The Price Normalisation Problem in General Equilibrium Models with Oligopoly Power: An Attempt at Perspective

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Abstract:
In general equilibrium models with oligopolistic firms, equilibrium outcomes may critically depend on the choice of numeraire. When firms have the power to influence prices strategically, different price normalisations entail objective profit functions which are generally not monotone transformations of each other. Hence, under the assumption of profit maximization an arbitrary change in the price normalisation rule amounts effectively to a change in the objective pursued by firms. Applied general equilibrium analysts using models with imperfect competition have largely ignored the price normalisation problem. In several recent contributions to the literature, applied modellers are explicitly criticized for their neglect to address the numeraire issue. The purpose of this paper is to assess the validity and practical relevance of these criticisms for applied policy analysis.

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1. Motivation

Economic theorists have long been aware of the fact that in general equilibrium models with oligopolistic firms, equilibrium outcomes may depend on the choice of numeraire. When firms have the power to influence prices strategically, different price normalizations entail profit functions which are generally not monotone transformations of each other. Hence, under the assumption of profit maximization an arbitrary change in the price normalization rule amounts effectively to a change in the objective pursued by firms and profit-maximizing behaviour does not unambiguously serve shareholders' interests (Gabszewicz and Vial, 1972, Dierker and Grodal, 1998).

Despite Ginsburgh's (1994) provocative numerical example, applied general equilibrium analysts using models with imperfectly competitive firm conduct have largely ignored the price normalization problem.[1] In several recent contributions to the literature, applied policy modellers are explicitly criticized for their neglect to address the numeraire issue. Kletzer and Srinivasan (1999) argue that

> " the dependence of equilibria on the choice of a numeraire is an important problem for theoretical models of international trade under imperfect competition and their empirical implementation. (...) Once it is established that equilibria are sensitive to the specification of the numeraire, it is a straightforward conclusion that estimates of the effects on welfare and resource allocation of changes in indirect or direct tax rates, tariff rates or quantitative restraints on international or national trade from computable general equilibrium models incorporating imperfect competition should be treated with suspicion. (...) The analyses of trade reforms using computable general equilibrium with monopolistically competitive or oligopolistic industries by Harris [1984], Cox and Harris [1985], de Melo and Roland-Holst [*1991*] and Devarajan and Rodrik [*1991*], among others, are all subject to the criticism that the results depend upon the arbitrary choice of price normalization made."[2]

Dierker and Grodal (1998) likewise emphasize the potential relevance of the price normalization problem for practical policy analysis:

> " Since imperfectly competitive markets abound in the real world, policy questions are often analyzed in models with strategically acting firms. However, the lack of a sound economic foundation for firms' payoffs ... often leads to confusion if such models are used as a basis for policy recommendations."

In a similar vein, Cordella (1998) suggests that

> " far from being a theoretical curiosity, the normalization problem ... has far-reaching implications in applied models".

The purpose of this paper is to address the price normalization problem from an applied modelling perspective. It is shown that existing applied general equilibrium models with imperfect competition actually sidestep the numeraire dependency problem by imposing plausible restriction on firms' perceptions of general equilibrium repercussions associated with their own strategic choices. In a literal sense the suggestion that the results of the studies cited above depend on arbitrary price normalization choices is technically invalid. A potentially valid

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[1] A recent exception is Hoffmann (2003). Burniaux and Waelbroeck (1993), Mercenier (1995), Kehoe and Prescott (1995) and Willenbockel (2004) mention the price normalisation problem *en passant*.

