OLIGOPSONY AND THE DISTRIBUTION OF WAGES

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ABSTRACT. A number of theories (search and efficiency wages) have been developed, in part, to explain why identically able workers are often paid different wages. However, when there is a minimum wage, they do not explain the resulting "spike" in the wage distribution. Our model's predictions are consistent with this evidence. We assume that workers are equally able but have heterogeneous preferences for non-wage characteristics, while employers have heterogeneous productivity characteristics. This results in a model of labor market oligopsony where "inside" and "outside" forces interact, producing wage dispersion as well as a spike at the minimum wage.

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1. Introduction

The empirical evidence on the structure of wages presents a serious challenge to the competitive theory of labor markets, where workers are paid their marginal products. Competitive theory predicts that wage rates should depend only on workers' abilities. This is in stark contrast to the empirical evidence, which finds large inter-industry wage differentials for workers with identical characteristics.[1] Even within industries, there is evidence that wages vary significantly (Dunlop, 1957; Groshen, 1991). There is also evidence that large establishments tend to pay substantial wage premiums (Brown and Medoff, 1989). Moreover, the effect on the wage distribution of an increase in the minimum wage reinforces these puzzles—rather than simply truncating the wage distribution, such a hike often raises wages, for minimum wage workers as well as those who are paid more than the minimum wage (Card and Krueger, 1995; Dolado *et al.*, 1997; Grossman, 1983). Further, a minimum wage produces a "spike" in the distribution of wages (Card and Krueger, 1995). While modifications of competitive theory can produce wage dispersion (e.g., sorting and compensating differentials), these modifications are limited in their explanatory power (see Katz (1986) and Krueger and Summers (1987) for surveys).

In response to this empirical evidence, a number of non-competitive theories have been developed. The leading examples are efficiency wage theory (Albrecht and Vroman, 1998; Bulow and Summers, 1986; Ramaswamy and Rowthorn, 1991; Stiglitz, 1985) and job search models (Burdett and Mortensen, 1998). We present an alternative model of the labor market whose predictions are consistent with the empirical evidence on the distribution of wages. Our model's key predictions are two-fold. First, workers of *identical* ability are paid different wages by different employers and the resulting distribution of wages is consistent with the observed wage distributions. Second, the imposition of a minimum wage raises the wages of minimum wage workers, has a spillover effect on the wages of higher-paid workers while compressing the wage distribution. Finally, our model also predicts the occurrence of a spike in the wage distribution at the minimum wage, the presence of which is contrary to the above models. Apart from its explanatory power, a key feature of our model is its tractability and simplicity, which makes it amenable for empirical analysis. We believe that our model provides

[1] Slichter (1950) was an early attempt to quantify the degree of wage dispersion and subsequently there have been numerous contributions. See for example Blackburn and Neumark (1992); Dickens and Katz (1987b); Gibbons and Katz (1992); Krueger and Summers (1988); Murphy and Topel (1987).

an alternative explanation for these empirical facts, which is, in some ways more persuasive—a comparison of the relative merits of these theories is deferred to Section 4, until we develop our theory and its implications.

Our model relies on two key assumptions. The first is that workers with *identical skills and abilities* have heterogeneous preferences over non-wage characteristics of employers (Bhaskar and To, 1998; Boal and Ransom, 1997). These include the actual job specification, hours of work, distance of the firm from the worker's home, the social environment in the workplace, etc. The importance of non-wage characteristics has been recognized in the theory of compensating differentials, which is a theory of vertical differentiation. Some jobs are good while other jobs are bad, and wage differentials compensate workers for these differences in characteristics. Our approach is one of horizontal job differentiation—we assume that different workers have different preferences over non-wage characteristics.[2] As long as the number of employers is finite, heterogeneous non-wage preferences ensure that wage setting employers have wage setting power. That is, we have what is classically referred to as *oligopsony*. The literature on oligopsony is sparse, consisting primarily of empirical evaluations of oligopsony power (Boal and Ransom, 1997, p. 91).[3]

The second important assumption is that the *marginal* product of labor varies between employers—note that this is well consistent with the average product of labor or profitability being the same across firms. Indeed, such heterogeneity is unavoidable if firms from different product markets compete in the same labor market. Given employer wage setting power, heterogeneity in their productive characteristics results in an equilibrium with dispersion in wages.

Our main results are as follows. Firms offer different wages in equilibrium with "high productivity" firms typically offering higher wages. Firms that offer high wages employ more workers and also tend to be more profitable. Finally, under a minimum wage, firms not directly affected by the minimum also raise their wages and a minimum wage introduces a spike in the wage distribution.

[2]McCue and Reed (1996) provide survey evidence of horizontal heterogeneity in worker preferences.

[3]Surprisingly, the early theoretical work on oligopsony appeared only recently in the agricultural economics literature (Chen and Lent, 1992). More recent theoretical treatments include Hamilton *et al.* (1998), Kaas and Madden (1999) and Naylor (1996).

