The wage-wage-...-wage-profit relation in a multisector bargaining economy*

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abstract

The equalization of profit rates across a multisector production economy subject to Nash bargaining over wages supports an industry wage structure like those that account for a large fraction of actual wage dispersion and a wage-wage-...-wage-profit surface on which the general profit rate can vary inversely or directly with the wage paid in a given industry. Institutional changes that compress or decompress the wage distribution depend for support on industrially specific cross-class coalitions of workers and capitalists. Technical changes that raise capitalists' profits in current prices can lower the equilibrium profit rate.

Wage-profit relation, prices of production, wage dispersion, bargaining, technical change.

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1 Introduction

Economists who spend time with wage regressions say that statistically visible differences between workers can explain no more than 30 percent of the variation in cross sections of the workers' wages (Mortensen, 2003). Industry-level wage differentials capture a great part of the residual dispersion, and orderings of industries by the wages paid in them are surprisingly resilient over time and across national economies (Gittleman and Wolff, 1993; Krueger and Summers, 1988). Students of income distribution in capitalist societies can do a lot worse than to study the industry structure of wages.

This paper is about the wage structures generated by bargaining in a Leontief circulating-capital economy. I suppose that wages are set in Nash bargains between transiently matched workers and capitalists, and I consider systems of goods prices and wages that equalize rates of profit over all sectors of production.

This idealization is interesting, I hope, from at least four different points of view. For one thing it's an easy place in which to make the point that if many goods are produced by goods and labor, a simple bargaining mechanism is enough to send indistinguishable workers home with different wages. In one-sector models equilibrium wage dispersion tends to be cooked up from some mixture of imperfect competition in the market for the one good, a dispersion of firms' technologies inside the frontier of efficient production, firm-level differences in the parameters of labor monitoring or training, and the strategic differentiation of wage offers by employers competing for scarce workers to fill vacant jobs. The capitalists of this paper are by contrast price takers with access to a common technology who face identically structured bargaining situations in which such head-hunting rivalries play no part.

The paper is also a development of the analysis of stationary price systems for linear production models, what Marx and Sraffa called *prices of production*. I think that, for all the pleasures of logical jousting with Walrasian capital theory that beckoned along the way, the original point of studying these objects was to better understand the institutionally variable joint determination of prices, wages, and profitability. If those pleasures were fleeting, the demand for understanding remains. To meet it calls for models of institutionally variable wage-setting mechanisms, and here I try out one mechanism like that.

Along with the classwide interests that arise from capitalist production relations, workers and capitalists have interests special to the industries in which they work or invest which pit them against their class fellows in other sectors. It's a truism of the class analysis of politics that inter-class conflict and intra-class conflict are each shaped by the specific ways in which they're combined. The third thing I do in the paper is to isolate some sharp albeit abstract instances of this interaction. I show that the equilibrium wages of workers in some industries can vary *inversely* with other workers' wages and *directly* with the uniform profit rate, I distinguish cases in which profitability bears an increasing relation to wage inequality from cases in which that relation is decreasing, and I identify conditions under which institutional changes that variously compress or decompress the equilibrium wage distribution and raise or lower the equilibrium profit rate might be championed by cross-class coalitions made up of particular sections of the two classes.

Finally the wage mechanism of this paper might matter to an economy's direction of technical change. I present one example in the line begun by Okishio (1961), pointing out that in this bargaining closure of the price-of-production system innovations that raise profits in current prices can result in a *lower* equilibrium rate of profit.

I hope it's obvious that my arguments offer only a limited explanatory handle on the world. For one thing we lack any general demonstration that prices of production are stable rest points of dynamical systems describing the motions of prices and production activity directed by capitalists' profit-minded trading and investment decisions. Even if stability in that sense were ensured, the process of convergence would be slow and nearly opaque to its participants, so it's hard to imagine that equilibrium comparisons of the kind that I'll discuss could guide workers' and capitalists' stances in class struggle and compromise. A far more interesting project than mine would show how stationary nondegenerate distributions of wages and (while we're at it) profit rates emerge from the joint evolution of prices, wages, capital allocation, and production technology, and would endogenize workers' and capitalists' collective action as a constituent adaptive process of the system that sustains them. Until that movie gets made, we're stuck with snapshots like the ones you'll see here.

2 Prices of production with Nash wage bargains

Consider some capitalists who run activities from a Leontief technology described by a couple (A, l). A, there, is a semipositive, indecomposable, productive $n \times n$ matrix whose jth column a^j lists the quantities of produced inputs needed to produce a unit of the jth good; l, a positive row n-vector whose jth coordinate l_j gives the jth activity's unit labor requirement. Each capitalist enters a production period owning stocks of commodities produced in the previous period, chooses a production plan that maximizes profits subject to a budget constraint in those stocks and prices p, and buys the required commodity inputs.

She also tries to hire the required labor in a market for costlessly enforceable one-period employment contracts with wages to be paid at the end of the period. This market closes after a single round of matching, so matched workers and capitalists who fail to agree on a wage rate are out of work or business for the period. Let p_j and w_j be the price of the jth good and the wage paid in the jth activity. A capitalist who plans to run that activity takes $p_j l_j^{-1} - w_j$ per worker if production goes ahead; if not, her fallback is the value of the inputs that she'd planned to tie up with the worker, $pa^j l_j^{-1}$. I assume that capitalistically unemployed workers receive a payment υ, the *outside wage*, which you can understand as an unemployment benefit or as income available from economic activity outside the capitalist sector. For some β in $[0, 1)$—call this weight *workers' power*, and let it take the same value in every activity—a generalized Nash bargain maximizes the weighted joint surplus over non-production

$$(w_j - \upsilon)^\beta \left(\left(p_j - pa^j\right) l_j^{-1} - w_j\right)^{1-\beta}.$$

If it's run, then, the jth activity pays each worker

$$w_j = \beta \left(p_j - pa^j\right) l_j^{-1} + (1 - \beta)\, \upsilon, \tag{1}$$

a weighted average of the capitalist's value added per worker and the outside wage.