[2] References to working papers in the original text have been updated to refer to more accessible final published versions where appropriate.

criticism is rather that oligopolists in these models do not act in full accordance with the rationality principle. It appears then reasonable to ask to which extent the quantitative predictions of a model with limited cognition actually deviate from the predictions of a corresponding model with complete cognition and fully rational behaviour. Common sense would seem to suggest that deviations are likely to remain negligible as long as the market shares of individual imperfectly competitive sectors in the economy remain small, as is indeed the case for modern economies with a diversified production and consumption structure.[3]

Since such casual appeals to common sense may be considered as too facile,[4] we compare the equilibria of a range of computable prototype models with and without full cognition of general equilibrium feedbacks in order to assess the quantitative relevance, or other, of the price normalisation problem under empirically plausible parameter choices. To develop a clear conception of the basic nature of the price normalisation issue, section 2 starts with a heuristic model of monopoly in general equilibrium while section 3 introduces oligopolistic interaction.

2. The Price Normalisation Problem I: Monopoly in General Equilibrium

Consider a closed economy which produces two consumption goods C_1 and C_2 using a single intersectorally mobile primary factor with linear production technologies. The economy is populated by numerous price-taking households with identical homothetic preferences represented by a CES utility function

$$U = [\delta C_1^{(\sigma-1)/\sigma} + (1-\delta) C_2^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}, \qquad (1)$$

where σ is the elasticity of substitution between the two goods. The factor market equilibrium condition is $L^s = C_1 + C_2$, where L^s denotes the aggregate exogenous factor endowment, which is evenly spread across households. Good 2 is produced by perfectly competitive firms so that $p_2 = w$, where p_i, $i \in \{1,2\}$, and w denote output prices and factor price respectively. In contrast, good 1 is supplied by a profit-maximizing monopolist. We assume initially that ownership titles to monopoly profits are evenly distributed. The demand function facing the monopolist is

$$C_1 = \delta^{\sigma} \theta^{\sigma-1} p_1^{-\sigma} Y \qquad (2)$$

where

$$\theta = [\delta^{\sigma} p_1^{1-\sigma} + (1-\delta)^{\sigma} p_2^{1-\sigma}]^{1/(1-\sigma)} \qquad (3)$$

[3] See Ruffin (2003) and Neary (2003) for recent arguments along these lines.

[4] See e.g. Cornwall (1977): “Of course, it is not realistic to assume that firms actually recognize that their production choices influence the consumption possibilities which are feasible for the firms’ owners and that the firms consequently choose non-profit-maximizing plans. However, it is equally clear that it is not enough to say that there are a lot of firms in a real world economy and therefore the assumption of profit-maximizing behaviour gives a good approximization. This is not enough of a justification because it is not clear what or how profit maximization approximates. ... It may well be a kind of second best rule is to maximize profits. However, this has not been shown”.

is the true price index dual to U and $Y = wL^s + \pi$ is aggregate household income including monopoly profits $\pi = (p_1\text{-}w)C_1$. As long as the monopolist is assumed to neglect the indirect general equilibrium repercussions of variations in its own decision variable on p_2, w and Y – i.e. as long as the firm is taken to act like a standard textbook partial equilibrium monopolist – no price normalization problem arises. In this case, subjectively optimal pricing behaviour is unambiguously characterised by the familiar Lerner condition

$$p_1(1-1/\varepsilon(.)) = w \quad , \tag{4}$$

where

$$\varepsilon = -\frac{\partial \ln C_1}{\partial \ln p_1} = \sigma + (1-\sigma)S \quad , \quad S \equiv \delta^\sigma \left[\frac{p_1}{\theta}\right]^{1-\sigma} = \frac{p_1 C_1}{Y} \tag{5}$$

is the perceived elasticity of demand. ε is homogeneous of degree zero in prices, and the optimal mark-up, and hence the general equilibrium of the two-sector economy, is independent of any price normalization rule a modeller may adopt to determine nominal variables.
The situation changes once the assumption of limited cognition is dropped and the monopolist is assumed to recognise his influence on prices in other markets and thus on aggregate income via factor price and profit feedback effects. With full recognition of the profit feedback effect on C_1 demand, monopoly profits can be expressed in the form

$$\pi = \frac{(p_1 - w)h(p_1, p_2)wL^s}{1-(p_1 - w)h(p_1, p_2)} \quad , \quad h(p_1, p_2) \equiv \delta^\sigma \theta^{\sigma-1} p_1^{-\sigma} = \frac{C_1}{Y}, \tag{6}$$

