Comparison of Mean-Variance theory and Expected-Utility theory through a Laboratory Experiment

Andrea Morone[1]

University of Bari

e-mail: a.morone@dse.uniba.it

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Abstract

In the 40's and early 50' two decision theories were proposed and have since dominated the scene of the fascinating field of decision-making. In 1944 – when von Neumann and Morgenstern showed that if preferences are consistent with a set of axioms then it is possible to represent these preference by the expectation of some utility function – Expected Utility theory provide a natural way to establish "measurable utility". In the early 50's Markowitz introduced the Mean-Variance theory that is the basis of modern portfolio selection theory. Even if both models were analyzed from virtually all possible point of view; although they were tested against several generalizations; even though they seams to be the most attractive theories of decision making, they were never testes gains each other. This paper will try to fill this gap. It investigates, using experimental data, which of these two models represent a better approximation of subjects' preferences.

Keywords: Expected utility, Mean variance, preference functional, pair wise choice, experiments.

[1] Dipartimento di Scienze Economiche, Università degli Studi di Bari, Via Camillo Rosalba 53, 70124 BARI, ITALY

1. Introduction

This essay is motivated by simple questions: why in two different branches of Economics do two different preference functionals dominate the scene? Does Expected Utility perform significantly better than mean variance? Do we have a loss of accuracy if we use Mean Variance instead of Expected Utility?

Expected Utility lead the filed of decision making in Economics because, since 1944 – when von Neumann and Morgenstern showed that if preferences are consistent with a set of axioms then it is possible to represent these preferences by the expectation of some utility function – Expected Utility provides a natural way to establish "measurable utility": it is a simple and elegant way to derive utility cardinality.

Mean Variance lead the filed of decision making in Financial Economics. It was developed in 50's and 60's by Markowitz, Tobin, Sharpe and Lintner among others. It is an important model of investment based on decision theory. Actually it is the simplest model of investment that is sufficiently rich to be directly useful in applied problems. And probably, more important, it does not need any assumptions on subjects utility function.

It is clear that both models have nice a desirable properties. It is, also, rather obvious that Expected Utility should perform better than Mean Variance. Indeed it is a more general model. And finally we should expect it is clear that using Mean Variance instead of Expected Utility we have to accept a loss in accuracy. But what is rather striking is that neither the presumed superiority of Expected Utility not the accuracy loss of Mean Variance has been systematically investigated. The aim of the present paper is to fill this gap. In a certain sense we are addressing three trivial questions, which until now have no answers.

In section 2 we briefly describe the data, which we used to estimate the three preference functionals. Section 3 illustrates the features of the preference functionals analysed. Section 4 discusses the estimation methods. Finally results are presented and discussed in section 5.

2. The data

Much effort has been expended to produce a better theory of decision making under risk than that provided by EU. Therefore, there is now an abundant literature that compares EU with a number of its generalizations (e.g. Harless and Camerer (1994), Hey and Orme (1994)). It seems fairly natural to follow their approach to compare MV and EU. Thus we need a set of pair wise choice questions. Each pair wise choice is composed by two lotteries, labeled "Left Gamble" and "Right Gamble". Each subject has to report his preference between the two lotteries. The incentive mechanism is that the preferred lottery is played for real.

The enormous activity of this branch of experimental economics make useless to run our own pair wise choice question experiment, since we can address our questions using a data set from a previous experiment[2].

The experiment took place in the EXEC laboratory at the University of York with 53 participants. Each participant had to attend five separate treatments, Set 1, Set 2, Set 3, Set 4 and Set 5. Each of the five treatments was composed by the same 100 pair wise choice questions, but varying chronological order, and also left/right positioning was randomised. The pair wise choice

[2] I am indebted to John Hey for allowing me using these data base.

questions were presented in the form of segmented circles, and subjects were asked to report, for each pair, their preference.

The 100 questions were composed by three of the following four outcomes: -£25, £25, £75, and £125. One of these four outcomes involves a negative pay-off, this would increase the incentive power of the experiment, but because we did not want any subject to experience a real monetary loss, we gave all subjects a participation fee of £25 for attending all the 5 sessions of the experiment.

In table 1 are reported the 100 pair wise choice questions.

3. Estimation procedure and preference functionals

Our estimation procedure is similar to the one used by Hey and Orme (1994) which is motivated by two fundamental observations. First, there is not necessarily one best preference functional for all subjects but the behavior of different subjects may be explained best by different functionals. Second, subjects make from time to time errors in their responses which demand a stochastic specification of preference functionals for our empirical test. To take into account the first observation we have estimated the models subject by subject. To take into account the second observation we have added an error term to each preference functional. We assume that errors are identically and independently distributed among subjects and questions.

In our analysis we will consider three preference functionals:

- Risk Neutral (RN)[3];
- Mean Variance (MV);
- Expected Utility (EU).

First some notation, let $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$ be the vector of outcomes; $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}$ is the probability vector of the Left Gamble and $\mathbf{q} = \{q_1, q_2, \ldots, q_n\}$ the probability vector of the Right

[3] RN will be a kind of low benchmark

Gamble. W denotes the subject's preference function. Therefore if $W(\mathbf{p}) > W(\mathbf{q})$ Left will de preferred to Right and if $W(\mathbf{p}) < W(\mathbf{q})$ then Right will be preferred to Left.

