Hide and Seek in Arizona[*]

RobertW. Rosenthal[†], Jason Shachat[‡], and Mark Walker[§]

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Abstract

Laboratory subjects repeatedly played one of two variations of a simple two-person zero-sum game of "hide and seek." Three puzzling departures from the prescriptions of equilibrium theory are found in the data: an asymmetry related to the player's role in the game; an asymmetry across the game variations; and positive serial correlation in subjects' play. Possible explanations for these departures are considered.

[†]Department of Economics, Boston University; deceased February 27, 2002

[‡]IBM TJ Watson Research Laboratory; jshachat@us.ibm.com

[§]Department of Economics, University of Arizona; mwalker@arizona.edu

1   Introduction

Barry O'Neill's insightful 1987 paper revived interest in using experimental methods to test the predictive power of mixed-strategy equilibrium. Following O'Neill's experiment, a number of other researchers have conducted new experiments based on two-person zero- sum games that have unique mixed-strategy equilibria.[1] The evidence for and against equilibrium theory has been mixed: while the theory seems to perform better than it did in the experiments done prior to O'Neill's,[2] there have nevertheless been consistent departures from the implications of equilibrium theory in each of the experiments. We report here on a new experiment that produced unexpected departures from the theory, and we speculate on possible explanations for these departures.

The experiment was designed for an unrelated research project, but the strikingly systematic nature of the departures from equilibrium behavior led us to refine the experiment to focus on these departures. The experiment was based upon two simple games of "hide and seek," one with deterministic payoffs and the other with stochastic payoffs, but otherwise strategically identical. Since the two games are in some respects simpler than the games used in most previous experiments, the theory might be expected to perform better than it had done before, and this throws the departures from theoretical predictions into sharp relief.

The two games – stochastic and deterministic – have the same representation as a 2×2 normal form game. Each of the players, the Pursuer and the Evader, can move either to the Left or to the Right. The game has a unique equilibrium, in which the Pursuer and the Evader each use the same mixture: each player plays his Left action with probability 2/3 and his Right action with probability 1/3.

Each of our subjects played only one of the two games, playing it many times against a single, unchanging opponent (another subject). The subjects' play in our experiment resembled, in some respects, the results that have been consistently observed in previous experiments: choice frequencies when aggregated across all subjects were relatively close to the equilibrium frequencies, there was substantially more variation in choice frequencies across subjects than the theory predicts, and

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[1]See, for example, Rapoport and Boebel (1992), Mookherjee and Sopher (1994, 1997), Bloomfield (1994), Ochs (1994), Shachat (2002), and Binmore, et al. (2001). See also Brown and Rosenthal (1990) and O'Neill (1991) for an analysis and discussion of the extent to which the data support the theory.

[2]For example, Suppes and Atkinson (1960), and Malcolm and Lieberman (1965).

there was significant serial correlation in choices. But there were also some departures from the theory that had not been observed in previous experiments. These departures were unexpected and were strikingly systematic. They were of three kinds:

(1) Asymmetric role behavior: Although the player roles' equilibrium mixtures are the same, nearly every Evader chose Left more often than his Pursuer opponent.

(2) A game asymmetry effect: Although the equilibrium mixtures are the same in both games, Pursuers and Evaders each played Left more often in the stochastic game than in the deterministic game.

(3) Strong positive serial correlation: In contrast to previous experiments, in which subjects tended to change their actions too frequently to be consistent with random choice, our subjects switched from one action to the other too rarely.

We describe the experiment in detail in Section 2 and we describe the data in Section 3. In Section 4 we investigate how well three prominent alternative models explain the observed deviations from equilibrium play. Each of the alternative models is able to accommodate asymmetric role behavior, but not at the magnitude observed in our data. None of the models explains other important features of the data. In Section 5 we offer some conjectures about what might be driving the main features of the data.

2   The Experiment[3]

At the beginning of each experimental session, the subjects were matched into pairs. Each pair of subjects then repeatedly played either the Stochastic game or the Deterministic game, in some cases for 100 repetitions, in other cases for 200 repetitions.

In the Stochastic Game, each player has two choices, Left and Right. If the players choose different moves (one Left and one Right), then the Evader has successfully avoided detection and no money changes hands. If the players choose the same move (both Left or both Right), then the Pursuer has "found" the Evader, and a random device then determines whether the Pursuer wins

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[3]All sessions of the experiment took place at the University of Arizona. The subjects were all Arizona undergraduates recruited from undergraduate economics courses. No subject participated in more than one session. Sample instructions are available online at http://www.u.arizona.edu/ mwalker/RSWDataBank.htm.

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money from the Evader: if both chose Left, then with probability 1/3 the Pursuer wins a fixed amount of money from the Evader; and if both chose Right, then with probability 2/3 the Pursuer wins the same amount from the Evader.

In the experiment, the subjects sat side-by-side at a table, separated from each other by an opaque screen, facing a human monitor who recorded all details of play.[4] The subjects' roles as Pursuers or Evaders were chosen by lot at the outset, and each subject retained both the same role and the same opponent during the entire course of play. Each subject had two cards, one with the label L on its face, the other with R on its face. In each round of play, each subject selected one of his two cards, placing it face down and sliding it toward the monitor. Both cards were then turned face up within sight of all three individuals. If the cards matched, the monitor rolled a single die; if the die came to rest with one or two spots showing after both subjects had played L, then the Evader transferred to the Pursuer a token with a known monetary value (to be described momentarily); and if both subjects had played R, then the Evader transferred a token when the die showed three or more spots. The Evader was given some working capital of tokens at the beginning of play and the Pursuer started with no tokens.

The Deterministic Game differs in only one respect from the Stochastic Game we have just described: in the Deterministic Game, when the two players choose the same direction (both Left or both Right) the Pursuer wins an amount from the Evader *for certain*, and he wins twice as much when both have chosen Right as when both have chosen Left.

Note that the expected monetary payoffs for the Pursuer in both games can be described by the same payoff matrix

<div align="center">

Evader

		Left	Right
	Left	1	0
Pursuer	Right	0	2

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The games are zero-sum in expected monetary payoffs. The utility payoffs associated with these monetary payoffs will be considered below.

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[4]Our intent was that each subject see his or her opponent, drawn from the same subject pool, at the beginning of play, so that any thoughts of the opponent's being a stooge or a computer program would be minimized. The opaque screen prevented each player from being able to key on any element of the opponent's behavior during the course of play, other than perhaps the rate at which the opponent played.

The monetary transfers in the experiment were effected by using push-pins as tokens. Each subject had a pad into which the pins could be easily pushed, and the pads were demarcated so that the subject could easily see both his current balance of pins and the dollar value of that balance. The pads were redeemed for cash at the end of the session at a monetary rate per pin that was stated at the beginning of the session. The monetary redemption rate for the Stochastic Game was a half-dollar per pin. For the Deterministic Game it was one-sixth of a dollar per pin. The redemption rates were selected so that at the equilibrium mixture the expected monetary payoffs were the same in both games. When the Pursuer and the Evader both play their equilibrium mixtures the monetary value of the game to the Pursuer is 1/9 of a dollar and the value to the Evader is $-1/9$ of a dollar.

