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RESEARCH REPORT

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NORMAL APPROXIMATION OF THE DISTRIBUTION OF THE EQUILIBRIUM PRICE IN MARKETS WITH RANDOM DEMAND AND SUPPLY

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Abstract

In the paper, the asymptotic distribution of the equilibrium price in markets with the random demand and supply is described. Two special cases - the one with smooth demand and supply curves and the one with jump demand and supply curves - are studied. It is found that in both the cases the fluctuations of the price vanish at the rate $O(n^{-1/2})$ as the number of the agents n tends to infinity. Finally, a normal approximation of the distribution of the equilibrium price is suggested.

Keywords: equilibrium price, random demand, random supply, asymptotic distribution, normal approximation

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Introduction

In many real situations, the demand and/or supply curves have to be taken as stochastic. Even if there exist models with stochastic demand and/or supply, I have neot found a work describing the distribution of the equilibrium price.

The reason of this absence could be that the equilibrium price is generally a non-linear transformation of random parameters, hence its distribution may be described only in special cases, and even there the resulting formulas are complex (cf. Šmíd [2004a,b]).

One of the possible ways how to overcome these difficulties is studying the asymptotic properties of the model instead of trying to infer "exact" distributions.

Until now, no work applying this approach is known to me except for my own research report (Šmíd [2004b]) dealing with the special case of linear demand and supply curves. The present paper generalizes the results of the report.

The model of a market with stochastic demand nad supply, defined in the present paper, assumes a finite number number of market participants and the individual demand and supply curves are taken as identically distributed independent random elements taking values in function spaces. In accordance with the standard economic theory, the equilibrium price is assumed to lie in the intersection of the aggregate demand and supply curves.

Two special cases of the general model are studied in the paper: the case of continuous twice differentiable demand and supply curves (which may be suitable for modeling markets with a divisible good) and the case of jump demand and supply curves taking integer values (which could be suitable for markets with an indivisible commodity). In both the cases, the asymptotic behavior of the equilibrium price, as the number of participants tends to infinity, is described.

In both the "continuous" and "jump" cases, the distribution of the price P_n converges to a constant, namely to the intersection of the expected demand curve and the expected supply curve, and the variance of P_n vanishes at the rate $O(n^{-1/2})$ where n is the number of participants.

The asymptotic distribution of $\sqrt{n}(P_n - p^\star)$, where $p^\star$ is the limit price, depends directly only on the derivatives of the expected aggregate demand and supply curves in the point $p^\star$, and on the variance of the curves in $p^\star$. Rather surprisingly, the asymptotic distribution is identical in both the "continuous" and "smooth" cases.

As a main result of the paper, a normal approximation of the equilibrium price distribution is suggested.

The paper is organized as follows. In Section 1, the general model is defined. In Section 2, the asymptotic behavior of the price in the "continuous" case is analyzed. In section 3, the same is done for the "jump" case. In Section 4, the normal approximation is suggested.

1 Definition of the Model

Assume a market peopled by n agents with the individual demand functions d_i, $i = 1, 2, \ldots, n$, and with individual supply functions d_i, $i = 1, 2, \ldots, n$, such that d_i is a random element taking values in the space of non-negative non-increasing real functions, and s_i is a random element taking values in the space of non-negative non-decreasing real functions for each $i = 1, 2, \ldots, n$. Let (d_i, s_i), $i = 1, 2, \ldots, n$, be identically distributed and let (d_i, s_i) be independent of (d_j, s_j) for each $i \neq j$. Moreover, let the expectations

$$\delta(p) \triangleq Ed_1(p),$$

and

$$\sigma(p) \triangleq Es_1(p)$$

exist and be finite for each $p \in \mathrm{R}$, and let there exist a unique $p^\star \in \mathrm{R}$ such that $\delta(p^\star) = \sigma(p^\star)$.