Faced with stationary prices p, capitalists who anticipate the wage bargains (1) are indifferent between committing capital to the different activities if and only if those prices satisfy

$$p_j = (1+r)\,pa^j + w_j l_j \tag{2}$$

$$= (1-\beta+r)\,pa^j + \beta p_j + (1-\beta)\,\upsilon l$$

$$= \left(1 + r\,(1-\beta)^{-1}\right) pa^j + \upsilon l_j \tag{3}$$

for some $r \geq 0$. So a price system that supports the production of all n goods by profitmaximizing capitalists and that takes a working-class consumption basket d as numéraire looks like

$$p^* = \left(1 + r^*\,(1-\beta)^{-1}\right) p^* A + \upsilon l, \tag{4}$$

$$p^* d = 1. \tag{5}$$

You can next flesh this out by showing that some vector of activity levels clears all the goods markets at these prices on one or another assumption about capitalists' and workers' tastes. I won't do this, though, since all the structure I need is in the wage and price system, $(1, 4, 5)$, which follows from profit-rate equalization under the Nash bargaining rule whatever the quantity relations you impose on it.

If this bargaining set-up seems rigged to rationalize the wage equation (1), you might prefer to skip the set-up and *start* the discussion at (1). The paper's topic is then just the question, What follows if every capitalist pays each of her workers a convex combination of her operated activity's value added per head and an outside wage? From this point of view (1) stands in for any bargaining process that splits the difference between *ceilings* given by capitalists' revenues net of material input costs and an economywide wage *floor*.[1] For

[1]Where υ is identified with a subsistence wage, (1) conjures up Sraffa's view of the "double character of the wage" as including "besides the ever-present element of subsistence ... a share of the surplus product."

example you might entertain this as a rough-and-ready representation of enterprise- or industry-level collective bargaining subject to a uniform dole or strike benefit υ.

I need, though, to flag one analytically crucial contrivance. My assumptions that employment is transient and that workers and capitalists can't return to the market to search for other production partners in the current period have the effect of insulating the wage bargain from the market's degree of tightness or slack. An obvious next step is to remove that insulation. But in opening the price-of-production system to endogenous wage dispersion an interesting *first* step is to choose the smallest changes that possibly accommodate this. It's in such a spirit of analytical gradualism that (1) preserves that system's signature decomposition between relative prices and macroeconomic quantities.

3 Relative wages and capital-labor ratios

One conclusion about the industry structure of wages is available right away. Substituting for p_j from (3) into (1) gives that

$$w_j^* = \beta \left(1 - \beta\right)^{-1} r^* \frac{p^* a^j}{l_j} + \upsilon. \tag{6}$$

For any two operated activities j and k, then,

$$w_j^* - w_k^* = \beta \left(1 - \beta\right)^{-1} r^* \left[\frac{p^* a^j}{l_j} - \frac{p^* a^k}{l_k}\right]. \tag{7}$$

Sraffa himself expresses this idea by working with an exogenous and uniform real wage measured in units of the economy's given net output—in effect a given share of wages in national income—which assumption leaves him no way in which "to separate the two components of the wage." (Sraffa, 1960, p.9) The wage bargains (1), which share out value added at the level of the individual sectors rather than economywide, bring the Sraffian "double character" of wages back to the surface. (See Franke (1981) and Burgstaller (1995, pp.78-87) for developments of Sraffa's wage-share closure. I thank Duncan Foley for suggesting that I spell out the connection to Sraffa.)

Wage differences in this bargaining equilibrium are proportional to differences in the activities' ratios of the value-of-produced-inputs to labor employed, so (7) is a starting point for explaining the positive estimates of coefficients on capital intensities that are an outstanding result of interindustry wage regressions (Gittleman and Wolff, 1993; Arai, 2003; and cf. Acemoglu, 1999).[2] I plan to pursue that explanation in another paper, but I think that its promise is reason enough to reconsider the comparative statics of income distribution in multisector economies on the assumption that wages are dispersed by bargaining as in (7).

Of course (7) does not say that relative wages fall out of the production technology alone. The capital intensities on its righthand side are creatures of the price system associated with a particular profit rate, workers' power, outside wage, and numéraire. So to understand the wage structure any further you have to close that system.

4 A two-parameter family of equilibria

Suppose that state policy or living standards in the noncapitalist sector peg the real outside wage in terms of the bundle d to a number υ. And with far greater violence to reality suppose that β is given by facts of the social and technical organization of production that are independent of the processes of price and wage formation. Then you can think of pairs (β, υ) as distributive parameters that define a family of price-of-production systems.

[2] Interindustry wage inequality in these casual labor markets is consistent with equal expected personal incomes: If every worker faces the same stationary probabilities of being hired for the different activities, every sufficiently long-lived worker can expect to pass through high- and low-wage jobs in the same proportions. To give wage dispersion some political bite, assume instead that a worker has a greater probability of being hired for some activity in a later period if she's employed on it now.

Prices of production normalized as in (5) satisfy

$$p^* = \left(1 + r^* (1-\beta)^{-1}\right) p^* A + \upsilon p^* dl \tag{8}$$

which shows itself to be an eigen equation when it's rewritten as

$$p^* A \left[I - \upsilon dl\right]^{-1} = \left(1 + r^* (1-\beta)^{-1}\right)^{-1} p^*. \tag{9}$$

It's readily checked that

$$\left[I - \upsilon dl\right]^{-1} = I + \frac{\upsilon}{1 - \upsilon ld} dl \tag{10}$$

where this inverse exists. For $\upsilon < (ld)^{-1}$ the matrix $A\left[I - \upsilon dl\right]^{-1}$ is semipositive and indecomposable. Let $\lambda_F(A, l, \upsilon, d)$ be its greatest eigenvalue, and suppress A, l, d to define

$$\rho(\upsilon) \equiv \lambda_F(A, l, \upsilon, d)^{-1} - 1.$$

Then the profit rate given by

$$r^* = (1-\beta)\rho(\upsilon) \tag{11}$$

and the corresponding lefthand eigenvector, scaled so as to satisfy (5), are the only candidate solution of (9) for $0 \leq \upsilon < (ld)^{-1}$; no other root of the matrix has a strictly positive eigenvector.

