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Systematics of Advanced Capital Market Models
based on Empirical Research by Gerhard Schroeder*

A. Introduction

There is an inbalance between mathematical-stochastic models[1] contributing interesting aspects to the analysis of capital markets and their products (refined assumptions) and their empirical validation which doesn't seem to keep pace[2] with the developement. The complex blue prints remain closed to systematic review. Particularly, when some authors of mathematical models can not or may not offer explicit solutions.[3]

The refined method of generating „courses“ randomly, proves that the Black and Scholes pricing formula causes significant pricing errors even though the fictitious underlying fits into a perferfectly distributed lognormal distribution.

This approach uses a rather simple exponential hyperbolic distribution and a range of effects like Mean Reversion, Volatility Clustering and others suggested by the theoretical models. In fact the results come closer to the real market figures. The effects, applied as rules, allow the generation of courses, clones, when the models are using the lognormal distribution.

But it requires a combination of all known effects to make a perfekt clone. Thus Jump Diffusion Processes[4] are an important contribution - in generating perfect stochastic properties rather than just visual representation.

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B. Methodology:

The generating of fictitious underlyings, socalled clones, can be seen as experimental research. That includes the usage of a given (assumed, suggested) distribution and a full set of rules derived of the theoretical models.

Intensity and timing in how to apply these rules has to be analyzed in order to make a perfect clone. The cloned quotations are then used to validate the evaluation formulas.

All courses are computed using rolling iterations for 10 years with the stochastic properties compared with the historical data.

The application of random machines is a complex mathematical subject in itself requiring careful analysis. But it is a well established discipline. Once reviewed for this purpose it can be provided as a standard function that allows a promising approach for market analysis.

B.I. Generation of fictitious „Courses“

The intention was to generate data covering 10 years or 522 weeks respectively. The distribution is either taken from a exponential hyperbolic distribution with best fit to the historical data or by taking the historical data themselves - rounded to 0.5 percent returns.

Actual returns are „drawn“ out of sample of 522 weekly returns and the next course computed accordingly. After 522 iterations the sample is done - the repository holding the sample is emty. Statistically it is a model „without returns“. However one could design of applications where the „draws“ (or thrown dices) are „returned“ and the repertoire maintains the level of 522 figures thoughout the sample.

B.II. Random Machines

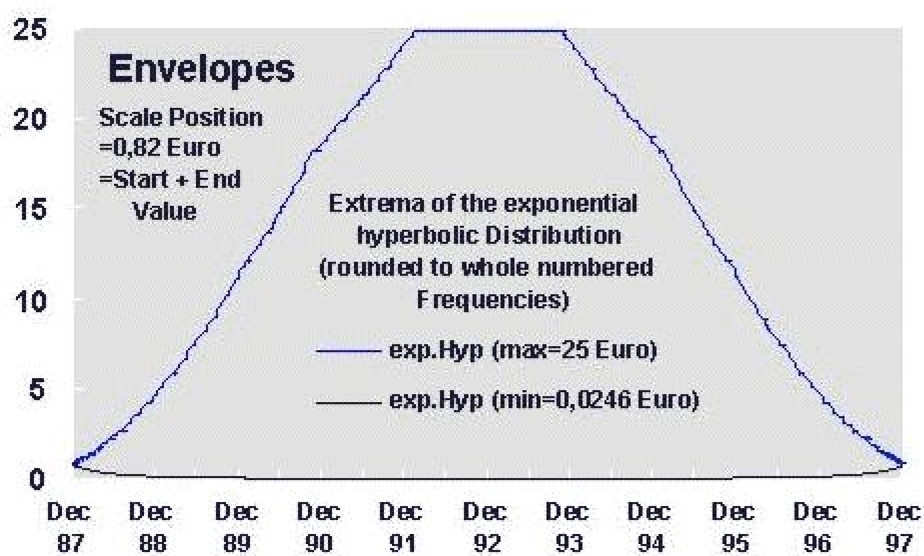
Radommachines use random numbers - from 1 to n (here with n = 522). The figures of weekly returns are grouped in 25 classes from -6%. -5.5%,... until +6%. The actual sequence of these classes is changed frequently. The class sequence is changed frequently - is "tossed" („geschüttelt“), to avoid potential bias of the randomness. Thus the random machine „draws“ a sequence of 522 weekly returns.

The game rule „without returning“ ensures that the repertoire of a given distribution is always fully applied. The sequence is random only. The rule „with returning“ is usefull for mass prediction when 100 or more trials are done to predict one week with the mean interpreted as the expected course.

B.III. Extrema of Probability Distributions

It is easy to prove thad a probability distribution does not fully describe a given underlying. When the weekly figures are sortet according to increasing or decreasing returns of the - here - exponential-hyperbolic distribution one gets the enveloping curves that describe any potential course may take if the sequence of the weekly returns would be fully at will:

Figure 1: Eveloping Curves



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Then a Dollar could run - totally unrealsitic - to an equivalent of 25 Euro or fall down to 0.0246 Euro. Both curves would cause a quite „volatile“ volatility (Max: 20% - 0% - 5% / Min: 5% - 0% - 5%).