Further, assume that the equilibrium price P_n "happens" in some point of the "intersection" of the aggregate demand curve $D_n = \sum_{i=1}^n d_i$ with the aggregate supply curve $S_n = \sum_{i=1}^{n} s_i$, i.e. it fulfils the condition

$$P_n^L \leq P_n \leq P_n^H \tag{1}$$

where

$$P_n^L = \inf\{p : D_n(p) \leq S_n(p)\},$$

and

$$P_n^H = \sup\{p : D_n(p) \geq S_n(p)\}.$$

2 The Case of Continuous Demand and Supply Functions

Assume, in the present section, that each realization of d_i, and each realization of s_i are twice differentiable for each $i = 1, 2, \ldots, n$.

Theorem 1 *If $E|d_1(p^\star)| < \infty$, $E|s_1(p^\star)| < \infty$, $0 < E[s_1'(p^\star) - d_1'(p^\star)] < \infty$, $E|d_1''(p^\star)| < \infty$, and $E|s_1''(p^\star)| < \infty$ then*

$$\sqrt{n}(P_n - p^\star) \overset{n\to\infty}{\longrightarrow} \mathcal{N}(0, V) \tag{2}$$

in distribution where

$$V = \frac{\operatorname{var}(s_1(p^\star) - d_1(p^\star))}{[E(s_1'(p^\star) - d_1'(p^\star))]^2}$$

independently on the choice of P_n.

If, moreover, there exists an open interval I, such that $p^\star \in I$, and a random variable g, $Eg < \infty$, such that

$$s_1'(p) - d_1'(p) \leq g \tag{3}$$

for each $p \in I$ almost sure then

$$V = \frac{\operatorname{var}(s_1(p^\star) - d_1(p^\star))}{[\sigma'(p^\star) - \delta'(p^\star)]^2}. \tag{4}$$

Proof. The proof is given in the Appendix A. □

3 The Case of Jump Demand and Supply Functions

Assume, in this section, that each realization of d_i, $i = 1, 2, \ldots, n$, is left-continuous with integer values, and that each realization of s_i, $i = 1, 2, \ldots, n$, is right-continuous with integer values. Further, assume that there exist twice differentiable functions $\hat{\delta}$ and $\hat{\sigma}$ such that

(a) $d_1(p) \sim \text{Po}(\hat{\delta}(p))$ for each $p \in \mathrm{R}$,

(b) $d_1(p) - d_1(p') \sim \text{Po}(\hat{\delta}(p) - \hat{\delta}(p'))$ for each $p < p'$,

(c) $s_1(p) \sim \text{Po}(\hat{\sigma}(p))$ for each $p \in \mathrm{R}$,

(d) $s_1(p) - s_1(p') \sim \text{Po}(\hat{\sigma}(p) - \hat{\sigma}(p'))$ for each $p' < p$.

Evidently, $\delta(p) = \hat{\delta}(p)$ and $\sigma(p) = \hat{\sigma}(p)$ for each $p \in \mathrm{R}$ under these assumptions.

Theorem 2 *If* $\sigma'(p^\star) - \delta'(p^\star) > 0$ *then*

$$\sqrt{n}(P_n - p^\star) \stackrel{n\to\infty}{\longrightarrow} \mathcal{N}(0, W) \tag{5}$$

in distribution where

$$W = \frac{\text{var}(s_1(p^\star) - d_1(p^\star))}{[\sigma'(p^\star) - \delta'(p^\star)]^2}.$$

If, moreover d_1 *is independent of* s_1*, it holds that*

$$W = \frac{\sigma(p^\star) - \delta(p^\star)}{[\sigma'(p^\star) - \delta'(p^\star)]^2}. \tag{6}$$

Proof. The proof in given in the Appendix B. □

4 Normal Approximation

Our asymptotic results indicate that, if the assumptions of Theorem 1 (together with (3)) or the assumptions of Theorem 2 are fulfilled, and if n is sufficiently large, we may write

$$P_n \dot{\sim} \mathcal{N}\left(p^\star, \frac{1}{n} \cdot \frac{\text{var}(s_1(p^\star) - d_1(p^\star))}{[\sigma'(p^\star) - \delta'(p^\star)]^2}\right).$$

Rewritten "in the language of aggregate functions", the formula sounds

$$P_n \dot{\sim} \mathcal{N}\left(p^\star, \frac{\text{var}(S_n(p^\star) - D_n(p^\star))}{[\Sigma'_n(p^\star) - \Delta'_n(p^\star)]^2}\right)$$

where $\Delta_n = ED_n$, $\Sigma_n = ES_n$.[1]