and the monopolist is aware that utility-maximising consumer behaviour in combination with the resource constraint entails that the relative price $P = p_1/p_2$ varies with the choice of monopoly output according to the objective inverse general equilibrium demand schedule

$$P(C_1) = \frac{\delta}{1-\delta}\left(\frac{L^s}{C_1} - 1\right)^{1/\sigma}, \tag{7}$$

and that profit-maximizing behaviour in the competitive sector entails p_2=w. Without a nominal anchor, the maximisation of nominal profits is now obviously an ill-defined problem. The choice of a numeraire, or more generally, a price normalisation rule is required *before* the optimal equilibrium mark-up can be characterised. Figure 1 shows the profit profile as a function of monopoly output described by (6) and (7) for the three normalisations p_1=1, p_2(=w)=1, and θ=1, thus measuring profits respectively in units of the monopoly good, in units of the competitive good (or in factor units), and in units of the consumption index U. Evidently the profit-maximizing output level does not remain invariant to a change in the numeraire - the objective general equilibrium profit functions under different price normalizations are not monotone transformations of each other. E.g., the maximisation of profits in terms of good 2 ($\pi^{(2)}$) and in terms of good 1 ($\pi^{(1)}$) are different objectives. Formally,

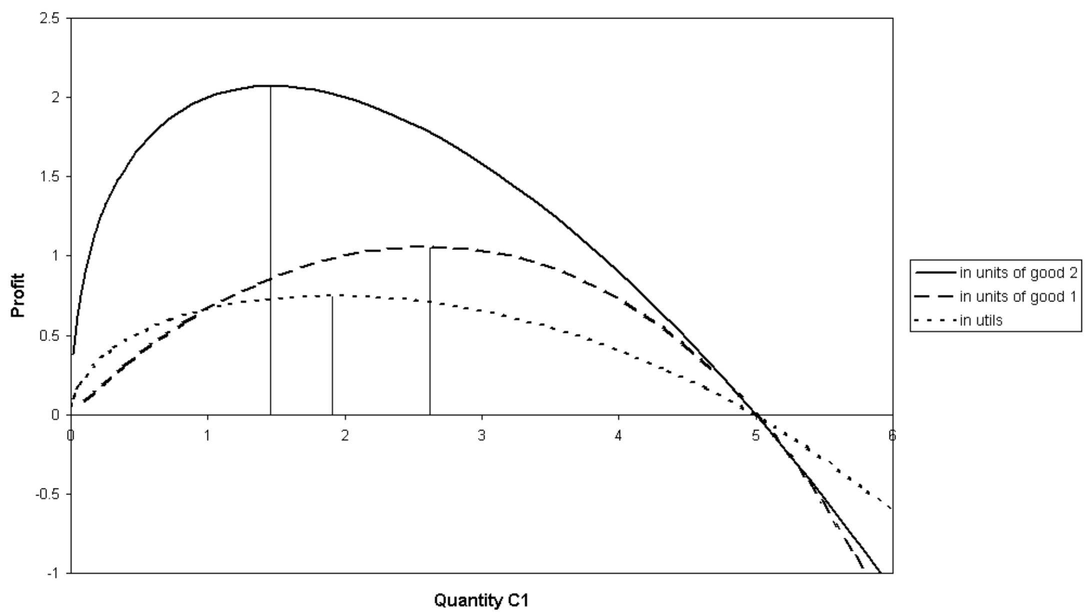
$$\pi^{(2)}(C_1) = \pi^{(1)}(C_1) \cdot P(C_1) \tag{8}$$

and hence the first-order condition for a maximum of $\pi^{(2)}$,

$$\frac{d\pi^{(2)}}{dC_1} = \frac{d\pi^{(1)}}{dC_1} P(C_1) + \pi^{(1)}(C_1)\frac{dP(C_1)}{dC_1} = 0\,, \tag{9}$$

differs from the first-order condition for a maximum of $\pi^{(1)}$ unless equilibrium profits are zero.