Altogether subjects' derived preferences are determined by:

$W(\mathbf{p}) - W(\mathbf{q}) + \varepsilon$,

where ε is an error term. We assume that ε is symmetric and has a mean of zero.

The first model we have estimated is RN given by

$$RN: \quad W(\mathbf{p}) - W(\mathbf{q}) + \varepsilon = k\sum_{i=1}^{4} p_i x_i - k\sum_{i=1}^{4} q_i x_i + \varepsilon .$$

For RN we have to estimate only the parameter k which is the relative magnitude of subjects' errors.

Let us now turn to MV where we have

$$MV: \quad W(\mathbf{p}) - W(\mathbf{q}) + \varepsilon = v\sum_{i=1}^{n} p_i x_i + w\sum_{i=1}^{n}\left(p_i x_i - \sum_{i=1}^{n} p_i x_i\right)^2 - v\sum_{i=1}^{n} q_i x_i + w\sum_{i=1}^{n}\left(q_i x_i - \sum_{i=1}^{n} q_i x_i\right)^2 + \varepsilon$$

Concerning MV we have to estimate v and w, which represent, respectively, the weight that each subject gave to the mean of the lottery and to its variance.

$$EU: \quad W(\mathbf{p}) - W(\mathbf{q}) + \varepsilon = \sum_{i=1}^{4} p_i u(x_i) - \sum_{i=1}^{4} q_i u(x_i) + \varepsilon .$$

For EU we estimated $u(x_i)$, we normalised $u(x_1)$ to zero, and the variance of the error term to one. Under this procedure a subject who makes relatively small errors will have relatively large values for $u(x_i)$ whereas a subject who makes relatively large errors will have relatively small values for $u(x_i)$.

4. The estimation results

The question we are trying to address is which, RN MV and EU, of the various preference functionals best explain subjects' behaviour. A very natural way to compare the performances of our three preference functionals is ranking them according to the Aikike Information Criterion (AIC). This is a measure of goodness of fit, which takes into account the model parsimony.

This analysis is reported in table 2. In table 3 it is reported the frequency of ranking first, second or third by the three models according the AIC[4].

	RN			MV			EU		
	1	2	3	1	2	3	1	2	3
Set 1	0	0	53	3	50	0	50	3	0
Set 2	0	0	53	5	48	0	48	5	0
Set 3	0	0	53	6	47	0	47	6	0
Set 4	0	0	53	4	49	0	49	4	0
Set 5	0	1	52	4	49	0	52	0	1

Table 3

Looking at table 2 and table 3 we have a very clear picture: EU rank first, MV second and RN third. This kind of analysis is essentially statistical; it tells us that using the AIC we have to prefer EU to MV.

This strength of this kind of analyses is that it gives us a complete ranking of the preference functionals, but it does not help to see how one preference functional is better than the other one. To investigate this particular aspect we can analyse the log-likelihood value. This value gives us the probability that a preference functional fit correctly the subject actual preferences, but it does not correct for the degree of freedom (that is, it does not penalize for the number of parameters).

Looking at table 4 we can immediately noticed, even if it is not directly related to the target of this work, that there is a learning process. In fact moving from Set 1 to Set 5 the accuracy of the preference functionals rise of 3-5%, except for RN for which it decreases.

In table 5 is reported the difference between the likelihood value of EU and the likelihood value on MV.

[4] When we calculated the average rankings two models got the same rank if they performed identical. If for example two models have the highest Aikike criterion they both get the first rank and the next model gets rank three. For this reason the average of the average ranks may differ from the rank average.

	up to 1%	1%-5%	5%-10%	10%-15%	more then15%
Set 1	28.30	32.08	28.30	7.55	3.77
Set 2	24.53	43.40	20.75	7.55	3.77
Set 3	30.19	35.85	24.53	9.43	0.00
Set 4	24.53	45.28	28.30	0.00	1.89
Set 5	35.85	28.30	30.19	3.77	1.89

Table: 5

From this table we have again a clear picture of the superiority of EU, but more important it gives us an indication on the loss of accuracy we have to be ready to accept if we use MV instead of EU.

This kind of analysis is only a statistical one, and even that we reach some important conclusion on the superiority of EU respect to MV and the loss of accuracy. But we are interested also to some economics analysis to measure the accuracy loss.

A very intuitive measure of this evaluates the distance between the real subjects' preferences and the estimated once. But unfortunately it is not obvious how to define a distance function. Should we consider only the number of times that the estimated preference does not matched with the actual preference or should we consider also the magnitude of the errors. It seams that the harmless mechanism should be counting how many mistakes are produced by a particular preference functional in the prediction of actual behaviour.