[1]Indeed,

$$\text{var}(S_n(p^\star) - D_n(p^\star)) = n\,\text{var}(s_1(p^\star) - d_1(p^\star)),$$

and

$$E(S'_n(p^\star) - D'_n(p^\star)) = n(\sigma'(p^\star) - \delta'(p^\star)).$$

Conclusion

In the paper, the asymptotic behavior of the equilibrium price in markets with random demand and supply, and with the independent behavior of its participants was studied. In particular, markets with smooth demand and supply curves, and markets with jump demand and supply curves were taken into account. As a main result, a normal approximation of the distribution of the equilibrium price, depending only on the aggregate demand and supply curves, was suggested.

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A Proof of Theorem 1

The core of the proof consists in the determination of the asymptotic distribution of the vector

$$\Xi_n \triangleq \sqrt{n}(P_n^H - p^\star, P_n^L - p^\star).$$

It could be shown analogously to e.g. Araujo and Giné [1980], p. 8, that

$$S_{c_1,c_2} = (-\infty, c_1] \times (-\infty, c_2), \qquad c_1 \in \mathrm{R}, c_2 \in \mathrm{R}, \tag{7}$$

is a convergence determining system of sets, i.e. it suffices to study the convergence on the sets (7).

Let us do it: From the continuity of D_n and S_n, it follows that

$$P_n^L \leq p \iff S_n(p) \geq D_n(p)$$

for each $p \in \mathrm{R}$. Using it and the Taylor expansions

$$d_i(p^\star + \Delta) = d_i(p^\star) + d_i'(p^\star)\Delta + d_i''(p^\star)\frac{(\xi_i^\Delta)^2}{2},$$

and

$$s_i(p^\star + \Delta) = s_i(p^\star) + s_i'(p^\star)\Delta + s_i''(p^\star)\frac{(\eta_i^\Delta)^2}{2}$$

where ξ_i^Δ and η_i^Δ are constants lying between 0 and Δ, we get

$$\begin{aligned}
\sqrt{n}(P_n^L - p^\star) \le c_1 &\iff P_n^L \le p^\star + \frac{c_1}{\sqrt{n}} \\
&\iff D_n\left(p^\star + \frac{c_1}{\sqrt{n}}\right) - S_n\left(p^\star + \frac{c_1}{\sqrt{n}}\right) \le 0 \\
&\iff \frac{1}{\sqrt{n}}\left[D_n\left(p^\star + \frac{c_1}{\sqrt{n}}\right) - S_n\left(p^\star + \frac{c_1}{\sqrt{n}}\right)\right] \le 0 \qquad (8) \\
&\iff A_n - c_1 B_n + R_n^{c_1} \le 0
\end{aligned}$$

where

$$A_n = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}[d_i(p^\star) - s_i(p^\star)], \qquad B_n = \frac{1}{n}\sum_{i=1}^{n}[s_i'(p^\star) - d_i'(p^\star)],$$

and

$$R_n^c \;=\; \frac{1}{2\sqrt{n}}\left[\sum_{i=1}^{n} d_i''(p^\star)(\xi_i^{c/\sqrt{n}})^2 - \sum_{i=1}^{n} s_i''(p^\star)(\eta_i^{c/\sqrt{n}})^2\right].$$

Since

$$P_n^H < p \iff S_n(p) > D_n(p)$$

we could prove, similarly to the case of P_n^L, that

$$\sqrt{n}(P_n^H - p^\star) < c_2 \iff A_n - c_2 B_n + R_n^{c_2} < 0. \qquad (9)$$

