The commons with capital markets[1]

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December 21, 2004

[1]The authors thank Ralph Bailey, Siddhartha Bandyopadhyay, Matthew Cole, Carl Devore, Chirantan Ganguly, Indrajit Ray and Celine Rochon for valuable comments.

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Abstract

We explore commons problems when agents have access to capital markets. The commons has a high intrinsic rate of return but its fruits cannot be secured by individual agents. Resources transferred to the capital market earn lower returns, but are secure. In a two period model, we assess the consequences of market access for the commons' survival and welfare; we compare strategic and competitive equilibria. Market access generally speeds extinction, with negative welfare consequences. Against this, it allows intertemporal smoothing, a positive effect. In societies in which the former effect dominates, market liberalisation may be harmful. We reproduce the multiple equilibria found in other models of competitive agents; when agents are strategic, extinction dates are unique. Strategic agents generally earn their surplus by delaying the commons' extinction; in unusual cases, strategic agents behave as competitive ones even when their numbers are small.

Key words: commons, capital markets, Washington Consensus, property rights

JEL classification numbers: C73, D91, O17, Q21

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1 Introduction

We analyse commons problems when agents have access to capital markets. We are particularly interested the effect of capital market access on the commons' survival and on welfare.

As, in almost any contemporary common access problem, those drawing on the resource also have access to capital markets, this research contributes to most commons analyses: fishermen may sell their catch and bank the proceeds; power plants may be financed against future profits earned, in part, by emitting into a common atmosphere.

The problem most directly motivating our interest is that considered in Tornell and Velasco (1992). It noted that capital might flee from capital scarce poor countries to capital abundant rich countries if property rights in the former were weak.[1] Its chief insight was to view property rights as entirely absent, treating the entire capital stock of poor countries as a communal endowment. Access to foreign capital markets then allowed agents to convert these insecure but high yield resources into privately appropriable assets, with reduced yields, held abroad.[2]

In addition to being of theoretical interest, this environment is the subject of policy debate. Free markets and strong property rights are both elements of the 'Washington Consensus' on development and transition policy.[3] While the case for each of its elements individually is often strong,[4] the Consensus implicitly outlines a theory of the first best. It is, however, well known that "it is *not* true that a situation in which more, but not all, of the optimum conditions are fulfilled is necessarily, or is even likely to be, superior to a situation in which fewer are fulfilled" (Lipsey and Lancaster, 1956).

In practice, as strengthening property rights requires enhancing state ca-

[1]Reinhart and Rogoff (2004) provide a recent discussion of this question, regarding sovereign debt default as a central example of weak property rights.

[2]In contrast, Thirsk (1967) notes that the classical English commons earned lower returns than did enclosed land. This was a consequence of management: aware of the problem of communal management, grazing of animals was not allowed on commons land, restricting its use to tillage.

[3]See Williamson (2000) for both a more detailed description and a caveat on the broad use of this term.

[4]Besley and Burgess (2003) assess the ability of commonly discussed policies for poverty reduction (e.g. foreign aid, debt relief) to meet the World Bank's stated aim of halving the number of poor by 2015. While pessimistic about the effectiveness of many of these policies, they find that strengthening property rights by half a standard deviation would suffice to halve global poverty. Roll and Talbott (2001) seek to uncover 'deep' determinants of wealth - those conditions realistically amenable to change. They conclude that over 80% of GNP differences are susceptible to change, and find property rights and black market activity have the highest level of significance.

pacity, doing so may take longer than implementing those policies involving reduced state controls - e.g. liberalising markets. Thus, it is possible that liberalising capital markets while property rights remain weak may encourage the pillaging of domestic assets and capital flight - impeding development. Rich 'oligarchs' with a comparative advantage in predation may even favour the persistence of weak property rights: this help explain some of Russia's immediate post-Soviet experience (Sonin, 2003; Hoff and Stiglitz, 2004).

The conclusion of Tornell and Velasco (1992) was more uniformly positive: access to foreign capital markets may "ameliorate the tragedy of the commons, and increase welfare" by putting "a floor on the common-access asset's rate of return and, thus, a ceiling on the appropriation rate".

They derived this result by modelling the problem as a continuous time, infinite horizon differential game. To ensure tractability, they restricted agents' extraction in two ways. First, extraction strategies are Markovian and stationary, thus dictated by the same function of the communal endowment in each instant. These are standard assumptions in infinite horizon differential games.

Second, extraction rates are restricted to be shares of the commons stock. This reduces the dimension of the problem, allowing each agent's strategy to be identified with a single fraction. In continuous time this precludes the possibility of extinguishing the resource in finite time. However, as access to the outside option of capital markets may increase the rate of return below which a project ceases to be viable, a central effect of market access may be to encourage extinction of the commons.

We, therefore, analyse the simplest version of this problem in order to impose as few restrictive assumptions on strategies as possible: agents make extraction and consumption decisions at two points in time. Thus, they may extinguish the communal endowment immediately, in the second period, or not at all. Without capital markets, consumption in each period cannot exceed extraction. Capital markets replace this set of budget constraints with a single, intertemporal constraint.

While capital market access improves welfare under a broad range of circumstances, we find that it may also reduce welfare: market access introduces two opposing effects. The welfare enhancing effect is the intertemporal smoothing of consumption and extraction plans allowed by markets. Against this, capital market access reduces incentives to preserve the commons, a depletion effect: consumption utility can still be earned without it. This effect is particularly strong in competitive equilibria, whose agents regard themselves as 'extinction date takers' - akin to price takers in a GE environment.

Competitive equilibria may yield multiple extinction dates, an effect found elsewhere in the literature (Kremer and Morcom, 2000; Dutta and Rowat, 2004). This reflects strategic complements in extraction arising from the competitive equilibrium concept: when other agents extract sufficiently quickly to extinguish the resource rapidly, rapid extraction is a best response; when they extract slowly enough to ensure its preservation, slow extraction is a best response. We present a sufficient condition for unique extinction dates; this places an upper bound on the slopes of agents' best response functions ensuring that best response functions, having intersected once, cannot do so again.

Under strategic equilibria, agents take the effect of their own extraction on the extinction date into account. This seems to eliminate the multiple equilibria found under 'extinction date taking'. As agents recognise that their conservation may yield them returns in later periods, they tend to preserve the commons more, earning their surplus by doing so. Thus, the welfare effects of capital market access may not be monotonic in the market rate. When the market rate is low, savings are insufficiently attractive to speed the commons' depletion, but nevertheless aid intertemporal smoothing - a gain. At higher market rates, the commons will be exhausted earlier, the efficiency loss outweighing the smoothing effect. As the market rate increases further, the smoothing gains may again come to outweigh the depletion loss.

Under competitive equilibria with market access, constant marginal extraction costs do not necessarily exhaust the commons. This contrasts to their consequences in Dutta and Rowat (2004) and Gaudet, Moreaux, and Salant (2002), where they produce 'jump extinctions', extinguishing the commons in single instants.

Two reasons explain these differences. First, with only two consumption periods, our agents may become satiated if extraction costs are positive. Second, in Gaudet et al. (2002), scarcity increases the market price of the extracted good to the point where it covers extraction costs.

Before proceeding, we review the related literature in Section 2. We introduce the model in Section 3. Section 4 analyses competitive equilibria in the general case with access to capital markets. Section 5 then analyses strategic equilibria. Section 6 revisits the analysis in the absence of capital markets. Section 7 discusses the results and concludes. Unless indicated otherwise, proofs are relegated to the Appendix A.

2 Related literature

Formal analyses of the commons problem has a long history, beginning with static formulations (Gordon, 1954). This assumed that free-entry would yield the competitive outcome, dissipating all rents. Much subsequent commons analysis has been dynamic, but intertemporally autarkic (Mirman, 1979; Levhari and Mirman, 1980; Benhabib and Radner, 1992; Dutta and Sundaram, 1993b; Dockner and Sorger, 1996; Sorger, 1998): agents may extract from the commons over time, but are forced to consume the goods upon extraction.