2. The Model

We now present our model of an imperfectly competitive labor market. Its central feature is that jobs differ in terms of their non-wage characteristics and employers differ in characteristics which affect the marginal revenue product of labor. To model horizontal differentiation in a simple and tractable way, we adapt the influential model of Salop (1979b). We assume that the job characteristic space is a circle of unit circumference (as in Figure 1). Workers of equal ability are uniformly distributed along all points of the circumference. Let there be n firms in the market. Following Salop, we do not model the location choices of firms, but assume that these firms are uniformly spaced around the circle.[4] In Figure 1 firms are located at points marked with an H or an L (ignore for the moment the particular interpretation associated with H and L). A worker who travels distance d to work in a firm incurs a transportation cost of td (i.e., this cost is linear in distance, and t is the unit transportation cost). In evaluating wage offers at two firms, a worker takes into account the wages offered as well as the transport cost incurred in working at each of these firms. Workers at different locations will evaluate job offers differently, since they will have different transport costs associated with work at any firm.

We allow for a diversity of workers' reservation wages, in the simplest possible way, by assuming that there is a unit mass of low reservation wage workers who are uniformly distributed along the circle, and a mass μ of high reservation wage workers who are similarly uniformly distributed. For simplicity we set the former's reservation wage to zero, and assume that the latter's reservation wage is $v > 0$. Our basic results extend to the general case where we have any arbitrarily large finite set of types of workers at each location.

A worker will choose to work as long as the wage less their transportation cost is at least their reservation wage. Our focus is on parameter values where, in equilibrium, all low reservation wage workers work and only some high reservation wage workers work. This ensures that there is competition for workers between firms and that total employment can vary.

[4]While job characteristics may be, to a large extent, exogenously determined (type of work, physical location, etc.), employers may be able to affect other characteristics (management style, the physical environment of the workplace, etc.) If we make the extreme assumption that all job characteristics are endogenously determined then employers will choose their job characteristics prior to choosing wages and equilibrium locations will be, as we have assumed, uniform about the circle.

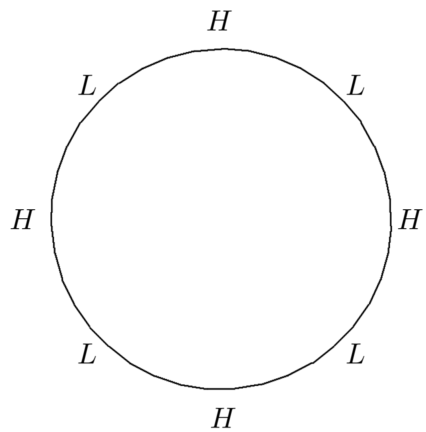

FIGURE 1. Example with $n = 8$
Alternating Firm Types

2.1. **Labor Supply.** We consider a model of oligopsony where there is no free entry or exit so that the number of firms, n, is fixed. Some of the implications of allowing firm entry and exit are explored in Bhaskar and To (1998).

With n firms in the market, the distance between firms is $1/n$. Suppose that firm i offers wage w_i and one of firm i's nearest rivals, j, is offering wage w_j. Consider a low reservation wage worker who is located between firms i and j at distance x from i and $1/n - x$ from j. Such a worker will work for firm i if $w_i - tx > w_j - t(1/n - x)$, and will work for i's rival if this inequality is reversed. A worker located at a distance $x^0 \in (0, 1/n)$ is indifferent between working for firm i and i's closest neighbor when:

$$w_i - tx^0 = w_j - t(1/n - x^0).$$

Solving for x^0 we see that $x^0 = (t/n + w_i - w_j)/2t$, provided that $|w_i - w_j| \leq t/n$. Since all workers located up to a distance of x^0 from firm i have lower transportation costs they will work for firm i. Similarly, all workers located farther than x^0 from firm i have higher transportation costs and will work for i's rival. Since there is a similar set of workers on the other side of firm i, firm i's supply of 0-reservation wage workers is

$$\frac{t/n + w_i - \bar{w}_i}{t}$$

where $\bar{w}_i$ is the mean wage offered by i's two nearest neighbors.

Consider now the supply of high reservation wage workers. A high reservation wage worker located at x will not work at all if $w_i - tx < v$ but will work for firm i if $w_i - tx > v$. Let $x^v \in (0, 1/n)$ be the distance at which a high reservation wage worker is indifferent between working for firm i and not working at all, i.e., $v = w_i - tx^v$. Solving for x^v yields,

$$x^v = \frac{w_i - v}{t}$$

provided that $w_i \geq v$. Again, all workers located between firm i and x^v work for firm i and those located farther than x^v do not work. Hence firm i's supply of v-reservation wage workers is $2\mu x^v$.