The results of this analysis are reported in tables 6, 7 and 8. It is clear from tables 6, 7 and 8 that RN does particularly badly at predicting behaviour but that the other functionals do much better. In general, the more general preference functionals make fewer mistakes in prediction, i.e. EU predicts better then MV

	up to 1	1-1.5	1.5-2	2-2.5	more then 2.5
Set 1	18.87%	35.85%	30.19%	5.66%	9.43%
Set 2	11.32%	39.62%	26.42%	3.77%	18.87%
Set 3	24.53%	30.19%	20.75%	15.09%	9.43%
Set 4	16.98%	37.74%	32.08%	3.77%	9.43%
Set 5	16.98%	41.51%	13.21%	15.09%	13.21%

Table 9

In table 9 is reported the percentage of ratio between the number of times EU's prediction is different from the actual subject preference and the number of times MV's prediction is different from the subject actual preference. From this table it is clear that MV performances are not particularly good. In fact only in 18-25% of the case its performance are better then EU. It is particularly surprising that for 10-19% of the subjects using MV instead of EU will produce an error more then 2.5 times bigger.

5.**Conclusion**

This article produces two important results, one in the experimental field and the other in the financial one. On one hand it covers the gap in the literature of decision under risk comparing the Expected Utility theory with Mean-Variance theory.

In terms of best-fitting preference functional EU emerges to perform better than its challenger (i.e. table 2). On the other hand it suggests that the loss of accuracy using MV instead of EU in terms of fitting is generally low (for more than the 50% of the subjects it is less then 5%). But from a non statistical analysis we learned that it is dangerous to use MV instead of EU because 10-19% of the subjects using MV instead of EU will produce an error more then 2.5 times bigger.

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Question	Choice 1				Choice 2				Question	Choice 1				Choice 2			
Number	p_1	p_2	p_3	p_4	q_1	q_2	q_3	q_4	Number	p_1	p_2	p_3	p_4	q_1	q_2	q_3	Q_4
1	0	0	875	125	0	125	0	875	51	0	750	0	250	250	375	0	375
2	0	0	875	125	0	125	0	875	52	0	750	0	250	375	125	0	500
3	0	0	875	125	0	125	500	375	53	0	750	0	250	625	0	0	375
4	0	0	875	125	0	375	0	625	54	0	875	0	125	250	375	0	375
5	0	0	875	125	0	375	125	500	55	0	875	0	125	375	125	0	500
6	0	0	875	125	0	375	250	375	56	0	875	0	125	500	250	0	250
7	0	0	875	125	0	625	0	375	57	0	875	0	125	625	0	0	375
8	0	125	500	375	0	375	0	625	58	0	875	0	125	625	125	0	250
9	0	125	500	375	0	375	125	500	59	125	750	0	125	250	375	0	375
10	0	125	875	0	0	375	0	625	60	125	750	0	125	375	125	0	500
11	0	125	875	0	0	375	125	500	61	125	750	0	125	500	250	0	250
12	0	125	875	0	0	375	250	375	62	125	750	0	125	625	0	0	375
13	0	125	875	0	0	375	500	125	63	125	750	0	125	625	125	0	250
14	0	125	875	0	0	625	0	375	64	125	875	0	0	250	375	0	375
15	0	125	875	0	0	875	0	125	65	125	875	0	0	375	125	0	500
16	0	250	750	0	0	375	0	625	66	125	875	0	0	500	250	0	250
17	0	250	750	0	0	375	125	500	67	125	875	0	0	625	0	0	375
18	0	250	750	0	0	375	250	375	68	125	875	0	0	625	125	0	250
19	0	250	750	0	0	375	500	125	69	125	875	0	0	750	125	0	125
20	0	250	750	0	0	375	500	125	70	125	875	0	0	875	0	0	125
21	0	250	750	0	0	625	0	375	71	125	875	0	0	875	0	0	125
22	0	250	750	0	0	875	0	125	72	250	375	0	375	375	125	0	500
23	0	375	500	125	0	625	0	375	73	500	250	0	250	625	0	0	375
24	0	125	875	0	0	250	750	0	74	500	250	0	250	625	0	0	375
25	0	375	125	500	0	375	250	375	75	0	750	0	250	125	750	0	125
26	0	0	500	500	125	0	250	625	76	0	750	250	0	125	0	875	0
27	0	0	500	500	125	0	250	625	77	0	750	250	0	125	375	500	0
28	0	0	875	125	125	0	250	625	78	0	750	250	0	375	125	500	0
29	0	0	875	125	125	0	625	250	79	0	750	250	0	375	250	375	0
30	0	0	875	125	375	0	375	250	80	0	750	250	0	500	0	500	0
31	0	0	875	125	500	0	0	500	81	0	750	250	0	500	125	375	0
32	0	0	875	125	750	0	0	250	82	0	1000	0	0	125	0	875	0
33	0	0	1000	0	125	0	250	625	83	0	1000	0	0	125	375	500	0
34	0	0	1000	0	125	0	625	250	84	0	1000	0	0	250	625	125	0
35	0	0	1000	0	375	0	375	250	85	0	1000	0	0	375	125	500	0
36	0	0	1000	0	500	0	0	500	86	0	1000	0	0	375	250	375	0
37	0	0	1000	0	750	0	0	250	87	0	1000	0	0	500	0	500	0
38	0	0	1000	0	750	0	0	250	88	0	1000	0	0	500	0	500	0
39	0	0	1000	0	750	0	125	125	89	0	1000	0	0	500	125	375	0
40	125	0	625	250	500	0	0	500	90	0	1000	0	0	750	125	125	0
41	250	0	750	0	375	0	375	250	91	250	625	125	0	375	125	500	0
42	250	0	750	0	500	0	0	500	92	250	625	125	0	375	250	375	0
43	250	0	750	0	750	0	0	250	93	250	625	125	0	500	0	500	0
44	250	0	750	0	750	0	125	125	94	250	625	125	0	500	125	375	0
45	375	0	375	250	500	0	0	500	95	375	250	375	0	500	0	500	0
46	375	0	625	0	500	0	0	500	96	375	250	375	0	500	0	500	0
47	375	0	625	0	750	0	0	250	97	375	625	0	0	500	0	500	0
48	375	0	625	0	750	0	125	125	98	375	625	0	0	500	125	375	0
49	250	0	750	0	375	0	625	0	99	375	625	0	0	750	125	125	0
50	750	0	0	250	750	0	125	125	100	375	125	500	0	500	125	375	0