Figure 1:
The Objective Profit Function under Alternative Normalizations



Parameters: δ=0.5, σ=2, L^s=10

The fact that the choice of price normalisation rule affects the equilibrium levels of real variables illustrated by this example is a generic feature of general equilibrium models with imperfectly competitive profit-maximising firms, given that these firms fully recognise their influence on the price system. Indeed Böhm (1994) and Grodal (1996) present oligopoly examples in which virtually every feasible production plan is an equilibrium for some normalisation rule. Other oligopoly examples in the literature demonstrate that an equilibium in pure strategies may exist for some price normalisation rules while other normalisations entail non-existence.[5] However,

[5] See Dierker and Grodal (1986). Böhm (1994) shows that the price normalisation problem can even arise in pure exchange economies with utility-maximising *price*-setting agents and argues correspondingly that the assumption of profit maximisation is not essential for the occurence of the numeraire dependency result. See Rasmussen (1996) for further discussion of this point.

here the goal of profit maximisation in combination with a normalisation rule that takes no account of firm owner's actual interests will generally not be consistent with the aims of shareholders in the imperfectly competitive setting, and is thus not a rational objective. As Dierker and Grodal (1998) put it, "if price normalization rules and hence firms' objectives fail to be based on economic considerations, only ill-founded, arbitrary conclusions can be drawn from such models". In competitive Arrow-Debreu economies with production, in which no agent can influence the price system strategically, on the other hand, the goal of profit maximisation is unambiguously in the interest of shareholders irrespective of the choice of price normalisation.

The present example serves to elaborate the point. Maintaining the assumption of an even spread of monopoly shares for a moment, it is immediately evident that the goal of monopoly profit maximisation is irrational or indeed schizophrenic under any normalisation rule. With full recognition of his control over the price system via (7), the monopolist as agent of shareholders is in effect in the position of an omniscient central planner and should mimic the perfectly competitive outcome by setting P equal to the marginal rate of transformation (MRT=1) in order to maximise shareholder welfare. The optimum is of course associated with zero profits under any normalisation. The selection of a relative price P>MRT along the general equilibrium price schedule (7) would generate positive profit income but would at the same reduce the purchasing power of factor income in terms of good 1 and entail a net welfare loss. In other words, the maximisation of "producer surplus" without regard to the consequences for shareholders' "consumer surplus" is generally not in the interest of firm owners. The example may appear trivial, since as a matter of course there is no room for strategic behaviour in what is effectively a single-representative-agent framework. Yet the key message that a rational, not self-defeating strategy for an imperfect competitor must take shareholders' preferences and endowments into account, as highlighted by this extreme example, carries over to settings with real scope for strategic behaviour.

Thus let us introduce income heterogeneity by decomposing the household sector into a monopoly shareholder group with income $Y_s=wL_s+\pi$ and a non-shareholder group with income $Y_n=wL_n$. Both household types have identical CES preferences as before, so that the aggregate demand function for the monopoly good (2) and the general equilibrium price schedule (7) still apply. The rational objective of the monopolist is to maximise

$$U(C_1^s, C_2^s) = V(p_1, p_2, Y_s) = \theta(p_1, p_2)^{-1} Y_s \,. \tag{10}$$

Since the indirect utility function on the RHS of (10) is homogeneous of degree zero in its arguments, the optimal supply strategy is independent of the choice of price normalization. Without loss of generality, we can normalise the true consumer price index θ at unity. Thus the rational objective of the monopolist can equivalently be expressed as maximisation of shareholders' *total* real income (in units of the consumption index),

$$Y_s = p_2 L_s + (p_1 - p_2) C_1 = \frac{p_2 L_s + (p_1 - p_2) h(.) p_2 L_n}{1 - (p_1 - p_2) h(.)} \;\; with \theta = 1. \tag{11}$$

