# Performance evaluation of ethical mutual funds in slump periods

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**Abstract.** In this paper we tackle the problem of the presence of negative average rate of returns in the computation of the performance of ethical mutual funds. The presence of these negative values raises problems both in the computation of the classical performance indicators and in DEA modeling. In this paper we propose a suitably adjusted DEA model which allows the presence of non negative outputs. The model is applied to data on the UK market of ethical mutual funds.

KEYWORDS. Performance evaluation, ethical mutual funds, data envelopment analysis.

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# 1 Introduction

The field of ethical mutual funds arises more and more interest in modern financial world.

However, few empirical studies investigate the performance of ethical funds by taking into account not only their returns and risks but also the investment costs and the ethical profile. Using a data envelopment analysis technique, [5] proposes a conveniently adjusted DEA model which enables to take all these features into account at the same time. This model is applied to Italian data in [4].

However, the DEA approach requires the assumption that all the input and output values are non negative, while in slump periods of the business cycle, the average rate of return of most stocks, and thus that of most mutual funds, is negative.

In this paper we tackle the problem of the presence of negative average rate of returns both in the computation of numerical performance indicators and from the point of view of its consequences on the significance of the outcomes of a DEA model.

The paper is organized as follows. Section 2 discusses the problem raised by the presence of negative mean returns in computing mutual fund performance

indicators. Section 3 analyzes the consequences on DEA modeling of the presence of negative values in an output variable. In section 4 we propose an adjustment to cope with this problem while in section 5 we propose a method to build an ethical measure for mutual funds. In Section 6 we formulate a DEA model for the evaluation of the performance of ethical mutual funds in the presence of non negative outputs and an exogenously fixed ethical level. Finally, in section 7 we present the results of an empirical analysis on UK data.

# 2 The problem of negative mean returns in computing mutual fund performance indicators

Let us consider a set of n mutual funds j = 1, 2, ..., n with risky rate of return  $R_j$  and assume to have to compare their performances. We denote by  $E(R_j)$  the expected rate of return and by  $\sigma_j = \sqrt{Var(R_j)}$  the standard deviation of the rate of return, often used as a risk indicator for a fund investment.

It is usual to compare the performance of mutual funds over past periods and use this performance measure in order to assess the ability of the fund managers. This is often done by substituting the average rate of return obtained by the funds in the period considered and the historical volatility of the returns for the expected rate of return  $E(R_j)$  and standard deviation  $\sigma_j$ .

For instance, let us assume to measure the performance of mutual fund j by using the well-known Sharpe ratio (see [9]) computed by considering the rates of return  $r_{j1}, r_{j2}, \ldots, r_{jT}$  obtained by this fund in the periods  $1, 2, \ldots, T$  (for example, we could consider the monthly rates of return of the last three years):

$$I_{j,Sharpe} = \frac{\overline{R}_j - r}{\overline{\sigma}_j},\tag{1}$$

where

$$\overline{R}_j = \frac{1}{T} \sum_{t=1}^T r_{jt} \tag{2}$$

is the average rate of return in the period considered, r is the rate of return of a riskless asset and

$$\overline{\sigma}_j = \frac{1}{T} \sum_{t=1}^T (r_{jt} - \overline{R}_j)^2 \tag{3}$$

is the historical volatility of fund j in the period under consideration.

Of course, the expected excess return  $E(R_j) - r$  for a risky asset actually traded in the market must be positive, in order that the asset is bought by risk averse investors. However, this is not necessarily true for all mutual funds on the market when the computations are based on historical data, i.e. for the average excess return computed *ex post*,  $\overline{R}_j - r$ : the observed excess return is positive only for funds which present a rate of return higher than the riskless interest rate.

On the other hand, for the funds that exhibit a negative excess return, the traditional performance ratios, such as Sharpe ratio, can be misleading. The situation is well depicted in figure 1, which shows the behavior of the Sharpe ratio as the the excess return  $\overline{R}_j - r$  and the standard deviation  $\sigma_j$  vary. It can be noticed that only when that excess return is positive, the value of the Sharpe ratio decreases with the risk indicator  $\sigma_j$ , as we would expect for a performance indicator; on the contrary, when the excess return is negative, the value of the Sharpe ratio increases with the value of the standard deviation. Hence, for the funds with a rate of return lower than the riskless interest rate, at a parity of the rate of return, it would be chosen the fund with the highest risk.