Dutta and Sundaram (1993a) and Brooks, Murray, Salant, and Weise (1999) compare results under different equilibrium concepts: the former show that strategic equilibrium dynamics can deviate considerably from those arising under the first best; the latter present a model in which, as the number of agents goes to infinity, one strategic equilibrium converges to the competitive, while the other yields rents (but violates a finite valuation condition). Clark (1973) considered when a single owner would drive a resource to extinction.

Tornell and Lane (1999) considered a variant on Tornell and Velasco (1992) with a high yield formal sector, subject to predatory taxation, and a low yield informal sector, hidden from taxation. A companion paper, to this one, Dutta and Rowat (2004), analyses competitive equilibria in an infinite horizon, continuous time model with stationary Markov extraction strategies. For low communal endowments, the depletion effect of capital market access outweighs the smoothing effect. At intermediate levels, there are generally three equilibria with capital market access: autarky is superior to some of these, but not to others. At high levels, the smoothing effect dominates the depletion effect.

Otherwise, analyses of the commons that do not force agents to consume their harvest immediately have focussed on competitive agents with a storage capability rather than full capital market access.

Kremer and Morcom (2000) studied poachers, who may kill elephants (an open access resource) and store their ivory tusks at the opportunity cost.[5] As we do, they found multiple equilibria: "if others poach, the animal will become scarce, and this will increase the price of the good, making poaching more attractive." Homans and Wilen (2001) allowed fish catches to be sold either immediately to the fresh market or over the rest of the year on the frozen market. They concluded that fishing industry rents both induce entry and shorten the fishing season, causing more fish to be sold on the inferior market. Finally, Gaudet et al. (2002), explored abstract private storage and found that, as average extraction costs become constant, 'jump extinctions' occur, similar to speculative attacks on currency.

[5]See Bulte, Horan, and Shogren (2003) and Kremer and Morcom (2003) for further discussion.

3 The model

An endowment of resources, k_1, is communally held. In other words, property rights are either not defined or not enforced over this stock. A finite set of N agents, indexed by i or j, take two actions at each of two points in time, $t \in \{1, 2\}$: agents *extract* from the commons; these decisions are denoted by x_{1i} and x_{2i}, respectively; agents also *consume* resources, denoted by c_{1i} and c_{2i}. We defer explanation of how agents make these decisions and how they are related, until the environment is fully described.

Collectively, agents' extraction causes the communal stock to evolve to k_2 at $t = 2$ according to:

$$k_2 = (1 + a) \left(k_1 - \sum_{i=1}^{N} x_{1i} \right); \tag{1}$$

where $a > 0$ is an exogenous constant.[6]

Agent i's utility depends on four arguments, as specified here:

$$U_i (\boldsymbol{c}_i, \boldsymbol{x}_i) = \frac{c_{1i}^{1-\alpha} + \beta c_{2i}^{1-\alpha}}{1 - \alpha} - \frac{x_{1i}^{1+\gamma} + \beta x_{2i}^{1+\gamma}}{(1 + \gamma)\,\theta}; \tag{2}$$

where $\alpha > 0$ makes consumption utility concave, $\beta \in [0, 1]$ is a (common) discount factor, and $\gamma \geq 0$ makes extraction disutility convex; $\theta \geq 0$ allows extraction costs to be further scaled.[7]

The concavity of consumption utility needs no further explanation. The curvature of extraction disutility implies diseconomies of scale in extraction. The case of $\gamma = 0$, linear extraction costs, may be interpreted as a situation in which there is a competitive market for the inputs (e.g. a labour market) into a CRS extraction function.

In contrast with the traditional literature, extraction and consumption need not be equal in any given period. Instead, the presence of capital markets merely requires satisfaction of an intertemporal budget constraint:

$$c_{1i} + \frac{c_{2i}}{1 + r} \leq x_{1i} + \frac{x_{2i}}{1 + r}. \tag{3}$$

[6] In continuous time, the analogous equation of motion is

$$\dot{k}(t) = ak(t) - \sum_i x_i(t).$$

Constraining extraction to be shares of the stock, so that $x_i(t) = f_i k(t)$, yields $\dot{k} = (a - f) k(t)$, where $f \equiv \sum_i f_i$. Thus, the stock either remains constant, grows exponentially or asymptotically approaches zero. These modelling assumptions therefore preclude extinction in finite time.

[7] We rule out $\alpha = 0$ to avoid bang-bang effects.

Thus, r is the exogenous market interest rate. As our society is a small one relative to the global, net lending or borrowing need not equal zero across our agents. We further assume that $a > r$ as this creates a trade-off between efficiency, which dictates that resources should remain in the commons, and weak property rights, which create incentives for removing them. In the enclosure interpretation, $a - r$ is the cost of protecting enclosed resources.

The structure of the problem is common knowledge, as is the endowment, k_1. At $t = 2$, the $t = 1$ extraction decisions also become common knowledge.[8]

Now consider how extraction and consumption are determined. We consider two different concepts of agents' behaviour, a competitive one and a strategic one. In both, capital markets allow agents' consumption problem to be treated decision-theoretically (Tornell and Velasco, 1992). We therefore divide agents' problems into two separate problems: optimal consumption subject to a budget constraint (defined by extraction); and optimal extraction subject to a feasibility constraint.

3.1 The consumption problem

Agents' consumption problems are decision rather than game theoretic problems. The consumption problem facing agent i is subject to a single intertemporal budget constraint:

$$\max_{c_{1i},c_{2i}\geq 0} \frac{c_{1i}^{1-\alpha} + \beta c_{2i}^{1-\alpha}}{1-\alpha} \text{ s.t. } c_{1i} + \frac{c_{2i}}{1+r} \leq y\left(r, \boldsymbol{x}_i\right);$$

where $y\left(r, \boldsymbol{x}_i\right) \equiv x_{1i} + \frac{x_{2i}}{1+r}$, the present value of the private benefits of extraction, is assumed fixed in this sub-problem.

Therefore

$$c_{ti}^{*} = \frac{\left[\left(1+r\right)\beta\right]^{\frac{t}{\alpha}}}{1 + \left(1+r\right)^{\frac{1-\alpha}{\alpha}} \beta^{\frac{1}{\alpha}}} y\left(r, \boldsymbol{x}_i\right). \tag{4}$$

Given these optimal consumption levels, the agent's utility function may be rewritten in terms of $\boldsymbol{x}_i$ alone as the value function

$$V_i\left(\boldsymbol{x}_i\right) = \phi \frac{y\left(r, \boldsymbol{x}_i\right)^{1-\alpha}}{1-\alpha} - \frac{x_{1i}^{1+\gamma} + \beta x_{2i}^{1+\gamma}}{\left(1+\gamma\right)\theta}; \tag{5}$$

where

$$\phi \equiv \left[1 + \left(1+r\right)^{\frac{1-\alpha}{\alpha}} \beta^{\frac{1}{\alpha}}\right]^{\alpha}. \tag{6}$$

Thus, the value function is strictly concave in the extraction levels.

[8]The game is therefore a continuous game of almost perfect information, in the sense of Harris, Reny, and Robson (1995): agents' actions sets are continua; information about past play is perfect but there are simultaneous moves each period.

3.2 Equilibrium concepts

Two equilibrium concepts are compared in this paper, a competitive and a strategic one. Before presenting these, we define a number of ancillary objects.

First, let

$$s_{ti} \equiv \max \left\{ 0, k_t - \sum_{j \neq i} x_{tj} \right\};$$

be the upper bound on i's extraction at time t. Then

Definition 1. *A* Markov strategy $\boldsymbol{\pi}_i$ *for agent i is a pair of functions,* $\{\pi_{1i}, \pi_{2i}\}$ *such that*

$$x_{ti} = \pi_{ti}(s_{ti}) \in [0, s_{ti}];$$

for each $t = 1, 2$. Let $\Pi_i(s_{ti})$ be the set of all Markov strategies available to agent i.