Table 1: The 100 Pairwise Choice Questions

AIC														
Set 1			Set 2			Set 3			Set 4			Set 5		
RN	MV	EU	RN	MV	EU	RN	MV	EU	RN	MV	EU	RN	MV	EU
2,384	0,982	0,452	2,442	0,414	0,228	2,422	0,567	0,284	2,429	0,433	0,181	2,448	0,263	-0,057
2,045	1,352	0,909	2,190	1,230	0,798	2,272	1,128	0,723	2,376	1,238	0,876	2,376	1,137	0,790
2,350	1,062	0,683	2,250	1,211	0,680	2,303	1,042	0,458	2,359	0,924	0,598	2,303	0,990	0,458
1,768	1,480	1,082	1,666	1,377	1,054	1,412	1,160	0,729	1,768	1,417	0,938	1,814	1,552	0,985
2,227	0,993	0,379	2,376	1,158	0,789	2,323	1,150	1,004	2,429	0,965	0,457	2,376	0,864	0,299
1,448	0,904	0,930	1,483	1,202	1,218	1,199	0,899	0,904	1,516	1,252	1,211	1,149	1,077	1,019
2,313	1,292	1,015	2,415	1,038	0,667	2,466	0,789	0,423	2,488	0,725	0,608	2,435	0,666	0,392
1,900	1,345	0,635	1,994	1,670	1,145	1,791	1,575	1,338	1,579	1,257	0,965	1,743	1,490	1,091
2,164	1,413	0,935	2,227	1,402	0,969	2,272	1,440	0,835	2,250	1,340	0,781	2,332	1,243	0,282
2,392	1,412	0,681	1,958	1,539	0,930	1,994	1,709	0,683	1,994	1,679	0,562	1,879	1,675	0,907
2,332	0,962	0,605	2,332	1,015	0,260	2,415	0,972	0,433	2,384	1,061	0,452	2,415	1,069	0,489
2,376	0,936	0,358	2,376	0,580	0,047	2,359	0,812	0,262	2,359	0,637	-0,057	2,384	0,603	0,120
1,036	0,941	0,637	1,199	1,158	1,106	1,036	0,969	0,802	0,972	0,869	0,873	1,412	0,847	0,760
1,692	1,164	0,748	1,579	1,375	0,731	1,920	1,307	0,929	1,791	1,215	0,935	2,045	1,330	1,124
0,902	0,700	0,564	1,638	1,016	0,806	1,958	1,127	0,967	1,976	0,972	0,750	1,516	0,779	0,525
1,976	1,110	0,768	2,122	0,862	0,531	2,250	0,852	0,702	1,879	1,067	0,857	2,177	1,450	1,244
1,199	1,155	0,789	1,609	1,480	0,798	1,579	1,407	0,864	1,743	1,153	1,036	2,177	2,044	4,928
1,448	1,018	0,807	2,012	0,939	0,790	1,920	0,908	0,676	1,994	1,141	0,685	2,150	0,996	0,533
2,177	1,527	1,417	1,994	1,509	1,438	1,879	1,348	1,202	1,483	0,877	0,781	2,164	1,330	1,235
1,958	1,667	1,610	2,238	1,360	1,173	2,429	1,427	1,482	2,384	1,348	1,315	2,238	1,397	1,416
1,837	1,212	1,059	1,483	0,913	0,919	1,448	0,949	0,952	1,448	1,021	1,010	1,483	0,530	0,445
1,412	0,524	0,515	1,483	0,742	0,723	1,692	1,072	1,038	1,149	0,723	0,593	1,412	0,688	0,634
1,791	1,087	0,921	1,548	0,659	0,593	1,448	0,954	0,809	1,448	0,914	0,875	1,638	0,721	0,665
2,177	1,347	0,874	2,164	1,370	0,835	2,313	1,444	0,919	2,092	1,564	1,439	2,122	1,434	1,224
2,190	1,928	1,330	2,202	1,951	1,409	1,920	1,764	1,484	2,107	1,784	1,357	2,332	1,398	0,730
1,291	0,895	0,813	1,149	0,920	0,947	1,291	1,126	1,050	1,814	0,810	0,440	2,261	1,265	0,706