**Fig. 1.** Behavior of the Sharpe ratio as the the excess return  $\overline{R}_j - r$  and the standard deviation  $\sigma_j$  vary.

The same problem is exhibited by the other numerical indexes of performance defined as ratios between a return and a risk indicator, such as the Treynor index (see [11]), the reward to half-variance and the reward to semivariance indexes (see [1]).

On the other hand, we could use a performance indicator which generalizes the numerical indexes of performance defined as ratios between a return and a risk indicator, such as the DEA performance indicators  $I_{DEA-1}$  and  $I_{DEA-1}$ proposed by Basso and Funari in [3]. This approach can be applied by using as return indicator either the average excess return or the average rate of return. By using directly the average rate of return, we get a performance measure for

all funds that exhibit a positive rate of return, even for the funds with a negative average excess return.

However, while the expected rate of return of a mutual fund, as that of all risky portfolios, must be positive, this is not necessarily true for all assets in all periods when the computations are based on historical data. In particular, when the period to which the historical data refer falls within a slump period of the business cycle, the average rate of return of most stocks, and thus that of most mutual funds, is negative.

Nevertheless, the DEA approach can only be applied under the assumption that all the input and output values are non negative. Indeed, when some output variables may take negative values, the DEA performance measure may give non satisfactory results, as will be highlighted in next section.

# 3 The problem of a negative output value in DEA modeling

It is common in classical DEA models to assume that all the input and output values are non negative (see for example [6]). This is indeed a crucial assumption in the measurement of performance with the DEA technique, and the reason can be seen from the analysis of the following stylized example.

Let us consider the problem of evaluating the performance of four decision making units (DMUs)  $U_1$ ,  $U_2$ ,  $U_3$ ,  $U_4$ , with one input x and two outputs  $y_1$  and  $y_2$ . Let the normalized values of the outputs of the four DMUs, with respect to the input value, be as follows:

$$U_1 = \left(\frac{y_{11}}{x_1}, \frac{y_{21}}{x_1}\right) = (5, 1) \tag{4}$$

$$U_2 = \left(\frac{y_{12}}{x_1}, \frac{y_{22}}{x_1}\right) = (3, 2) \tag{5}$$

$$U_3 = \left(\frac{y_{13}}{x_1}, \frac{y_{23}}{x_1}\right) = (2, 3) \tag{6}$$

$$U_4 = \left(\frac{y_{14}}{x_1}, \frac{y_{24}}{x_1}\right) = (-1, a),\tag{7}$$

with  $a \in \mathbf{R}^+$ .

It is known that the DEA performance measure for DMU  $j_0$ , with  $j_0 \in \{1, 2, 3, 4\}$ , is the optimal value of the following linear fractional programming problem

$$\max_{v,u_1,u_2} \frac{u_1 y_{1j_0} + u_2 y_{2j_0}}{v x_{j_0}} \tag{8}$$

s.t.

$$\frac{u_1 y_{1j} + u_2 y_{2j}}{v x_j} \le 1 \qquad \qquad j = 1, 2, 3, 4 \tag{9}$$

$$v, u_1, u_2 \ge \varepsilon, \tag{10}$$

where  $v, u_1, u_2$  are the weights associated to the input and output variables, respectively, and  $\varepsilon$  is a non-Archimedean constant (see for example [6]).

The optimal solution of the fractional problem (8)-(10) can be found by solving the following equivalent linear programming problem

$$\max_{v,u_1,u_2} u_1 y_{1j_0} + u_2 y_{2j_0} \tag{11}$$

s.t.

$$vx_{j_0} = 1 \tag{12}$$

$$u_1 y_{1j} + u_2 y_{2j} \le v x_j \qquad \qquad j = 1, 2, 3, 4 \tag{13}$$

$$v, u_1, u_2 \ge \varepsilon, \tag{14}$$

which is equivalent to the reduced linear problem

$$\max_{u_1, u_2} u_1 y_{1j_0} + u_2 y_{2j_0} \tag{15}$$

s.t.

$$u_1 y_{1j} + u_2 y_{2j} \le \frac{x_j}{x_{j_0}} \tag{16}$$

$$u_1, u_2 \ge \varepsilon, \tag{17}$$

in which we have set  $v = \frac{1}{x_{j_0}}$ .