The theoretical models suggest some approaches, how to smooth the volatility and thus adjust the base model better to reality:

- Jump Diffusion Processes of returns.
- Stochastic Volatility (SV) including phenomena like Mean Reversion and Volatility Clustering etc.
- Mean Reversion of interest rates
- Correlation or Non-Correlation of interest rates or volatility and underlying
- Bandwidth Control
- Sign Bias Effects
- Implementing real or fictitious pricing or market rules

These approaches (e.g. Duffie, Pan, Singleton 1999 [6]) have in common, that the mathematical complexity is increasing. On the other hand the number of solutions of ODEs and PDEs found is increasing as well. The theory of Martingales (Elliot 1982 and many others) plays a central role. The classic Fourier-Transformations experience a new zenith. Quite a number of Working Papers are devoted to new solution possibilities. Other solutions can be taken from different disciplines and applied to capital markt models.

On the other hand complex mathematical functions can be approximated by the Monte-Carlo-Method or related techniques. The so-called implied volatility is „computed" by the reverse formula of Black and Scholes.

The broad development of mathematical models has two causes at last: The empirical research shows(?) an increasing number of phenomena that cannot be explained by the Black and Scholes model sufficiently. The volatility e.g. is quite a complex but dependant function and not as stable as assumed by the model. On the other hand financial market research is requesting more realistic models as complex as they may be.

B.IV. Hyperbolic Exponentialfunctiones

Disadvantages of the lognormal distribution as a base of most of the theoretical models are: the typical lean peak (leptocurtosis) around scale position „0%“ and the the „fat tales“ are not properly represented. The pricing error are related to the fact that frequencies of options „at-the-money“ and during course jumps are too low.

The hyperbelic function

$b^2 x^2 - a^2 y^2 = a^2 b^2$ (crossing the X-axis at +/- a) [1]

is mirrowed at the axis 'X=Y'. The the lower negative curve is a exponential curve („fitted hyperbolic densities“) ein. By replacing one gets:

$f(x) = C_2$ * EXP (-SQARE ROOT ($C_3 + (x/C_1)$^2). [1b]

Since this curve can be varied sufficiently without the constant C_3 one could simplify like:

$f(X) = C_2$ * EXP (-ABSOLUT (1 - X/C_1)) (with C_2 to standardize by 1) [1c]

This surprisingly simply formula allows to describe todays exchange rate returns.[7]. For shares and indexes it is meore efficient to use historical quotations over the last 10 years rounded along a scale with 0.5 per cent units.

C. Results

When setting the model instructions the outcome must be reviewed in a way that effects shouldn't be „overdone“. E. g. a run could produce a perfect sequence at the beginning as long as the „supply“ of drawn returns is sufficient. However, at the end there may be a lack of returns to continue and finish at the same level of perfectness. To avoid such effects the model instructions must be either softened or applied stochasticly by allowing each 'n'-th draw only and again randomly. In other words a balanced repertoire should be ensure throughout the complete run.

C.I. Performace Expectations and Saisonnal Effects

Hyperbolic distributions are strongly symetric and because of this fact always wrong when the underlying has a positive trend (drift) On the other hand it is questionnable to use the skewness of a model distribution for forecast a performance of a given underlying. The DOW and related stock might have a performance of around one percent eventually. Typically they show a performaance of 11 Percent and more. Exchange rates on the other hand don't have any performance expectation.

The modelling described implies a weekly saisonnal component, that can be „loaded" also to carry a yearly performance effect.

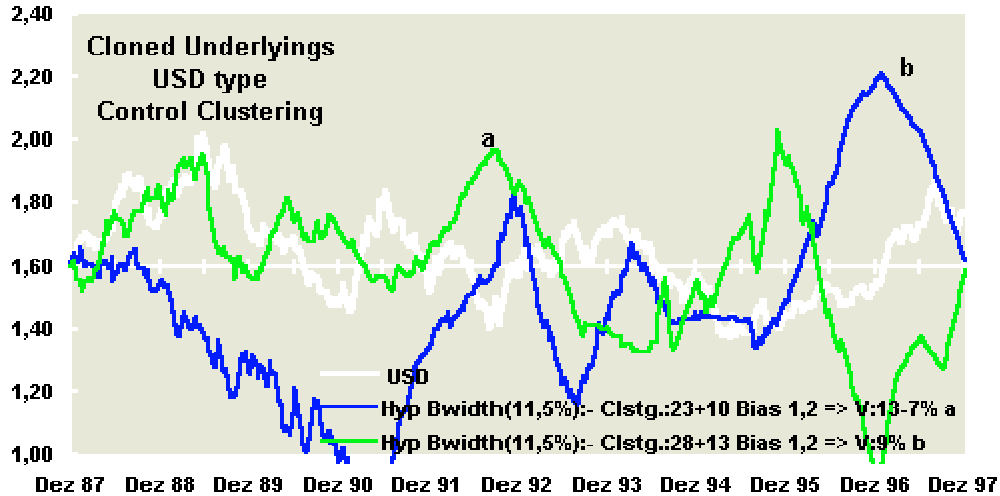
C.II. Clustering

Clustering[8] as a modelling instruction is understood, that in case of volatility excess during the n-th draw that return out of a series of „m" is selected that shows a minim deviation of the previous volatility. The length of phases with and without clustering can be varied.