1,900	1,566	1,179	1,548	1,168	1,136	1,483	1,203	1,148	1,448	1,308	1,160	1,199	1,162	1,107
2,250	1,123	0,497	2,332	0,982	0,243	2,400	1,551	1,265	2,384	0,853	0,358	2,323	1,104	0,663
2,045	0,969	0,803	2,045	1,101	0,529	2,261	1,318	0,514	2,392	1,097	0,482	2,376	1,067	0,430
1,958	1,280	0,921	2,028	1,298	1,030	2,122	1,226	1,283	2,303	1,149	0,747	2,190	1,189	0,897
2,272	1,092	0,848	2,150	1,024	0,903	2,448	0,263	-0,057	2,368	0,869	0,720	2,368	0,817	0,827
2,341	1,264	0,330	2,368	1,379	0,364	2,368	0,950	0,391	2,332	1,035	0,273	2,341	1,083	0,334
2,313	1,442	1,000	2,092	1,141	0,678	2,215	0,822	0,618	2,376	1,434	1,155	2,250	1,239	0,886
0,621	0,436	0,420	0,902	0,662	0,690	0,621	0,221	0,189	0,621	0,170	0,197	0,621	0,221	0,189
2,435	1,633	0,286	2,384	1,515	0,305	2,359	0,988	0,376	2,238	1,238	0,646	2,303	1,272	0,801
2,293	1,347	0,614	2,215	0,725	0,118	2,341	0,903	0,258	2,368	0,619	0,086	2,368	0,415	0,272
2,293	1,420	0,980	2,442	1,430	0,965	2,472	0,850	0,490	2,461	0,888	0,488	2,472	1,193	0,865
2,261	1,449	0,909	2,272	1,782	1,513	2,077	0,828	0,682	2,250	1,171	0,679	2,272	0,953	0,292
1,666	1,003	0,711	1,638	0,780	0,704	1,791	0,526	0,714	1,666	0,333	0,373	1,837	0,423	0,576
1,858	1,437	1,061	1,768	1,141	0,705	1,858	1,348	0,784	1,939	1,418	0,826	1,837	1,369	0,648
1,333	1,201	1,185	1,483	1,162	1,151	1,333	1,029	0,971	1,516	1,380	1,082	1,579	0,608	0,408
2,368	1,560	1,574	2,215	1,207	0,999	1,768	0,957	0,811	2,092	1,256	1,140	2,028	1,119	0,827
1,692	0,913	0,648	1,768	0,839	0,721	1,920	0,660	0,483	1,994	0,794	0,863	2,202	0,582	0,425
2,407	0,934	0,440	2,341	0,977	0,298	2,341	1,045	0,325	2,384	1,038	0,439	2,429	0,999	0,477
2,359	1,097	0,586	2,455	0,619	0,193	2,466	0,531	2,942	2,488	0,468	0,189	2,520	0,351	0,206
2,466	0,551	0,318	2,516	0,602	1,160	2,503	0,242	-0,057	2,503	0,242	-0,057	2,503	0,242	-0,057
2,164	1,259	0,948	1,994	1,378	1,166	2,136	1,432	1,234	2,045	1,419	1,231	1,837	1,375	1,106
1,791	1,007	0,772	2,164	0,445	0,284	2,350	0,796	5,205	2,283	0,590	0,556	2,384	0,597	0,491
1,958	1,267	0,915	1,994	1,008	0,929	2,238	0,892	0,578	2,077	0,828	0,425	2,164	0,638	0,226
2,407	1,355	0,728	2,455	1,523	0,516	2,341	1,306	0,308	2,384	1,570	1,320	2,442	1,596	0,959
2,215	1,094	0,785	2,272	0,891	0,505	2,107	0,947	0,306	1,976	0,905	0,733	1,976	0,705	0,502
1,814	1,435	1,422	1,579	1,200	1,112	1,743	1,240	1,065	1,692	1,092	0,968	1,483	1,030	1,009
2,107	1,584	0,943	2,215	1,394	0,447	2,238	1,335	0,603	2,150	1,591	1,005	1,858	1,383	0,531