If we restrict the analysis to the set of DMUs  $U_1, U_2, U_3$ , we have a classical DEA problem in which all the input and output values are positive. The efficiency frontier of such an instance can be represented as in figure 2, where the cartesian axes represent the normalized output values  $\frac{y_{1j}}{x_i}$  and  $\frac{y_{2j}}{x_i}$ .

The efficient frontier is the upper-right line which connects the efficient DMUs, i.e. the DMUs with a DEA performance measure equal to 1. Figure 2 shows that DMUs  $U_1$  and  $U_3$  are efficient, while  $U_2$  is inefficient, since its DEA performance measure,  $E_{DEA,U_2}$ , equal to the ratio

$$E_{DEA,U_2} = \frac{dist(O, U_2)}{dist(O, P_2)} = 0.923,$$
(18)

is less than 1. The point  $P_2$  represents the virtual unit which has the same input and output orientation as  $U_2$  and lies on the efficient frontier. This virtual unit suggests that the inefficient unit  $U_2$  might improve its output values while keeping the input value fixed, by moving along the dashed line  $\overline{OP_2}$  towards the efficient frontier, till its reaches efficiency.

If we include in the analysis also DMU  $U_4$ , which has a negative value of output 1, puzzling results can be obtained, so that the DEA fractional problem (8)-(10) does not give a reasonable efficiency measure any longer.

Table 1 displays the DEA efficiency measures  $E_{DEA,U_j}$  for the four DMUs for different values of the second output of  $U_4$ , a; figures 3–6 show the efficient frontier obtained in some relevant cases.



**Fig. 2.** Efficient frontier of the DEA problem for DMUs  $U_1, U_2, U_3$ . The cartesian axes are associated to the normalized output values  $\frac{y_{1j}}{x_j}, \frac{y_{2j}}{x_j}$ .

a	$E_{DEA,U_1}$	$E_{DEA,U_2}$	$E_{DEA,U_3}$	$E_{DEA,U_4}$
	1	0.000	1 000	0.999
1	1	0.923	1.000	0.333
2	1	0.923	1.000	0.667
3	1	0.923	1.000	1.000
4	1	0.923	1.000	1.000
5	1	0.923	1.000	1.000
6	1	0.871	0.903	1.000
7	1	0.833	0.833	1.000
8	1	0.805	0.780	1.000
9	1	0.783	0.739	1.000
10	1	0.765	0.706	1.000

**Table 1.** DEA efficiency scores  $E_{DEA,U_j}$  for DMUs  $U_1, U_2, U_3, U_4$  for different values of the second output of  $U_4$ , a.



**Fig. 3.** Efficient frontier of the DEA problem for DMUs  $U_1, U_2, U_3, U_4$  in the case  $U_4 = (-1, 2)$ . The cartesian axes are associated to the normalized output values  $\frac{y_{1j}}{x_j}, \frac{y_{2j}}{x_j}$ .



**Fig. 4.** Efficient frontier of the DEA problem for DMUs  $U_1, U_2, U_3, U_4$  in the case  $U_4 = (-1, 3)$ . The cartesian axes are associated to the normalized output values  $\frac{y_{1j}}{x_j}, \frac{y_{2j}}{x_j}$ .

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**Fig. 5.** Efficient frontier of the DEA problem for DMUs  $U_1, U_2, U_3, U_4$  in the case  $U_4 = (-1, 4)$ . The cartesian axes are associated to the normalized output values  $\frac{y_{1j}}{x_i}, \frac{y_{2j}}{x_i}$ .

As can be seen from table 1 and figures 3 and 4, in the cases with  $a \leq 3$  the inclusion in the analysis of DMU  $U_4$  does not modify the part of the efficient frontier which envelops  $U_1, U_2, U_3$ : this part is exactly the same as that of the case without  $U_4$ ; this is important because it entails that the efficiency scores of  $U_1, U_2, U_3$  does not change either. For a < 3 (see figure 3)  $U_4$  does not lie on the efficient frontier and therefore it is not efficient, while for  $a = 3 U_4$  reaches the efficient frontier, as shown in figure 4, and therefore it becomes efficient.

In the cases with  $3 < a \leq 5$ , represented in figure 5, the displacement of  $U_4$  upwards does modify the efficient frontier; however this shift does not alter the section of the efficient frontier that determines the efficiency scores of  $U_1, U_2, U_3$ , so that their performance measures do not change. On the other hand, for  $a \geq 3$   $U_4$  lies on the efficient frontiers and hence it is efficient.