Further, it follows from the Central Limit Theorem for the sums of i.i.d. variables that

$$A_n \overset{n\to\infty}{\longrightarrow} U, \qquad (10)$$

where $U \sim \mathcal{N}(0, \mathrm{var}(d_1(p^\star) - s_1(p^\star)))$, in distribution. Further, due to the Law of Large Numbers,

$$B_n \overset{n\to\infty}{\longrightarrow} E(s_1'(p^\star) - d_1'(p^\star)) \qquad (11)$$

in probability, hence in distribution. Finally, since

$$|R_n^c| \;\le\; \frac{1}{\sqrt{n}}\left(\frac{1}{n}\sum_{i=1}^{n}\left[\frac{c^2|d_i''(p^\star)|}{2} - \frac{c^2|s_i''(p^\star)|}{2}\right]\right),$$

we may use the Law of Large Numbers to get that

$$R_n^c \overset{n\to}{\longrightarrow} 0 \qquad (12)$$

in distribution.[2] By a combination of (8), (9), (10), (11) and (12) and by using the Continuous Mapping Theorem (cf. Pollard [2002], p.175), we get that

$$\begin{aligned}
&\lim_{n\to\infty} \mathcal{P}\left\{\Xi_n \in (-\infty, c_1] \times (-\infty, c_2)\right\} \\
&\quad = \mathcal{P}\left\{U - c_1 E[s_1'(p^\star) - d_1'(p^\star)] \le 0, U - c_2 E[s_1'(p^\star) - d_1'(p^\star)] < 0\right\} \\
&\quad = \mathcal{P}\left\{\frac{U}{E[s_1'(p^\star) - d_1'(p^\star)]} \le c_1, \frac{U}{E[s_1'(p^\star) - d_1'(p^\star)]} < c_2\right\} \\
&\quad = \mathcal{P}\left\{\frac{U}{E[s_1'(p^\star) - d_1'(p^\star)]} \le c_1, \frac{U}{E[s_1'(p^\star) - d_1'(p^\star)]} \le c_2\right\},
\end{aligned} \tag{13}$$

i.e.

$$\Xi_n \overset{n\to\infty}{\longrightarrow} \frac{1}{E[s_1'(p^\star) - d_1'(p^\star)]}(U, U)$$

in distribution.

Now that we have the asymptotic distribution of Ξ_n, it is easy to prove the rest. It follows from (13) that

$$\sqrt{n}(P_n^L - p^\star) \overset{n\to\infty}{\longrightarrow} \mathcal{N}(0, V)\,. \tag{14}$$

in distribution, and, by the Continuous Mapping Theorem, that

$$\begin{aligned}
\left|\sqrt{n}(P_n - P_n^L)\right| &\overset{(1)}{\le} \left|\sqrt{n}(P_n^H - P_n^L)\right| = \left|\sqrt{n}(P_n^H - p^\star) - \sqrt{n}(P_n^L - p^\star)\right| \\
&\overset{n\to\infty}{\longrightarrow} \frac{1}{E[s_1'(p^\star) - d_1'(p^\star)]}|U - U| = 0
\end{aligned}$$

in distribution, which, together with (14), gives

$$\sqrt{n}(P_n - p^\star) = \sqrt{n}(P_n - P_n^L) + \sqrt{n}(P_n^L - p^\star) \overset{n\to\infty}{\longrightarrow} \mathcal{N}(0, V)$$

by the Continuous Mapping Theorem.

Ad (4): The formula follows from the well known theorem on the exchangeability of an integral with a derivative (see e.g. Lukeš and Malý [1995], par. 9.2.).

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[2]Indeed, if X_1, X_2 are i.i.d. such that $E|X_1| < \infty$ then

$$\begin{aligned}
\mathcal{P}\left\{n^{-1/2}|\bar{X}_n| \ge \epsilon\right\} &\le \mathcal{P}\left\{n^{-1/2}(|\bar{X}_n - EX_1| + |EX_1|) \ge \epsilon\right\} \\
&\le \mathcal{P}\left\{|\bar{X}_n - EX_1| \ge \epsilon/2\right\} + \mathcal{P}\left\{n^{-1/2}|EX_1| \ge \epsilon/2\right\} \overset{n\to\infty}{\longrightarrow} 0
\end{aligned}$$

from the Law of Large Numbers.