Markov strategies are history independent. By defining them over a domain, s_{ti}, that depends on the simultaneous actions of the remaining agents, we have specified a generalised game (Debreu, 1952). Doing this allows us to avoid specification of arbitrary allocation rules in the event that agents collectively seek infeasible extraction from the commons.[9] In what follows, we focus on symmetric equilibria, consistent with egalitarian allocation rules.

As our model only has two time periods, we do not need to impose the further requirement of stationarity for analysis to remain tractable.

The competitive concept is:

Definition 2. *A strategy profile,* $\{\boldsymbol{\pi}\}_{i=1}^N$ *is a* rational expectations equilibrium *(RE) if, for each i, (π_{1i}, π_{2i}) maximise objective function 5 subject to*

$$x_{1i} \leq s_{1i}; \text{ and } x_{2i} \leq s_{2i}.$$

Thus, the intertemporal feasibility constraint is not faced by any agent individually: agents do not take the effect of their extraction on the capital stock into account. They may therefore be thought of as capital takers, analogously to price takers in competitive equilibrium. Where the latter interact through their collective effect on prices, our agents interact through their collective determination of the extinction date. This equilibrium concept seems more reasonable when agents are numerous and small.

The strategic equilibrium concept is:

[9] See Dutta and Sundaram (1993a) for a discussion.

Definition 3. *A strategy profile* $\{\boldsymbol{\pi}_i\}_{i=1}^N \in \times_{i=1}^N \Pi_i$ *is a* Markov perfect equilibrium *(MPE) if, for each* i*:*

1. $\pi_{2i}(s_{2i})$ *maximises objective function 5 given* $\pi_{1i}(\cdot)$*; and*

2. $\pi_{1i}(s_{1i})$ *maximises objective function 5 given* $\pi_{2i}(\cdot)$*.*

Subgame perfect equilibria, of which MPE form a subset, need not exist in games of almost perfect information, even in mixed strategies: counter-examples are presented in both Harris et al. (1995) and Dutta and Sundaram (1993a). Harris et al. (1995) prove that public randomisation restores existence. In our case, MPE exist in pure strategies.

As the communal resources' extinction date is a central interest of ours, we categorise equilibria, whether RE or MPE, by these:

1X is an equilibrium in which the resource is instantly extinguished, at $t = 1$: $\sum_i x_{1i} = k_1, x_{2i} = 0 \forall i$.

2X is an equilibrium in which the resource is extinguished at $t = 2$: $\sum_i x_{1i} < k_1, \sum_i x_{2i} = k_2$;

NX is an equilibrium in which the resource is not extinguished: $\sum_i x_{1i} < k_1, \sum_i x_{2i} < k_2$;

In 1X and 2X RE equilibria, extraction is generally indeterminate: if the endowment is scarce, any division of it may be an equilibrium. Had we not specified a generalised game, a rule for resolving infeasible extraction would be the standard approach for ensuring determinacy. Instead, we pin them down by considering only symmetric equilibria.

4 Rational expectations equilibria

We fully characterise necessary and sufficient conditions for the rational expectations equilibria.

The lemma doing so then allows this section's main result, which provides necessary and sufficient conditions on k_1 for extinction dates.

Lemma 1. *Extraction in a symmetric RE equilibrium is characterised by:*

1X $x_{1i} = \frac{k_1}{N}$ *and* $x_{2i} = 0$

2X

$$x_{1i}^{\gamma}\left[\frac{1+a}{1+r}\frac{k_1}{N}+\frac{r-a}{1+r}x_{1i}\right]^{\alpha}=\phi\theta; \tag{7}$$

$$x_{2i}=(1+a)\left(\frac{k_1}{N}-x_{1i}\right).$$

NX

$$x_1^{NX}\equiv(\phi\theta)^{\frac{1}{\alpha+\gamma}}\left[\frac{(1+r)^{\frac{1+\gamma}{\gamma}}\beta^{\frac{1}{\gamma}}}{(1+r)^{\frac{1+\gamma}{\gamma}}\beta^{\frac{1}{\gamma}}+1}\right]^{\frac{\alpha}{\alpha+\gamma}}; \tag{8}$$

$$x_2^{NX}\equiv\frac{(\phi\theta)^{\frac{1}{\alpha+\gamma}}}{[(1+r)\beta]^{\frac{1}{\gamma}}}\left[\frac{(1+r)^{\frac{1+\gamma}{\gamma}}\beta^{\frac{1}{\gamma}}}{(1+r)^{\frac{1+\gamma}{\gamma}}\beta^{\frac{1}{\gamma}}+1}\right]^{\frac{\alpha}{\alpha+\gamma}}. \tag{9}$$

If symmetry is not imposed, extraction under the NX equilibrium is unaltered: agents are already extracting to their glut points. Before addressing the condition under which the above equilibria arise, we take two preliminary steps.

First, differentiate x_{1i}, as defined by equation 7, with respect to s_{2i} for:

$$-1<\frac{dx_{1i}}{ds_{2i}}=-\frac{\alpha x_{1i}}{(1+r)\left[\alpha x_{1i}+\gamma y\left(r,x_{1i},s_{2i}\right)\right]}<0. \tag{10}$$

Second, define:

Definition 4. *Threshold capital stocks κ_1 and κ_2 are defined by*

$$\kappa_1\equiv N\left(\phi\theta\right)^{\frac{1}{\alpha+\gamma}};$$

$$\kappa_2\equiv N\left[x_1^{NX}+\frac{x_2^{NX}}{1+a}\right].$$

Thus, costless extraction (Tornell and Velasco, 1992; Tornell and Lane, 1999), which sets $\theta=\infty$, also sets $\kappa_1=\kappa_2=\infty$.

The main result of this section is:

Theorem 1. *Necessary and sufficient conditions for symmetric RE equilibria are:*

1X $k_1\leq\kappa_1$*;*

2X $k_1\in(\kappa_1,\kappa_2]$ *or* $k_1\in[\kappa_2,\kappa_1)$*, as appropriate;*

NX $k_1 > \kappa_2$.

It therefore follows that:

Corollary 1. *A sufficient condition for the coexistence of 1X, 2X and NX equilibria is* $\kappa_2 < \kappa_1$.

Multiple extinction dates may be avoided by ensuring that $\kappa_1 < \kappa_2$:

Proposition 1. *A sufficient condition for* $\kappa_1 < \kappa_2$ *is*

$$\frac{1+r}{1+a} \geq \frac{\alpha}{\alpha+\gamma}. \tag{M}$$

The proof, in the Appendix, shows that Condition M ensures that tighter constraints at $t = 2$ reduce the present value of the extraction path. The condition holds when extraction costs are sufficiently non-linear. If we interpret $\gamma = 0$ as corresponding to perfectly competitive labour markets, the condition requires sufficient imperfections. We now show that $\gamma = 0$ yields unique extinction dates in spite of violating Condition M; we then provide an intuition for the condition itself.

Example 1. *Consider the case of constant marginal extraction costs,* $\gamma = 0$. *When* $a > r$, *this violates condition* M. *Nevertheless, substitution of* $\gamma = 0$ *into equations 8 and 9 reveal that*

$$x_1^{NX} = \left\{ \begin{array}{ll} (\phi\theta)^{\frac{1}{\alpha}} & \text{when } (1+r)\beta > 1 \\ 0 & \text{otherwise} \end{array} \right\}; \text{ and} \tag{11}$$

$$x_2^{NX} = \left\{ \begin{array}{ll} 0 & \text{when } (1+r)\beta > 1 \\ (\phi\theta)^{\frac{1}{\alpha}} & \text{otherwise} \end{array} \right\}; \tag{12}$$

so that

$$\kappa_1 = N(\phi\theta)^{\frac{1}{\alpha}}; \text{ and} \tag{13}$$

$$\kappa_2 = \left\{ \begin{array}{ll} N(\phi\theta)^{\frac{1}{\alpha}} & \text{when } (1+r)\beta > 1 \\ \frac{N}{1+a}(\phi\theta)^{\frac{1}{\alpha}} & \text{otherwise} \end{array} \right\}. \tag{14}$$

Therefore, when $\gamma = 0$, *extinction dates are unique when* $(1+r)\beta > 1$.