Table 2: AIC

exp(-ll/100)														
Set 1			Set 2			Set 3			Set 4			Set 5		
RN	MV	EU	RN	MV	EU	RN	MV	EU	RN	MV	EU	RN	MV	EU
0.526	0.756	0.874	0.518	0.878	0.927	0.521	0.844	0.914	0.520	0.874	0.939	0.517	0.914	1.000
0.576	0.685	0.774	0.554	0.708	0.797	0.542	0.727	0.813	0.527	0.706	0.781	0.527	0.725	0.799
0.531	0.740	0.822	0.545	0.711	0.823	0.538	0.744	0.873	0.530	0.767	0.841	0.538	0.754	0.873
0.620	0.662	0.739	0.637	0.680	0.745	0.681	0.721	0.812	0.620	0.673	0.768	0.612	0.650	0.759
0.549	0.753	0.891	0.527	0.721	0.799	0.535	0.723	0.755	0.520	0.759	0.873	0.527	0.780	0.910
0.675	0.771	0.770	0.668	0.713	0.713	0.721	0.772	0.775	0.662	0.703	0.715	0.730	0.737	0.752
0.536	0.696	0.753	0.522	0.745	0.826	0.515	0.795	0.881	0.512	0.809	0.839	0.519	0.822	0.888
0.598	0.686	0.833	0.584	0.630	0.727	0.616	0.646	0.691	0.651	0.703	0.763	0.624	0.660	0.738
0.558	0.674	0.769	0.549	0.676	0.762	0.542	0.669	0.790	0.545	0.687	0.801	0.534	0.705	0.914
0.525	0.674	0.822	0.589	0.652	0.770	0.584	0.623	0.822	0.584	0.628	0.849	0.602	0.629	0.775
0.534	0.760	0.839	0.534	0.749	0.920	0.522	0.758	0.878	0.526	0.740	0.874	0.522	0.738	0.865
0.527	0.765	0.896	0.527	0.841	0.973	0.530	0.790	0.919	0.530	0.828	1.000	0.526	0.835	0.954
0.752	0.764	0.832	0.721	0.721	0.735	0.752	0.758	0.796	0.765	0.779	0.782	0.681	0.783	0.805
0.632	0.720	0.808	0.651	0.681	0.812	0.595	0.693	0.770	0.616	0.710	0.769	0.576	0.689	0.731
0.780	0.814	0.848	0.641	0.749	0.796	0.589	0.727	0.762	0.586	0.758	0.807	0.662	0.797	0.857
0.586	0.730	0.804	0.564	0.780	0.856	0.545	0.782	0.818	0.602	0.739	0.785	0.556	0.667	0.709
0.721	0.722	0.799	0.646	0.662	0.797	0.651	0.675	0.784	0.624	0.722	0.749	0.556	0.570	0.267
0.675	0.748	0.795	0.581	0.764	0.799	0.595	0.771	0.824	0.584	0.724	0.822	0.560	0.753	0.855
0.556	0.654	0.677	0.584	0.657	0.673	0.602	0.686	0.716	0.668	0.777	0.801	0.558	0.689	0.710
0.589	0.630	0.643	0.547	0.684	0.722	0.520	0.671	0.665	0.526	0.686	0.695	0.547	0.677	0.677
0.609	0.711	0.744	0.668	0.769	0.772	0.675	0.762	0.765	0.675	0.748	0.754	0.668	0.852	0.876
0.681	0.853	0.859	0.668	0.805	0.813	0.632	0.738	0.748	0.730	0.809	0.842	0.681	0.817	0.833
0.616	0.735	0.772	0.657	0.823	0.842	0.675	0.761	0.795	0.675	0.769	0.781	0.641	0.810	0.826
0.556	0.686	0.781	0.558	0.682	0.790	0.536	0.669	0.772	0.569	0.648	0.673	0.564	0.670	0.712
0.554	0.588	0.692	0.552	0.585	0.678	0.595	0.614	0.665	0.566	0.611	0.688	0.534	0.677	0.812
0.703	0.773	0.794	0.730	0.768	0.766	0.703	0.727	0.746	0.612	0.791	0.877	0.544	0.701	0.817
0.598	0.647	0.721	0.657	0.719	0.729	0.668	0.713	0.727	0.675	0.693	0.724	0.721	0.721	0.735
0.545	0.728	0.864	0.534	0.756	0.924	0.524	0.650	0.704	0.526	0.782	0.896	0.535	0.732	0.826
0.576	0.758	0.796	0.576	0.732	0.856	0.544	0.691	0.860	0.525	0.733	0.867	0.527	0.739	0.879
0.589	0.698	0.772	0.578	0.695	0.750	0.564	0.708	0.701	0.538	0.723	0.808	0.554	0.715	0.777
0.542	0.734	0.787	0.560	0.747	0.775	0.517	0.914	1.000	0.529	0.778	0.814	0.529	0.789	0.791
0.532	0.701	0.903	0.529	0.680	0.895	0.529	0.762	0.888	0.534	0.745	0.916	0.532	0.736	0.902
0.536	0.669	0.756	0.569	0.724	0.823	0.551	0.788	0.836	0.527	0.670	0.725	0.545	0.706	0.779
0.840	0.873	0.881	0.780	0.822	0.820	0.840	0.925	0.937	0.840	0.937	0.935	0.840	0.925	0.937
0.519	0.636	0.913	0.526	0.656	0.909	0.530	0.754	0.892	0.547	0.706	0.830	0.538	0.700	0.797
0.539	0.686	0.837	0.551	0.809	0.955	0.532	0.772	0.920	0.529	0.832	0.963	0.529	0.878	0.917
0.539	0.673	0.760	0.518	0.671	0.763	0.514	0.782	0.865	0.516	0.775	0.866	0.514	0.715	0.783
0.544	0.668	0.774	0.542	0.611	0.660	0.571	0.787	0.822	0.545	0.719	0.823	0.542	0.761	0.912
0.637	0.751	0.816	0.641	0.797	0.818	0.616	0.853	0.815	0.637	0.897	0.892	0.609	0.876	0.846
0.605	0.670	0.744	0.620	0.724	0.817	0.605	0.686	0.800	0.592	0.673	0.791	0.609	0.682	0.830
0.695	0.713	0.720	0.668	0.720	0.726	0.695	0.746	0.762	0.662	0.680	0.740	0.651	0.834	0.884
0.529	0.648	0.649	0.551	0.712	0.756	0.620	0.761	0.795	0.569	0.703	0.728	0.578	0.729	0.791
0.632	0.770	0.830	0.620	0.785	0.814	0.595	0.823	0.867	0.584	0.794	0.784	0.552	0.840	0.880
0.523	0.765	0.877	0.532	0.757	0.910	0.532	0.743	0.904	0.526	0.744	0.877	0.520	0.752	0.868
0.530	0.733	0.843	0.517	0.832	0.936	0.515	0.852	0.452	0.512	0.866	0.937	0.508	0.893	0.933
0.515	0.847	0.906	0.508	0.836	0.724	0.510	0.919	1.000	0.510	0.919	1.000	0.510	0.919	1.000
0.558	0.702	0.766	0.584	0.680	0.723	0.562	0.671	0.710	0.576	0.673	0.711	0.609	0.681	0.735
0.616	0.751	0.803	0.558	0.871	0.914	0.531	0.794	0.248	0.541	0.838	0.850	0.526	0.837	0.865
0.589	0.701	0.773	0.584	0.750	0.770	0.547	0.774	0.845	0.571	0.787	0.880	0.558	0.828	0.928
0.523	0.685	0.812	0.517	0.655	0.859	0.532	0.693	0.908	0.526	0.647	0.694	0.518	0.642	0.764
0.551	0.734	0.800	0.542	0.774	0.862	0.566	0.763	0.908	0.586	0.771	0.811	0.586	0.813	0.862
0.612	0.670	0.676	0.651	0.713	0.734	0.624	0.706	0.743	0.632	0.734	0.762	0.668	0.746	0.754
0.566	0.644	0.767	0.551	0.677	0.875	0.547	0.688	0.840	0.560	0.643	0.755	0.605	0.679	0.856