Where

$$\upsilon = \upsilon_{\max} \equiv \frac{1}{l\left[I - A\right]^{-1} d},$$

$(4, 5)$ is solved with $r^* = 0$. An indecomposable semipositive matrix's maximum eigenvalue is strictly increasing in its elements, so

$$\rho'(\upsilon) < 0 \tag{12}$$

for $0 \leq \upsilon \leq \upsilon_{\max} < (ld)^{-1}$. It follows that for any (β, υ) in $[0,1) \times [0, \upsilon_{\max}]$ there's a unique price-of-production bargaining equilibrium with a nonnegative profit rate that has

$$\frac{\partial r^*}{\partial \beta} = -\rho(\upsilon) < 0 \tag{13}$$

and

$$\frac{\partial r^*}{\partial \upsilon} = \rho'(\upsilon) < 0. \tag{14}$$

Equilibrium profitability goes to zero as β approaches 1 or as υ goes to $\upsilon_{\max}$, while at $\beta = \upsilon = 0$ it reaches a maximum equal to -1 plus the reciprocal of A's greatest root—the maximum profit rate familiar from square production models with uniform wages.[3]

This negative dependence of profitability on workers' power and the outside wage recalls the inverse wage-profit relation that holds across the equal-profit-rate equilibria of those uniform-wage models. However I have not yet said anything about *wages* here. Turning to them, I have to consider the entire collection of equilibrium solutions for n activity-specific real wages

$$w_j^* = \beta\rho(\upsilon)\, p^* a^j l_j^{-1} + \upsilon, \qquad j = 1, ..., n, \tag{15}$$

picked out by the possible pairs (β, υ). In relation to the equilibrium profit rate these describe, not a curve in the plane, but a surface in an $n+1$-dimensional space, a wage-wage-...-wage-profit relation. I claim next that as workers' power β increases at a

[3]The last two paragraphs are indebted to the analysis of that uniform-wage case in Kurz and Salvadori (1995, pp. 100-101).

constant value of υ so that the profit rate falls, all these activity-specific real wages increase. Section 6 shows that no similar conclusion is available in the case of a pure variation in the outside wage. An increase in υ, even as it calls for a lower equilibrium profit rate, can require *lower* real wages in some of the activities. Capital-labor antagonism is tangled up with and its monotonicity possibly disrupted by the distinct relation of labor to labor under study here.

5 Variation of workers' power

A particular value of υ picks out a single version of the eigen problem (9) whose solution immediately gives the equilibrium prices for that economy. Prices are invariant with respect to β, then, and it follows by differentiation of (15) that

$$\frac{\partial w_j^*}{\partial \beta} = \rho(\upsilon) \frac{p^* a^j}{l_j} > 0. \tag{16}$$

Wages increase, and the profit rate falls, as workers' power rises—a classical insight is borne out. Moreover along the continuum of equilibria swept out by the variation of β at a constant value of υ

$$\frac{dw_j^*}{dr^*} = \frac{\partial w_j^* / \partial \beta}{\partial r^* / \partial \beta} = -\frac{p^* a^j}{l_j} = \text{constant}$$

since they all solve the same problem (9). In this limited respect the linear wage-profit relation of Ricardo's corn model is recovered without resort to a standard commodity or a labor theory of value.

If the capitalists run the various production activities at levels x that are independent of wage and profit rates, the national income $p^* [I - A] x$ is also invariant with respect to β, so from (16) wages' share in the national income is increasing in that parameter.

Consider next the wage structure (15). A pure increase in bargaining power amplifies any existing wage differentials since in

$$\frac{\partial\left(w_j^*/w_k^*\right)}{\partial\beta} = \frac{\upsilon\rho\left(\upsilon\right)}{w_k^{*2}}\left[\frac{p^*a^j}{l_j} - \frac{p^*a^k}{l_k}\right] \tag{17}$$

the righthand side is positive or negative according as j is more or less capital-intensive than k in the going prices.

Where the composition of output x is again taken as constant, the jth sector's wage differential with respect to an economywide mean wage,

$$\theta_j \equiv \frac{w_j}{w} - 1;$$

$$w \equiv \frac{\sum_k l_j x_j w_j}{lx} = \beta\rho\left(\upsilon\right)\frac{pAx}{lx} + \upsilon, \tag{18}$$

has

$$\text{sign}\ \frac{\partial\theta_j}{\partial\beta} = \text{sign}\ \left[\frac{pa_j}{l_j} - \frac{pAx}{lx}\right] = \text{sign}\ \theta_j \tag{19}$$

as can be seen by substituting for a^k and l_k in (17) the composite activity (Ax, lx). So any measure of wage inequality that's increasing in the absolute values of these differentials is increasing in β.

The invariance of prices with respect to β entitles you to conclude from the *local* comparative statics (17) that wage differentials are in fact absolutely *globally* increasing in β for a given technology, numéraire, and outside wage. Greater bargaining power for workers implies higher wages across the board but also greater inequality among workers, and this direction of parametric change supports a downsloping relation between the profit rate and wage inequality.