Figure 6 shows that for a > 5 the raising of  $U_4$  moves the efficient frontier away from DMUs  $U_2$  and  $U_3$ , causing a worsening of their efficiency scores; this shift makes  $U_3$  become inefficient.

Hence, a sufficiently high value of the second output can compensate for the negative value of the first output, in such a way as to make  $U_4$  become efficient when the value of the second output is high enough.

On the other hand, let us keep the values of both the input the second output constant while decreasing the value of the first (negative) output. In particular, let us analyze the behavior of the efficiency score of DMU  $U_4$  as the value of the

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**Fig. 6.** Efficient frontier of the DEA problem for DMUs  $U_1, U_2, U_3, U_4$  in the case  $U_4 = (-1, 6)$ . The cartesian axes are associated to the normalized output values  $\frac{y_{1j}}{x_j}, \frac{y_{2j}}{x_j}$ .



**Fig. 7.** Distance from the efficient frontier of DMU  $U_4(-k, a)$  for different values of k and a = 2. The cartesian axes are associated to the normalized output values  $\frac{y_{1j}}{x_j}, \frac{y_{2j}}{x_j}$ .

negative output worsens. In such a case, a good performance measure should exhibit a decreasing efficiency score for  $U_4$  as the negative output value worsens. However, this is not what happens with the DEA model.

Actually, let us consider  $U_4(-k, a)$  for  $0 < a \leq 3$  as k > 0 increases; figure 7 shows the situation for k = k' = 1 and k = k'' = 2 and a = 2. The DEA efficiency measure of  $U_4$  coincides with the distance ratio

$$\frac{dist\left(O,U_{4}\right)}{dist\left(O,P_{4}\right)}.$$
(19)

It is easy to see that the Cartesian coordinates of the virtual unit  $P_4$  on the efficient frontier are the following

$$P4\left(-\frac{3k}{a},3\right),\tag{20}$$

so that the DEA efficiency score of  $U_4$  turns out to be constant

$$\frac{dist(O, U_4)}{dist(O, P_4)} = \frac{\sqrt{(-k^2) + a^2}}{\sqrt{\left(-\frac{3k}{a}\right)^2 + 3^2}} = \frac{a}{3},$$
(21)

no matter the value of the first output.

This means that the efficiency measure of  $U_4$  is the same for all values of the negative input, independently of the actual value taken, and thus the value of the second output (and that of the input) is the only thing that matters.

In the context of the measurement of the performance of ethical mutual funds, this fact has an unrealistic consequence, that does not satisfy the usual economic assumptions on the investors preferences. Actually, if the first output represents the average rate of return of the mutual fund and the second output is an indicator of its ethical level, this entails that when the average rate of return is negative, its value is indifferent for investors, wether it is only slightly less than zero or imply a heavy loss: in this case only the ethical level would be relevant. This is clearly in contrast with the economic principle that, all other things equal, a higher expected value is always preferred.

### 4 How to have a positive return indicator

We have seen in section 2 that in the evaluation of mutual fund performance we do encounter the problem of the occurrence of negative values of the output which represents the return indicator.

Actually, if we use in the analysis as return indicator the average excess return observed in the period considered, its value is negative for all funds which obtain a rate of return lower than the riskless interest rate. On the other hand, if we use as return indicator in the DEA analysis the average rate of return, this often turns out to be negative for many mutual funds in the slump periods of the business cycle. In order to solve the problem, we might change the definition of the return indicator in such a way as it is always positive under all circumstances and thus it can be directly used as an output variable in a DEA model. To this purpose, it would be sufficient to use a suitable DEA model which is translation invariant.

A model is said translation invariant if the optimal value of the objective function, which represents the DEA efficiency measure, is invariant for translations of the original input and output values which are the consequence of an addition of a constant to the original data.

A DEA model which has such a property is the additive model (on additive DEA models see e.g. [6], section 4.3), and actually this model is often used in order to tackle the problem of negative data in DEA analysis. In particular, it can be proved (see [7] and [8]) that the additive model is indeed translation invariant, while the basic CCR DEA model is not.

However, an additive DEA model discriminates between efficient and inefficient DMUs, but it cannot gauge the depth of eventual inefficiencies: indeed, the efficiency measure given by an additive model does not provide a scalar efficiency measure such as that given by the basic CCR model. Also another approach, proposed in [10], treats the problem of negative data in DEA models by modifying the efficiency measure used, but neither this approach is directly connected to radial efficiency.