Note that Y_s, which can be expressed as a function of C_1 by using (7) in (11) – is synonymous with the general equilibrium profit function (6) for the normalisation θ=1 in Figure 1 if L_s=0. Thus only if shareholders' only income source is monopoly dividends, is the maximisation of profits in combination with the specific class of normalisation rules θ=constant>0 a fully rational objective, i.e. an objective that is in complete agreement with the interests of shareholders.[6]

Profit maximisation together with a specific normalisation rule - namely $p_2=1$ - would also be totally consistent with shareholder preferences if these preferences take the form $U_s=u(C_2^s)$, $u'>0$, so that shareholders don't consume the output of their own firm.[7]

This extreme case suggests the conjecture, that the practical relevance of the numeraire problem may be negligible if the share of monopoly output in agents' total consumer expenditure is sufficiently small. But how small is sufficiently small? Table 1 provides a tentative answer.

The Table compares the general equilibria of the two-sector model when the monopolist has respectively limited and full cognition of the equilibrium consequences of his price-setting behaviour for alternative values of the preference intensity parameter δ, which governs the market share of the monopolistic sector in total consumer expenditure. In the limited cognition model, the monopoly mark-up is determined in partial-analytical fashion via (4) and (5), i.e. the monopolist ignores his influence on Y and pays no attention to the true interests of shareholders as consumers in his price setting decision. Not surprisingly, when the monopoly sector is small in the economy, the actual general equilibrium income feedback effect is indeed negligible, so that the limited cognition model provides an almost perfect approximation of the equilibrium with an omniscient rational monopolist. More interestingly, the deviation remains moderate even under empirically unreasonable values for the share of a single firm in GDP.

Table 1: Deviations of Limited Cognition Model from Full Cognition Monopoly Model

δ	*Monopoly Share* S_{Full} *%*	*Monopoly Share* $S_{Limited}$ *%*	*Price* ΔP *%*	*Monopoly Output* ΔC_1 *%*	*Welfare* ΔU *%*
0.1	0.6	0.6	+0.3	-0.0	-0.00
0.2	3.0	3.0	+1.5	-3.2	-0.00
0.3	8.4	8.1	+4.2	-8.0	-0.06
0.4	18.1	16.8	+9.1	-14.6	-1.45
0.5	32.7	29.3	+17.3	-23.4	-4.43
0.6	50.9	44.5	+29.8	-31.5	-10.13
0.7	69.5	60.6	+48.0	-38.9	-18.36
0.8	84.7	75.7	+77.7	-42.4	-27.97
0.9	95.1	89.0	+138.4	-45.7	-38.06

Model parameter values: $\sigma=2$, $L_s=0$, $L_n=10$.

[6] However, once preference heterogeneity among shareholders is introduced, the very notion of shareholders' preferences becomes an elusive concept due to Arrow's impossibility theorem. See however Dierker and Grodal (1998,1999)'s approach to the formulation of a rational firm objective in the presence of heterogeneous shareholders. A separate literature analyses shareholder voting equilibria – see Yalcin and Renstrom (2003) for further reference.

[7] The island model of Hart (1985) can be seen as an extension of this observation to a multi-sector multi-agent setting.

Table 1 may be seen to provide a first indication of the practical irrelevance of the price normalisation problem for quantitative policy analysis. However, since imperfectly competitive sectors in computable general equilibrium models are typically oligopolies rather than monopolies, the next section extends the analysis to a setting with strategic interaction among firms.