B Proof of Theorem 2

It follows from the definition of P_n^L that

$$P_n^L \le p \iff S_n(p^+) \ge D_n(p^+).$$

which is equivalent to

$$P_n^L \le p \iff S_n(p) \ge D_n(p^+).$$

due to the right-continuity of S_n. Moreover, since

$$\mathcal{P}\left\{D_n(p) - D_n(p^+) > 0\right\} \overset{\text{(b)}}{=} 0, \tag{15}$$

it holds that

$$\begin{aligned}
\mathcal{P}\left\{P_n^L \le p\right\} &= \mathcal{P}\left\{S_n(p) \ge D_n(p^+)\right\} \\
=&\mathcal{P}\left\{S_n(p) \ge D_n(p)\right\} - \mathcal{P}\left\{[S_n(p) \ge D_n(p^+)] \wedge [D_n(p) - D_n(p^+) > 0]\right\} \\
\overset{(15)}{=}&\mathcal{P}\left\{S_n(p) \ge D_n(p)\right\}
\end{aligned}$$

i.e.

$$P_n^L \le p \iff S_n(p) \ge D_n(p) \quad \text{almost sure.} \tag{16}$$

Similarly we get that

$$P_n^H \ge p \iff S_n(p) \le D_n(p) \quad \text{almost sure}$$

which is equivalent to

$$P_n^H < p \iff S_n(p) > D_n(p) \quad \text{almost sure.} \tag{17}$$

Using it, we are getting

$$\begin{aligned}
&\mathcal{P}\left\{\sqrt{n}(P_n^L - p^\star) \le c_1, \sqrt{n}(P_n^H - p^\star) < c_2\right\} \\
&\quad=\mathcal{P}\left\{P_n^L \le p^\star + \frac{c_1}{\sqrt{n}}, P_n^H < p^\star + \frac{c_2}{\sqrt{n}}\right\} \\
&\quad\overset{(16)(17)}{=}\mathcal{P}\left\{\frac{1}{\sqrt{n}}\left[D_n\left(p^\star + \frac{c_1}{\sqrt{n}}\right) - S_n\left(p^\star + \frac{c_1}{\sqrt{n}}\right)\right] \le 0,\right. \\
&\qquad\qquad\left.\frac{1}{\sqrt{n}}\left[D_n\left(p^\star + \frac{c_2}{\sqrt{n}}\right) - S_n\left(p^\star + \frac{c_1}{\sqrt{n}}\right)\right] < 0\right\}.
\end{aligned} \tag{18}$$

Let us expres the aggregate curves similarly to the proof of Theorem 1: Thanks to the the differentiability of $\delta(\bullet)$, we may expand

$$\delta(p^\star + \Delta) = \delta(p^\star) + \delta'(p^\star)\Delta + \delta''(p^\star)\frac{(\xi^\Delta)^2}{2}$$

where ξ^{Δ} lies somewhere between 0 and Δ. Hence, and thanks to the divisibility of the Poisson distribution, we may write

$$d_i\left(p^\star + \frac{c}{\sqrt{n}}\right) = a_i + b_i^c + r_i^c \tag{19}$$

where

$$a_i \sim \text{Po}(\delta(p^\star)), \quad b_i^c \sim \text{Po}\left(\delta'(p^\star)\frac{c}{\sqrt{n}}\right), \quad r_i^c \sim \text{Po}\left(\delta''(p^\star)\frac{(\xi^{c/\sqrt{n}})^2}{2}\right).$$

Similarly, we can write

$$s_i\left(p^\star + \frac{c}{\sqrt{n}}\right) = \alpha_i + \beta_i^c + \rho_i^c \tag{20}$$

where

$$\alpha_i \sim \text{Po}(\sigma(p^\star)), \quad \beta_i^c \sim \text{Po}\left(\sigma'(p^\star)\frac{c}{\sqrt{n}}\right), \quad \rho_i^c \sim \text{Po}\left(\sigma''(p^\star)\frac{(\eta^{c/\sqrt{n}})^2}{2}\right)$$

for some $\eta^{c/\sqrt{n}}$ lying between 0 and $c/\sqrt{n}$.