We now interpret Condition M. In the case of 2X equilibria, equation 10 and $\frac{ds_{2i}}{d\sum_{j\neq i} x_{1j}} = -(1+a)$ allow us to write

$$\frac{dx_{1i}}{d\sum_{j\neq i} x_{1j}} = \frac{1+a}{1+r}\frac{\alpha x_{1i}}{\alpha x_{1i} + \gamma y(r, x_{1i}, s_{2i})} > 0.$$

Thus, the problem exhibits strategic complementarity in $t = 1$ extraction: larger aggregate extraction by all $j \neq i$ induces larger extraction by i. The derivative takes on its maximum value when $s_{2i} = 0$:

$$\frac{dx_{1i}}{d\sum_{j\neq i} x_{1j}} \leq \frac{1+a}{1+r}\frac{\alpha}{\alpha+\gamma}.$$

Condition M therefore ensures that the slope of i's best response function is bounded above by unity in 2X equilibria. Equally, the slope of the aggregate best response of all $j \neq i$ is bounded above by $N - 1$.

Similar logic applies to 1X and NX equilibria. In 1X, $\frac{dx_{1i}}{d\sum_{j\neq i} x_{1j}} < 0$ by the resource constraint. In NX, $\frac{dx_{1i}}{d\sum_{j\neq i} x_{1j}} = 0$ as the resource is sufficiently abundant that x_{1i} is independent of others' extraction. Thus, in both cases, the slopes of the best response functions are bounded above by zero.

Thus, if there is a fixed point in $\left(x_{1i}, \sum_{j\neq i} x_{1j}\right)$ space, the upper bound on best responses ensure that there are no further intersections at higher values of $\sum_{j\neq i} x_{1j}$.[10]

This interpretation also explains Example 1: $\gamma = 0$ ensures linear best responses functions, which intersect once.

Finally, interpreting condition M in terms of best responses allows economic intuitions for it. We mention two. First, as $\frac{1+r}{1+a}$ increases, i's best response to $j \neq i$ becomes flatter: as the disadvantage of private savings relative to use of the commons is reduced, more extraction becomes appealing independently of others' actions. Second, play also becomes less responsive as γ increases: the increased curvature of the extraction costs makes i less responsive to increases in 'unclaimed' communal resources.

5 Markov perfect equilibria

Value function 5 is strictly concave in its arguments. Therefore, given any $\boldsymbol{x}_{-i}$, agent i has a unique best response in each period; this means that only pure strategies need be considered.

Before presenting our main result we define

$$\kappa_1^- \equiv \kappa_1 \left[1 - \frac{1+a}{(1+r)\,N}\right]^{\frac{1}{\alpha+\gamma}}$$

[10] This argument also holds for asymmetric equilibria. Generally, extraction is indeterminate for 1X and 2X equilibria. In these cases, the monotonic mapping between s_{2i} and x_{1i} means that a different s_{2i} could be chosen, leading to a different, unique, intersection in (x_{1i}, x_{1j}) space.

This section's main result parallels that of the previous section:

Theorem 2. **1X** *a necessary condition for a 1X MPE is that* $k_1 \leq \kappa_1$. *A sufficient condition is that*

$$k_1 \leq \kappa_1^-.$$

2X *a necessary condition for a 2X MPE is that* $k_1 \leq \kappa_2$. *A sufficient condition for a 2X MPE is that*

$$\kappa_1^- < k_1 \leq \kappa_2.$$

NX *a necessary and sufficient condition for an NX MPE is that* $k_1 > \kappa_2$.

Figure 1: Symmetric RE and MPE equilibria with market access

Figure 1 compares the conditions presented in Theorems 1 and 2. As the proofs of the sufficiency conditions presented above rely on symmetric examples, it is the sufficiency conditions that are displayed.

When the endowment is abundant, the competitive and strategic concepts yield identical extinction dates, regardless of the number of agents. This reflects the problem's finite horizon: strategic and competitive agents alike know that resources left in the commons at $t = 2$ will be lost. Thus, they face the same incentives to conserve at $t = 2$, namely none.

When the communal resource is scarce, however, strategic and competitive agents think differently about the consequences of conservation: while a strategic agent assumes that he will receive $\frac{1}{N}$ of the returns to extraction foregone now, a competitive agent assumes that he will later receive none. This explains strategic agents' ability to obtain 2X equilibria for k_1 levels at which competitive agents attain 1X equilibria.

The difference in k_1 sufficient to achieve a 2X outcome is proportional to $\frac{1+a}{(1+r)N}$. The statics on the wedge are intuitively appealing: it increases with returns to the commons, a, and decreases with returns to private saving, r, and the number of agents, N. Thus, as N increases, the gap closes, so that the MPE converges to the RE.

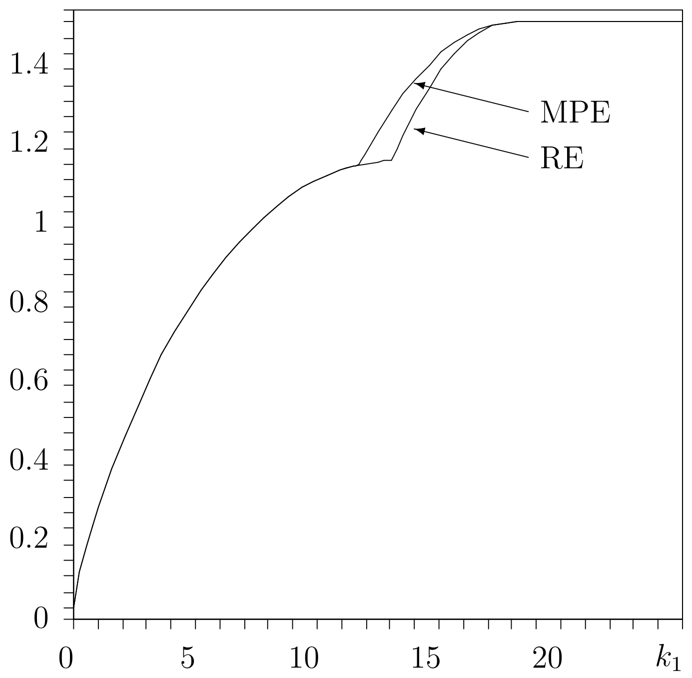

Figure 2: Welfare with capital market access when $(N, a, r, \alpha, \beta, \gamma, \theta) = \left(10, \frac{40}{100}, \frac{35}{100}, \frac{2}{10}, \frac{97}{100}, \frac{8}{10}, 1\right)$ under symmetric strategies.

Strategic agents' more sophisticated consideration of the consequences of conservation provide them with a surplus relative to competitive agents. An example is plotted in Figure 2. When the resource is non-scarce, or when it is sufficiently scarce to yield a 1X under either equilibrium concept, welfare is identical. For intermediate endowments, strategic agents' returns to moving to 2X at lower k_1 are clearly displayed. When both concepts yield 2X, the difference owes to strategic agents' reducing their $t = 1$ extraction relative to competitive agents.

Under some circumstances, a 2X may be guaranteed for all scarce k_1:

$$N \leq \frac{1+a}{1+r}; \qquad (S')$$

ensures that the boundary between the 1X and 2X sufficiency conditions of Theorem 2 is non-positive. Thus, when k_1 is insufficient for NX, a symmetric

2X always exists. Intuitively, the left hand side of condition S' is a measure of the benefits of commonly held assets; its right hand side is a measure of the costs of leaving them there.