Therefore, when $|w_i - w_j| \leq 1/n$ and $w_i \geq v$, firm i's total labor supply is:

$$L_i = \frac{t/n - 2v\mu + (1 + 2\mu)w_i - \bar{w}_i}{t}. \tag{1}$$

Equation (1) shows that labor supply is increasing in the firm's own wage, w_i, but decreasing in the wages paid by other firms, $\bar{w}_i$.[5] However, due to variations in the participation rate (due to the presence of high reservation wage workers), the former effect is larger than the latter, so that a unit increase in both w_i and $\bar{w}_i$ leads to increased labor supply for firm i. This also implies that the elasticity of labor supply for the individual firm exceeds the elasticity of industry labor supply. Thus the situation differs from both monopsony and perfect competition—under monopsony there is no distinction between the two elasticities and under perfect competition, labor supply is infinitely elastic at the level of the firm.

2.2. **Firm Profit Maximization.** We now turn to the firm's output decisions, which affect labor demand. Firm i's output is given by the homogeneous of degree one production function:

$$Y_i = L_i f_i(K_i/L_i) \tag{2}$$

where K_i is i's capital input and f_i is assumed to be twice differentiable, increasing and concave, so that $f_i'' < 0$. Since different workers have identical skills and abilities, they enter the production function uniformly.

Firm i's profits can be written as follows:

$$\pi_i = p_i L_i f_i(K_i/L_i) - rK_i - w_i L_i. \tag{3}$$

[5]That is, one firm's wage setting decision has an externality effect on other firms' labor supply. This externality will have an important effect on the equilibrium wage distribution.

where p_i is the price of firm i's output and r is the capital rental rate. Product prices may differ due to product differentiation or because firms competing within the same labor market may be selling different goods.

Firm i's first order condition with respect to K_i is

$$p_i f_i'(k_i) - r = 0 \tag{4}$$

where $k_i = K_i/L_i$ is the capital-labor ratio. This implies that in equilibrium, the capital-labor ratio is a function of r/p_i only, and is therefore constant if r and p_i are fixed. Furthermore, given the assumption of concavity, an increase in the market price or a fall in the capital rental rate results in an increase in the capital labor ratio.

Using (4) we see that profits (3) can be rewritten as

$$\pi_i = \phi_i(p_i, r)L_i - w_i L_i \tag{5}$$

where $\phi_i(p_i, r) = p_i[f_i(k_i(r/p_i)) - f_i'(k_i(r/p_i))k_i(r/p_i)]$. We call ϕ_i firm i's *net revenue product of labor* which differs from its marginal revenue product in that firm i is optimally adjusting its capital labor ratio.

The first order condition with respect to the wage for firm i is

$$\begin{aligned}\frac{\partial \pi_i}{\partial w_i} &= (\phi_i - w_i)\frac{\partial L_i}{\partial w_i} - L_i \\ &= \frac{(1+2\mu)\phi_i - t/n + 2\mu v + \bar{w}_i - 2(1+2\mu)w_i}{t} = 0.\end{aligned}$$

This first order condition yields the firm's optimal wage as a function of the mean wage set by its nearest rivals:

$$w_i = \alpha_i + \beta \bar{w}_i \tag{6}$$

where

$$\alpha_i = \frac{(1+2\mu)\phi_i - t/n + 2\mu v}{2(1+2\mu)}$$

$$\beta = \frac{1}{2(1+2\mu)}.$$

Observe that the individual firm's optimal wage, w_i, is an increasing function of the wage set by other firms, $\bar{w}_i$. For example, in Figure 2, if i's nearest rivals offer a mean wage of $\bar{w}_i$, i's optimal wage is w_i. However, if i's rivals raise their mean wage to $\bar{w}_i'$, i must similarly raise its wage to w_i' in order to continue maximizing profits. This implies that we have a situation of *strategic complementarity* in wage setting. As we shall see later, this has important implications for the effects of minimum wage legislation upon firms which are initially paying wages

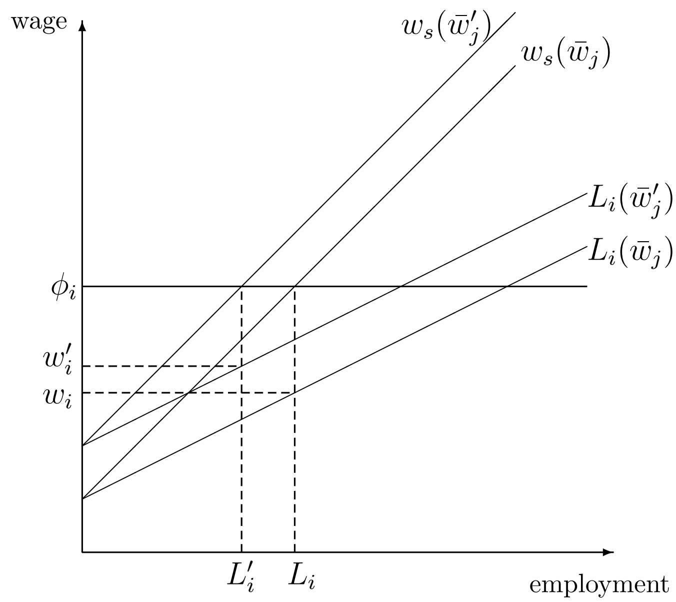

FIGURE 2. Wage Setting Oligopsonists

above the minimum wage, and would therefore seem to be unaffected by such legislation.