Table 4

RN									
Set 1		Set 2		Set 3		Set 4		Set 5	
# wrong	# right	# wrong	# right	# wrong	# right	# wrong	# right	# wrong	# right
30	51	32	49	32	49	32	49	33	48
21	60	26	55	26	55	28	53	28	53
29	52	24	57	26	55	29	52	26	55
13	68	12	69	10	71	14	67	14	67
24	57	27	54	27	54	30	51	28	53
11	70	10	71	8	73	11	70	6	75
26	55	32	49	33	48	33	48	32	49
19	62	18	63	16	65	13	68	12	69
23	58	22	59	25	56	23	58	27	54
29	52	19	62	21	60	23	58	20	61
27	54	27	54	29	52	29	52	29	52
28	53	30	51	29	52	29	52	30	51
6	75	8	73	6	75	4	77	10	71
14	67	12	69	18	63	15	66	19	62
3	78	11	70	15	66	17	64	10	71
19	62	22	59	25	56	17	64	20	61
7	74	14	67	14	67	16	65	18	63
10	71	18	63	20	61	20	61	21	60
23	58	20	61	18	63	11	70	24	57
21	60	24	57	31	50	30	51	25	56
16	65	10	71	10	71	11	70	11	70
10	71	10	71	12	69	6	75	10	71
14	67	12	69	11	70	11	70	13	68
23	58	21	60	25	56	20	61	19	62
25	56	23	58	17	64	21	60	26	55
8	73	7	74	8	73	18	63	25	56
17	64	11	70	11	70	13	68	8	73
25	56	27	54	29	52	29	52	25	56
20	61	21	60	26	55	30	51	28	53
17	64	19	62	21	60	26	55	23	58
26	55	21	60	33	48	30	51	28	53
28	53	27	54	28	53	27	54	28	53
25	56	21	60	24	57	27	54	24	57
1	80	2	79	1	80	1	80	1	80
31	50	30	51	28	53	25	56	25	56
25	56	25	56	27	54	29	52	30	51
26	55	33	48	32	49	33	48	32	49
28	53	29	52	22	59	24	57	26	55
15	66	13	68	15	66	14	67	16	65
18	63	15	66	17	64	21	60	19	62
10	71	11	70	10	71	12	69	12	69
28	53	24	57	13	68	21	60	20	61
15	66	14	67	16	65	18	63	24	57
30	51	28	53	28	53	29	52	30	51
29	52	32	49	33	48	34	47	36	45
33	48	36	45	35	46	35	46	35	46
19	62	17	64	17	64	15	66	14	67
16	65	25	56	30	51	28	53	30	51
17	64	18	63	24	57	22	59	22	59
32	49	32	49	28	53	27	54	31	50
25	56	26	55	21	60	18	63	21	60
13	68	12	69	14	67	14	67	9	72
23	58	25	56	26	55	26	55	19	62