On the other hand, we could exploit the financial meaning of the variable involved in order to choose a different return indicator as output variable which is financially significant and, at the same time, guarantees non negativity in all circumstances.

Such an output variable can be found in the capitalization factor  $\overline{U}_j = 1 + \overline{R}_j$ , which gives the final value of a unit initial investment at the end of a unit period. This quantity cannot become negative since in the worst case we may at most lose all the capital invested in a mutual fund.

#### 5 An ethical measure for mutual funds

In order to evaluate the performance of ethical mutual funds we need to build an ethical measure which can be used as an output variable to be taken into account together with the return indicator.

On the other hand, various consultant agencies and research institutes analyze the ethical nature of mutual funds. For example, in the 'SRI Fund Service' The European Social Investment Forum (EUROSIF) together with Avanzi rating agency and Morningstar, give some basic information regarding the socially responsible profile of European ethical mutual funds. Such information is organized in various sections; in particular, the funds are analyzed on the basis of the most important questions taken into consideration in order to define negative and positive ethical screening.

Actually, one of the most important strategies applied by socially responsible mutual funds is ethical screening. According to such a strategy, the assets included in the mutual fund portfolios are selected on the basis of social and

environmental grounds. The selection can be carried out either with a negative screening, by excluding from the portfolios the assets of the companies with a profile that is bad for socially responsible criteria, or with a positive screening, by including in the portfolio investments in companies which are selected on the ground of their ethically and socially behaviour.

The most important information on the ethical screening used by the SRI fund Service takes into consideration a set of features which can be either present or absent in the ethical profile of each fund:

- a. Negative screening issues: 1. firearms; 2. weapons and military contracting; 3. nuclear energy; 4. tobacco; 5. gambling; 6. human rights and ELO fundamental conventions violations; 7. child labour; 8. oppressive regimes; 9. pornography; 10. alcohol; 11. animal testing; 12. factory farming; 13. furs; 14. excessive environmental impact and natural resources c.; 15. GMO; 16. products dangerous to health/environment; 17. others.
- b. Positive screening issues: 1. products beneficial for the environment and quality of life; 2. customers, product safety, advertisement competition; 3. environmental services and technologies; 4. environmental policies, reports, management systems; 5. environmental performances; 6. employees policies, reports, management systems; 7. employees performances; 8. suppliers and measures to avoid human rights violations; 9. communities and bribery; 10. corporate governance; 11. others.

Another important information on the ethical behaviour of mutual funds is the presence or absence of an ethical committee which has the function of defining the guidelines of the socially responsible investments and controlling the actions of the fund management in this respect.

We have used such information in order to define an ethical measure by assigning each ethical feature a weight and then computing their weighted sum.

More precisely, let  $n^N$  and  $n^P$  be the number of negative and positive screening issues taken into account, respectively, and  $n_j^N$  and  $n_j^P$  be the number of negative and positive screening features presented by fund j. Then

$$N_j = \frac{n_j^N}{n^N} \quad \text{and} \quad P_j = \frac{n_j^P}{n^P} \tag{22}$$

represent the quota of the positive and negative screening issues which are present in the ethical profile of fund j, respectively. Moreover, let

$$C_{j} = \begin{cases} 1 & \text{if fund } j \text{ has an ethical committee with full powers} \\ 1/2 & \text{if fund } j \text{ has an ethical committee with partial powers} \\ 0 & \text{if fund } j \text{ does not have an ethical committee.} \end{cases}$$
(23)

An ethical measure defined in the real interval [0, L] can be computed as follows:

$$e_j = \omega^N N_j + \omega^P P_j + \omega^C C_j \tag{24}$$

where  $\omega^N$ ,  $\omega^P$  and  $\omega^C$  are positive weights assigned to the negative and positive screening and to the ethical committee, respectively, and  $L = \omega^N + \omega^P + \omega^C$ .

By construction, fund j has a zero ethical measure if and only if it has no ethical profile, so that  $e_j = 0$  for non ethical funds.

# 6 A DEA model for the performance evaluation of ethical funds with non negative outputs and an exogenously fixed ethical level

In the previous section we have defined a real measure of the ethical level for mutual funds. With regard to this, it is important to observe that the ethical level is usually chosen by investors a priori and cannot be arbitrarily worsened. Therefore the output  $e_j$  has to be considered as exogenously fixed, beyond the discretionary control of managers of fund j.