By summing the individual curves, we obtain

$$\frac{1}{\sqrt{n}}\left(D_n\left(p^\star + \frac{c}{\sqrt{n}}\right) - S_n\left(p^\star + \frac{c_1}{\sqrt{n}}\right)\right) = A_n - B_n^c + R_n^c \tag{21}$$

where

$$A_n = \frac{1}{\sqrt{n}}\sum_{i=1}^n [a_i - \alpha_i], \quad B_n^c = \frac{1}{\sqrt{n}}\sum_{i=1}^n [\beta_i^c - b_i^c], \quad R_n^c = \frac{1}{\sqrt{n}}\sum_{i=1}^n [r_i^c - \rho_i^c].$$

Therefore and due to (18),

$$\begin{aligned}\mathcal{P}&\left\{\sqrt{n}(P_n^L - p^\star) \le c_1, \sqrt{n}(P_n^H - p^\star) < c_2\right\} \\ &= \mathcal{P}\left\{A_n - B_n^{c_1} + R_n^{c_1} \le 0, A_n - B_n^{c_2} + R_n^{c_2} < 0\right\}.\end{aligned}$$

Finally, since

$$A_n \stackrel{n\to\infty}{\longrightarrow} \mathcal{N}\left(0, \text{var}(d_1(p^\star) - s_1(p^\star))\right),$$

in distribution,

$$B_n \stackrel{n\to\infty}{\longrightarrow} c(\sigma'(p^\star) - \delta'(p^\star))$$

in distribution[3] for each $c \in \mathrm{R}$, and since

$$R_n^c \stackrel{n\to\infty}{\longrightarrow} 0$$

in distribution[4] for each $c \in \mathrm{R}$, we may use the Continuous Mapping Theorem to get that

$$A_n - B_n^c + R_n^c \stackrel{n\to\infty}{\longrightarrow} \mathcal{N}\left(0, \operatorname{var}(d_1(p^\star) - s_1(p^\star))\right) - c(\sigma'(p^\star) - \delta'(p^\star))$$

in distribution for each $c \in \mathrm{R}$. The rest of the proof of (5) is identical to the proof of Theorem 1.

Ad. (6). The formula follows from basic rules for computing variances.

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J. Lukeš and J. Malý. *Measure and Integral.* Matfyzpress, Prague, 1995.

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[3]Indeed, if $X_1, X_2, \ldots, X_n$ are i.i.d. such that $X_i \sim \mathrm{Po}(d/\sqrt{n})$ for each $i = 1, 2, \ldots, n$ then there exist independent random variables $Y_1, Y_2, \ldots, Y_n, Z_n$ such that $Y_i \sim \mathrm{Po}(d)$ and $Z_n \sim \mathrm{Po}(\gamma_n)$, $\gamma_n < d$, and that

$$\sum_{i=1}^{n} X_i = \sum_{i=1}^{\lfloor\sqrt{n}\rfloor} Y_i + Z_n.$$

Consequently,

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i = \frac{1}{\lfloor\sqrt{n}\rfloor} \sum_{i=1}^{\lfloor\sqrt{n}\rfloor} Y_i + \left(\frac{1}{\sqrt{n}} - \frac{1}{\lfloor\sqrt{n}\rfloor}\right) \sum_{i=1}^{\lfloor\sqrt{n}\rfloor} Y_i + \frac{Z_n}{\sqrt{n}}.$$

Since the limit of the first summand is d due to the Law of Large Numbers, and the variance of the second and the third summand goes to zero, the limit of the whole sum is d.

[4]The convergence follows from the fact that

$$\operatorname{var}(R_n^c) = \operatorname{var}(r_i^c - \rho_i^c) \stackrel{\text{Schwarz}}{\leq} \left(\sqrt{\operatorname{var}(r_i^c)} + \sqrt{\operatorname{var}(\rho_i^c)}\right)^2$$

$$= \left(\xi^{c/\sqrt{n}} \sqrt{\frac{\delta''(p^\star)}{2}} + \eta^{c/\sqrt{n}} \sqrt{\frac{\sigma''(p^\star)}{2}}\right)^2 \leq \frac{c^2}{n} \left(\sqrt{\frac{\delta''(p^\star)}{2}} + \sqrt{\frac{\sigma''(p^\star)}{2}}\right)^2 \to 0$$

for each $c \in \mathrm{R}$.

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