The 20 exzessive changes (more than +/- 4% change in one week) „drawn" out of the exponential hyperbolic sample of a total of 522 cause „noise" or increasing volatility of the generated course developement. Clustering therfore must allow interrupts simply because of the limitatitions of the repository.

Isolated application of clustering could cause wide ranging course moves that were never observed in reality:

Figure 2: Clustering



The courses achieved by clustering are partly realistic. However the bandwidth is unrealistic often. However, when the course is too „tame“ the effect could be used to provoke strong deflections. Clustering doesn’t control the volatility efficiently. Volatilität.

C.III. Mean Reversion (MR)

Mean Reversion is the tendency of a variable to approach ist mean from any excess. That‘s a tautology since that derives from the definition of any mean. However in case of cloning and forecasting MR can make sure that a given (assumed, predicted) mean will be achieved. MR is mainly used as a volatility effect but is used for interest rates and for the course as well. MR is intended to solve the problem of unstable volatility. Some complex mathematical are include MR[9] in a system of solvable PDEs/ODEs.

Mean Reversion can be realized by returning „draws“ of returns that create or keep excessive levels of volatility.MR controls the volatility efficiently. Therefore it must be interrupted stochasticly to avoid untypical courses.

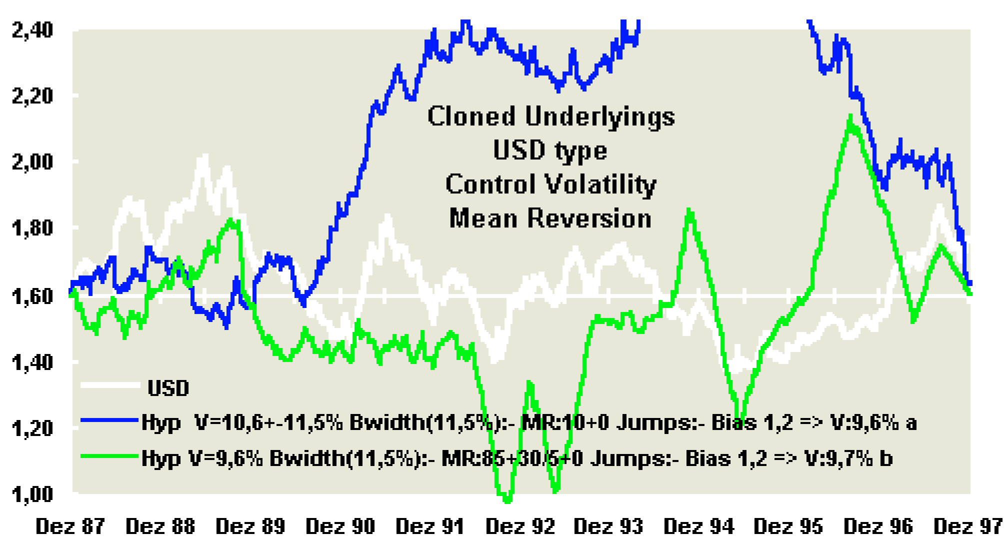
Experimentally it is possible to measure rolling volatilities and trigger MR whenever a limit is exceeded. But setting limits is a kind of deus-ex-machina technique: Who knows the limits of a given underlying? One could interpret any base distribution as an oscillation with a normal distributed oscillation on top thus smoothing the volatilatility without changing the base distribution.

The cloning instructions could be related to different volatility measures: A history volatility measured over each last 13 weeks and the total volatility measured from the beginning for a better judgement of any current situation. Referring to the USD/DEM[10]-exchange rate one could target the 10 years mean of 9.6 percent and respect historical minimum /maximum values of 4.4 or 19.5 pervcent.

However, this way one could exclude potential trends eventually. While the the USD/DEM-exchange rate might have trends situatively in case of the DAX the volatility exceeded in 1998 the limit of 40 Percent first time. Within the previous 10 years a limit of 30 percent was never exceeded.

The 'Mean Reversion'-Instruction seems an important contribution to models since it controls volatility very effectively.

Figure 3: Mean Reversion



Both cloned courses match the intended volatility perfectly. However, the bandwidth is several times unrealistic

C.IV. Jump Processes

Jump processes are subprocesses that trigger excessivity in a base process. Related to the Dollar-exchange rate during 1988 til 1997 one could consider 30 weeks (or 6 per cent of the weeks) with returns greater than (absolute) 2.8 percent as subprocesses. Since historical or theoretical distributions contain already excess returns they are triggered by the base distribution. When excessive returns are „drawn“ they trigger a four week subprocess with a smoothing that can be interpreted aa a Poisson-distribution or an exponential smoothing. There is a revers return in week t+1 and returns less than 1 percent in the following four weeks.