Equations similar to (6) have been popular in some recent empirical work in labor economics (Abowd and Lemieux, 1993; Blanchflower *et al.*, 1996; Nickell and Wadhwani, 1990, for example). This literature estimates a wage equation where the wage depends upon "inside" factors such as firm profitability and upon the outside wage $\bar{w}$. Our theoretical wage equation has a similar form, since α_i captures the inside factors while β is the coefficient upon the outside wage. Note however that the theoretical model says that the outside wage is *firm-specific,* i.e., $\bar{w}_i$. Hence, to re-interpret these wage equation estimates, this implies that the outside wage in such empirical work is subject to measurement error. In particular, the estimated coefficient on the outside wage will be biased downward. Moreover, this also implies that the estimate of the role of inside factors in such empirical work will also be biased. The biases of insider factors will have the further problem that both the direction and the magnitude of these biases are unknown. In other words, our model suggests that one should be careful in proxying the outside wage, especially for firms which operate in a spatially separated environment.

Now returning to solving the model, a Nash equilibrium is given by the simultaneous solution to (6) for $i = 0, \ldots n-1$. This can be reformulated as a matrix equation of the form $\mathbf{w}^* = B^{-1}\boldsymbol{\alpha}$ where

$$B = \begin{bmatrix} 1 & -\frac{\beta}{2} & 0 & \cdots & 0 & -\frac{\beta}{2} \\ -\frac{\beta}{2} & 1 & -\frac{\beta}{2} & & 0 & 0 \\ 0 & -\frac{\beta}{2} & 1 & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & -\frac{\beta}{2} & 0 \\ 0 & 0 & & -\frac{\beta}{2} & 1 & -\frac{\beta}{2} \\ -\frac{\beta}{2} & 0 & \cdots & 0 & -\frac{\beta}{2} & 1 \end{bmatrix}. \tag{7}$$

After solving for B^{-1}, we show that each employer's equilibrium wage is a positive weighted sum of all α_j's where the impact of rival j on firm i's wage declines in j's distance from i. Furthermore, i's own α_i has the greatest weight. This suggests that, although it need not be the case (see p. 15), high productivity firms will typically offer high wages and low productivity firms will typically offer low wages. It should now be apparent that as long as $\alpha_i \neq \alpha_j$ for some i and j, we will have a dispersed wage equilibrium.

Define $[\cdot]$ to be the greatest integer function (i.e., $[x] = \max\{i \in \mathbb{I} \mid i \leq x\}$). The above results, stated more formally,

Proposition 1. *For $i = 0, \ldots n-1$, $w_i^* = \sum_{j=0}^{n-1} q_j \alpha_{(i+j) \bmod n}$ where $q_j = q_{n-j}$ for $j = 1, \ldots [n/2]$ and $q_0 > q_1 > \ldots > q_{[n/2]} > 0$.*

Proof. See Appendix.

In order to better understand the properties of our dispersed wage equilibrium, we now restrict ourselves to a few simple, symmetric examples. One particularly simple example is when $\phi_i = \phi_j$ for $i \neq j$. In this case, all firms offer the same equilibrium wages and hire the same number of workers.

$$w_i^* = \frac{\alpha}{1-\beta} \tag{8}$$

$$L_i^* = \frac{1}{n} + \frac{2\mu}{t}\left(\frac{\alpha}{1-\beta} - v\right) \tag{9}$$

Using slightly more complicated symmetric examples, we now explore the implications of oligopsony in more detail.

3. Wage Dispersion

In order to have wage dispersion, we must have firms differing in some respect. We choose to model this by assuming that some firms are of type H, having high net revenue product, ϕ_H, while others are of type L, having low net revenue product, ϕ_L. These differences in net revenue product can be for a number of reasons. For example, if firms in this labor market are in different industries then product prices and production functions will differ, giving rise to differences in net revenue product. On the other hand, firms within the same product market may have different production functions because of firm specific characteristics such as, differing managerial talent, different production techniques, different access to assets with varying productivities (fertile vs infertile land), etc.[6] As we will see, this implies that on average, higher productivity firms will pay higher wages than lower productivity firms. However, the extent of wage dispersion depends upon the extent to which the two types of firms compete for workers. To examine this effect we consider two types of interaction.

To start with, consider a model where high productivity firms only compete with low productivity firms for workers. Let there be an even number of firms n, of which $n/2$ are of type H, having high productivity, and $n/2$ are of type L, with low productivity. We assume that the n firms are evenly spaced along the circle, and the types alternate in location, as in Figure 1. This implies that a high type firm's immediate neighbors are of low type, and vice-versa. Nash equilibrium wages are given by:

$$w_H^* = \frac{\alpha_H + \beta\alpha_L}{1-\beta^2} \tag{10}$$

$$w_L^* = \frac{\alpha_L + \beta\alpha_H}{1-\beta^2} \tag{11}$$

This implies that the equilibrium wage differential is given by:

$$w_H^* - w_L^* = \frac{\alpha_H - \alpha_L}{1+\beta} = \frac{(\phi_H - \phi_L)(1+2\mu)}{3+4\mu} \tag{12}$$

We illustrate the optimal wages of both types of firms in Figure 3. Equilibrium wages are given by the intersection of the two reaction functions—this intersection is above the 45° line, reflecting the higher wages paid by the more productive firms. Accordingly, employment is higher in the high productivity firms than in the low productivity firms with the more productive firms employing more of both types of workers.