The Evaders began with \$25 worth of tokens in the 100-play sessions and with \$50 worth of tokens in the 200-play sessions.[5] Under equilibrium play, the Pursuer's expected earnings were a little more than \$11 in the 100- play experiments and a little more than \$22 in the 200-play sessions. The Evader could expect on average to keep about \$14 of the \$25 he started with in the 100-play experiments and about \$28 of the \$50 he started with in the 200-play sessions. Each experimental session lasted for about one hour.

Twenty pairs of subjects played the Stochastic Game repeatedly, and another twenty pairs played the Deterministic Game. In each case, fourteen of the pairs played the game 100 times and the remaining six pairs played 200 times.

The natural prediction from game theory is that the subjects in the experiment will play a Nash equilibrium of the repeated game (the 100- play game, or the 200-play game), which might or might not consist of the players playing the equilibrium of the stage game at each of the 100 or 200 plays. And even in the 2×2 stage game the equilibrium may depend upon the players' risk attitudes.

O'Neill and others explicitly avoided risk attitudes in their stage games by devising games with only two outcomes (for example, in O'Neill's experiment every action profile led to one player or the

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[5]By having one subject pay the other from an initial endowment, we provide stronger incentives (i.e., larger payoff differences) in each round per dollar paid to the subjects, as compared with a design in which both subjects are paid nonnegative amounts in each round. The disadvantage of this design element is that it raises the theoretical possibility an Evader could go bankrupt. Players were notified that if that were to happen then the game would terminate at that time. In practice, no Evader's balance ever fell below \$6.50.

other winning a nickel). The Wooders-Shachat (2001) Theorem ensures that for such two-outcome stage games the only equilibrium of the finitely repeated game consists of the players playing the stage-game equilibrium at every stage. Thus, the two-outcome design introduced by O'Neill provided a much sounder connection between equilibrium theory and mixed-strategy equilibrium experiments.

But O'Neill's elimination of theoretical concerns about the subjects' preferences came at some cost: because the games incorporated only deterministic payoffs, four or more actions for each player were needed in order to produce a game that had only two outcomes and in which the "right" way to play was not transparent.[6] In our Stochastic Game, however, each player has only two actions, but the Nash equilibria in both the stage game and the repeated game nevertheless remain independent of players' risk attitudes if the players' preferences satisfy the compound lottery reduction axiom of decision theory. Thus, if we assume that the subjects prefer first- order stochastically dominating lotteries and that they also satisfy the compound lottery reduction axiom, then the players' stage-game utility payoffs in the Stochastic Game are simply their expected monetary payoffs and there is a unique Nash equilibrium in which both the Pursuer and the Evader play the mixture that places 2/3 probability on Left and 1/3 probability on Right in each stage game.

The Deterministic Game has three outcomes, not two. If the players are risk neutral, then the monetary payoffs in the stage game can again be taken to be their utilities, and the Deterministic Game is game- theoretically equivalent to the Stochastic Game. For most of our analysis, we will assume that our experimental subjects were indeed risk neutral at each play of the game, since they were playing a succession of 100 or 200 plays of the game for relatively small stakes. In Section 5.1, however, we will take some account of the effect of risk attitudes, and we will find that Prospect Theory may provide a partial explanation of differences in play between the Stochastic and Deterministic Games.

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[6]The only mixed- equilibrium games where theory does seem to square with experimental results are games with so much symmetry that the equilibrium consists of each player mixing uniformly over his actions – games such as "matching pennies" or "rock, paper, scissors." It is generally agreed that the experimental results in these games may be driven by the symmetric frame and are therefore not useful for testing equilibrium theory in general. Cf. Mookherjee and Sopher (1994).

3 The Data

Individual subjects' play, aggregated over all 100 or 200 of the subjects' action choices, is presented in Tables 1(a) and 1(b) and in Figure 1. Each row of Table 1 describes the play of one pair of subjects. The first twenty pairs of subjects played the Stochastic Game: the pairs labeled #1 to #14 played 100 repetitions each (*i.e.*, there were 100 "plays," indicated in the second column of Table 1); and the six remaining pairs (#15 to #20) played 200 repetitions each. The twenty pairs labeled #21 to #40 played the Deterministic Game; again, fourteen of these pairs played 100 repetitions and six pairs played 200 repetitions. The four columns labeled "Action Profile" in Table 1 report the frequencies with which each of the four joint action profiles LL, LR, RL, and RR were played by each pair. The next two columns, "Pur L" and "Eva L," report the frequencies with which the Pursuer subject or the Evader subject played his Left action (*i.e.*, the Pur L entry in any row is the sum of the LL and LR entries, and the Eva L entry is the sum of the LL and RL entries). Figure 1 depicts these Pur L and Eva L frequencies for each pair of subjects in a scatter diagram: each of the forty points in Figure 1 represents the frequencies of Left play by a single pair of subjects.

We describe several noteworthy features of the data, organized around the following questions: Did the subjects play Left and Right in the equilibrium proportions? Did Pursuers and Evaders play the same way, as the symmetric equilibrium predicts? Was play the same in both the Stochastic and Deterministic Games, as their identical normal form representation predicts? And were the subjects playing randomly, with no serial correlation, as equilibrium theory predicts?

3.1 Equilibrium Frequencies

The first thing one sees in the data in Table 1 and in Figure 1 is that the theoretical equilibrium has some qualitative saliency, but that the equilibrium's predicted mixtures do not provide a quantitatively accurate description of the subjects' play. The equilibrium mixture for each player in each game calls for Left with probability 2/3, so we should see substantially more Left play than Right. And indeed, every Evader and almost every Pursuer did play Left more often than Right, and most of them played Left substantially more often. But it is clear in Figure 1 that the Evaders, on average, played Left significantly more often than the equilibrium rate of two-thirds, and further

that there was a great deal of subject-to-subject variation. And the Pursuers, while indeed playing Left about two-thirds of the time on average, also displayed substantial variation from subject to subject. Column #9 in Table 1 presents the p-value of the chi-square test that compares the joint action frequencies (columns #3 to #6 in the table) with the Nash equilibrium frequencies. The equilibrium frequencies are rejected at the 5% significance level for 14 of the 20 pairs in each game.

However, since the games in our experiment are zero-sum, both players will receive their equilibrium (or minimax) payoffs even if only one of the players plays his part of the mixed-strategy equilibrium. It is therefore natural to ask whether *either* of the players in a fixed pair is playing the equilibrium strategy. If neither is, then we have a stronger rejection of the theory: neither player is ensuring himself his minimax value and both players' play is exploitable. We assess whether each subject is playing the equilibrium proportion of Left play with a simple binomial hypothesis test. Column #10 in Table 1 reports the value of $Pr(x; n, p)$, where $Pr(\cdot; \cdot, \cdot)$ is the cumulative distribution function for the binomial distribution, x is the number of periods the Pursuer chose Left, n is the total number of stage games, and p is the Nash equilibrium mixture probability, 2/3. Column #11 reports the same calculation for the Evaders. If we conduct a two-tailed test at the 5% level of significance, we reject equilibrium play for probabilities less than .025 and greater than .975. The Nash equilibrium frequencies are rejected at the 5% level for 45 of our 80 subjects. In 12 of the 40 pairs, we reject that *either* of the subjects' play is consistent with equilibrium.

