069 | 0.569 |
| Import | 0.193 | 0.598 |
| Export | 0.114 | 0.577 |
| External Reserves | 0.168 | 0.822 |
| CPI, %/mo | 0.168 | 0.822 |
| p3 | 0.096 | 0.870 |
| p2 | 0.096 | 0.870 |
| p1 | 0.047 | 0.980 |
| Unemployment | 0.019 | 0.979 |
| Payable of enterprises | 0.056 | 0.975 |

## 7. Conclusions

It is obtained a new class of the sector's production functions described by the production capacity's distribution on technologies. The class of production functions describes the output as a function of labour quality, precisely, the function of the skilled and unskilled labour.

The new production function, which was identified for the non-growing Russian economy in 1996-1998, has shown a capacity for work. This capacity is illustrated on the example of the modified PPSh's model for transition Russian economy. The results of identification show that the model is good enough to make scenario calculations, which can give the qualitatively good results.

That all gives explanation for increase of unemployment in Russian economy in 1996-1998, when workers' assignment is described according to the wage levels. It is demonstrated that unemployment of unskilled workers is not structural one (absence of working places) but it is voluntary unemployment (absence of places with enough wage level.) The skilled labour unemployment is defined by the absence of sufficient investments (and therefore by the low demand on the industrial sector's production) and by the low transfer pace of skilled workers from the military industrial complex to another sector.

The absence of requisite capital investments causes destruction of the middle class (skilled workers). It gives one of the similarities between Russia today and the Weimar republic that preceded Hitler's ascension to power in 1933. So that, Russia is needed in international help to recover and to build a strong regulatory system at first.

Without the strong regulatory system all tries to reform Russian economy (price liberalisation, enterprise privatisation) did not give the needful results in recover from slump. Now Russia has a quasi-feudalism system when most transactions are handled by barter or are not paid at all. It is unprofitable for foreign capital and for Russian capital from abroad to come into the country when property rights and real estate are not sufficiently respected in Russia.

The labour resources separation on castes by the places of residence reduces the impact of human capital on development. In conditions, when we practically have the absence of capital investments in Russia, we cannot use the effect of learning-by-doing for increase of labour productivity. But in any case the large human capital stocks install hope for renewal of economic growth.

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## Annex: *Graphical results of identification and verification*

The aim of this annex is to give an example of diagrammatic representation for estimation results. In the Figures A1-A4 you may see matching of some calculated and statistical curves for a base scenario.

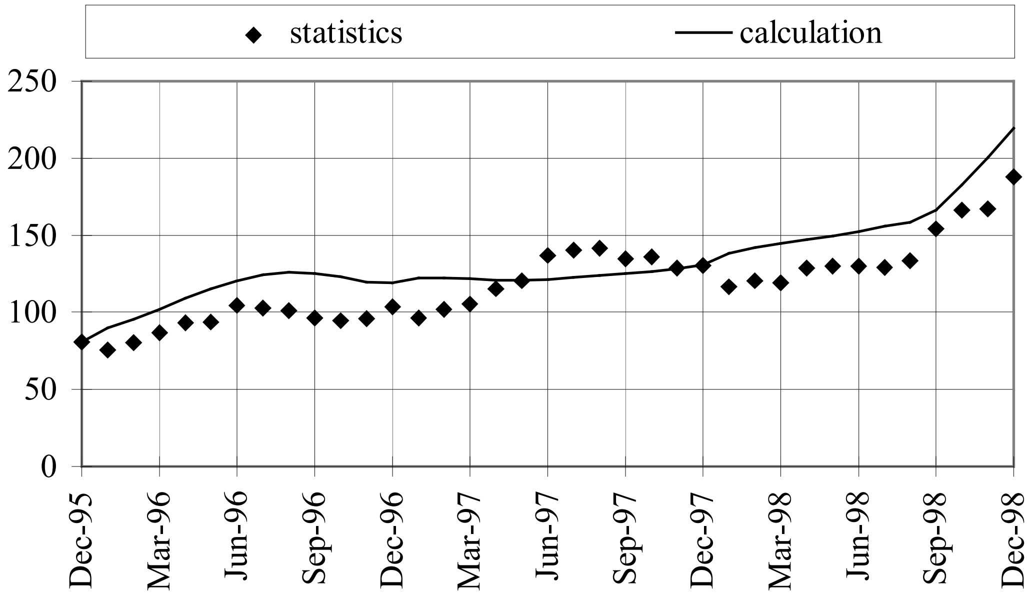


Figure A1. Money supply, M0 [Rouble milliards]