In a DEA model, it is known that the solution provides a virtual unit which represents an efficient benchmark for an inefficient unit (see e.g. [6]). In order to guarantee that the ethical level of this benchmark is not lower than the actual ethical level of the original fund, in [5] Basso and Funari propose an exogenously fixed output DEA model which exploits Banker and Morey's suggestion (see [2]) to keep the level of the exogenously fixed variables constant at their current value.

To this aim, we may explicitly impose in the DEA optimization model the constraint that the ethical level of a virtual unit is not lower than the ethical level of the fund under evaluation.

In particular, we may conveniently modify the exogenously fixed output model suggested by [5] in order to cope with the problem of negative rates of return, by using the capitalization factor suggested in section 4 instead of the usual return indicator.

In detail, we include the amount of initial investment, set equal to  $C_0 = 1$  for all funds, among the model inputs and the final value of the investment in fund j,  $\overline{U}_j$ , among the outputs. The ethical level  $e_j$  is considered as an exogenously fixed output variable, while the model includes the standard deviation of the rate of return  $\sigma_j$  among the inputs, as a risk indicator.

In addition, the model can take into account among the inputs also the initial and exit fees  $f_j^I$  and  $f_j^E$  usually required by an investment in mutual funds.

The DEA model that we propose in order to evaluate the performance of ethical mutual funds can be written as follows:

$$\max_{\{u_r, v_i\}} \quad \frac{u_1 \overline{U}_{j_0}}{v_1 C_0 + v_2 \sigma_{j_0} + v_3 f_{j_0}^I + v_4 f_{j_0}^E - u_2 e_{j_0}} \tag{25}$$

subject to

$$\frac{u_1 \overline{U}_j}{v_1 C_0 + v_2 \sigma_j + v_3 f_j^I + v_4 f_j^E - u_2 e_j} \le 1 \qquad \qquad j = 1, 2, \dots, n$$
 (26)

$$u_1 \ge \varepsilon, \quad u_2 \ge 0 \tag{27}$$

$$v_i \ge 0$$
  $i = 1, 2, 3, 4.$  (28)

The DEA performance measure for fund  $j_0$ ,  $I_{j_0,Ethic}$ , is the optimal value of the objective function (25).

The solution of the DEA fractional programming problem (25)-(28) can be more conveniently computed by solving the following equivalent linear programming problem which can be found as the dual of the output-oriented linear problem equivalent to the original fractional problem:

$$\max \quad z_0 + \varepsilon \sum_{i=1}^4 s_i^- + \varepsilon s_1^+ \tag{29}$$

subject to

$$\sum_{j=1}^{n} C_0 \lambda_j + s_1^- = C_0 \tag{30}$$

$$\sum_{j=1}^{n} \sigma_j \lambda_j + s_2^- = \sigma_{j_0} \tag{31}$$

$$\sum_{j=1}^{n} f_j^I \lambda_j + s_3^- = f_{j_0}^I \tag{32}$$

$$\sum_{j=1}^{n} f_j^E \lambda_j + s_4^- = f_{j_0}^E \tag{33}$$

$$z_0 \overline{U}_{j_0} - \sum_{j=1}^n \overline{U}_j \lambda_j + s_1^+ = 0$$
(34)

$$-\sum_{j=1}^{n} e_j \lambda_j + s_2^+ = -e_{j_0} \tag{35}$$

$$\lambda_j \ge 0 \qquad \qquad j = 1, 2, \dots, n \tag{36}$$

$$i_{i} \geq 0$$
  $i = 1, 2, 3, 4$  (37)

$$s_r^+ \ge 0$$
  $r = 1, 2,$  (38)

where  $z_0$  is the dual variable associated with the equality constraint,  $\lambda_j$  are the dual variables associated with the mutual funds constraints and  $s_i^-$  and  $s_r^+$  are the dual variables connected with the input and output weight constraints, respectively.

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A different optimization problem of kind (29)–(38) has to be solved for each fund  $j_0 = 1, 2, ..., n$ , in turn, and the DEA performance measure  $I_{j_0,Ethic}$  can be computed as the inverse of the optimal value of  $z_0$ .