Since our subjects all played at least 100 times, and some 200 times, the question arises whether their behavior changed over time, and in particular whether their play could have been converging to an equilibrium. Figure 2 provides some evidence on this question: it shows the frequency of Left play in successive time blocks of twenty periods each. The figure depicts in separate panels the subjects who played 100 periods and those who played 200; in the top half of the figure is the play in the Deterministic Game, and in the bottom half the play in the Stochastic Game. In none of the four panels is there any indication that play is moving toward equilibrium (the dotted line, for both player roles).

3.2   Symmetry of Player Roles

Another feature of the data that stands out sharply is that nearly every one of the points in Figure 1 lies above the 45°- line: the subjects who played the Pursuer role and those who played the Evader

role were not playing the same, despite the fact that the equilibrium mixtures for Pursuer and Evader are the same in each game.[7] In 35 of the 40 pairs the Evader played Left more often than the Pursuer, in many cases much more often. In only 3 pairs (#16, 19, and 22) did the Pursuer play Left more often than the Evader, and in each case it was only slightly more often. Aggregated across all forty pairs, the Pursuers played Left 64% of the time, the Evaders 74% of the time. We refer to this strong asymmetry between Pursuers' and Evaders' play as *asymmetric role behavior*.

For a formal test whether Pursuers and Evaders were playing the same or whether Evaders were playing Left with higher probability we conduct a nonparametric sign test. Under the null hypothesis the probability that a given Evader's realized number of Left choices will exceed the Left choices by his Pursuer opponent is one-half. (In fact, it is slightly less than one half, since there is a non-zero probability they will play Left the same number of times.) Thus, the number of pairs in which the Evader will play left more often than the Pursuer has a binomial distribution $Pr(x; n, p)$, where n is the number of pairs in which the Evader and Pursuer play Left an unequal number of times, x is the number of those pairs in which the Evader plays Left more often, and $p = 1/2$ (the conditional probability of such a "success," given unequal Left play, if the two players are playing the same). In our data 35 of the 40 Evaders played Left more than their opponents; the probability of 35 or more such "successes" is 6.9×10^{-7} under the null hypothesis. It is clear that asymmetric role behavior is a statistically significant feature of our data.

3.3   Behavior Across the Two Games

Comparing play in the Stochastic Game with play in the Deterministic Game reveals another systematic feature of the subjects' behavior that is not accounted for by equilibrium theory. Here we see a *game asymmetry effect*: despite the fact that both games have the same profile of equilibrium mixtures, both the Pursuers and the Evaders played Left (on average) more often in the Stochastic Game than in the Deterministic Game. The last two rows of Tables 1(a) and 1(b) present the choice frequencies aggregated across all pairs in each of the two games. The Pursuers played Left 67% of the time in the Stochastic Game and only 61% of the time in the Deterministic Game. The Evaders played Left 76% of the time in the Stochastic Game and only 72% of the time in the

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[7]In Binmore, et. al. (forthcoming) subjects played a game with the same normal form, under considerably different conditions. A similar role asymmetry appears in their data but is unremarked by them.

Deterministic Game. The hypothesis that the observed joint action frequencies in the Stochastic and Deterministic Games are generated by the same distribution is stongly rejected: the chi-square test statistic's value is 36.437, for which the p-value is less than 10^{-7}. While the game asymmetry effect is not as striking as the asymmetric role behavior, it is nevertheless statistically convincing.

3.4 Serial Correlation

In many previous experiments with mixed-equilibrium games,[8] the subjects have exhibited significant negative serial correlation: they tended to change their actions too often to be consistent with independent play. A similar behavioral phenomenon – subjects switching too often – has been consistently observed by experimental psychologists when they have asked subjects to simulate random sequences. [9] But the subjects in our experiment exhibited the opposite behavior: they tended to switch between Left and Right much less than they would have done if their choices were independent.

In order to evaluate whether the experimental subjects were changing their actions from play to play either too much or too little to be consistent with random play, we conduct a runs test for serial independence on each subject's sequence of choices. The test's null hypothesis assumes that a subject's choices are generated by a fixed mixed strategy which is independent across time. Larger realized values of the test statistic (and thus larger p-values) are produced by sequences with more (and thus shorter) runs – $i.e.$, sequences with more switching of choices.

Table 3 reports the test statistic's p-value for each of our subjects. With a two-sided hypothesis test and a 5% significance level, we reject the assumption of serial independence for 31 of our 80 subjects: these are the bold-face entries in Table 3. There are 27 subjects for whom rejection is the product of too little switching and only four for whom rejection is the result of too much switching. (Note that if all the subjects had truly been playing randomly, the expected number of subjects for whom this 5% test would yield a rejection is four – two subjects on average would appear to switch too often and two too rarely.) Overall, then, play for many of our subjects was characterized overwhelmingly by positive serial correlation.

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[8]For example, O'Neill (1987), Mookherjee and Sopher (1994), Rapoport and Boebel (1992).

[9]See, for example, Tune (1964), Wagenaar (1972) and Lopes (1982). Rapoport and Budescu (1992) found, however, that negative serial correlation was less pronounced in subjects' action choices in a mixed-equililbrium game than in parallel simulation tasks.

4   Alternative Models

Many previous studies[10] have demonstrated the inability of mixed strategy Nash equilibrium to provide a comprehensive description of human behavior in laboratory settings. Alternative models have therefore been proposed to replace Nash equilibrium as positive theories of game playing. We use our experimental data to estimate three prominent alternative models, and then to evaluate the models' success in explaining the anomalies in our data.

4.1   Quantal Response Equilibrium

The Quantal Response Equilbrium (QRE) introduced by McKelvey and Palfrey (1995) has successfully organized the aggregate empirical frequencies of action choices in many experimental games.[11] Like Nash equilibrium, it is a static concept, or more specifically, a parametrized family of static equilibrium models.

The key assumption of the QRE model is that each player calculates the expected value of each of his available actions given his (correct, in equilibrium) belief about his opponent's choice probabilities, and that he attempts to respond optimally, but he makes random errors in the process. In our games this assumption implies that the Pursuer calculates the expected utility of playing Left and Right as follows, assuming that the Evader plays Left with probability q:

$$E_L(q) = q + \varepsilon_{PL} \text{ and} E_R(q) = 2(1-q) + \varepsilon_{PR},$$

where ε_{ij} is a random payoff disturbance for player i and action j. If the $\{\varepsilon_{ij}\}$ are independent random variables having identical Log-Weibull distributions with variance parameter $1/\lambda$, which is the hypothesis of the logistic QRE, then the Pursuer will play Left with probability

$$p = \frac{e^{\lambda q}}{e^{\lambda q} + e^{2\lambda(1-q)}}.$$

Similar calculations yield the Evader's probability of Left:

$$q = \frac{e^{-\lambda p}}{e^{-\lambda p} + e^{-2\lambda(1-p)}}.$$

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[10]For example, Brown and Rosenthal (1990), Rapoport and Boebel (1992).