6 Variation of the outside wage

I turn to comparisons of the equilibria picked out by different values of the outside wage at a constant value of workers' power. From (15)

$$\frac{\partial w_j^*}{\partial \upsilon} = l_j^{-1}\beta \left[\rho'(\upsilon)\, p^* a^j + \rho(\upsilon) \left\langle \frac{\partial p^*}{\partial \upsilon} \right\rangle \cdot a^j \right] + 1; \tag{20}$$

defining z^* by

$$z^* \equiv p^* A \left[I - (1 + \rho(\upsilon))\, A\right]^{-1}, \tag{21}$$

(20) becomes

$$\frac{\partial w_j^*}{\partial \upsilon} = \frac{\beta}{\upsilon z^* d} \frac{1}{l_j} \left[-p^* a^j + \rho(\upsilon)\left((z^* d)\, p^* - z^*\right) a^j\right] + 1 \tag{22}$$

after a little algebra in the appendix. You can then use (22) to take

$$\lim_{(\beta,\upsilon)\to(1,\upsilon_{\max})} \frac{\partial w_j^*}{\partial \upsilon} = \frac{1}{\upsilon z^* d} \frac{-p^* a^j}{l_j} + 1. \tag{23}$$

And it must be that

$$\min_j \frac{p^* a^j}{l_j} < \upsilon z^* d < \max_j \frac{p^* a^j}{l_j} \tag{24}$$

wherever the outermost expressions are not in fact equal.[4] Barring the fluke of equal equilibrium capital intensities, then, the righthand side of (23) is strictly negative for at least one activity. So for (β, υ) in some $\left[\bar{\beta}, 1\right) \times [\bar{\upsilon}, \upsilon_{\max}]$ the equilibrium wage in the sector with the locally greatest value of capital per head is decreasing in the outside wage. And since the profit rate is everywhere decreasing in υ, it follows that in this parametric

[4]Suppose the second inequality in (24) were false and the first true. Then $z^* d\upsilon l \geq p^* A$. (Here and throughout $x \geqq y \Leftrightarrow x_i \geq y_i$, all i; $x > y \Leftrightarrow x_i > y_i$, all i; $x \geq y \Leftrightarrow x \geqq y$ and $x \neq y$.) Postmultiplying by the strictly positive $\left[I - (1 + \rho(\upsilon))\, A\right]^{-1}$, you have $z^* d p^* > z^*$. But dotting both sides into d produces the contradiction $z^* d > z^* d$. A symmetrical argument shows that the first inequality can't fail to hold if the second holds.

region it's true of at least one sector that the sector-specific wage-profit relation induced by a pure variation in the outside wage slopes up.

This ambiguous behavior of individual wages nonetheless washes out of the comparative statics of the aggregate wage share. From (1) the economywide wage bill when capitalists run activities at the intensities x is

$$\sum_j w_j l_j x_j = \beta p^* [I - A] x + (1 - \beta) \upsilon l x, \tag{25}$$

and wages' share in the national income is

$$\omega \equiv \frac{\sum_j w_j l_j x_j}{p^* [I - A] x} = \beta + (1 - \beta) \frac{\upsilon l x}{p^* [I - A] x}. \tag{26}$$

If x were again supposed constant, it would follow that

$$\frac{\partial \omega}{\partial \upsilon} = (1 - \beta) \frac{l x}{p^* [I - A] x} \left\{ 1 - \frac{\upsilon}{p^* [I - A] x} \left\langle \frac{\partial p^*}{\partial \upsilon} \right\rangle \cdot [I - A] x \right\} \tag{27}$$

which you can rewrite as

$$\frac{\partial \omega}{\partial \upsilon} = (1 - \beta) \frac{(l x) z^* [I - A] x}{(z^* d) (p^* [I - A] x)^2} > 0 \tag{28}$$

after a substitution from (57) in the appendix: The wage share is increasing in the outside wage.

Now consider wage differentials. For any j and k,

$$\begin{aligned} & \frac{\partial \left(w_j^* / w_k^*\right)}{\partial \upsilon} \\ = \quad & \beta w_k^{*-2} \left\{ \left(\rho\left(\upsilon\right) + -\upsilon \rho'\left(\upsilon\right)\right) \left[\frac{p^* a^k}{l_k} - \frac{p^* a^j}{l_j} \right] + \rho\left(\upsilon\right) \left\langle \frac{\partial p^*}{\partial \upsilon} \right\rangle \cdot \left[w_k^* a^j - w_j^* a^k \right] \right\}. \end{aligned} \tag{29}$$

If the partial derivatives of the prices with respect to υ were to vanish, you could infer that the righthand side of (29) is negative if and only if j has the greater value of capital per head and hence a greater wage. And evidently the same inference is good for price partials in a small-enough neighborhood of zero. Let "C" hold place for the unknown conditions on technology, the outside wage, and the numéraire that confine these derivatives to that neighborhood. Given C wage differentials with respect to an employment-weighted mean wage are also absolutely decreasing in υ. (Again this follows by letting k in (29) stand for the composite activity (Ax, lx).) By (14), then, a pure variation in the outside wage picks out, under C, an *upsloping* relation between capitalist profitability and an appropriate index of working-class inequality.[5]

Collecting the partial derivatives calculated in this section and the last two, you can finally write

$$dr^* = -\rho\left(\upsilon\right) d\beta - \frac{(1-\beta)}{\upsilon z^* d} d\upsilon \tag{30}$$

and

$$dw_j^* = \rho\left(\upsilon\right) \frac{p^* a^j}{l_j} d\beta + \left\{ \frac{\beta}{\upsilon z^* d} \frac{1}{l_j} \left[-p^* a^j + \rho\left(\upsilon\right) \left(\left(z^* d\right) p^* - z^*\right) a^j \right] + 1 \right\} d\upsilon \tag{31}$$

and use these to approximate the displacements of equilibrium required by small changes

[5]By the same token if C is unavailable because the "price-Wicksell effect" terms in the derivative (29) are large and uncooperative, you can say nothing systematic about wage dispersion's dependence on the outside wage. Analysis is frustrated by the same arbitrary behavior of relative prices that is the heart of the "Cambridge" problems in capital theor

in (β, υ). They imply that there's an interval of directions of parametric change under which the jth wage and the uniform profit rate increase together, though of course this possibility is just a generic property of pairs of functions on the plane. The economically interesting increasingness result is this section's earlier conclusion that the profit rate can vary directly with some industry's wage even where just one parameter, υ, is perturbed.