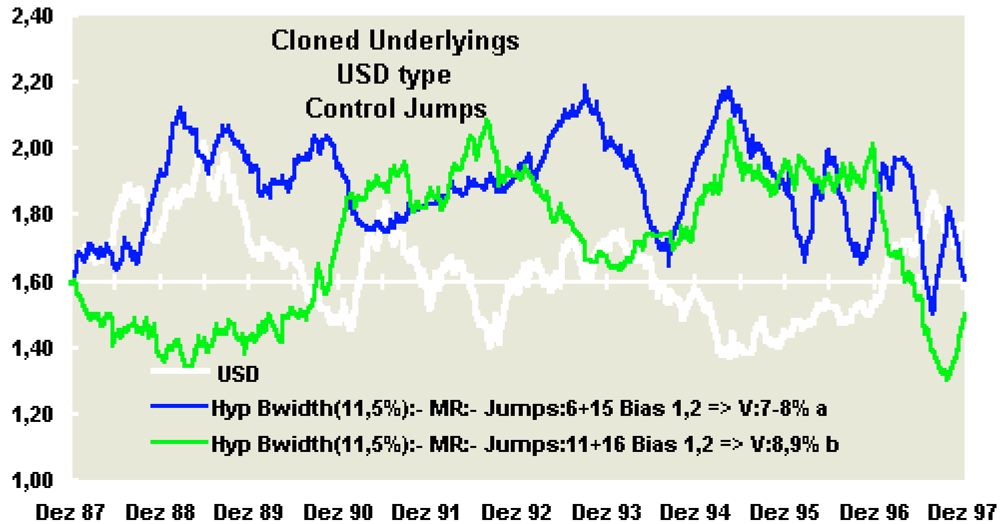
Figure 4: Jump Processes Dollar

Excessive if	typical course after excessive returns (∅ value 1980-1998).						
	Frequency	x-1	Jump week t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
x > 2.8%	2,9%	-0.32%	3.84%	-0.58%	-0.25%	0.25%	0.05%
x < -2.8%	3,1%	0.11%	-3.63%	0.25%	-0.06%	-0.20%	-0.42%

Despite of the length jumps should determine only weeks t til t+2. Otherwise 30 * 5 = 150 weeks out of 522 weeks in total.

Jump processes can be considered as a kind of contra effect ensuring the smoothing whenever an excess jump happens (see Figure 5: Jumprocesses)

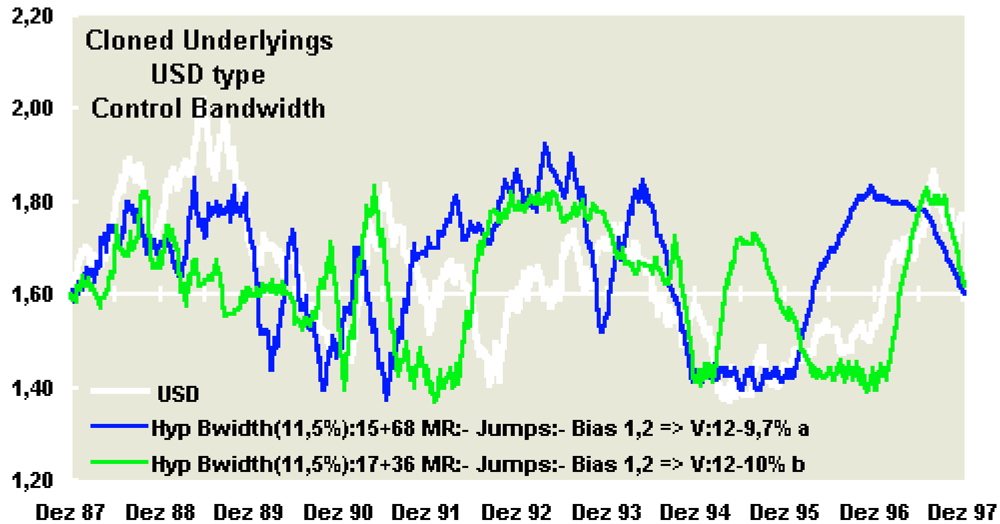
Figure 5: Jumprocesses



C.V. Bandwidth

The model instructions discussed thus far do not ensure balanced courses for exchange rates for which „horizontal trends“ (no trend or trend zero) can be assumed. This can be done by limiting the course to a bandwidth. In case underlyings maintain a trend balancing can be defined as a channel arround a liniear trendline or drift. To avoide unrealistic extremous the control of bandwidth[11] seems to be necessary.