[11]For example, McKelvey, et.al. (2000), Anderson, et. al. (1998), Capra, et. al. (1999).

For any fixed value of λ, a logistic QRE (*i.e.*, an LQRE) for the Pursuer-Evader game is a mixed strategy pair $(p^*(\lambda), q^*(\lambda))$ satisfying

$$p^*(\lambda) = \frac{e^{\lambda q^*(\lambda)}}{e^{\lambda q^*(\lambda)} + e^{2\lambda(1-q^*(\lambda))}} \quad \text{and} \quad q^*(\lambda) = \frac{e^{-\lambda p^*(\lambda)}}{e^{-\lambda p^*(\lambda)} + e^{-2\lambda(1-p^*(\lambda))}}.$$

Notice that as λ goes to zero the variance of the errors goes to infinity, and the LQRE has each player playing each action with equal probability; and as λ goes to infinity, the variance of the errors goes to zero, and the LQRE coincides with the Nash equilibium.

Figure 3 depicts the graphs of the $p^*(\cdot)$ and $q^*(\cdot)$ functions that uniquely solve the pair of equations above, *i.e.*, it depicts the LQRE for each value of λ. Note that $p^*(\lambda) < q^*(\lambda)$ for all $\lambda \in (0, \infty)$: the $p^*(\cdot)$ graph lies significantly below the $q^*(\cdot)$ graph, so the LQRE robustly (and strikingly) predicts the role asymmetry effect.

Next we estimate separately, for each of our two games, the value of λ that maximizes the likelihood of the data from that game.[12] These estimated λ-values, along with their corresponding LQRE choice probabilities of Left play – *i.e.*, the model's predictions – are presented in Figure 3. All four predictions are consistent with two of the features we've identified in the data: each player in each game is predicted to play Left substantially more often than he plays Right, as the subjects did (on average), and as Nash equilibrium also predicts; and the Evader is predicted to play Left substantially more than the Pursuer, as nearly all of our subject pairs did, and which the Nash equilibrium does not predict.

Note, however, that all four predictions of Left play are nevertheless significantly lower than the corresponding actual play of Left by our subjects: .568 predicted vs. .675 actual for Pursuers in the Stochastic Game, and .698 predicted vs. .762 actual for the Evaders; and .543 predicted vs. .610 actual for the Pursuers in the Deterministic Game, and .692 predicted vs. .717 actual for the Evaders. For each game, the multinomial distribution over joint actions predicted by the maximum likelihood LQRE is overwhelmingly rejected by the chi-square goodness- of-fit test.

A QRE explanation of the game asymmetry effect would require that a single value of the parameter λ yield different play in our two games, but that cannot happen: the QRE is defined in terms of a game's normal form, and our two games have the same normal form. The QRE also cannot account for the serial correlation in our data, because it is a static equilibrium concept.

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[12]In each game's data set, each of the $14 \times 100 + 6 \times 200$ observations was weighted equally.

4.2   Learning Models

Another class of alternative models assumes adaptive behavior by individuals as they repeatedly play a game. The motivation for these models comes partly from the results of laboratory experiments, in which behavior is typically not stationary. We consider two such learning models, the Reinforcement Learning model introduced by Erev and Roth (1998) and the Experience Weighted Attraction (EWA) model introduced by Camerer and Ho (1999).

4.2.1   Reinforcement Learning

Erev and Roth introduced a simple three-parameter reinforcement- learning dynamic which they used to predict play in games that have a unique mixed-strategy equilibrium. In their model each player maintains a score for each of his actions as the game is played repeatedly. At each play, the player selects actions probabilistically; actions with a higher current score are chosen with higher probability. Specifically, let $R_{ij}(t)$ denote player i's score for his jth action prior to the game at stage t ; let $\sigma_{ij}(t)$ denote the probability that i chooses j at stage t; and let X_i denote the set of player i's possible stage- game payoffs. The two initial conditions for the dynamical system are (a) that at the initial stage, each of a player's actions has the same probability of being selected (thus, in our setting, $\sigma_{ij}(1) = .5$ for each player i and each action j) and (b) that

$$R_{ij}(1) = \sigma_{ij}(1)S(1)\overline{X_i},$$

where $S(1)$ is an unobservable strength parameter and $\overline{X_i}$ is the average of player i's payoffs across all action profiles less his minimum possible payoff for any action profile, or $\min\{X_i\}$.

After the play at each stage, each score is updated according to the rule

$$R_{ij}(t+1) = (1-\phi)R_{ij}(t) + ((1-\varepsilon)I_{(a_i(t)=j)} + \frac{\epsilon}{2})(\pi_i(j, a_{-i}(t)) - \min\{X_i\}),$$

where ϕ is an unobservable parameter that discounts past scores; where $I_{(a_i(t)=j)}$ is an indicator function for the event that player i selected action j in period t; where the unobservable parameter ε determines the relative impacts on the scores of the selected vs. the unselected action; and where $\pi_i(j, a_{-i}(t))$ is i 's payoff when he plays action j against the opponent's stage-t action $a_{-i}(t)$. (Player i's minimum possible payoff for any action profile, $\min\{X_i\}$, is subtracted from $\pi_i(j, a_{-i}(t))$

to avoid negative scores.) The model is completed by assuming that

$$\sigma_{ij}(t) = \frac{R_{ij}(t)}{\sum_k R_{ik}(t)}.$$

Notice that the range of X_i values varies across the two games even for fixed i: the different monetary rewards and the move by nature both enter explicitly in the transition function for the Stochastic Game.

We use the same estimation procedure used by Erev and Roth (1998): we estimate the values of the three unobservable parameters $S(1), \phi$, and ε by minimizing the mean square error of the model's predicted proportions of Left play in 20-period blocks, as depicted in Figure 4.[13] More specifically, for each fixed triple of parameter values from a discrete grid, we proceeded as follows: For each of the two games we simulated the play of 500 pairs playing 200 stage games, and then we calculated separately the frequency of Left play by the 500 Pursuers and by the 500 Evaders in each 20-period block. These frequencies are the model's predictions for that triple of parameter values. The true values of the parameters should of course not depend upon which game the subjects are playing,[14] so the parameters were estimated using the data from both games together. Within the three-dimensional parameter-value grid, the triple of parameter values that minimizes the mean square error, given the data, turns out to be

$$(S(1), \phi, \varepsilon) = (3, .47, .09).$$

In the center panels of Figure 2 we present the average of 10,000 simulations using these estimated parameter values. It is evident in Figure 2 that the model does predict that the Evaders will play Left more often than the Pursuers, as our subjects did. Comparison of the left and center panels of Figure 2 (the data, and the model's predictions) reveals that the model underpredicts the magnitude of the asymmetry, just as the QRE model does. This is particularly so in the first 5 time blocks, which contain most of the data.