7 Working-class cleavage

An old radical tradition holds that it's possible for privileged workers to join the capitalists in taking a surplus from the working class as a whole. In all but the final paragraph of this section I use a less conceptually fraught *counterfactual* strategy for classifying working-class privilege. Instead of trying to map the disposition of a surplus within a single equilibrium position, I compare how different groups of workers would fare in moving from that position to some interesting benchmarks.[6]

Take first an economy described by a (β, υ) with $\beta > 0$ and consider parametric changes that sustain profitability at its equilibrium value, $r(\beta, \upsilon)$. The point of this constraint might be that changes which violate it are doomed to draw political resistance from the capitalists. Or perhaps this profit rate is necessary for constant unemployment given capitalists' saving decisions and the growth rate of the working population. Now let

$$\bar{w}(\beta, \upsilon) \equiv \frac{1}{l\left[I - (1 + r(\beta, \upsilon)) A\right]^{-1} d}$$

so that

$$r(0, \bar{w}(\beta, \upsilon)) = r(\beta, \upsilon);$$

[6] Compare Wright (1997, pp. 14-6).

$\bar{w}(\beta, \upsilon)$ is the greatest *uniform* wage that meets the profitability constraint. For any $\beta > 0$, there are j and k such that

$$w_j(\beta, \upsilon) > \bar{w}(\beta, \upsilon) > w_k(\beta, \upsilon)$$

except where wages are already equal in the equilibrium for (β, υ).[7] So the project of equalizing wages without depressing the general profit rate necessarily cleaves the class into two opposed sections: one group who would gain from it and a second group of losers.

For a second counterfactual classification of working-class cleavage, suppose that wages are to be equalized at the value,

$$\bar{w}_{\max} = \frac{1}{l[I-A]^{-1}d},$$

that sends the profit rate to zero. In the actual equilibrium

$$w_j(\beta, \upsilon) l_j = \beta\left(p_j - pa^j\right) + (1-\beta)\upsilon l_j, \tag{32}$$

so the vector of unit labor costs is

$$\gamma(\beta, \upsilon) \equiv (w_1 l_1, w_2 l_2, ..., w_n l_n) = \beta p[I-A] + (1-\beta)\upsilon l \tag{33}$$

from which

$$\gamma(\beta, \upsilon)[I-A]^{-1} d = \beta + (1-\beta)\frac{\upsilon}{\bar{w}_{\max}} \tag{34}$$

[7] Suppose to the contrary it were possible that $w_j(\beta, \upsilon) \leq \bar{w}(\beta, \upsilon)$ for all j with $w_k(\beta, \upsilon) < \bar{w}(\beta, \upsilon)$ for some k. Then from (3)

$$p(\beta, \upsilon) - (1 + r(\beta, \upsilon)) p(\beta, \upsilon) A \leq \bar{w}(\beta, \upsilon) l;$$

dot both sides into the strictly positive vector $[I - (1 + r(\beta, \upsilon)) A]^{-1} d$ to get the contradiction

$$p[I - (1 + r(\beta, \upsilon)) A][I - (1 + r(\beta, \upsilon)) A]^{-1} d = 1 > \bar{w}(\beta, \upsilon) l [I - (1 + r(\beta, \upsilon)) A]^{-1} d = 1.$$

A symmetrical argument dismisses the remaining possibility.

and therefore

$$\lim_{\beta \to 1} \gamma(\beta, \upsilon)[I - A]^{-1} d = 1 = \bar{w}_{\max} l [I - A]^{-1} d. \quad (35)$$

If there's wage inequality at any $\beta < 1$ then certainly

$$\lim_{\beta \to 1} \left(\max_j w_j(\beta, \upsilon) - \min_j w_j(\beta, \upsilon) \right) > 0. \quad (36)$$

By (35) and (36), then,

$$\lim_{\beta \to 1} \max_j w_j(\beta, \upsilon) > \bar{w}_{\max} > \lim_{\beta \to 1} \min_j w_j(\beta, \upsilon) \quad (37)$$

except where wages are equal by a fluke. It follows that for any $\upsilon < \bar{w}_{\max}$ there's a $\bar{\beta}(\upsilon) < 1$ such that

$$\beta \geq \bar{\beta}(\upsilon) \Rightarrow \exists j, k : w_j(\beta, \upsilon) > \bar{w}_{\max} > w_k(\beta, \upsilon). \quad (38)$$

Workers in at least one industry are better off in a status quo marked by positive profits and dispersed wages than after an egalitarian *fan shen* that abolishes profits while levelling all wage differentials. Were they to try to maximize their equilibrium wages subject to the model of this paper, these workers would side with the capitalists against the remaining workers.

At this point my warning from the start of the paper kicks in. You can't step too heavily on these kinds of equilibrium comparisons when you go to explain the course of struggle over institutional change. Even if these bargaining equilibria were asymptotically stable in a price and investment dynamics, people would be too harried to compute the comparisons, and they would likely put a lot of weight on how they might fare in the transition. The next section steps around these difficulties by considering workers and capitalists who take a less Olympian view of their economic interests.