Alternatively, assume that n is divisible by four and consider the location pattern depicted in Figure 4, where firms are evenly spaced,

[6]Differences in worker productivities does not necessarily drive out firms that are less productive. As it is commonly argued—even with a perfectly competitive product market—assets which lead to higher productivities will command a higher price. Hence firm profits may not be higher in firms with greater producitivity.

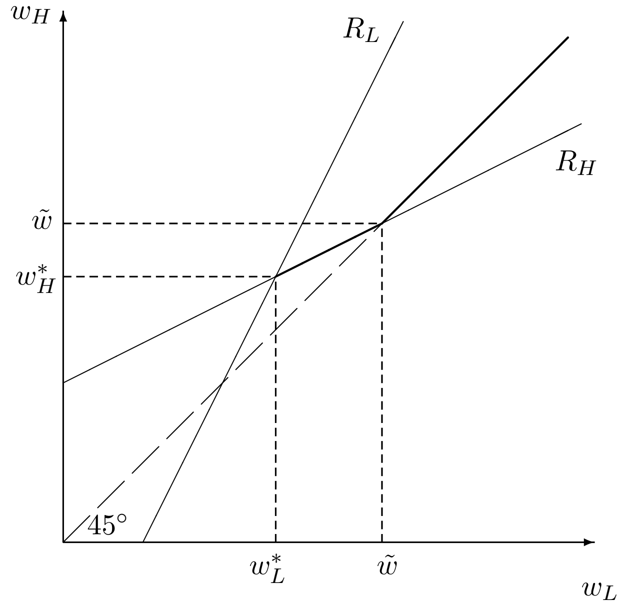

FIGURE 3. Equilibrium with Asymmetric Firms

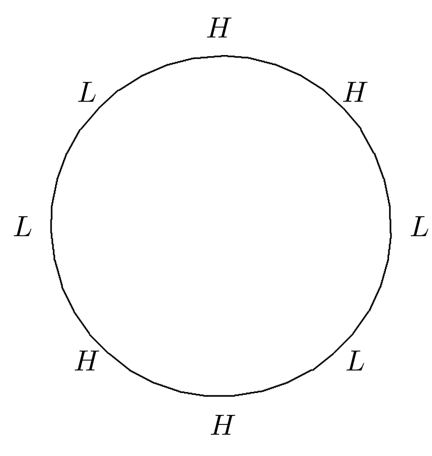

FIGURE 4. Alternating pairs of firm types

and we have two H type firms, followed by two L type firms, followed by two H type firms, and so on. In this configuration each firm has one L type neighbor and one H type neighbor. Equilibrium wages, and the wage differential are now given by:

$$w_H^* = \frac{2\alpha_H - \beta(\alpha_H - \alpha_L)}{2(1-\beta)} \tag{13}$$

$$w_L^* = \frac{2\alpha_L + \beta(\alpha_H - \alpha_L)}{2(1-\beta)} \tag{14}$$

$$w_H^* - w_L^* = \alpha_H - \alpha_L \tag{15}$$

The wage differential in this case is higher than the case with alternating locations. Clearly, the magnitude of wage differentials depend on the degree to which high and low productivity firms interact and we conjecture that the greater the degree of interaction, the smaller the differential.

Since firms in different industries have different production techniques and face different prices, their net revenue products will differ (i.e., $\phi_H > \phi_L$). Thus these results are consistent with inter-industry wage differentials. Furthermore, firms within the same industry can have different production functions, and as a result different productivities. Thus our results are consistent with the existence of intra-industry wage differentials. To the extent that prices and production techniques are likely to be more similar within an industry, we expect that measured intra-industry wage differentials should typically be smaller than measured inter-industry wage differentials. Finally, notice that these wage differentials can be quite large. With our first example, the magnitude of the wage differential ranges from a sixth to a quarter of the difference in employer productivity while with the latter example the wage differential is equal to a half this difference.

Similarly, for each of these examples, we can compute the size differential of high productivity firms and in comparison to low productivity firms:

$$L_H^* - L_L^* = \frac{2(1+\mu)}{t}(w_H^* - w_L^*) \tag{16}$$

$$L_H^* - L_L^* = \frac{(1+4\mu)w_H^* - w_L^*}{2t}. \tag{17}$$

Thus employers that offer higher wages employ more workers. That is, the "employer size–wage effect" (Brown and Medoff, 1989) arises naturally in this model. Furthermore, Dickens and Katz (1987a) found that industries which paid well in one occupation also tended to pay highly in other occupations. Suppose that an industry's marginal revenue products are correlated over the occupations it employs. For example, it would be unusual for a high tech computer manufacturer to provide its secretaries with manual typewriters rather than modern computers and software whereas in a small corner shop, although unusual, it might not be that surprising. If marginal revenue products are correlated within an industry we would find that industries which tend to pay high wages for one occupation also tend to pay high wages in other occupations. That is, a high wage firm is likely to have high marginal revenue products for all occupations that it hires and as a result it will offer higher wages than its rivals.