3. The Price Normalisation Problem II: Oligopoly in General Equilibrium

We now assume that sector 1 is populated by n symmetric firms and characterised by horizontal product differentiation a la Dixit and Stiglitz (1977). Consumer preferences over the composite output of sector 1 and the competitive good C_2 are Cobb-Douglas with share parameters α_i, where

$$C_1 = \left[\sum_{v}^{n} x_v^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \qquad (12)$$

x is output per firm, and $\sigma>1$ the elasticity of substitution between firm-specific varieties. Thus the demand function facing an individual oligopolist takes the form

$$x = \alpha_1 p^{-\sigma} P_1^{\sigma-1} Y, \qquad (13)$$

where p is the price of an individual variety and

$$P_1 = \left[\sum_{v=1}^{n} p_v^{1-\sigma} \right]^{1/(1-\sigma)} \qquad (14)$$

is the consistent price index dual to C_1. On the production side, we maintain the assumption of linear single-factor technologies but add a recurrent fixed factor requirement per firm in sector 1 to introduce increasing returns to scale. This setting is a stylised two-sector closed-economy version of typical multi-sectoral open-economy computable general equilibrium models as employed in the studies cited by Kletzer and Srinivasan (1999) above.[8]

Supply behaviour in sector 1 depends on the assumed form of strategic interaction among firms. Most applied studies assume either Bertrand or Cournot competition and the individual firm perceives to have no influence on Y, factor prices and prices in other sectors. Under Bertrand competition, the perceived elasticity of demand, which determines the equilibrium mark-up via (4) is then

$$\varepsilon = -\frac{\partial \ln x}{\partial \ln p} = \sigma + (1-\sigma)/n \qquad (15)$$

while Cournot competition entails

[8] See Willenbockel (1994, 2004) for further references to applied policy studies of this type.

$$\frac{1}{\varepsilon}=\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\frac{1}{n} \quad (16)$$

In both cases the equilibrium mark-up is independent of the choice of price normalisation. In order to determine firm behaviour under full cognition of general equilibrium feedbacks, the price normalisation problem re-appears, since the elasticity of Y with respect to p is indeterminate without a normalisation rule. In analogy to the previous section, the appropriate normalisation rule is to normalise the true consumer price index
dual to U, i.e.

$$\theta=\prod_{i=1}^{2}\left(\frac{P_i}{\alpha_i}\right)^{\alpha_i} \quad (17)$$

at unity (or any positive scalar constant), provided that shareholders of any firm receive only profit income and do not hold shares of other firms. The price elasticity of x_s with full cognition of general equilibrium feedbacks including factor price effects, which governs the Nash equilibrium mark-ups via the first-order condition $p_s(1-1/\varepsilon)=w+(x_s+F)dw/dx_s$ under normalization $\theta=1$, must obey (see appendix)[9]

$$\varepsilon \;=\sigma+(1-\sigma)\frac{1}{n}-\left(\frac{\alpha_1}{n}-\frac{wx}{Y}\right)\left(\sigma(n-1)-n\varepsilon_B\right) \quad (18a)$$

in the Bertrand-Nash case, and

$$\frac{1}{\varepsilon}=\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\frac{1}{n}-\left(\frac{\alpha_1}{n}-\frac{wx}{Y}\right) \quad (18b)$$

in the Cournot-Nash case. Note that the term (α_1/n –wx/Y) equals the equilibrium share of an individual firm's operating profit in GDP, (p-w)x/Y. In the limit for $\alpha_1 \rightarrow 0$ this term vanishes, i.e. for a decreasing market share of the oligopolistic sector in the economy the elasticity under full cognition converges to the limited cognition elasticity (15) or (16). Thus, as long as the GDP share of an individual oligopolistic industry within an applied general equilibrium model remains sufficiently small, the 'error' incurred by neglecting general equilibrium feedbacks will remain negligible.