Before leaving (38), though, I should point out that it invites a second, old-school gloss. The quantity $l\left[I-A\right]^{-1}d$, the reciprocal of the maximum uniform wage measured in d-units, is also just the labor embodied in one unit of d bundles. And (38) says that for great enough β, there are j and k with

$$w_j\left(\beta,\upsilon\right)l\left[I-A\right]^{-1}d>1>w_k\left(\beta,\upsilon\right)l\left[I-A\right]^{-1}d,$$

workers who have respectively more and less labor embodied in their wage bundles than they contribute to production. If β is big enough, bargaining partitions the class into a Marxianly exploited stratum and a stratum of Marxian exploiters.

8 Institutional innovation

Consider an economy that's in the price-of-production bargaining equilibrium for some (β,υ) and suppose that workers and capitalists can act to secure small perturbations of those parameters which they believe will leave the prices of produced commodities unchanged. Such myopia excuses people from working out the equilibrium effects of their decisions; it also creates the possibility of interesting conflict among the capitalists, whose interests in *equilibrium* changes are identical by definition.

From (6) a myopic break-even line for wages or profits in the jth activity is

$$d\upsilon=-\left(1-\beta\right)^{-1}\rho\left(\upsilon\right)\frac{p^{*}a^{j}}{l_j}d\beta;\qquad(39)$$

capitalists in the jth sector should accept or reject a small change in the direction given by $(d\beta, dv)$ according as it lies below or above this line, and the sector's workers should strike the opposite stance.

Evidently no change draws unanimous support in the two classes. Suppose therefore that within each class *political weights* adding up to 1 are distributed over the sectors and that a coalition of sectors all of whose members accept some deal under the myopic rule (39) can impose it on the remaining members of their class if their weights sum to more than .5. A given assignment of weights picks out a unique classwide break-even line such that only deals to one side of that line are possibly imposed. I'll say that a class has a more or less capital-intensive political center of gravity according as this classwide break-even line has absolutely greater or lesser slope.

For any weighting that assigns distinct centers of gravity to the two classes, there's a set of institutional changes that dominant coalitions in both classes would opt to impose on everyone else. Imposable deals have β increasing and v decreasing if the workers' political center of gravity is more capital-intensive than the capitalists' center, and they show the opposite profile in the opposite case. In particular the political alignment most favorable to a social-democratic Great Compression of wage rates puts all the workers' weight on the most labor-intensive sector and all the capitalists' weight on the most capital-intensive sectors, while the inverse polarization promotes wage-dispersing exchanges of greater workers' power for a lower outside wage.

From (30) the break-even line for the *equilibrium* profit rate is

$$dv = -(1-\beta)^{-1}\rho(v)[vz^*d]\,d\beta. \tag{40}$$

By (24) this must lie between the myopic break-even lines (39) for the activities that are most and least capital-intensive in the equilibrium prices. If an omniscient executive committee of the bourgeoisie were distributing weights over the myopic sectoral actors to maximize the induced increase in the equilibrium profit rate, it would assign to capitalists an intermediate center of gravity that coincides with (40), and it would send all the workers' weight either to the most-capital-intensive activities or to the most-labor-intensive ones so that all imposable deals raise the equilibrium profit rate. Profitability is best served by the combination of this middle-of-the-road capitalist coalition with either a capital- or a labor-intensive worker coalition, and it's served worst by the opposite scenario: Where *workers*' weight centers on the break-even line for equilibrium profitability and *capitalists* lean toward one or the other sectoral extreme, imposable deals always bring down the general profit rate.

Though it's hard to say more at this level of abstraction, these claims give some idea of the explanatory payoff to political-economy arguments that cross the two classes with n sectors of production. The dependence of directions of change on sectoral political alignments that shows up here might help to account for the differential evolution of wage-setting systems.[8] And since political arrangements that depress profitability are especially vulnerable to disruption, these alignments' induced effects on the general profit rate can help to explain their differential longevity.

[8]Ferguson (1984) and Swenson (2002) explain inter- and post-war capital-labor accords as the projects of specific sectoral coalitions, and Swenson argues that differences in the terms of these compromises in Sweden and the US are explained in part by differences in the coalitions' industrial compositions.

9 Collectively self-defeating technical change

Apart from regulating institutional evolution in these ways, the bargaining arrangements of this paper impinge on an economy's direction of *technical* change. This section presents one example of the difference bargaining can make.

In a circulating-capital model closed by an exogenously constant uniform real wage, the introduction of activities that raise capitalists' profits in current equilibrium prices necessarily induces a new equilibrium with a strictly greater rate of profit (Okishio 1961). Wages and the profit rate are both endogenous to the equilibria I'm discussing, so this constant-wage experiment is unavailable to me. What I can consider, though, are the displacements of equilibrium brought about by profitmaximizing technical change holding constant the distributive parameters (β, υ). I'll show that there's a class of technical changes whose adoption increases individual profits in the old prices yet lowers the equilibrium profit rate if bargaining power and the outside wage are unchanged.

In this bargaining economy the technological upshot of the profit motive depends on the timing of wage bargains and technological learning. Making an opportunistic choice from the wealth of plausible scenarios, I assume that innovation follows bargaining and is unanticipated by it.[9]

Suppose that a capitalist enters a period planning to run the activity $\left(a^j, l_j\right)$. She signs a costlessly enforceable contract for a worker's labor at the wage given by (1) for the

[9] I mean that I'm choosing these over the alternatives because they have the possibly interesting implication that profitmaximizing innovation can induce a lower equilibrium profit rate. You will want to keep reading the section if (a) you believe that capitalist growth has included episodes of declining profitability that invite a technological-cum-social explanation, or (b) you enjoy hearing stories about other people's self-defeating behavior. I should point out that the myopia of the capitalists of the text is in the spirit of Okishio (1961), which studied the technological implications of current-period profit maximization. My discussion follows Roemer (1981)'s version of the Okishio argument.