Figure 6: Bandwidth

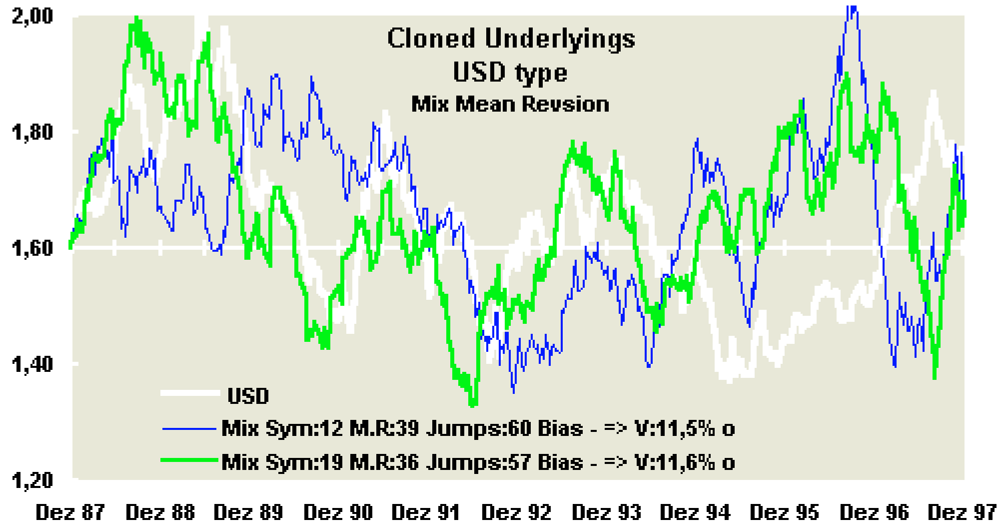


In both cases, „a“ and „b“ the control of bandwidth is overdone: The maintain the level of 1.45 DEM strating Dec 1994 for more than a year. In addition course „b“ has not enough peaks. In both cases the volatility is unvbalanced.

C.VI. Combined Approaches

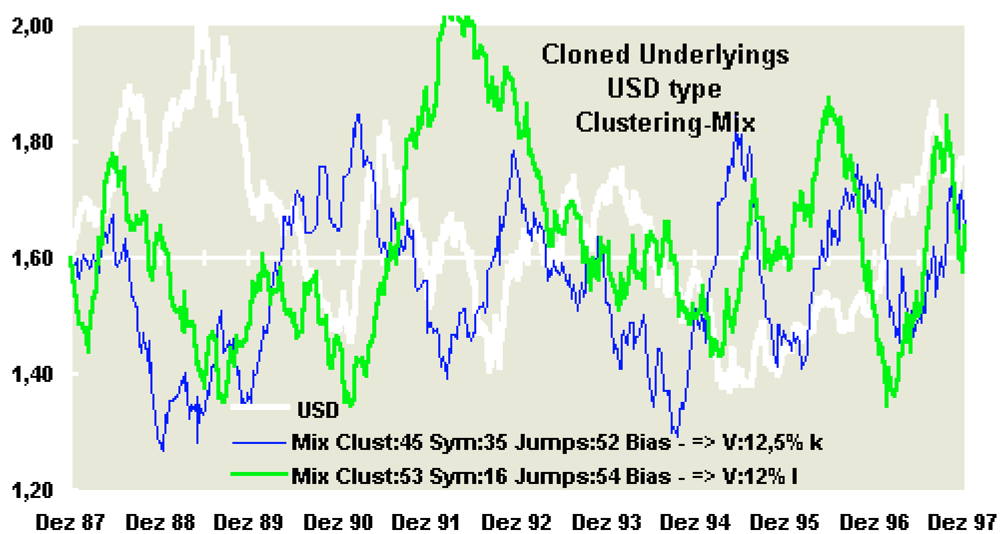
By studying the effects of isolated model instructions one can start to combine different approaches to prodduce more realistic courses.The first sample combines enforced Mean Reversion, a few jumps and a spering bandwidth control.

Figure 7: Rule Mix based on Mean Reversion



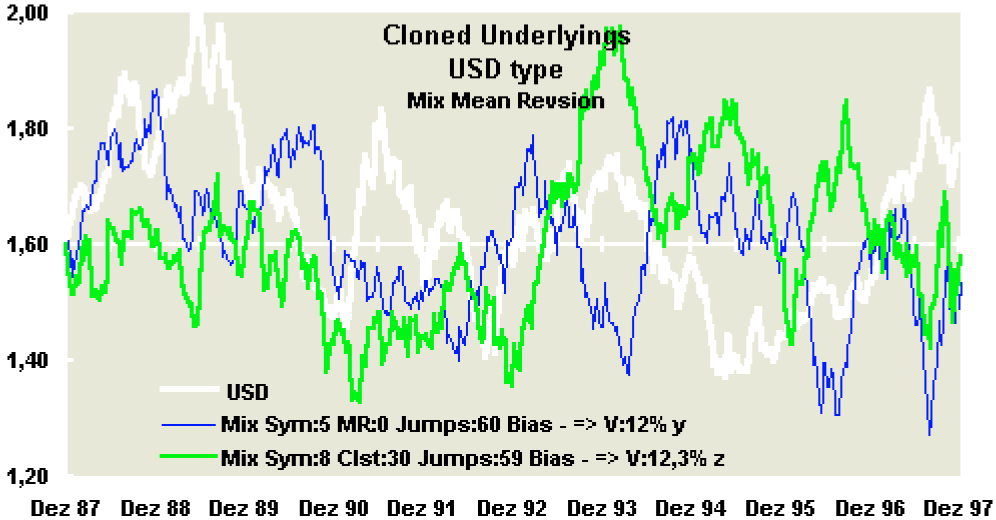
The frequencies count how often the rules are triggered. That is not identical with the number of jumps or MRs. That is decided when a rule is applied.