How small is 'sufficiently small'? Figure 2 plots the percentage deviation of equilibrium aggregate welfare and firm output levels between the limited and complete cognition models for varying market shares of an individual oligopolistic sector in the economy. For empirically relevant ranges of the relative size of an individual oligopolistic sector producing similar products within the economy as a whole, the limited cognition equilibria are almost perfect approximations of the corresponding equilibria with fully rational oligopolists. More

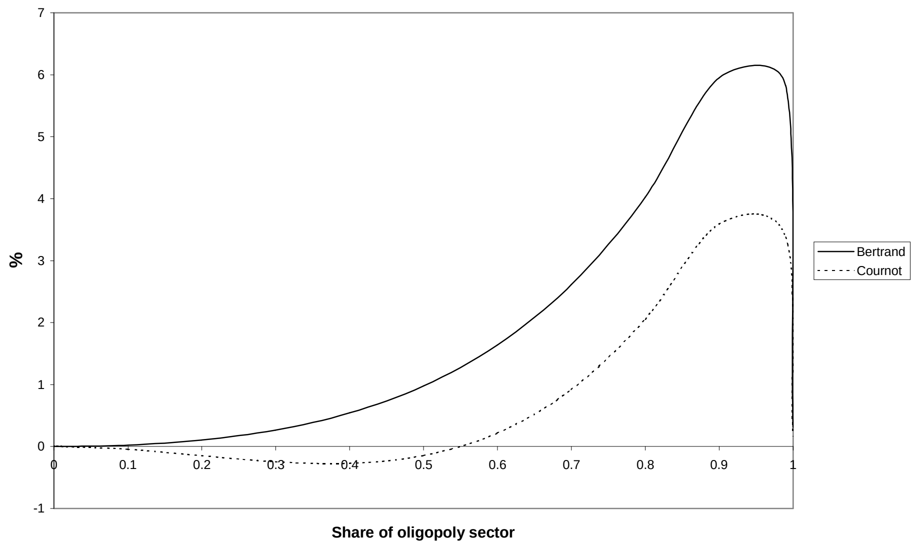
[9] d'Aspremont *et al.* (1996) derive the corresponding perceived elasticity expression for the normalization w=1 but do not address the dependency of the result on this particular numeraire choice.

interestingly, the deviations remain moderate even for empirically unlikely α_2 values. The tenor of Figure 2 is robust to parameter variations, and the argument carries over to large-scale multisectoral models with multiple oligopolistic industries, which together may comprise a large fraction of the economy, as long as each individual industry is small in relation to aggregate income.

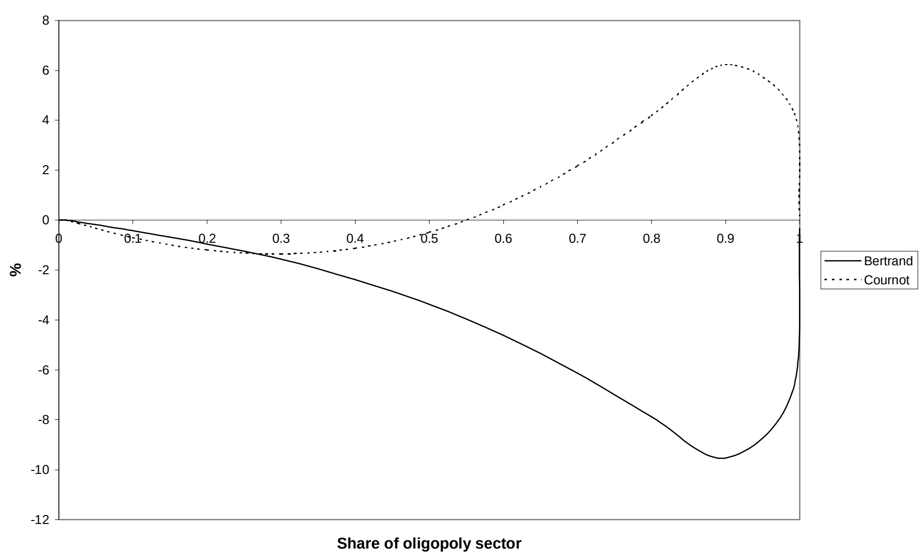
Figure 2:

Equilibrium Deviations of Limited Cognition from Full Cognition Model

(a) Deviation of aggregate welfare level



(b) Deviation of output per firm in oligopoly sector



Parameters: σ=2, n=5, L=2, F=0.001.