inherited technology. Before production can begin, however, she draws a prospective new activity $\left(\bar{a}^j, \bar{l}_j\right)$ and must decide whether to substitute it for $\left(a^j, l_j\right)$ given the contractual value of the wage and the current equilibrium commodity prices p^*. Her unit labor cost for the activity $\left(\bar{a}^j, \bar{l}_j\right)$ is

$$\beta\left(p_j^* - p^* a^j\right) l_j^{-1} \bar{l}_j + (1-\beta)\, \upsilon \bar{l}_j,$$

and the new activity pays a higher return if and only if

$$\left(1 - \beta l_j^{-1} \bar{l}_j\right) p_j^* - (1+r^*)\, p^* \bar{a}^j + \beta l_j^{-1} \bar{l}_j p^* a^j - (1-\beta)\, \upsilon \bar{l}_j > 0. \tag{41}$$

Next define the *collective cost of production* of an activity $\left(a^j, l_j\right)$ as

$$\alpha\left(a^j, l_j\right) \equiv (1 + \rho(\upsilon))\, p^* a^j + \upsilon l_j. \tag{42}$$

This cost, which for the original jth activity just equals the equilibrium price of the jth good, values produced inputs at prices marked up by the factor $1 + \rho(\upsilon)$ and labor at the outside wage υ. This is in fact the cost that capitalists would minimize who were able to choose production activities *before* they bargain over wages. But it's irrelevant to the current profit maximization problem. Its importance and my point in calling it "collective" instead emerge from behind the backs of the capitalists in the following argument.

From the definition, $\alpha\left(\bar{a}^j, \bar{l}_j\right) > \alpha\left(a^j, l_j\right)$ if

$$\upsilon\left(l_j - \bar{l}_j\right) < (1 + \rho(\upsilon))\, p^* \cdot \left\{\bar{a}^j - a^j\right\}. \tag{43}$$

In the appendix I show that for given values of the distributive parameters and a given initial technology you can always construct a positive-measure set of new activities that satisfy both inequalities (41) and (43) in the associated equilibrium prices. A switch to any of the activities in this set raises the value of capital requirements while reducing labor requirements,

$$l_j - \bar{l}_j > 0 > p^* \cdot \left\{a^j - \bar{a}^j\right\}, \tag{44}$$

which is to say that they fit the profile of "Marx-biased" technical change.

Suppose that (41) and (43) hold for some pair of activities $\left(a^j, l_j\right), \left(\bar{a}^j, \bar{l}_j\right)$, write the social technology formed by replacing a^j with $\bar{a}^j$ and l_j with $\bar{l}_j$ in (A, l) as $\left(\bar{A}, \bar{l}\right)$, and let α and $\bar{\alpha}$ be the vectors of collective costs in current prices corresponding to the two technologies. Innovation increases the collective cost of producing the jth good, so

$$(1 + \rho(\upsilon))\, p^* \bar{A} + \upsilon \bar{l} = \bar{\alpha} \geq \alpha = p^*. \tag{45}$$

Rearrange this using the numéraire condition to get

$$(1 + \rho(\upsilon))\, p^* \bar{A} \geq p^* - \upsilon p^* d\bar{l}. \tag{46}$$

Provided that $\upsilon \bar{l} d \leq 1$, it follows in light of (10) that

$$p^* \bar{A} \left[I - \upsilon d\bar{l}\right]^{-1} \geq \lambda_F (A, l, \upsilon, d)\, p^*. \tag{47}$$

(If instead $\upsilon \bar{l} d > 1$, the new technology doesn't support a nonnegative profit rate, so it's immediate that innovation decreases equilibrium profitability.) Let $\bar{m}^i$ be the ith column of $\bar{A}\left[I - \upsilon d\bar{l}\right]^{-1}$. Then

$$\text{for all } j,\ \frac{p^* \bar{m}^j}{p_j^*} \geq \lambda_F (A, l, \upsilon, d)\,;\ \text{for some } i,\ \frac{p^* \bar{m}^i}{p_i^*} > \lambda_F (A, l, \upsilon, d)\,. \tag{48}$$

But it's a theorem on square matrices (Roemer, 1981, p. 110) that for any positive indecomposable A and positive q, either A's maximum eigenvalue λ_F satisfies

$$\max_i \frac{qa^i}{q_i} > \lambda_F > \min_i \frac{qa^i}{q_i} \tag{49}$$

or these three expressions are equal. So (48) implies that if $p^*\bar{m}^i/p_i^* > p^*\bar{m}^j/p_j^*$ for some i and j, then

$$\lambda_F\left(\bar{A}, \bar{l}, \upsilon, d\right) > \min_j \frac{p^*\bar{m}^j}{p_j^*} \geq \lambda_F\left(A, l, \upsilon, d\right),$$

and that if instead $p^*\bar{m}^i/p_i^* = p^*\bar{m}^j/p_j^*$ for all i and j, then

$$\lambda_F\left(\bar{A}, \bar{l}, \upsilon, d\right) = \frac{p^*\bar{m}^i}{p_i^*} > \lambda_F\left(A, l, \upsilon, d\right).$$

It follows in either case that $\lambda_F\left(\bar{A}, \bar{l}, \upsilon, d\right) > \lambda_F\left(A, l, \upsilon, d\right)$ and therefore that

$$\bar{r}^* < r^*; \tag{50}$$

the capital-using, labor-saving technical changes that satisfy (41) and (43), though they raise profits in the old prices, lower the equilibrium profit rate.[10] Bargaining drives the crucial wedge between individual and collective cost criteria, and in the appendix I describe a condition under which the probability of self-defeating technical change is increasing in workers' power β.

This argument leaves open the possibility that capitalists might revert to a discarded activity because its costs are lower than those of the adopted activity in the *new* prices.

[10] I haven't given any reason to suppose that technical changes will indeed satisfy those inequalities over time, so this conclusion does not establish a tendency for the profit rate to fall.

However in the numerical economies that I've looked at, though some innovations that lower the equilibrium profit rate are unsustainable in that sense, others are indeed sustained in the new equilibrium even by profitmaximizing capitalists who remember their technological pasts.