Figure 8: Rule Mix based on Clustering



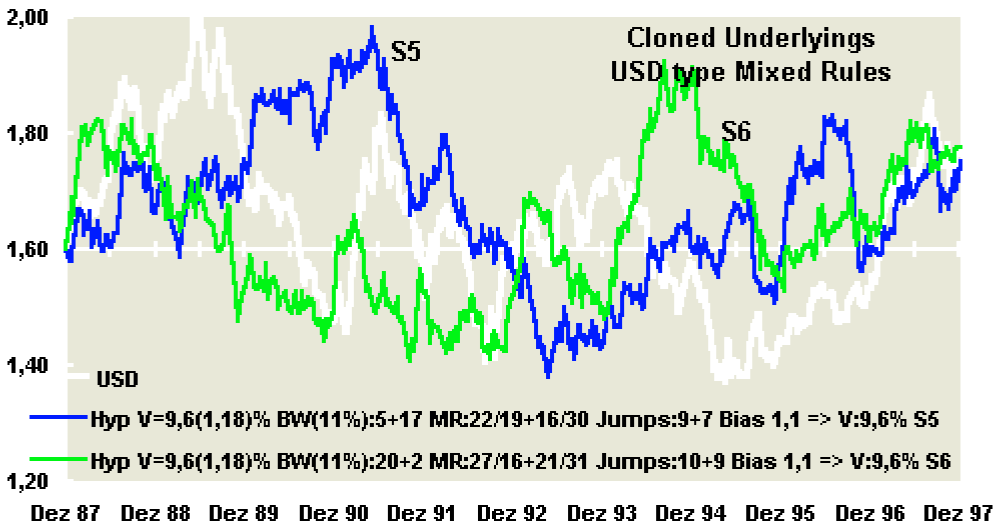
Both courses show a the stochastic properties of a Dollar-DEM exchange rate. Only the volatility 2 percent higher than the historic mean.

Figure 9: Compare MR and Clustering:



The best result was achieved without Clustering.

Figure 10: Optimum Rule Mix



A parallel trial without jump processes (no figure) show the same visible quality. However the stochastic mathematical properties are suffering.

D. Judgements and Suggestions

All rules discussed can be used for the construction of fictitious realistic courses like they can be observed in reality. The Stochastic Volatility (SV) seems to be a set of important rules to ensure realistic formats. A sparing use of MR[12] allows to generate precisely an intended volatility.

D.I. „Definition" of the Dollar Exchange Rate

A model for the Dollar exchange rates can be defined in the following way:

A weekly USD-DEM[10] exchange rate is generated when based on an approximated hyperbolic distribution returns are drawn randomly while applying the following rules:

1. When the volatility is excessive those returnes are selected and applied for the following n_1 weeks that ensure mean reversion.
2. When the bandwidth is excessive those returnes are selected and applied for the following n_2 weeks that reduce the bandwidth best.
3. The n_3 weeks after an excessive return (jump) make a Poisson process.

The application of the rules is not determined strongly but rather „soft" and controlled randomly how often and how long they should be applied in a particular situation. The rules require the following parameter:

Bandwidth, number of repetitive draws in case of excessivity, mean (expected value) of volatility (being a measure of bandwidth also), factor for sign bias effects and duration parameters: For simplification $n_1 = n_2 = n_3$ can be assumed.

For tuning purposes the rules and parameters can be tried out with the history quotations. In case of forecasting weekly seasonal effekts should be considered.

That's a way to describe but also to create or clone a fictitious USD-DEM exchange rates or important indexes. By increasing or decreasing positive returns (and negative ones vice versa) hausse and baisse effecs can be enabled.

D.II. The Brownian Motion as a Model Blueprint

Professional microscopes allow to observe the Brownian Motion. The surrounding molecules of the liquid preparing a particle for examinations under the microscope move in random directions and are knocking the pollen grain in random directions. Since it is uniformly surrounded by this bombardment, the effects tend to neutralize over time. The idea is that stock orders create similar impulses regarding the course[13] being the equivalent to the particle however in an one dimensional process[14] over time while the original Brownian motion is two dimensional.

Financial markets processes with timing patterns and phases for order gathering and course fixing are different - not to mention special rules for opening and closing. The order size varies[15] and doesn't correspond to the homogenuous molecules of the preparation liquid. And there are quite significant saisonal and pseudo-saisonal properties as documented in F Appendix - Pseudo Saisonality.

The Brownian motion allows jump processes also within the range of the lognormal distribution but less frequent than in reality. In financial markets jumps are probably reactions to news and exogene foundet respectively. The timing is event driven and not predictable by formulas. Actual course are better represented by leptocurtic courses than by the lognormal distribution.

To describe the „laws" of financial markets in theoretical modell it requires obviously stochastic subprocesses with the SV processes beeing the most important ones in controlling the volatility and the bandwidth.

The key weakness of the Black und Scholes formula is due to the fact that the future volatility is required but unknown and that the volatility is considered a constant but is heteroscedastic instead.

D.III. Stochastic vs Implied Volatility

Studying SV has implications for the usage of implied but recent volatility as a predictor for the future volatility. The only rational is based on the clustering effect. But this is not a continuous effect. In 75 percent of all cases the volatility related to week x+1 changes less than 0.25 percent from the volatility in week x. During 210 weeks with excessive volatility (2.5% deviation from mean) MR happens in 47 weeks - in 20 weeks the volatility is becoming more excessive. But excessive values suggest predictions being too high.