# 7 An empirical application to the UK market of ethical mutual funds

We have applied the DEA model for the evaluation of the performance of ethical mutual funds with non negative outputs and an exogenously fixed ethical level

Fund name	Initial	Std.	Initial	Exit	Final	Ethic.	DEA
	capital	Dev.	fee	fee	value	level	score
Aberdeen Ethical World Fund A Acc	100	15.34	4.25	0.00	97.83	2.09	0.9026
AEGON Ethical Fund A Acc	100	17.03	5.50	0.00	100.35	2.76	0.9173
Allchurches Amity Fund A inc	100	14.92	5.00	0.00	99.44	4.05	0.9333
AXA Ethical Fund A Acc	100	19.14	5.00	0.00	95.95	2.03	0.8702
AXA Ethical Fund B Acc	100	19.14	5.00	0.00	95.44	2.03	0.8656
Banner Real Life Fund A Acc	100	16.12	5.50	0.00	97.94	2.64	0.8936
CIS Sustainable Leaders Trust Inc	100	14.54	5.00	0.00	96.53	3.50	0.8924
Credit Suisse Fellowship Fund R Inc	100	16.03	5.25	0.00	97.12	3.63	0.8990
F&C Stewardship Growth Fund Share	100	15.21	5.00	0.00	101.18	3.98	0.9414
F&C Stewardship Income Fund Share	100	10.94	5.00	0.00	107.48	3.98	1.0000
F&C Stewardship Intern. Fund Share	100	15.31	5.00	0.00	94.40	3.98	0.8783
Family Charities Ethical Trust Acc	100	15.12	5.00	0.00	99.01	1.18	0.8882
Framlington Health Fund Acc	100	21.02	5.50	0.00	89.96	1.45	0.8071
Halifax Ethical Fund A Inc	100	15.98	4.00	0.00	94.56	3.13	0.8895
Henderson Ethical Fund A Inc	100	16.67	5.00	0.00	90.12	4.22	0.8660
Henderson Global Care Growth	100	16.89	4.50	0.00	90.55	4.70	0.9328
Henderson Global Care Income	100	13.92	4.50	0.00	97.07	4.70	1.0000
Insight Investment European Ethical	100	14.39	5.25	0.00	100.22	3.19	0.9218
Insight Investment Evergreen Fund	100	15.12	5.25	0.00	96.02	2.89	0.8793
Jupiter Ecology Fund Inc	100	19.10	5.00	0.00	92.83	4.70	0.9563
Jupiter Environmental Opportunities	100	16.33	5.25	0.00	100.53	4.35	0.9837
Legal & General Ethical Trust Acc	100	18.03	0.00	0.00	97.97	2.03	1.0000
Morley Sust. Future Absolute Growth	100	19.73	4.00	0.00	94.41	3.62	0.8966
Morley Sust. Future European Growth	100	15.89	4.00	0.00	98.16	3.62	0.9322
Morley Sust. Future Global Growth	100	15.47	4.00	0.00	89.26	3.62	0.8477
Morley Sustainable Future UK Growth	100	16.52	4.00	0.00	97.73	3.62	0.9282
Old Mutual Ethical Fund A Acc	100	16.30	4.00	0.00	99.15	2.95	0.9304
Prudential Ethical Trust Acc	100	16.09	4.75	0.00	99.35	1.29	0.8974
Scottish Widows Environmental Invest.	100	17.68	5.00	0.00	89.18	2.85	0.8175
Scottish Widows Ethical Fund Acc	100	13.76	5.00	0.00	95.97	2.24	0.8728
Sovereign Ethical Fund Inc	100	16.74	5.50	0.00	96.44	2.41	0.8771
Standard Life UK Ethical Fund R Acc	100	15.08	4.00	0.00	100.24	3.92	0.9809
AEGON UK Equity Fund A Acc	100	14.07	5.50	0.00	101.26	0.00	0.9165
Allchurches UK Equity Growth Fund	100	14.16	5.00	0.00	105.47	0.00	0.9539
AXA UK Equity Income A Acc	100	15.36	5.00	0.00	106.32	0.00	0.9517
CIS UK Growth Trust Inc	100	13.86	5.00	0.00	101.38	0.00	0.9193
Credit Suisse UK Mid 250 Fund R Inc	100	16.94	5.25	0.00	105.39	0.00	0.9306
F&C UK Growth & Income Fund Share	100	14.57	5.00	0.00	102.59	0.00	0.9246
F&C UK Equity Income Share Class 1	100	13.82	5.00	0.00	103.10	0.00	0.9352
F&C UK Global Growth Fund Share	100	15.23	5.00	0.00	94.86	0.00	0.8501
Family Asset Trust Acc	100	14.20	6.00	0.00	102.14	0.00	0.9234
Halifax International Growth Fund	100	16.67	4.00	0.00	98.96	0.00	0.8934
Henderson International Fund A Acc	100	16.95	5.00	0.00	96.66	0.00	0.8542