Imagine that some capitalists draw one of these sustainable, profitability-depressing innovations. And suppose that price-of-production equilibria are asymptotically stable and the convergence to them fast. Then the capitalists would all be better off were they all to consult the criterion of collective cost and discard the innovation rather than maximize their own current profits by implementing it. But then it's also true that, whatever the other capitalists do, each does better by maximizing those profits and innovating. Technical change has here the prisoners'-dilemma flavor that's often attributed to Marx's own arguments but that's proven so difficult to establish in his terms.[11]

5750 words including the appendix, references, and notes.

[11] Compare the argument of Foley (1986, pp. 136-9) and Franke (1999) that, where Okishio's constant real wage gives way to a constant wage share, cost-reducing technical change can bring down the general profit rate. Instead of appealing to an aggregate boundary condition, the new argument follows the one-sector model of Skillman (1997) by making a specific bargaining mechanism responsible for the relevant changes in real wages.

Appendix

Derivation of (22).

From (8) the vector of partials of the prices with respect to the outside wage satisfies

$$\left\langle \frac{\partial p^*}{\partial v} \right\rangle = \rho'(v)\, p^* A + (1+\rho(v)) \left\langle \frac{\partial p^*}{\partial v} \right\rangle A + l \tag{51}$$

which can be written as

$$\left\langle \frac{\partial p^*}{\partial v} \right\rangle = \{\rho'(v)\, p^* A + l\}\, M(v). \tag{52}$$

where

$$M(v) \equiv [I - (1+\rho(v))\, A]^{-1} \tag{53}$$

is defined for a given A, l, and d. Dotting both sides of (52) into the numéraire d, you get

$$\left\langle \frac{\partial p^*}{\partial v} \right\rangle d = 0 = \{\rho'(v)\, p^* A + l\}\, M(v)\, d. \tag{54}$$

From the numéraire condition, the fact that prices satisfy

$$p^* = vlM(v), \tag{55}$$

and the definition

$$z^* \equiv p^* A\, [I - (1+\rho(v))\, A]^{-1}$$

it follows that

$$\rho'(v) = -v^{-1} (z^* d)^{-1}. \tag{56}$$

Substitution from (56) into (52) shows that

$$\begin{aligned} \left\langle \frac{\partial p^*}{\partial \upsilon} \right\rangle &= \left(-\upsilon^{-1} \left(z^* d \right)^{-1} p^* A + l \right) M \left(\upsilon \right) \\ &= \upsilon^{-1} \left(- \left(z^* d \right)^{-1} z^* + p^* \right), \end{aligned} \tag{57}$$

and substitution from (56) and (57) into (20) yields (22).

Satisfaction of the inequalities (41) *and* (43)

Substitute for p_j from (3) into (41) to write

$$\left(1 - \beta l_j^{-1} \bar{l}_j\right) \left[\left(1 + (1-\beta)^{-1} r^* \right) p^* a^j + \upsilon l_j \right] - (1 + r^*) \, p^* \bar{a}^j$$

$$+ \beta l_j^{-1} \bar{l}_j p a^j - \upsilon (1 - \beta) \bar{l}_j > 0.$$

Putting

$$\eta \equiv \frac{\bar{l}_j}{l_j} \text{ and } \mu \equiv \frac{p^* \bar{a}^j}{p^* a^j},$$

this becomes

$$\frac{\upsilon l_j (1 - \eta)}{p^* a^j} > \mu (1 + r^*) - \beta \eta - (1 - \beta \eta) \left(1 + (1 - \beta)^{-1} r^* \right)$$

or

$$\mu < \mu_1 (\eta) \equiv \frac{\upsilon l_j (1 - \eta)}{(1 + r^*) \, p^* a^j} + \frac{1 + (1 - \beta\eta) (1 - \beta)^{-1} r^*}{1 + r^*}. \tag{58}$$

On the other hand you can rewrite (43) as

$$\left(1 + (1 - \beta)^{-1} r^* \right) (\mu - 1) > \frac{\upsilon l_j (1 - \eta)}{p^* a^j}.$$

or

$$\mu > \mu_0(\eta) \equiv \frac{\upsilon l_j (1-\eta)}{\left(1 + (1-\beta)^{-1} r^*\right) p^* a^j} + 1. \tag{59}$$

Evidently $\mu_1(\eta)$ is greater or less than $\mu_0(\eta)$ according as η is less or greater than 1. So for any $0 < \eta < 1$, there's an interval $(\mu_0(\eta), \mu_1(\eta))$ with $\mu_0(\eta) > 1$ such that for μ in that interval (μ, η) satisfies both inequalities.

Also for any $0 < \eta < 1$, $\mu_1(\eta) - \mu_0(\eta)$ is strictly increasing in β, and therefore so must be

$$\int_0^1 (\mu_1(\eta) - \mu_0(\eta))\, d\eta,$$

the area of the closure of the region in which both inequalities hold. Because equilibrium commodity prices are independent of β, there is associated with every (A, l, υ, d) a mapping ϕ from the set of regions T of μ, η space to the set of sets of prospective new activities

$$\phi T = \left\{ \left(\bar{a}^j, \bar{l}_j\right) \middle| \eta = \frac{\bar{l}_j}{l_j}, \mu = \frac{p^* \bar{a}^j}{p^* a^j} \text{ for some } j \text{ and for some } \mu, \eta \text{ in } T \right\}$$

such that a switch to an activity in ϕT, evaluated in the equilibrium prices for (A, l, υ, d), yields proportional rates of labor-productivity and capital-cost change that live in T. If at (A, l, υ, d) the probability measure describing the distribution of prospective new activities assigns a greater probability to ϕT, the greater the area of T, then the probability of drawing an innovation that satisfies (41) and (43) is increasing in β.

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