Further, extraction under condition S' may be efficient:

Lemma 2.

$$\phi\theta\left[\frac{1+a}{1+r}\frac{k_1}{N}\right]^{-\alpha}\left[1-\frac{1+a}{(1+r)\,N}\right]+\beta\left(\frac{1+a}{N}\right)^{1+\gamma}k_1^{\gamma}\leq 0$$

is necessary and sufficient for $x_{1i}=0\forall i$.

The Lemma's condition is the derivative of the value function (q.v. equation 21 in the Appendix) at $x_{1i}=0$. Thus, its negativity ensures the corner solution. This is efficient as it allows society to use the high growth rate, a, rather than the lower market rate, r. For any parameter values satisfying condition S' there is a value of θ above which it holds: as extraction costs fall, so do the advantages of smoothing extraction.

Less optimistically, plausible values of N, a and r do not satisfy condition S'. Even the easiest case, $N=2$, requires $a\geq 1+2r>2r$, which seems strong.

The complementary condition,

$$N>\frac{1+a}{1+r};\qquad (S)$$

seems more usual. When it holds, the 1X-2X sufficiency boundary is positive, allowing 1X. If extraction is costless, the boundary (and κ_2) become infinite; then, 1X is the unique MPE for all finite k_1.

6 No capital markets

This section re-analyses the results of the preceding two sections in the game without capital market access. This is the standard approach taken in the dynamic commons literature. The comparison allows us to address our main question, that of the consequences of market access.

Analysis proceeds as before, with two exceptions. First, the intertemporal budget constraint is replaced by two separate budget constraints.

Second, c_{ti} is equated to x_{ti}: the consumption and extraction problems cannot be disentangled. This reduces value function 5 to

$$V_i^{\emptyset}\left(\boldsymbol{x}_i\right)=\frac{x_{1i}^{1-\alpha}+\beta x_{2i}^{1-\alpha}}{1-\alpha}-\frac{x_{1i}^{1+\gamma}+\beta x_{2i}^{1+\gamma}}{(1+\gamma)\,\theta}.\qquad (15)$$

This differs in three ways from equation 5, all pertaining to consumption utility: ϕ has been replaced by $1 \leq \phi$; the consumption term is now the sum of concave functions, instead of being a concave function of a sum; and the market discount rate, $\frac{1}{1+r}$, has been replaced by the subjective discount rate, β.

These effects do not work in the same direction. Thus, whether extraction under markets exceeds that under autarky will depend on parameter values, including the level of k_1.

As before, we define an endowment level that will help distinguish extinction dates:

$$\kappa_2^{\emptyset} \equiv \frac{2+a}{1+a} N\theta^{\frac{1}{\alpha+\gamma}}.$$

Following the comparison of equations 5 and 15, this may be greater or less than κ_2: the inequality does not seem to reduce to a simple expression.

6.1 Rational expectations equilibria

	1X	2X	NX
k_1	$\left[0, N\theta^{\frac{1}{\alpha+\gamma}}\right]$	$\left(N\theta^{\frac{1}{\alpha+\gamma}}, \kappa_2^{\emptyset}\right]$	$\left(\kappa_2^{\emptyset}, \infty\right)$
x_{1i}	$\frac{k_1}{N}$	$\theta^{\frac{1}{\alpha+\gamma}}$	$\theta^{\frac{1}{\alpha+\gamma}}$
x_{2i}	-	$(1+a)\left(\frac{k_1}{N} - \theta^{\frac{1}{\alpha+\gamma}}\right)$	$\theta^{\frac{1}{\alpha+\gamma}}$

Table 1: Symmetric RE EQ under autarky

Without access to capital markets, extinction dates are unique. Table 1 presents necessary and sufficient conditions for symmetric RE under autarky. Although marginal consumption utility is infinite at $x_{2i} = 0$, 1X RE exist for some k_1: competitive agents do not believe that conservation at $t = 1$ will gain them access to consumption at $t = 2$.

The effects of market access on extinction dates may be seen by comparing Theorem 1 to Table 1. Autarky attains symmetric 2X for (weakly) lower endowment levels than do capital markets. This follows directly from the Inada consumption utility specification: marginal utility at $c_{2i} = 0$ is infinite; under autarky, this can only be obtained by preserving the commons.

Whether market access or autarky achieves NX at lower k_1, however, is parameter dependent. When $\beta = \frac{1}{1+r}$, κ_2 and $\kappa_2^{\emptyset}$ are identical: if the market discount rate and the subjective discount rates are the same, access to a market rate does not alter the problem. Otherwise:

Proposition 2. $\beta \in \left[\frac{1}{(1+r)^{1+\gamma}}, \frac{1}{1+r}\right) \Rightarrow \kappa_2 < \kappa_2^{\emptyset}$ *while* $\beta > \frac{1}{1+r} \Rightarrow \kappa_2 > \kappa_2^{\emptyset}$.

Intuitively, the higher interest rate allows society to consume more by increasing the efficiency of capital markets.

Theorem 3. *For any calibration, symmetric RE under market access are Pareto superior to those under autarky for at least some values of* k_1*.*

Proof. Comparing welfare under the 1X equilibria shows that market access outperforms autarky by a factor of ϕ. Thus, $\phi > 1$ suffices for the result. $\square$

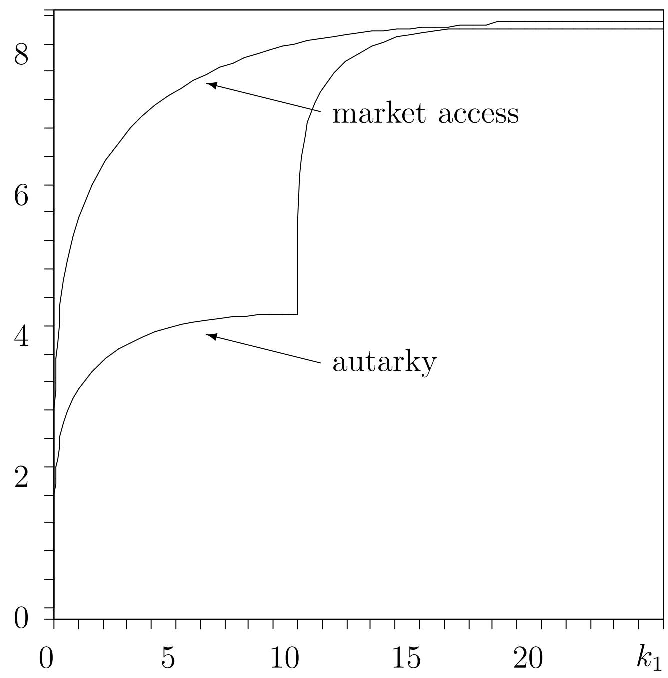

Figure 3: Welfare under symmetric RE when $(N, a, r, \alpha, \beta, \gamma, \theta) = \left(10, \frac{40}{100}, \frac{35}{100}, \frac{8}{10}, \frac{97}{100}, \frac{2}{10}, 1\right)$.

Theorem 3 leaves two possibilities: markets may outperform autarky for all k_1; and, markets may outperform autarky for some k_1. Figures 3 and 4, respectively, illustrate these cases. In both cases, condition M is satisfied; under market access, this guarantees unique extinction dates. The two cases differ only in their choices of α and γ.