The model does not account for the game asymmetry effect: indeed, the model actually makes the opposite prediction, that Left will be played more frequently in the Deterministic Game than

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[13]Because there were only six pairs in each of the later time blocks, and twenty pairs in each of the early blocks, we weighted each mean square error by the square root of the number of pairs observed for that block.

[14]But because the realized monetary payoffs that are used to update the scores R_{ij} are generated differently in our two games, the same parameter values will typically generate different predictions in the two games.

in the Stochastic Game. With regard to the data's dynamic features, comparison of the left and center panels of Figure 2 suggests that the model might be consistent with the upward trend of Left play in the Deterministic Dame, but it does not capture the (stronger) upward trend in the Stochastic Game.

Using the parameter values we have estimated, the Erev and Roth model generates positive serial correlation in action choices: whenever an action is chosen its score is increased by more than the unchosen action's score, and its probability of being selected in the next stage game therefore increases. But the model also predicts positive serial correlation in mixed-equilibrium games in which subjects actually exhibit negative serial correlation.

4.2.2 The Experience Weighted Attraction Model

The second dynamic model we consider is EWA. In this model subjects choose stage-game actions probabilistically according to the expression

$$\sigma_{ij}(t) = \frac{e^{\lambda R_{ij}(t)}}{\sum_k e^{\lambda R_{ik}(t)}},$$

where at stage t player i chooses action j with probability $\sigma_{ij}(t)$; where λ is a parameter that plays the same role here as in the Logistic QRE model; and where $R_{ij}(t)$ is a scoring function, as in the reinforcement model, but is defined (i.e., updated) differently than in the reinforcement model. The updating of $R_{ij}(t)$ involves a "discounting" factor $N(t)$, which is updated according to

$$N(t+1) = \rho N(t) + 1 \text{ for } t \geqq 1,$$

where ρ is an unobservable discount parameter and $N(1)$ is an unobservable parameter, interpreted as the strength of experience prior to the beginning of play. The score $R_{ij}(t)$ is then updated as follows:

$$R_{ij}(t+1) = \frac{N(t)\phi R_{ij}(t) + ((1-\varepsilon)I_{(a_i(t)=j)} + \frac{\varepsilon}{2})\pi_i(j, a_{-i}(t))}{N(t+1)},$$

where $\pi_i(j, a_{-i}(t)), \phi$, and ε have the same interpretations as in the model of Erev and Roth. The initial scores, $R_{ij}(1)$ for each i and j, are additional unobservable parameters.

As we did in our reinforcement-learning estimation, we estimated the EWA model from the entire data set, a process in which given paramater values generate distinct predictions for the

Stochastic and Deterministic Games. The data from our experiment yield the following maximum-likelihood estimate of the nine-tuple of unobservable EWA parameters:[15]

$$(N(1), R_{PL}(1), R_{PR}(1), R_{EL}(1), R_{ER}(1), \rho, \phi, \varepsilon, \lambda)$$
$$= (.833, .657, 0, -1.863, 0, .993, .998, 1, .578).$$

We used our estimated EWA model to simulate 10,000 pairs playing 200 periods of each game. The rightmost panels in Figure 2 report the simulations' averages in each block of twenty periods. First note that the model's predictions quickly move toward the Nash equilibrium, but that the asymmetric role behavior does appear: the predictions consistently have the Pursuer playing Left slightly less than the Evader. A comparison of the upper and lower right-hand panels reveals that the model does not predict the game asymmetry effect.

As in the Erev and Roth model, with our estimated parameter values the EWA model generates positive serial correlation in the players' choices, but the same caveat applies here as in the Erev and Roth model: the EWA model predicts positive serial correlation even in many games in which subjects actually exhibit negative serial correlation.

Summarizing, each of the three alternative models generates, at least qualitatively, the asymmetric role behavior observed in our data: the Evaders consistently playing Left more often than the Pursuers. In the QRE model this effect is present for all possible values of the model's parameters; in the learning models the effect is less pronounced than in the QRE model, and its presence depends upon the values of the models' parameters. In all three models the magnitude of the effect is considerably less than what is observed in the data. None of the models captures the game asymmetry effect, and while the two learning models do predict positive serial correlation, we have indicated why this prediction is not very convincing.

5 Discussion

If the three alternative models we have considered aren't able to rationalize the data generated by our experiment, then how do we explain the behavior we have observed? We discuss several possibilities, each of which may be playing some role in our experiment.

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[15]$R_{PR}(0)$ and $R_{ER}(0)$ are arbitrarily fixed at zero. This is a normalization.

5.1 Risk Attitudes

Perhaps the most important insight that led O'Neill to reject prior mixed-strategy experiments was that the experiments used games with three or more possible outcomes, and the results of the experiments could therefore have been affected by the subjects' risk attitudes. Like O'Neill's game, our Stochastic Game has only two possible outcomes for each player, and play should therefore be unaffected by risk attitudes. The Deterministic Game, on the other hand, has three outcomes, so risk attitudes might play a role in that game.

Incorporating risk attitudes into the Deterministic Game is straightforward. Since for each player there are only three possible outcomes, a player's preferences over lotteries among the outcomes can be completely described by his relative utility for the intermediate outcome (i.e., the outcome $+1$ for the Pursuer and -1 for the Evader). To make things precise, normalize each player's utility so that his worst outcome (0 for Pursuer, -2 for Evader) gives him utility 0 and his best outcome ($+2$ for Pursuer, 0 for Evader) gives him utility 2. Let π_P denote the utility the Pursuer obtains from his intermediate outcome, $+1$; and let π_E denote the utility the Evader obtains from his intermediate outcome, -1. Then a player is risk-neutral over his three possible outcomes if $\pi = 1$; he is risk averse if $\pi > 1$; he is risk-preferring if $\pi < 1$; and in general he is more risk averse as the value of his π is larger.

It is easy to verify that the Pursuer's and Evader's Nash equilibrium mixtures place probability

$$\sigma_P = \frac{2}{4 - \pi_E} \text{ and } \sigma_E = \frac{2}{2 + \pi_P}$$

on play of Left. If each player is risk-neutral, then we have $\sigma_P = \sigma_E = 2/3$, as we already know. If both are risk averse, then we have $\sigma_E < 2/3 < \sigma_P$. But in virtually every one of our subject pairs the Evader played Left more often than the Pursuer. So risk aversion is simply not consistent with our subjects' behavior. Indeed, the data suggest that $\sigma_P < \sigma_E$.

Prospect Theory (Kahneman and Tversky (1979)) holds that while people are indeed risk averse for gains, they are instead *risk preferring* when facing losses. This would mean that the π-values of players in the Deterministic Game would satisfy $\pi_P > 1$ and $\pi_E < 1$, and the equilibrium in the Deterministic Game would therefore satisfy the inequalities $\sigma_E < 2/3$ and $\sigma_P < 2/3$. Since the equilibrium in the Stochastic Game is unaffected by risk attitudes, each player in that game uses a mixture that places 2/3 probability on playing Left. Thus, if people's risk attitudes are correctly

described by Prospect Theory, then both Pursuers and Evaders should (in equilibrium) play Left less often in the Deterministic Game than in the Stochastic Game – precisely the game asymmetry effect that appears in our data. However, average play of Left was *m*ore than 2/3 by subjects playing both roles in both games.