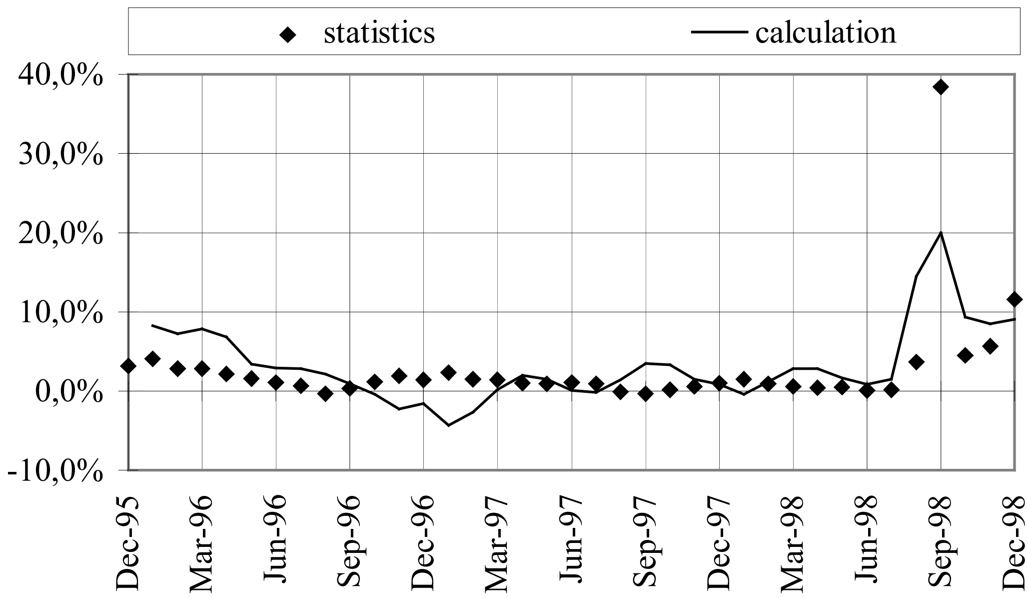


Figure A2. Consumer price index, total, *(dp/dt)/p* [*%/mo*]

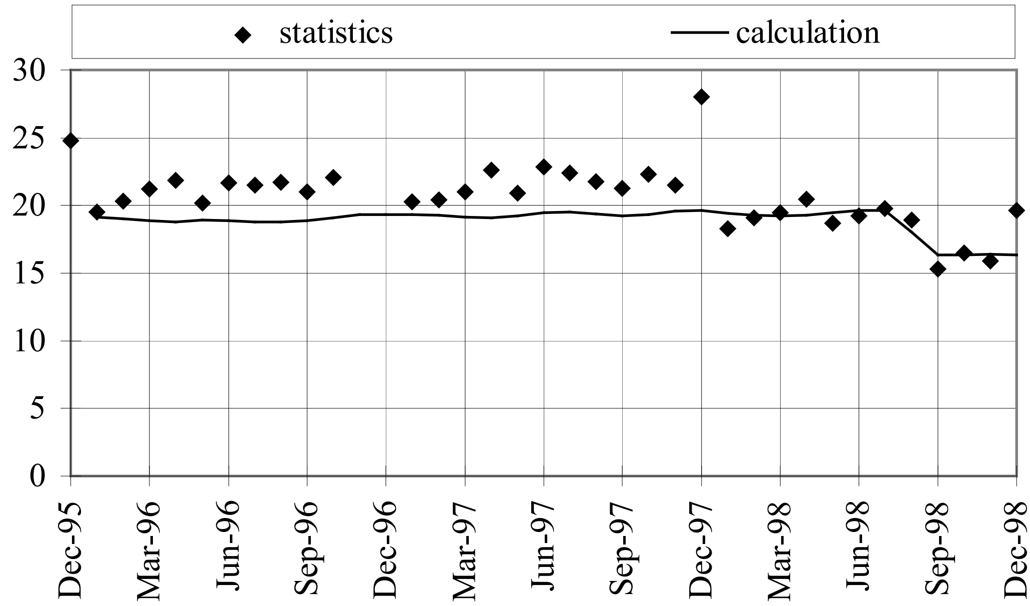


Figure A3. Real Personal Income, [Rouble1990, milliards]

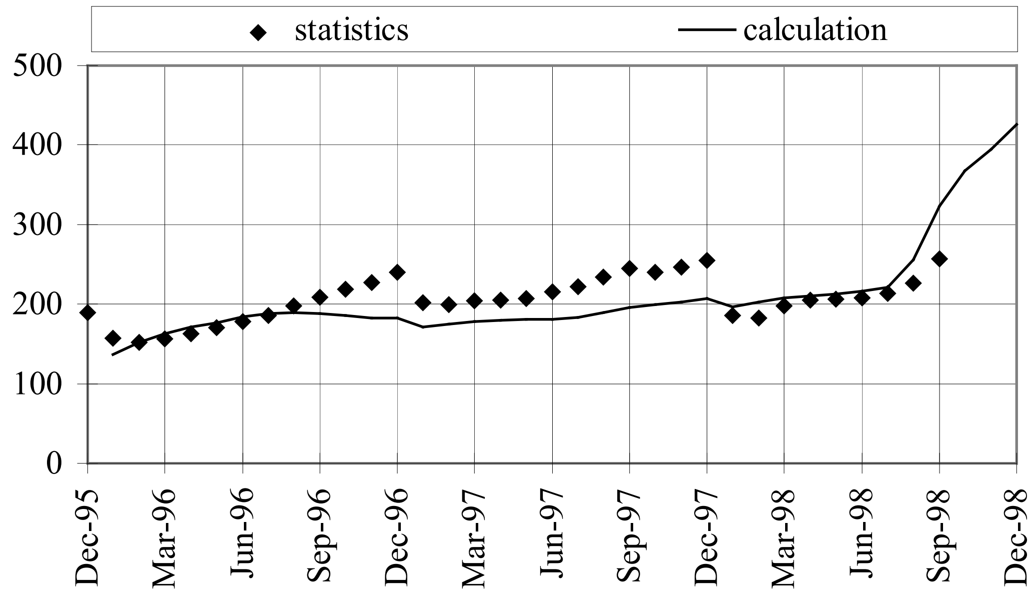


Figure A4. Nominal Gross Domestic Product [Rouble milliards/mo]