In Figure 3, the curvature of consumption utility is greater than that of extraction disutility. Thus, consumption smoothing is important relative to extraction smoothing. The relative unimportance of extraction sequencing

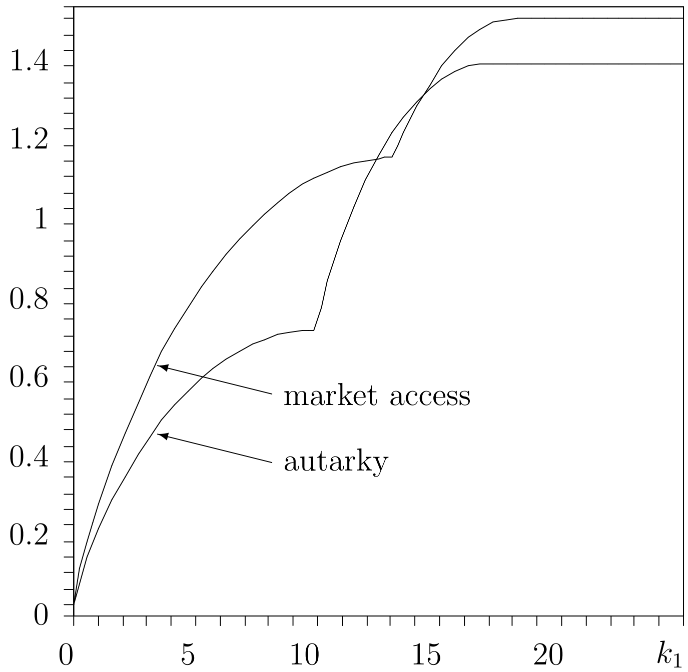

Figure 4: Welfare under symmetric RE when $(N, a, r, \alpha, \beta, \gamma, \theta) = \left(10, \frac{40}{100}, \frac{35}{100}, \frac{2}{10}, \frac{97}{100}, \frac{8}{10}, 1\right)$.

does not seem to convey much advantage: knowledge of the extinction date already imposes controls on extraction. For high levels of extraction, the difference in consumption utility between budget constraints every period and a single intertemporal constraint is reduced: in both cases, marginal consumption utility is small.

In Figure 4, the relative importance of consumption and extraction smoothing is reversed. Nevertheless, the welfare curves resemblance each other more closely than do their counterparts in Figure 3. This may owe to the RE concept already encouraging extraction smoothing in both Figures.

The higher levels of utility in Figure 3 reflect the lower extraction costs.

The mixed welfare results illustrate clearly the two effects of market access. First, a consumption effect, whereby agents may optimise their consumption paths. This increases the value of assets in the capital markets, as indexed by ϕ. It is a positive effect, even without concave consumption utility.

The second effect is an extraction effect, whereby the higher value of assets in the capital markets prompts the more rapid depletion of the commons. This effect is negative, and seems to grow stronger as ϕ increases above unity.

The general benefits to market access may not be surprising under RE: as agents do not regard themselves as able to alter the extinction date, one of the main concerns about the ability to take resources drawn from the commons to the capital markets disappears.

Dutta and Rowat (2004) also found that autarky outperformed market access for some levels of communal endowment. For low endowment levels, autarky dominated market access; at intermediate levels, market access yielded multiple equilibria, some of which outperformed autarky. Here, when autarky is superior, it is so for intermediate rather than low endowment levels.

In both cases, the results reflect the autarkic environment's superior ability to preserve the commons. Here, this requires attaining a 2X outcome which requires, in turn, $k_1 > N\theta^{\frac{1}{\alpha+\gamma}}$. In Dutta and Rowat (2004), time is continuous; thus, autarky delays extinction relative to the market for all low levels of k_1. In both cases, market access outperforms autarky when the communal resource is non-scarce (NX, here).

6.2 Markov perfect equilibria

The parallel result to the main result of Section 5 is:

Theorem 4. *For an NX MPE under autarky, $k_1 > \kappa_2^{\emptyset}$ is necessary and sufficient; the complement is necessary and sufficient for a 2X MPE. No finite k_1 is consistent with a symmetric 1X MPE under autarky.*

Thus, as under RE, extinction dates are unique. Now, however, symmetric 1X equilibria are eliminated: strategic agents expect their conservation at $t = 1$ to yield returns at $t = 2$. Thus, autarky's ability to preserve the commons is strengthened under MPE.

Asymmetric 1X MPE do, however, exist for some values of k_1. Consider a candidate equilibrium in which some subset of agents is to divide k_1 between them; out of equilibrium, a distinct subset will divide whatever is produced by deviation at $t = 1$. A member of this first subset now effectively faces a single extraction period: while he may wish to take advantage of the infinite marginal consumption utility at $t = 2$, he cannot.

The MPE transition between 2X and NX is the same as that for RE.

An exact parallel to Theorem 3 holds, for the same reasons:

Theorem 5. *For any calibration, symmetric MPE under market access are Pareto superior to those under autarky for at least some values of* k_1.

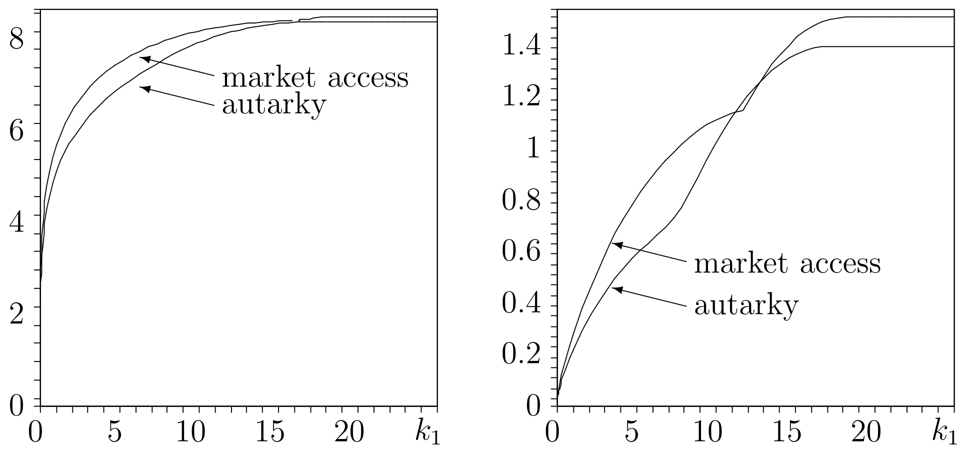

Figure 5: Welfare under symmetric MPE. Parameters as in Figures 3 and 4.

Figure 5 provides the MPE analogs of Figure 3 and 4 in its left and right panels, respectively. In both cases, the deep scallops present in the autarkic RE outcomes have been reduced by the elimination of 1X MPE. This is particularly dramatic in the former calibration. The shallow scallop in the second calibration around $k_1 \approx 8$ therefore does not reflect a 1X to 2X transition but the relative extraction levels, x_{1i} and x_{2i}: while x_{1i} grows almost linearly in k_1 for low endowments, x_{2i} remains near zero until $k_1 \approx 8$, when it begins to grow quickly.

Thus, as under RE, market access conveys both consumption and extraction effects. Under MPE, however, agents' additional foresight allows them to mitigate substantially autarky's harmful effects.

As before, the MPE equilibria converge to the RE as $N \to \infty$. As this causes $N\theta^{\frac{1}{\alpha+\gamma}} \to \infty$, 1X becomes the unique autarkic RE for all finite k_1. For autarkic MPE, 2X remains the unique equilibrium, but $\frac{k_2}{N}$ vanishes, making the outcome effectively a 1X one.

7 Discussion

Example 1 presented an example of unique extinction dates for RE equilibria which violated Condition M. The example is also interesting as it produces jump extinctions in Dutta and Rowat (2004) and Gaudet et al. (2002). In the former, the extinction date went to zero as γ did. In the latter, when reserves reached a critical level after being sold at a constant price, instant depletion would occur when average extraction costs were constant. This is not the case here: κ_1 and κ_2 are finite, meaning that there are finite k_1 that yield 1X, 2X (when $k_1 = \kappa_1 = \kappa_2$) and NX RE equilibria.

We now explain these differences.

In Gaudet et al. (2002), the non-renewable resource is sold at its marginal - also the average - extraction cost until stocks are reduced to a level at which the price begins to rise at the rate of interest, r: the standard Hotelling result. Instant depletion occurs at that point. Thus, market demand ensures that prices will eventually rise to the point where they cover finite, constant marginal extraction costs.

In Dutta and Rowat (2004), the continuous time formulation means that there are an infinite number of consumption periods. Infinite marginal consumption utility at $c_{ti} = 0$ then ensure that finite, constant marginal extraction costs are covered.