An additional component of Prospect Theory is loss aversion: The magnitude of the change in utility from a loss exceeds the magnitude of the change in utility from a gain of the same size. For the payoffs in the Deterministic Game this is equivalent to the inequality $1-\pi_E > \pi_P-1$, which is in turn equivalent to $\pi_P+\pi_E < 2$, and the latter inequality is equivalent to $\sigma_P < \sigma_E$. Prospect Theory therefore provides, at least qualitatively, an equilibrium explanation of the observed asymmetric role behavior in the Deterministic Game. Furthermore, its explanatory power could be tested by modifying our design so that both Pursuer and Evader are always paid nonnegative amounts (cf. footnote 5).

Prospect Theory is clearly not the whole story however: it does nothing to account for the strong asymmetric role behavior in the Stochastic Game.[16] Further, a significant number of the Evader subjects in the Deterministic Game played Left on substantially more than 2/3 of their plays, which requires that $\pi_P < 1$ – i.e., it requires that these Evaders believed (correctly) that their opponents were risk preferring for gains, while the other Evader subjects believed (also correctly) that their opponents were risk averse for gains. This seems unlikely. So while risk attitudes may have contributed to the observed behavior, they fall short of fully accounting for the behavior.

5.2 Objective vs. Subjective Uncertainty

In the Introduction we indicated that since the Stochastic Game is simpler than the games typically used in previous two-outcome mixed strategy experiments (each player has only two pure strategies instead of four or more), the theory might be expected to perform better here than in the previous experiments. But the Stochastic Game might not actually be so simple. In two of the four cells of the payoff matrix the outcome is a lottery, an uncertain prospect. So while the game has far fewer strategy combinations than a 4×4 or larger game, the reduction in the strategy sets might have been more than offset by replacing outcomes that are certain with ones that are uncertain.

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[16]A non-linear probability weighting function, which is often adopted in Prospect theory, could potentially account for the asymmetric role effect in the Stochastic Game.

For example, in some of our experimental sessions we asked the subjects, upon completing the experiment, to write down for us any strategy(s) they were using as they played the game. Several of the subjects who played the Stochastic Game indicated that they were paying more attention to the behavior of the die than to their opponent's behavior. One subject (an Evader) wrote "I tried to guess the probability of when a 1 or 2 was 'due' up on the dice, and sneak a right card in. It is not very easy to figure out when the other person is going to change their 'favorable' left or right card." A Pursuer subject wrote "Sometimes I would try to guess what the die roll would be, and didn't even think about what the evader was going to do." Altogether, nearly half of the Stochastic Game subjects indicated that on at least some of their plays they were trying to predict the outcome of the die roll.

These comments suggest that some of the subjects did not view the die rolls as a serially independent random process. While it's not likely that they treated the die, or Mother Nature, as literally a third player in the game, they may have viewed the opponent's actions and the die's outcomes as similar processes, each generating unpredictable "actions" that affected their own payoff.

How would this view of the lottery outcomes affect behavior? We know from the Ellsberg Paradox, and similar examples, that people are typically more comfortable with objective risks than with subjective ones. Of the two kinds of uncertainty faced by a subject in the Stochastic Game, uncertainty about what one's opponent will do is surely the more subjective, and uncertainty about the die roll more objective. A subject might therefore view the die-uncertainty as more "focal" and devote more of his attention to "playing against the die" than to playing against his opponent – exactly the approach described by the subjects quoted above.

Consider the extreme situation in which the players in the Stochastic Game are playing *only* against the die, and placing no weight on the opponent's likely action. Then a Pursuer would play Right, and an Evader would play Left. This is of course not an equilibrium (at least not a Nash equilibrium): while the Pursuer is using his best response to the die's "mixture," he is using his *worst* response to the Evader's choice of Left. The Evader's play of exclusively Left, on the other hand, is a best response to both the die's "mixture" *and* his opponent's play.

Because the Pursuer does so badly here, this pattern of play could not be expected to persist, and indeed we observe every Stochastic Game Pursuer play Left on at least 40% of his plays. But

the additional uncertainty of the die in the Stochastic Game, which is more objective than the strategic uncertainty about one's opponent, could be expected to reinforce a tendency toward Left play by Evaders. Moreover, greater Left play by Evaders in the Stochastic Game should eventually bring forth greater Left play by Pursuers as well, as in the game asymmetry effect that our data exhibits.

5.3 Initial Beliefs and the Dynamics of Play

Each of the subjects in our experiment was surely a novice at the game he or she was playing: none of the subjects had seen or played either game before. So there is no reason to expect equilibrium play at the outset of a match. This leads us to consider how play might progress as players play the game repeatedly. We have already discussed reinforcement and EWA learning models, each of which predicts a path of play that depends partly upon assumptions about the model's initial conditions. Here we focus directly on the idea that players respond to their beliefs about how their opponents will play, and we ask what players' initial beliefs might have been and how they might have influenced observed play.

In belief-based learning models[17] a player has a *belief* about how his opponent is going to play, and chooses actions for himself that he thinks will do well against his opponent's play. As play progresses, a player observes his opponent's actual play and uses that information to update or revise his belief, which produces a corresponding change in his own play in order to do well against the revised belief.

The left panels of Figure 4 show the time series of play, aggregated across all subjects, and aggregated across time into twenty-play blocks. In the earliest stages of play the Evader subjects, on average, played Left somewhat more often than the equilibrium prescribes, while the Pursuer subjects played Left less often than called for in equilibrium. This pattern of play at the outset would be consistent with beliefs by Evader and Pursuer subjects alike that their opponents would play Left less often than the 2/3 rate prescribed by equilibrium.

Can we say anything directly about the beliefs our subjects actually did hold at the outset of an experimental session? We did not ask our subjects for any information prior to their play. But the beliefs of another, similar set of subjects provide some evidence on this question. One of the

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[17]For example, Milgrom and Roberts (1991) and Fudenberg & Levine (1998).

authors presented the Stochastic Game instructions to the 44 students in his game theory course. This was done at the first meeting of the course, before the students had learned anything about game theory; thus, they were drawn from roughly the same subject pool as the subjects in our experiment. The students were asked to predict the average number of Left plays, out of 100 plays, by people who had played this game in an experiment. (A five dollar reward was promised for the prediction that came closest to the actual average of play.) The average prediction about Evader play was 58 Left plays out of 100; the average prediction about Pursuer play was 49 Left plays out of 100. Only five of the 44 student respondents predicted more than 67 Left plays by Pursuers, and only eight respondents predicted more than 67 Left plays by Evaders.

Thus, it seems very likely that the predominant belief of our experimental subjects was that one's opponent would play Left less than 2/3 of the time. As we have described above, this would lead, in a belief-based model of play, to play in the early stages that would be similar to our data.