The existence of SV disallows the application of the formula required to convert a daily volatility into a yearly figure:

$$V_{year} = V_{day} * \text{SQUARE ROOT } (1/365) \text{ (or 1/252 for 252 bank days)} \quad [X]$$

The equation requires a stable volatility![16] Thus the daily or even hourly DAX volatility computations to scale a year volatility are methodolically outrageous[17]. The volatility is an indicator for actual market heat but not a predictor of a future development.

D.IV. Suggested Systematics of advanced Capital Market Models

To cover all known properties a model should reflect the following stochastic properties:

Figure 11: Overview of advanced Market Properties

<table>
<tr><td>B&S</td><td colspan="2">Base Distribution Process</td><td colspan="2">lognormal, exp.-hyperbolical, historical or any other</td><td rowspan="10">Sign Bias Effects</td></tr>
<tr><td></td><td colspan="4">Subprocesses:</td></tr>
<tr><td></td><td>Stochastic Volatility</td><td>excessive .
Volatility
non excessive.</td><td colspan="2">Clustering
↓ Mean Reversion | Non MR ↑
Volatiliy within Mean Range</td></tr>
<tr><td></td><td>Jump Diffusion</td><td colspan="2">Conformity</td><td>Non Conformity</td></tr>
<tr><td></td><td>Stalagmites[18]</td><td colspan="2">Reduced Formats</td><td>„White Noise“</td></tr>
<tr><td></td><td colspan="4">Parameters or Subprocesses</td></tr>
<tr><td>B&S</td><td>Interest Rates</td><td colspan="3">Could be as complex as to Volatility</td></tr>
<tr><td></td><td>Bandwidth</td><td colspan="3">stochastic limit, related to Volatility or not</td></tr>
<tr><td></td><td>Drift / Trend</td><td colspan="3">only linear makes sense</td></tr>
<tr><td></td><td>Saisonal Effects</td><td colspan="3">yearly, monthly, weekly, daily, over night pattern</td></tr>
</table>

	saisonal or pseudo saisonal (see Ch. F)	
Trading Rules	Rules covering the price fixing process stock exchange rules and index definitions	

The Black and Scholes Model (B&S) covers the base process and interest rates only and with given fix volatility and interest rate. This may explain the limited power in representing real markets.

SV, Clustering and Jump Processes are important model contributions and could be considered as ‘overtones’. Jump subprocesses have a smoothing effect, while during Stalagmite phases returns above normal can be observed. Mean Reversion is an important rule with respect to option pricing since it dims the volatility as a key parameter for most of the evaluation formulas. Jump processes are smoothing the course immediately after the jump.

They are subject of the advanced models yet until today there is no mathematical model able to handle all these phenomena in common. The implementation of two or three rules already causes quite some mathematical complexity. To avoid unrealsistic courses a bandwidth control based also on the volatility and reflecting trends (drifts) in case of indices should be included.

This is a chance for experimental research to test and optimize a model concept even befor a mathematical formula is available.

D.V. Stochatics of Model Properties of Indexes and X-rates

Except the volatility percentages all other figures indicate how many weeks out of 52 weeks relate to the particular effect. Except the non-excess figures all other figures are not exclusive. I. e. a bandwidth violation would very probably coincide with a jump etc.

Figure 12: Frequencies of different SV Phänomena

	Jump	Double	MR	Clustering	Bandwidth	Non Excess	Average
Definition	> **2,8%**	Jump	**> 0,25%**	**< 0,25%**	Xt-26 +hVol	**2,00%**	Volatility
USD	**1,4**	**0,35**	**1,8**	**4,5**	**6,7**	**10,1**	**10,1%**

DOW	**3,4**	**1,65**	**2,1**	**4,2**	**5,9**	**8,7**	**14,5%**
NIKKEI	**4,3**	**2,17**	**2,8**	**4,8**	**6,3**	**6,2**	**15,7%**
DAX	**4,8**	**2,25**	**2,5**	**4,2**	**6,9**	**7,8**	**15,8%**
NASDAQ	**2,5**	**1,38**	**2,1**	**2,3**	**4,5**	**5,0**	**16,4%**

Some particularities with indexes are documened and analyzed in F Appendix - Pseudo Saisonality.

E. Summary

Experimental research is an easy way to study the behaviour of financial markets. Any underlying can be basically described by a probability function of periodical returns. Thus *cloned* underlyings can be derived from historical or theoretical quotations by random machines. To avoid excessive behavior, additional properties such as Stochastic Volatility (SV) suggested by various mathematical models should be applied as rules.

This method allows those who are not familiar with ultimate mathematics[19] to understand current financial market models and stochastic properties can be even analyzed before solutions of ODEs or PDEs[20] are available.

The Black and Scholes formula has the disadvantage that its key variable, the (future) volatility. is not known. In fact, what is known is that the volatility is volatile itself and the assumtion of a stable volatility is violated. The socalled advanced models try to model the stochastic volatility. However, this still implies assumptions how a particular volatility may (or may not) develope until a given point of time.

While the focus of this paper was on methodology some fllowing exprimental results may be of interest as well:

F. Appendix - Pseudo Saisonality

One could say "DAX and FTSE make their money in the sleep". The average open performance of both indexes exceeds the overall index performace - in other words, the news since the last evening , night and early morning have a positive effect while the in day performance is less positive and in case of the DAX negative.