In the present paper, however, there are only two consumption periods. Thus, agents may collectively reach their glut points - points at which the marginal extraction costs exceed the marginal consumption benefits - with finite k_1.

Intriguingly, the resource may survive even under more consumption periods. When the Uzawa consumption condition,

$$(1+r)^{1-\alpha}\beta < 1; \qquad (UC)$$

holds, ϕ and κ_1 converge to finite values even as the number of consumption periods goes to infinity. When this condition is violated, κ_1 becomes infinite, allowing $1X$ depletion for any initial commons stock. The condition is violated at $\beta = 1$, making sense of why such a condition is not relevant in Dutta and Rowat (2004). In that, there are an infinite number of consumption periods even in the first interval of time. Here, the infinite consumption periods are spaced out over an infinite length of time.

The results for rational expectations generalise naturally to games with more stages. When $T = \infty$, $V_i(\boldsymbol{x}_i)$ may be infinite for some $\boldsymbol{x}_i$. Problems associated with this may be avoided by imposing Uzawa conditions to ensure finite valuations. The Uzawa consumption condition has already been formally introduced, in equation UC. The corresponding extraction condition, equation UX, is present in the Appendix, although not introduced as such.[11]

Generalising results for MPE to longer games is not as easy. The difficulty reflects the need, in general, to consider two state variables, both k_t and a

[11]This condition appears in equations 11 and 12.

measure of private savings or debt. This latter becomes relevant as agents must consider the possibility of deviating at every subgame. As private savings or debt is generally payoff relevant, this dimension must generally be considered.

The model used here can be extended in a number of ways. We consider four possibilities. First, the commons' growth rate might, if it represents biological growth, be a function of the stock. Second, we have not considered the possibility of default. Third, the capital stock is not an argument in the utility function in this model. This seems more consistent with an interpretation of k as physical capital than as natural capital. In the latter case, k might provide eco-system services directly. Allowing k to enter directly into the utility function would allow re-analysis of the problem as the marginal rate of substitution between k and c_i varied. This might contribute to the 'weak' and 'strong' sustainability debate.

Finally, it seems of interest to consider weak, rather than entirely absent, property rights. In addition to seeming to cover more plausible cases, it would allow closer engagement with the empirical literature, which uses interior measures of the rule of law (q.v. Acemoglu, Johnson, and Robinson, 2001). Technically implementing weak property rights might require tracking N state variables rather than the single k_1. This would both allow sensitivity analysis of the results obtained under the present extreme assumption and allow engagement with the empirical literature.

A Appendix

Proof of Lemma 1. The 1X extraction is trivial. Next consider the NX extraction. In an NX equilibrium, each agent regards the extraction constraints as non-binding. The first order conditions from value function 5 produce

$$x_{ti}^{NX} = \left[\frac{\phi\theta}{(1+r)^{t-1}\beta^{t-1} y\left(r, \boldsymbol{x}_i^{NX}\right)^{\alpha}} \right]^{\frac{1}{\gamma}} \text{ for } t = 1, 2. \tag{16}$$

The definition of $y\left(r, \boldsymbol{x}_i^{NX}\right)$ allows this to be manipulated for the result.

Finally, consider 2X. The $t = 2$ extraction follows from symmetry and definition of the case. The $t = 1$ optimisation problem yields first order condition

$$\frac{\partial V_i\left(x_{1i}, x_{2i}\right)}{\partial x_{1i}} = \frac{\phi}{y\left(r, \boldsymbol{x}_i\right)^{\alpha}} - \frac{x_{1i}^{\gamma}}{\theta};$$

which produces the result. □

Proof of Theorem 1. First consider the 1X case. Agent i maximises value function 5 subject to $x_{2i} = 0$ and $x_{1i} \leq [s_{1i}]^+$. Doing so yields

$$x_{1i}^{1X} = \min\left\{(\phi\theta)^{\frac{1}{\alpha+\gamma}}, [s_{1i}]^+\right\}. \tag{17}$$

To satisfy the 1X requirement that $\sum_i x_{1i}^{1X} = k_1$ the lemma's condition is therefore necessary. As the value function is concave in x_{1i} and the single constraint convex, equation 17 is the unique maximiser; the lemma's condition is therefore also sufficient.

Now turn to the NX case. Given strategies $\{x_{1i}, x_{2i}\}_{i=1}^N$, this requires

$$\sum_i x_{1i} < k_1; \text{ and}$$
$$\sum_i \left[x_{1i} + \frac{x_{2i}}{(1+a)}\right] < k_1. \tag{18}$$

As the first of these is less strict than the second, only the second need be considered. As the objective function is concave in x_{1i} and x_{2i} and unconstrained, this condition is also sufficient.

Finally, consider the 2X case. When

$$\frac{\partial y\left(a, x_{1i}, s_{2i}\right)}{\partial s_{2i}} \geq 0; \tag{19}$$

the least k_1 consistent with 2X sets $s_{2j} = 0 \forall j \neq i$ and $s_{2i} > 0$. Were $s_{2i} = 0 \forall i$, a 1X would result, in which $x_{1i} = x_{1i}^{1X} \forall i$. For an $s_{2i} > 0$ it must be that $x_{1i}^{1X} = (\phi\theta)^{\frac{1}{\alpha+\gamma}}$. Otherwise agents $j \neq i$ would increase their extraction. Thus, $k_1 > \sum_i y\left(a, (\phi\theta)^{\frac{1}{\alpha+\gamma}}, 0\right) = \kappa_1$.

Similarly, when the social present value of extraction increases with s_{2i}, the greatest k_1 consistent with 2X sets $s_{2i} = x_2^{NX} \forall i$. As extracting x_2^{NX} at $t = 2$ implies extracting x_1^{NX} at $t = 1, k_1 \leq \sum_i y\left(a, \boldsymbol{x}^{NX}\right) = \kappa_2$.

These arguments are reversed when $y\left(a, x_{1i}, s_{2i}\right)$ decreases in s_{2i}. In this case, the least k_1 consistent with 2X sets $s_{2i} = x_2^{NX} \forall i$ while the greatest k_1 sets $s_{2j} = 0 \forall j \neq i$ and $s_{2i} > 0$.

As value function 5 is concave in x_{1i} and x_{2i}, each of which is subject to a convex constraint, the necessary conditions above are also sufficient. □

Proof of Proposition 1. Let $y\left(a, x_{1i}, s_{2i}\right)$ be the present value of the social costs of agent i's extraction in a 2X equilibrium. Condition M, ensures that inequality 19 holds. As $y\left(a, x_{1i}, s_{2i}\right)$ is continuous at $s_{2i} = x_2^{NX}, \kappa_2 = \sum_i y\left(a, \boldsymbol{x}_i^{NX}\right)$. The result follows from $\kappa_1 = \sum_i y\left(a, x_{1i}, 0\right)$. □

Proof of Theorem 2. First consider 1X MPE. The candidate x_{2i} that prompts the greatest extraction at x_{1i} is zero. Given this $t = 2$ extraction, no agent i has a unilateral incentive to set $x_{1i} > (\phi\theta)^{\frac{1}{\alpha+\gamma}}$. The necessary condition follows by summation.

To establish sufficiency, use candidate strategies $\hat{x}_{1i} = \frac{1}{N}k_1, f_i(k_2) = \frac{1}{N}k_2 \forall i$. The payoff to candidate play is

$$\frac{\phi}{1-\alpha}\left(\frac{k_1}{N}\right)^{1-\alpha} - \frac{1}{(1+\gamma)\theta}\left(\frac{k_1}{N}\right)^{1+\gamma};$$

while that to deviating by $\delta \in \left(0, \frac{k_1}{N}\right]$ is

$$\frac{\phi}{1-\alpha}\left[\frac{k_1}{N} - \delta + \frac{1+a}{1+r}\frac{\delta}{N}\right]^{1-\alpha} - \frac{1}{(1+\gamma)\theta}\left\{\left(\frac{k_1}{N} - \delta\right)^{1+\gamma} + \beta\left[\frac{(1+a)\delta}{N}\right]^{1+\gamma}\right\}.$$

Denote the difference between deviant play and candidate play by $\Delta(\delta)$. If $\frac{d\Delta}{d\delta} \leq 0$ at $\delta = 0$ then deviation is never profitable: the convex instantaneous disutility of extraction ensures that marginal extraction savings at $t = 1$ and marginal extraction costs at $t = 2$ are greatest at $\delta = 0$.