Figure 2 shows that subsequent play in both the Stochastic and Deterministic Games was at least qualitatively consistent with belief-based learning: Pursuers began by playing Left "too little," and Evaders correspondingly increased their rate of Left play; Evaders began by playing Left "too often," and Pursuers correspondingly increased *their* rate of Left play.[18]

6   Concluding Remarks

We have described several puzzling features of the data generated by our experiment, but we have not identified or developed a comprehensive theory that accounts for them all. Perhaps future experiments will provide sharper versions of these phenomena, which will in turn suggest a unified theory. Or perhaps one of our readers will see a pattern in this data set that has eluded us and that will reveal all.

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[18]There is perhaps some additional evidence that our subjects were responding directly to the play of their opponents (rather than responding to it indirectly, via the reinforcement their own earnings provided). About half of the responses to our debriefing question indicated the respondent was trying to anticipate his opponent's play, or to outwit his opponent, or that he had played as he did in order to do well against the play he had observed by his opponent.

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Table 1(a)
Pair Level Data Analysis: Stochastic Game

Pair #	Stages	Action Profile (Pursuer, Evader)				Pur. L	Eva. L	NE test P-value	Binomial Pur. L	Binomial Eva. L
		LL	LR	RL	RR					
1	100	.34	.07	.34	.25	.41	.68	.000	.000	.647
2	100	.84	.05	.09	.02	.89	.93	.000	1.000	1.000
3	100	.39	.25	.29	.07	.64	.68	.203	.320	.647
4	100	.34	.16	.46	.04	.50	.80	.000	.000	.999
5	100	.62	.07	.26	.05	.69	.88	.000	.723	1.000
6	100	.71	.08	.14	.07	.79	.85	.000	.998	1.000
7	100	.21	.19	.33	.27	.40	.54	.000	.000	.006
8	100	.30	.25	.30	.15	.55	.60	.028	.010	.097
9	100	.47	.10	.33	.10	.57	.80	.007	.028	.999
10	100	.52	.20	.20	.08	.72	.72	.457	.893	.893
11	100	.41	.16	.28	.15	.57	.69	.181	.028	.723
12	100	.59	.06	.29	.06	.65	.88	.000	.398	1.000
13	100	.71	.11	.13	.05	.82	.84	.000	1.000	1.000
14	100	.64	.10	.17	.09	.74	.81	.001	.954	1.000
15	200	.64	.15	.17	.04	.79	.81	.000	1.000	1.000
16	200	.47	.25	.18	.11	.72	.65	.382	.955	.281
17	200	.46	.17	.28	.10	.63	.74	.123	.121	.985
18	200	.56	.16	.25	.05	.71	.80	.000	.917	1.000
19	200	.48	.25	.20	.07	.73	.68	.191	.977	.680
20	200	.62	.10	.26	.02	.73	.88	.000	.968	1.000
Aggregate	2600	.52	.15	.24	.09	.67	.76	.000	.810	1.000

Table 1(b)
Pair Level Data Analysis: Deterministic Game

Pair #	Stages	LL	LR	RL	RR	Pur. L	Eva. L	NE test P-value	Binomial Pur. L	Binomial Eva. L
			Action Profile (Pursuer, Evader)							
21	100	.36	.07	.48	.09	.43	.84	.000	.000	1.000
22	100	.36	.26	.23	.15	.62	.59	.304	.188	.066
23	100	.32	.23	.27	.18	.55	.59	.032	.010	.066
24	100	.56	.20	.22	.02	.76	.78	.013	.984	.995
25	100	.50	.09	.36	.05	.59	.86	.000	.066	1.000
26	100	.55	.11	.27	.07	.66	.82	.013	.481	1.000
27	100	.36	.17	.31	.16	.53	.67	.038	.003	.566
28	100	.47	.15	.23	.15	.62	.70	.274	.188	.791
29	100	.54	.13	.24	.09	.67	.78	.093	.566	.995
30	100	.27	.26	.32	.15	.53	.59	.004	.003	.066
31	100	.41	.14	.28	.17	.55	.69	.047	.010	.723
32	100	.38	.18	.31	.13	.56	.69	.137	.017	.723
33	100	.23	.19	.32	.26	.42	.55	.000	.000	.010
34	100	.31	.15	.35	.19	.46	.66	.000	.000	.481
35	200	.43	.21	.25	.13	.63	.67	.741	.153	.566
36	200	.66	.16	.16	.03	.82	.82	.000	1.000	1.000
37	200	.37	.21	.27	.16	.58	.64	.037	.004	.190
38	200	.62	.14	.19	.06	.76	.81	.000	.997	1.000
39	200	.38	.13	.38	.12	.51	.75	.000	.000	.996
40	200	.49	.19	.26	.07	.68	.74	.128	.625	.990
Aggregate	2600	.44	.17	.28	.11	.61	.72	.000	.000	1.000

Table 2: Runs Tests' P-Values

(Boldface indicates a rejection of independence at the 5% level of significance.)