Table 2. Input and output data and DEA performance measures for the UK ethical and non ethical mutual funds obtained with model (25)–(28).

Fund name	Initial	Std.	Initial	Exit	Final	Ethic.	DEA
	$\operatorname{capital}$	Dev.	fee	fee	value	level	score
Insight Invest.UK Equity Income Port.	100	12.55	5.25	0.00	107.15	0.00	0.9828
Insight Investment Global Equity Fund	100	17.82	5.25	0.00	97.73	0.00	0.8592
Jupiter Global Opportunities Fund Inc	100	14.44	5.25	0.00	108.51	0.00	0.9789
Jupiter UK Smaller Companies Fund	100	17.46	5.25	0.00	113.75	0.00	1.0000
Legal & General Equity Trust (E) Acc	100	14.52	0.00	5.00	100.91	0.00	1.0000
Morley European Focus Fund 1 Acc	100	18.75	5.00	0.00	105.51	0.00	0.9325
Morley Global Theme Fund 1 Acc	100	14.49	5.00	0.00	97.27	0.00	0.8772
Morley UK Focus Fund 1 Acc	100	18.01	5.00	0.00	107.59	0.00	0.9508
Old Mutual UK Equity Acc	100	13.63	3.00	0.00	104.66	0.00	0.9657
Prudential UK Growth Trust Inc	100	14.85	5.50	0.00	99.98	0.00	0.8988
Scottish Widows UK Equity Income Acc	100	12.01	5.00	0.00	105.13	0.00	0.9689
Scottish Widows UK Growth Fund Acc	100	12.53	5.00	0.00	103.86	0.00	0.9528
Sovereign UK Growth Fund Inc	100	15.71	5.50	0.00	102.12	0.00	0.9112
Standard Life UK Equity Growth	100	14.34	4.00	0.00	103.12	0.00	0.9319
FTSE100	100	20.24	0.20	0.20	97.94	0.00	0.9723
Zero coupon bond yield	100	5.54	0.30	0.00	101.93	0.00	1.0000

(25)–(28) to the set of ethical mutual funds of United Kingdom, observed in the three-year period 31/01/2002 to 31/01/2005.

We found 32 UK ethical mutual funds present in the 'SRI Fund Service' data base with complete data for all the period considered. In addition, we have included in the set of funds analyzed also a non ethical fund with analogous features for each ethical fund considered, each time one such non ethical fund was offered by the same fund company. In this way we can compare the performance obtained by ethical and non ethical funds run by the same company. In order to consider also a common benchmark and the comparison with the rate of return of a riskless asset, we have included in the analysis also the the FTSE100 London Stock Exchange index and the yield of the 5 year Real Zero Coupon British Government Securities. On the whole, the set is made up of 59 DMUs.

The rate of returns and the volatilities of the funds have been computed on an annual base; the input and output data are reported in table 2 as per cent values, with an initial invested capital set equal to 100 for all investments. The ethical measure (24) has been computed by using the weights  $\omega^N = \omega^P = 2$ and  $\omega^C = 1$ , thus stressing the screening activity of the ethical funds. The non ethical funds in table 2 are those with a null ethical level.

The last column of table 2 shows the DEA performance measure obtained by all the funds analyzed. By examining the DEA scores we may observe that among the efficient funds we find 3 ethical and 2 non ethical funds; in addition, the riskless asset is efficient, too. If we rank the funds according to their DEA scores, and count the number of ethical and non ethical funds which rank among the first 10, 20 and 30 positions, we find that among the first 10 positions 5 funds are ethical and 4 are non ethical, while in the first 20 positions we find 7 ethical and 11 non ethical funds; in the first 30 positions we find 12 ethical and 16 non ethical funds.

From these results many ethical funds seem to show good results but many others seem to be overcome by non ethical funds.

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