As

$$\left.\frac{\partial\Delta}{\partial\delta}\right|_{\delta=0} = \phi\frac{(1+a)-(1+r)N}{(1+r)N}\left(\frac{N}{k_1}\right)^{\alpha} + \frac{1}{\theta}\left(\frac{k_1}{N}\right)^{\gamma};$$

the lemma's condition guarantees that the egalitarian candidates considered here survive deviations.

Now consider the sufficiency component of the NX MPE. Let x_{1j} and x_{2j} be defined by equations 8 and 9, respectively, for all $j \neq i$. When the condition holds, agent i faces an unconstrained optimisation problem and chooses identically to agents $j \neq i$.

Now consider necessity. At $t = 2$, no agent has an incentive to extract less than x_2^{NX}, as defined by equation 9. This must be strictly feasible for an NX equilibrium. Thus, it must be that $k_2 > Nx_2^{NX}$. Consequently, the marginal utility obtained by each agent at $t = 2$ is zero. Thus, no agent will sacrifice positive marginal utility at $t = 1$ to obtain it. Indeed, to support an NX equilibrium, each necessarily extracts up to x_1^{NX} at $t = 1$. The strict inequality of the condition distinguishes this from a 2X equilibrium.

The 2X necessary condition follows directly from the upper bounds on desirable extraction in each period that result from the concave objective function.

Finally, the symmetric profile, $\left\{\hat{x}_1, \frac{k_2}{N}\right\}_{i=1}^{N}$ establishes the sufficiency condition for the 2X MPE. At $t = 2$, upward deviations to 1X are no longer possible. Downward deviation to NX is avoided when $\frac{k_2}{N}$ is less than each agent's glut point. By the intertemporal budget constraint, this is a function of $\hat{x}_1$:

$$\left(\frac{k_2}{N}\right)^{\gamma}\left(\hat{x}_1+\frac{k_2}{(1+r)\,N}\right)^{\alpha}\leq\frac{\phi\theta}{(1+r)\,\beta}$$
$$\Leftrightarrow\frac{(1+a)^{\gamma}(1+r)^{1-\alpha}\beta}{\phi\theta N^{\alpha+\gamma}}(k_1-N\hat{x}_1)^{\gamma}\left[(1+r)\,N\hat{x}_1+(1+a)\,(k_1-N\hat{x}_1)\right]^{\alpha}\leq 1. \tag{20}$$

Now consider behaviour at $t=1$. The candidate $t=2$ play reduces value function 5 to

$$\begin{aligned}V_i(x_{1i})=&\frac{\phi}{1-\alpha}\left[x_{1i}+\frac{1+a}{(1+r)\,N}\left(k_1-(N-1)\,\hat{x}_1-x_{1i}\right)\right]^{1-\alpha}\\&-\frac{1}{(1+\gamma)\,\theta}\left\{x_{1i}^{1+\gamma}+\beta\left[\frac{(1+a)}{N}\left(k_1-(N-1)\,\hat{x}_1-x_{1i}\right)\right]^{1+\gamma}\right\}.\end{aligned} \tag{21}$$

When play is symmetric, the first order necessary condition for an interior solution is

$$\begin{aligned}\left[(1+r)\,N\hat{x}_1+(1+a)\,(k_1-N\hat{x}_1)\right]^{\alpha}&\left[\hat{x}_1^{\gamma}-\beta\left(\frac{1+a}{N}\right)^{1+\gamma}(k_1-N\hat{x}_1)^{\gamma}\right]\\&\times\frac{\left[(1+r)\,N\right]^{1-\alpha}}{\phi\theta\left[(1+r)\,N-(1+a)\right]}=1.\end{aligned} \tag{22}$$

A 2X MPE requires that first order condition 22 and the 1X and NX inequalities hold. The 1X inequality requires $k_1>N\hat{x}_1$; rearranged, this produces the lower bound on k_1 in the statement of the lemma. The latter requires that $\hat{x}_1$ and k_1 satisfy inequality 20 with equality as well as equation 22. As each defines a contour in (k_1,x_1) space, we solve for their intersection. Taking advantage of the term common to each, produces, by cancellation,

$$k_1-N\hat{x}_1=\frac{N\hat{x}_1}{(1+a)\left[(1+r)\,\beta\right]^{\frac{1}{\gamma}}}; \tag{23}$$

and, eventually,

$$\hat{x}_1=\frac{(1+a)\left[(1+r)\,\beta\right]^{\frac{1}{\gamma}}}{(1+a)\left[(1+r)\,\beta\right]^{\frac{1}{\gamma}}+1}\frac{k_1}{N};$$

an original ray.

These define the locus of all intersections of the two surfaces, not just those of their unit contours. Substituting equation 23 back into first order condition 22 yields $\hat{x}_1=x_1^{NX}$. As this produces $x_{2i}=\frac{k_2}{N}=\frac{1+a}{N}(k_1-N\hat{x}_1)=x_2^{NX}$, the k_1 that is just extinguished in 2X by these extraction plans is κ_2. □

Proof of Proposition 2.

$$\frac{\partial \kappa_2}{\partial r} = N\theta^{\frac{1}{\alpha+\gamma}} \left[1 + f_\alpha(\beta, r)\right]^{\frac{\alpha}{\alpha+\gamma}} \left[\frac{f_\gamma(\beta, r)}{1 + f_\gamma(\beta, r)}\right]^{\frac{\alpha}{\alpha+\gamma}}$$
$$\times \left\{ \frac{(1-\alpha) f_\gamma(\beta, r) \gamma \left[1 + \frac{r + a f_\alpha(\beta, r)}{1 + f_\alpha(\beta, r)}\right] + \alpha (1+\gamma) \left[1 + \frac{r + a f_\gamma(\beta, r)}{1 + f_\gamma(\beta, r)}\right]}{(\alpha+\gamma)(1+r)} - 1 \right\};$$

where

$$f_\alpha(\beta, r) \equiv (1+r)^{\frac{1-\alpha}{\alpha}} \beta^{\frac{1}{\alpha}}; \text{ and}$$
$$f_\gamma(\beta, r) \equiv (1+r)^{\frac{1+\gamma}{\gamma}} \beta^{\frac{1}{\gamma}}.$$

A necessary and sufficient condition $\frac{\partial \kappa_2}{\partial r} > 0$ is that the term in braces be positive. This is guaranteed by

$$\beta > \frac{1}{(1+r)^{1+\gamma}}. \qquad (UX)$$

As $\kappa_2^{\emptyset}$ is insensitive to r, the result follows. □

Proof of Theorem 4. As the proof of the NX result parallels that of Theorem 2, we merely prove the latter elements. First consider the 1X case. A symmetric equilibrium implies that $\hat{x}_{1i} = \frac{k_1}{N} \forall i$. A deviation yields $k_2 > 0$. As each agent i has infinite marginal consumption utility but only finite marginal extraction disutility at $x_{2i} = 0$, each will wish to consume at least some of this. While some agents may be allocated no share of k_2 in a candidate equilibrium, at least one will be. That agent has both the ability to deviate downwards from $\hat{x}_{1i} = \frac{k_1}{N}$ and the incentive to do so. A 2X MPE results as long as not all agents exceed their glut points.

Whether the MPE is a 2X and the NX then depends on whether k_1 is sufficiently high to allow all agents to reach their glut points in each period. □

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