3.1. **Profitability and Wages.** There is also evidence that firms that are more profitable tend to offer higher wages (Blanchflower *et al.*, 1996; Dickens and Katz, 1987a; Pugel, 1980). However, as we argued earlier (footnote 6), assets which increase worker productivity will command a higher price. Thus in order to draw any conclusions in this direction requires more than the heterogeneity which we've assumed. One possibility is that there is a fixed cost of production, c_i, which is correlated with the marginal revenue product of labor. For illustrative purposes, suppose that this correlation is perfect and that $c_i = \gamma\phi_i$. In this case, profits can be rewritten as:

$$\pi_i = \phi_i(L_i - \gamma) - w_i L_i \tag{18}$$

Although it is not necessarily true that that high productivity firms offer higher wages (see page 15), it will be true *on average*. In this case, higher productivity employers will typically earn higher profits *and* pay higher wages. Note that even though the correlation between c_i and ϕ_i is perfect, the correlation between π_i and w_i will be imperfect and depends on how employers are distributed in relation to one another.

Alternatively we could assume that there is rent sharing between firms and the owners of scarce resources used in production. The Nash bargaining solution would predict that $c_i = \zeta(\phi_i - w_i^*)L_i^*$ where ζ is the resource owner's "bargaining power." In this case, profits can be rewritten as $\pi_i^* = (1 - \zeta)(\phi_i - w_i^*)L_i^*$. Provided that the owners of these resources do not hold all of the bargaining power and bargaining powers are similar across these owners, our model would predict a positive relationship between firm profitability and wages. This is particularly interesting because the existence of rents can have effects even for employees who do not share in the rents.

In sum, once we include fairly reasonable assumptions regarding the fixed cost of production, our model can explain the observed correlation between profitability and wages. The degree of correlation depends on the nature of interaction between employers and on the degree of correlation between productivity and fixed production costs.

3.2. **The Effect of Minimum Wages on the Distribution of Wages.** We now consider the effect of minimum wages on the distribution of wages. Empirical work on minimum wages has noted that minimum wages tend to reduce wage dispersion, and can also cause a "spike" in the wage distribution at the minimum wage. We examine the effects of minimum wages upon wage dispersion and upon firms who pay more than the minimum.

In the case where high and low productivity firms alternate in location (as in Figure 1), consider the effect of a minimum wage, starting at w_L^*, the equilibrium wage of the less productive firms. An increase in the minimum wage above w_L^* affects the wage paid by the less productive firms one-for-one. The effect on wages paid by the high type firms is given by their reaction function, as long as this wage is above the minimum (Figure 3). Thus although the minimum wage is not binding on the high wage firms, there is a spillover effect which is due to the strategic complementarity in wage setting. Finally, at the point $\tilde{w}$, where the reaction function intersects the 45° line, the minimum wage becomes binding for the high productivity firms as well and they also start paying exactly the minimum wage.

Consider the implications of our analysis for wage dispersion. A minimum wage reduces wage dispersion, but to a smaller extent than if there were no interaction, since wages also rise in the high wage firms. Note that the difference between the wages paid by the two types of firms at any minimum wage is given by the vertical distance between the high productivity firm's reaction function and the 45° line. This declines with the minimum wage, and finally shrinks to zero at $\tilde{w}$.

Now consider again the configuration as illustrated in Figure 4. The wage of the high productivity firms, as a function of the wage of low productivity firms, is given by:

$$w_H = \frac{2\alpha_H}{2-\beta} + \frac{\beta}{2-\beta} w_L.$$

Hence a minimum wage which binds only on low wage firms raises the wages of high productive firms by a factor $\beta/(2-\beta)$. This effect is positive, but less than the effect in the case of alternating firms since $\beta < 1$. Indeed, one can show that high wage firms here are hurt less than in the former case.

These examples show that when high wage firms interact directly with only low wage firms, there is less wage dispersion and a minimum wage reduces this dispersion quickly. When high wage firms interact

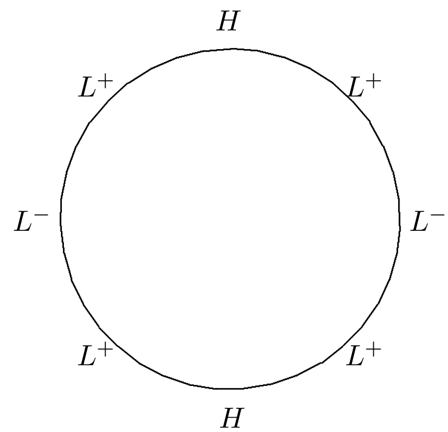

FIGURE 5. One H firm alternating with 3 L firms

with both low wage and high wage firms, there is greater wage dispersion and a minimum wage compresses this dispersion at a slower rate.