4. Conclusion

As long as imperfectly competitive firms are large in their own markets but small in the economy – as is typically the case for modern diversified economies – the so-called price normalization problem appears to be of negligible quantitative relevance for applied policy analysis. The suggestion that the results of typical applied general equilibrium models are contingent on arbitrary numeraire choices is invalid. Rather, these models bypass the issue by assuming that oligopolists have limited cognition of general equilibrium feedback effects and do not act in full accordance with shareholder objectives. However, simulations of a range of prototype models with shareholder homogeneity suggest, that for empirically relevant firm market share ranges the limited cognition approach generates close approximations to the corresponding equilibria with full rationality and perfect cognition.

Technical Appendix

The general equilibrium demand elasticity for a Bertrand oligopolist:

Log-differentiation of the demand function (13) faced by an individual Bertrand oligopolist i yields

$$\varepsilon \;= -\frac{d\ln x_i}{d\ln p_i} = \sigma + (1-\sigma)\frac{d\ln P_1}{d\ln p_i} - \frac{d\ln Y}{d\ln p_i} \quad . \tag{A-1}$$

The Bertrand assumption entails the perceptions $d\ln P_1 = d\ln p_i/n$ and $d\ln x_{-i} = (\sigma - \varepsilon_B)d\ln p_i$. Differentiation of $Y = wL + p_i x_i - w(x_i + F) + (n-1)(p_{-i}x_{-i} - w(x_{-i} + F)$ in combination with the aggregate resource constraint $C_1 = L - \Sigma_s(x_s + F)$ yields

$$\frac{d\ln Y}{d\ln p_i} = \alpha_1 \frac{d\ln w}{d\ln p_i} + \left(\frac{\alpha_1}{n} - 1 + \frac{wx_i}{Y}\right)\varepsilon_B + \frac{(n-1)(p_{-i} - w)x_{-i}}{Y}(\sigma - \varepsilon\ ) . \tag{A-2}$$

Since $d\ln w = d\ln P_1$, the normalization $\theta=1$ entails (see (17))

$$\frac{d\ln w}{d\ln p_i} = \frac{-\alpha_1}{\alpha_2 n} \ . \tag{A-3}$$

Using (A-2) and (A-3) in (A-1) yields (18a).

The general equilibrium demand elasticity for a Cournot oligopolist:

Log-differentiation of the inverse demand function $p_i = \alpha x_i^{-1/\sigma} C_1^{(1-\sigma)/\sigma} Y$ for an individual Cournot oligopolist yields

$$\frac{1}{\varepsilon} = -\frac{d\ln p_i}{d\ln x_i}\Big|_{dx_{-i}=0} = \frac{1}{\sigma} + \left(1 - \frac{1}{\sigma}\right)\frac{1}{n} - \frac{d\ln Y}{d\ln x_i}, \tag{A-4}$$

where

$$\frac{d\ln Y}{d\ln x_i} = \alpha_2 \frac{d\ln w}{d\ln x_i} + \frac{\alpha_1}{n}\left(1 - \frac{1}{\varepsilon_C}\right) + \frac{\alpha_1(n-1)}{n}\frac{d\ln p_{-i}}{d\ln x_i} - \frac{wx_i}{Y} . \tag{A-5}$$

Now $d\ln p_{-i} = (1/\sigma - 1/\varepsilon)d\ln x_i$, and under normalization $\theta=1$

$$\frac{d\ln w}{d\ln x_i} = -\frac{\alpha_1}{\alpha_2}\left(\frac{1}{n}\frac{d\ln p_i}{d\ln x_i} + \frac{n-1}{n}\frac{d\ln p_{-i}}{d\ln x_i}\right) = \frac{\alpha_1}{\alpha_2}\left(\frac{1}{n}\frac{1}{\varepsilon} + \frac{n-1}{n}\left(\frac{1}{\varepsilon} - \frac{1}{\sigma}\right)\right). \tag{A-6}$$

Using (A-5) and (A-6) in (A-4) yields (18b).

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