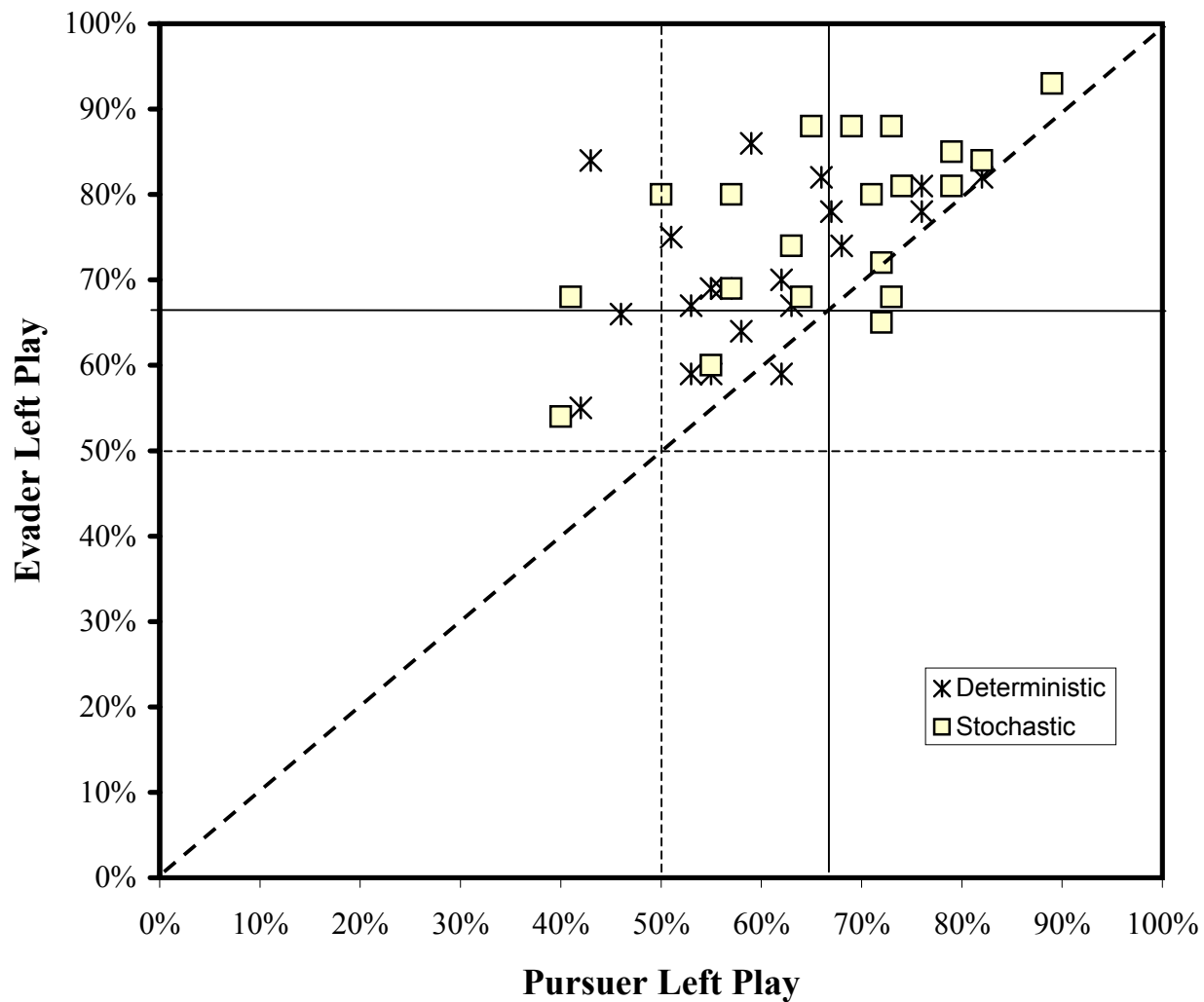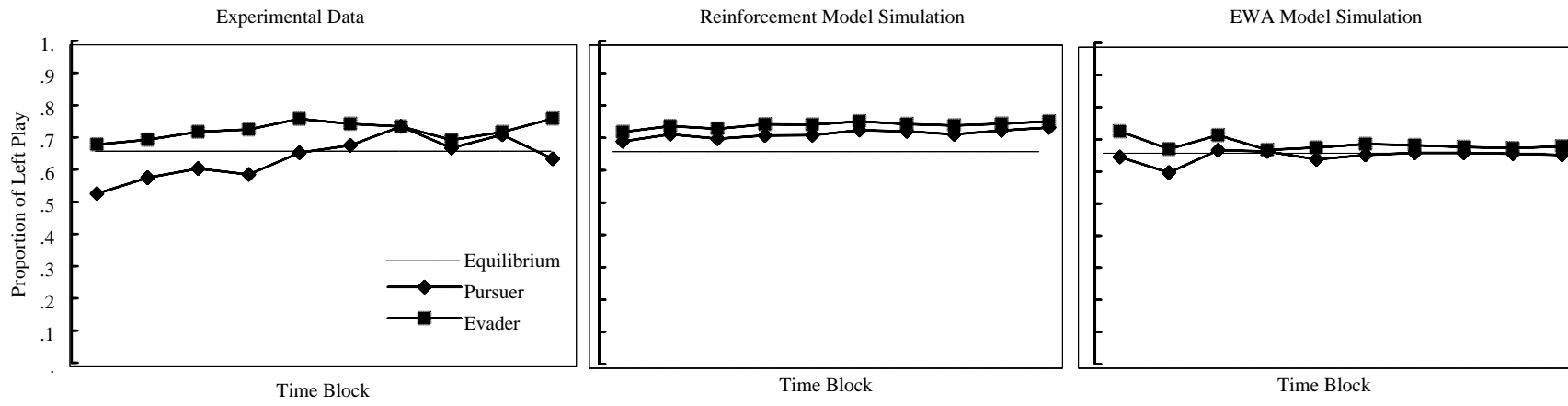| | Stochastic Game | | | Deterministic Game | |
| | Pursuer | Evader | | Pursuer | Evader |
Pair #	P-value	P-value	Pair #	P-value	P-value
1	.092	.790	21	**.993**	.664
2	.031	.209	22	.026	.632
3	.662	.850	23	.180	**.009**
4	.420	.172	24	**.000**	.953
5	.521	.294	25	.063	.487
6	**.014**	.840	26	**.001**	.694
7	.735	**.989**	27	**.003**	.657
8	.064	.799	28	.731	.684
9	.846	.172	29	.050	.689
10	.567	.092	30	.993	.551
11	.205	.338	31	.380	.521
12	**.018**	**.000**	32	.046	**.006**
13	**.000**	**.001**	33	.604	**.016**
14	.181	**.000**	34	.172	**.023**
15	.057	**.007**	35	**.000**	**.023**
16	**.001**	**.012**	36	**.000**	.045
17	.940	**.002**	37	**.005**	**.979**
18	**.004**	.588	38	.222	**.013**
19	**.000**	.685	39	**.002**	.935
20	.378	**.000**	40	**.000**	**.992**

Figure 1: Evader vs. Pursuer Left Play

Figure 2: Times Series of Proportion of Left Play: Actual and Predicted

Deterministic Game

Stochastic Game
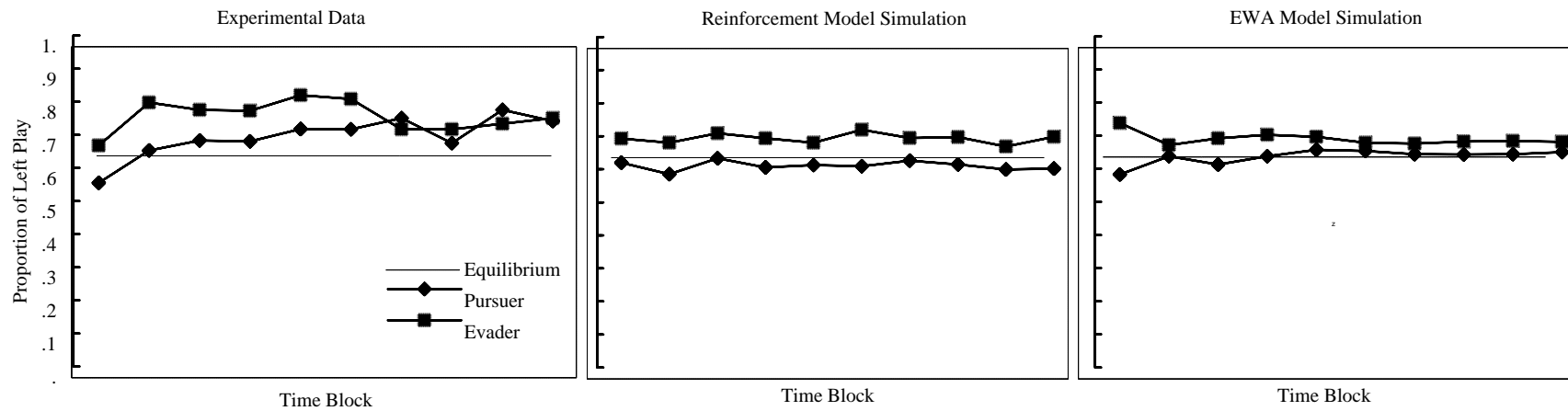
Figure 3: Logistic QRE for Pursuer-Evader Gane