To see how a spike in the wage distribution arises in our model, consider yet a third configuration. Suppose again that n is divisible by four but firms are located following a pattern like that in Figure 5 where three type L firms are followed by one type H firm, followed by three type L firms, etc. In this configuration, L^- firms have two L type neighbors, L^+ firms have one L type neighbor and one H type neighbor and H firms have two L type neighbors. The equilibrium wages in this case are:

$$w_H = \frac{2\alpha_H + 2\beta\alpha_L - \beta^2(\alpha_H - \alpha_L)}{2(1-\beta^2)} \tag{19}$$

$$w_L^+ = \frac{2\alpha_L + \beta(\alpha_H + \alpha_L)}{2(1-\beta^2)} \tag{20}$$

$$w_L^- = \frac{2\alpha_L + 2\beta\alpha_L + \beta^2(\alpha_H - \alpha_L)}{2(1-\beta^2)}. \tag{21}$$

It is easy to show that $w_H > w_L^+ > w_L^- > 0$. Thus the equilibrium wage distribution has 1/4 of all firms paying w_H, 1/2 of all firms paying w_L^+ and 1/4 of all firms paying w_L^-. Note that even though L^+ and L^- employers have identical net revenue products of labor, they offer different equilibrium wages. This is a result of the fact that L^+ firms are competing with both H and L type firms but L^- firms are only competing with L type firms. It should be apparent from this example that even with just two types of employers, more arbitrary location patterns will generate fairly complicated equilibrium wage distributions.

If we introduce a minimum wage which is binding on all of the L type firms but not binding on the H type firms (i.e., $w^m \geq (2\alpha_L+\beta\alpha_H)/(2-\beta-\beta^2)$ but $w^m < \alpha_H/(1-\beta)$) then 3/4 of all firms will be paying the minimum and 1/4 of all firms will be paying more than the minimum. In other words, there is now a spike in the wage distribution at the minimum. More generally with arbitrary n and ϕ_i's, the equilibrium will result in some finite set of wages (as many as but no more than n). A minimum wage will be binding on those employers offering the lowest wages and will have a spillover effect on those offering higher wages.

As a brief aside, note that with many types, one might conjecture that there is a monotonic relationship between net revenue product, ϕ_i, and the equilibrium wage, w_i. This intuition is incorrect as can be easily demonstrated. Suppose there are 3 productivity levels, ϕ_L, ϕ_M and ϕ_H, and four employers. Suppose that firms 0 and 3 are of type L and H and that both firms 1 and 2 are of type M. Let $\phi_M = \phi_L + \Delta\phi$. When $\Delta\phi$ is small, equilibrium wages will be close to those from the previous example with $n = 4$, implying that $w_3 > w_2$ even though $\phi_2 > \phi_3$.

4. Alternative Theories

We now compare our modeling approach to alternative non-perfectly competitive models of the distribution of wages.

Efficiency wage theory has been offered as one explanation for wage dispersion. Ramaswamy and Rowthorn (1991) consider an efficiency wage model where firms have heterogeneous production functions, and assume that effort in each firm is a function of the wage—the micro-foundations behind this effort decision are not specified. Each firm sets the wage to satisfy a generalized Solow condition and this gives rise to wage dispersion. Since effort does not depend upon outside wages, there are no spillover effects, and hence minimum wages would not affect high wage firms. For the same reason, this model can also accommodate a spike in the wage distribution caused by a minimum wage.

Albrecht and Vroman (1998) consider an efficiency wage model with homogeneous firms where workers differ in their disutility of effort, so that there is adverse selection in addition to moral hazard. For any given wage, the set of employees of the firm is partitioned into shirkers (those with a relatively high disutility of effort) and non-shirkers. When there is a continuous wage distribution, firms face a smooth trade-off, where a higher wage reduces the set of shirkers, and increases aggregate effort. When the wage distribution has a mass point, however, a firm at the mass point has an incentive to offer a slightly higher wage because by doing so it can discontinuously increase the proportion of non-shirkers amongst its new hires. This is because with a mass point, the distribution of unemployed workers is discontinuous and workers with a high disutility of effort are overrepresented. Thus the equilibrium in this model must not only involve wage dispersion but the distribution of wages must be atomless. As a result, there cannot be a spike in the wage distribution.[7]

An alternative approach is the literature on job search. Workers must search in order to know about wage offers, and this gives employers market power. Burdett and Mortensen (1998) analyze wage dispersion in a model with a fixed number of firms where workers search both when employed and unemployed. The equilibrium wage distribution is atomless and lies below the marginal product of labor with larger firms offering higher wages. Firms are indifferent between all wages in its support, since they attract more workers by offering higher wages. A minimum wage shifts the distribution upward so that there is a spillover effect. However, like Albrecht and Vroman (1998), it must also remain atomless, because otherwise a firm would be able to discontinuously increase its labor supply by a small increase in the wage. Hence this model does not explain the observed spike in the wage-distribution induced by minimum wages.