Table 6

MV									
Set 1		Set 2		Set 3		Set 4		Set 5	
# wrong	# right	# wrong	# right	# wrong	# right	# wrong	# right	# wrong	# right
10	90	6	94	7	93	5	95	4	96
14	86	15	85	16	84	17	83	16	84
13	87	17	83	15	85	11	89	14	86
20	80	19	81	17	83	21	79	22	78
13	87	14	86	15	85	13	87	10	90
10	90	16	84	14	86	18	82	14	86
19	81	13	87	7	93	6	94	8	92
17	83	23	77	19	81	18	82	18	82
19	81	21	79	20	80	20	80	17	83
20	80	16	84	20	80	22	78	27	73
12	88	13	87	14	86	14	86	15	85
12	88	6	94	10	90	7	93	7	93
15	85	15	85	15	85	14	86	11	89
14	86	19	81	18	82	17	83	17	83
7	93	12	88	16	84	13	87	12	88
15	85	12	88	11	89	14	86	19	81
18	82	20	80	20	80	12	88	25	75
12	88	13	87	10	90	13	87	12	88
17	83	16	84	17	83	12	88	12	88
23	77	18	82	14	86	13	87	16	84
16	84	12	88	11	89	13	87	5	95
8	92	11	89	11	89	9	91	11	89
13	87	8	92	11	89	14	86	8	92
20	80	17	83	21	79	23	77	17	83
28	72	28	72	25	75	24	76	21	79
10	90	13	87	14	86	10	90	15	85
20	80	16	84	15	85	21	79	8	73
15	85	13	87	17	83	10	90	16	84
13	87	14	86	18	82	14	86	13	87
15	85	16	84	13	87	17	83	16	84
14	86	12	88	4	96	10	90	9	91
16	84	18	82	12	88	14	86	13	87
22	78	15	85	11	89	16	84	16	84
2	98	2	98	1	99	2	98	1	99
23	77	20	80	11	89	15	85	20	80
19	81	9	91	10	90	7	93	4	96
20	80	19	81	8	92	12	88	13	87
17	83	23	77	9	91	15	85	10	90
11	89	8	92	8	92	5	95	5	95
17	83	12	88	16	84	16	84	17	83
18	82	10	90	16	84	18	82	7	93
18	82	16	84	11	89	16	84	14	86
11	89	10	90	9	91	10	90	7	93
12	88	14	86	13	87	13	87	14	86
13	87	9	91	5	95	4	96	4	96
5	95	4	96	3	97	3	97	3	97
17	83	20	80	22	78	22	78	20	80
10	90	6	94	8	92	6	94	6	94
15	85	10	90	12	88	10	90	9	91
18	82	22	78	18	82	20	80	22	78
14	86	11	89	10	90	12	88	8	92
12	88	18	82	16	84	17	83	12	88
19	81	18	82	17	83	22	78	16	84

Table 7

EU									
Set 1		Set 2		Set 3		Set 4		Set 5	
# wrong	# right	# wrong	# right	# wrong	# right	# wrong	# right	# wrong	# right

7	93	3	97	3	97	3	97	0	100
13	87	10	90	10	90	10	90	14	86
7	93	8	92	6	94	6	94	7	93
16	84	15	85	12	88	13	87	14	86
5	95	9	91	10	90	7	93	3	97
12	88	14	86	13	87	14	86	13	87
8	92	8	92	6	94	5	95	6	94
9	91	16	84	19	81	15	85	15	85
14	86	9	91	10	90	10	90	3	97
7	93	13	87	6	94	4	96	9	91
9	91	5	95	7	93	7	93	7	93
5	95	2	98	5	95	0	100	1	99
8	92	14	86	8	92	14	86	11	89
7	93	11	89	12	88	12	88	16	84
8	92	11	89	11	89	7	93	8	92
9	91	7	93	6	94	11	89	16	84
10	90	13	87	13	87	12	88	26	74
12	88	12	88	8	92	10	90	5	95
16	84	17	83	15	85	11	89	11	89
22	78	19	81	14	86	13	87	14	86
15	85	10	90	11	89	13	87	7	93
6	94	11	89	11	89	9	91	10	90
13	87	5	95	13	87	13	87	7	93
12	88	9	91	9	91	23	77	15	85
15	85	16	84	16	84	19	81	7	93
9	91	12	88	15	85	5	95	11	89
15	85	14	86	16	84	17	83	18	82
9	91	5	95	13	87	5	95	7	93
7	93	8	92	7	93	7	93	6	94
12	88	14	86	13	87	10	90	11	89
12	88	11	89	0	100	8	92	4	96
5	95	5	95	5	95	4	96	6	94
12	88	9	91	8	92	7	93	11	89
2	98	2	98	1	99	2	98	1	99
3	97	4	96	5	95	9	91	11	89
3	97	2	98	4	96	2	98	2	98
12	88	17	83	8	92	9	91	13	87
12	88	20	80	7	93	10	90	4	96
9	91	10	90	8	92	5	95	5	95
13	87	9	91	10	90	13	87	8	92
19	81	9	91	13	87	16	84	6	94
19	81	11	89	11	89	11	89	10	90
8	92	10	90	6	94	11	89	8	92
7	93	4	96	6	94	6	94	7	93
8	92	5	95	4	96	2	98	3	97
5	95	1	99	0	100	0	100	2	98
12	88	18	82	18	82	20	80	16	84
11	89	3	97	9	91	5	95	5	95
13	87	9	91	7	93	6	94	3	97
11	89	6	94	3	97	13	87	14	86
9	91	7	93	6	94	8	92	7	93
16	84	16	84	13	87	15	85	14	86
9	91	5	95	8	92	14	86	8	92

Table 8