Furthermore, the papers by Albrecht and Vroman (1998) and Burdett and Mortensen (1998) have very strong empirical predictions regarding the shape of the wage distribution. In particular, the density function over equilibrium wages in Albrecht and Vroman (1998) must be monotonically increasing in the wage rate. In contrast, Burdett and Mortensen (1998) predicts that the density function should be monotonically decreasing in the wage rate. That is, for identically able workers, the relative frequency of a wage offer is a monotonic function of the wage—either case seems implausible as a general rule. Burdett and Mortensen (1998) provide an extension which—like the current paper—requires employers to be heterogeneous in terms of their productivity. Once employer productivity differences are allowed for, non-monotonic

[7] The literature on employee turnover Salop (1979a) is also related. This is motivated by the notion that workers are unsure of employer characteristics prior to employment and only learn about them gradually, however, turnover is determined exogenously and lacks microfoundations (i.e., worker quit decisions are left unmodeled).

equilibrium wage distributions can emerge. However, as before, wage distributions must remain atomless and therefore even with employer heterogeneity, Burdett and Mortensen (1998) is unable to explain the existence of a spike at the minimum wage.

5. Concluding Remarks

The predictions of our model regarding the distribution of wages and how they are affected by minimum wages are consistent with the empirical evidence. In particular, a spike in the wage distribution is a natural feature in our model but is inconsistent with other models. The intuition is that preference heterogeneity implies that firms command "loyalty" among some of their workers. This in turn implies that wage offers do not need to increase at the same rate as the minimum wage so that as the minimum wage increases, some firms which previously paid more than the minimum will offer exactly the minimum wage. Our basic assumption (of less than perfect homogeneity of jobs) is realistic and the resulting is model is tractable.

Search and efficiency wages are no doubt also important to our understanding of the functioning of labor markets, however, these models, which are the most popular explanations for wage dispersion, fail on what would seem to be a fundamental point. That is, when the market is perturbed with a binding minimum wage, the resulting wage distribution fails to behave in accordance with the empirical evidence. A robust theory of wage dispersion would ideally explain not only the existence of wage differentials but should also be able to predict how these equilibrium wage distributions change in response to an important and commonly applied policy such as a minimum wage. Under such a criterion, our theory is highly successful.

Appendix

Proof of Proposition 1. Letting $Q = B^{-1}$, employer i's equilibrium wage is given by $w_i^* = \sum_{k=0}^{n-1} q_{ik}\alpha_k$. Since B is a symmetric *circulant*[8] matrix, Q is also a symmetric circulant matrix. Circulant matrices can be defined by their first row so let $\mathbf{q} = (q_0, q_1, \dots q_{n-1}) = (q_{0,0}, q_{0,1}, \dots, q_{0,n-1})$. Noting that $QB = I$, it is easy to see that $\mathbf{q}$ must solve:

$$q_0 - \frac{\beta}{2}q_1 - \frac{\beta}{2}q_{n-1} = 1 \tag{22}$$

[8]A square matrix C is *circulant* if the elements of each row of C are identical to those of the previous row, but are moved one position to the right and wrapped around (Davis, 1979).

$$-\frac{\beta}{2}q_t + q_{t+1} - \frac{\beta}{2}q_{t+2} = 0 \tag{23}$$

for $t = 0, 1, \ldots n-3$ and

$$-\frac{\beta}{2}q_0 - \frac{\beta}{2}q_{n-2} + q_{n-1} = 0 \tag{24}$$

Notice that (23) is a second order linear difference equation with characteristic roots:

$$\lambda = \frac{1}{\beta} - \sqrt{\left(\frac{1}{\beta}\right)^2 - 1} \tag{25}$$

$$\mu = \frac{1}{\beta} + \sqrt{\left(\frac{1}{\beta}\right)^2 - 1} \tag{26}$$

and since $\beta < 1/2$, it follows that $0 < \lambda < 1 < \mu$. The general solution to (23) is therefore

$$q_t = A\lambda^t + B\mu^t \tag{27}$$

for arbitrary constants A and B. Substituting this into (22) and (24) results in a system of two equations with two unknowns, A and B. Solving yields:

$$A = \frac{1}{(1-\lambda^n)\sqrt{1-\beta^2}} \tag{28}$$

$$B = \frac{1}{(\mu^n - 1)\sqrt{1-\beta^2}}. \tag{29}$$

These are both positive and therefore $q_t > 0$ for all $t = 0, \ldots n-1$.

Since Q is symmetric and circulant, it must be the case that $q_t = q_{n-t}$ for $t = 1, \ldots [n/2]$. Furthermore, since $\lambda < 1$ and $\mu > 1$, if q_t is non-monotonic in t, it must first be declining and then be rising. But because Q is symmetric, it must be the case that $q_0 > q_1 > \ldots > q_{[n/2]}$. ■

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