Since the DJ is performing in day twice as strong as overnight one could say the information processing at the Wallstreet is done faster or more synchronously.

Comparing 3500 days it can be said that 49 to 51 per cent the DAX open and close follow the the previous DJ close. However, it can be said also that at 50 per cent the DJ open follows the DAX close! Thus it is pretty random. Looking at weekly formats (see F.I) one could say that the Tuesday and Friday DAX open follow the previous DJ close. But these are only two out of 10 significant open and close quotations per week.

F.I. Particularities with important Indexes

Only the overnight performance (considering the previous close and the open result[1]) bring the DAX in the plus. The DAX returns in the day are rather low or negative - taking the yield from open until close - is typically negative in the weekly total and on average of several years - except Monday.

The DJ the night performance is moderately except Thursday. On Thursday a negative impulse from the previous night refelected in the open course is compensated in the day until close. In London it is different: The performance of the previous night lies over those, the previous day. The numbers suggest that the NYSE represented by the DOW is more efficient in the "processing" of global messages than Frankfurt or London - the foreign-exchange trading excluded. Or in other words: The troubles, the brokers make themselves and the field surrounding the

[1] Up to Jan. 1998 the open quotation of historical DJ Yahoo quotes was identical with the previous close quotation (with a few exceptions - probably correctons). Since then the overnight return is described by [{Open - (prev.) Close} divided by (prev.) Close]

stock exchange in London and Frankfurt during the day, are rather detrimental to the performance.

The DAX and FTSE overnight performance exceed the in the day performance. This effect becomes recognizable only if one splits the in day return up from close price tons closing price into A DAILY and into A night component. This is only determined into the opening auction.

That gives a lead to institutional investors. The DJ at night is actually moderate. The values are up to Wednesday under the in day ones. The DAX works itself hig-hly at night, also the FTSE is better at night than in the day. This effect becomes recognizable only if the daily total return is split up from closing price to closing price into a in day and into an overnight component. This is only determined in the opening auction.

Figure 13: Mean Differential Returns

DAX, DOW and FTSE 1990 - 2002	Over-night * DOW	In day DAX	In day FTSE	Over-night FTSE	Over-night DAX	In day DOW
CET-View	**Differential Returns** (Means**)					
Monday	2,4%	1,2%	2,1%	2,3%	4,0%	4,6%
Tuesday	0,6%	-1,9%	-0,2%	-0,02%	3,9%	1,3%
Wednesday	0,7%	-5,3%	-2,6%	0,5%	3,6%	0,7%
Thursday	-3,0%	-3,3%	0,9%	1,3%	1,7%	3,7%
Friday	0,6%	-0,7%	0,0%	3,3%	2,4%	-1,6%
Day:	t***	t		t+1	t+1	t

Quotes by Commodity Systems, Inc. (CSI).via Yahoo © 2000-2002

* US view - the DAX is already in the day by then.
** Scaled to yearly values.
*** "t" = Central European Time (CET).

How does it come to these differences? The european morning media focuse on the question whether the Wallstreet and Tokyo closed "more firmly" or "more weakly". A qualified course analysis of the night is at the disposal to some stock exchange professionals only. That gives a lead to institutional investors. European indices are stronger on Monday because of the "message distance" of the weekend.

F.II. Comparing Index Volatilities

The DAX achieves a performance of only 1.2 per cent in the plus on Mondays. The DJ open achieves some 2.4 per cent and in the day 4.6 per cent.

The scientific discussion focuses on the market efficiency: Which market is more efficient? Which market processes the global information more correctly, more timely and who limps after?

Figure 14: Index Volatilities

from Jan 00	til Jul 02	ex ante
V DJ ex a.	20,6%	computed
V Ftse ex a.	21,3%	//
V DAX ex a.	27,1%	//
V DAX	24,2%	quoted

The Wallstreet is still distant by the ideal also. However, the actual DAX volatility (ex post measured) is on average around per cent points higher than the DJ volatility in the last four years even up to 20 per cent points.

The FTSE volatility is against it about 21,3%. The best efficiency is reached with the dollar euro-rate of exchange with approximately 10 to 14 per cent. That the volatility of the indexes is higher compared with the dollar volatility refers to the fact that trading at the stock exchanges is interrupted during night and weekend. On the other hand the daily volumes in Frankfurt and in London respectively do not allow to carry the price formation out toward the actual intraday performance of many years. That happens during night and during the opening auction in particular.

With the daily DAX, more exactly with the DAX after the opening course, a "trailer effect" is to be observed: If the "engine car" DJ gets around a half meter off the course, the trailer DAX oscillates around a whole meter. (With modern electronic control devices this problem is strongly moderated).

The time lags and the interruptions are disturbing for the reaching of the efficiency. Investors should observe the evening development at the Wallstreet and place sales rather for the opening actions. For purchases one can wait into the afternoon, until the first Wallstreet tendencies are reported.

The transparency of sequential quotations is not available as basic information at night. Only institutes can recognize tendencies by the order book. Perhaps one should brake the sequential quotations - approximately by higher minimum conversions -. But it would be possible to make the pending order books public. The night rest of investors, who live after the Central European time, might be past. But nightshifts have to be done in other industries also.

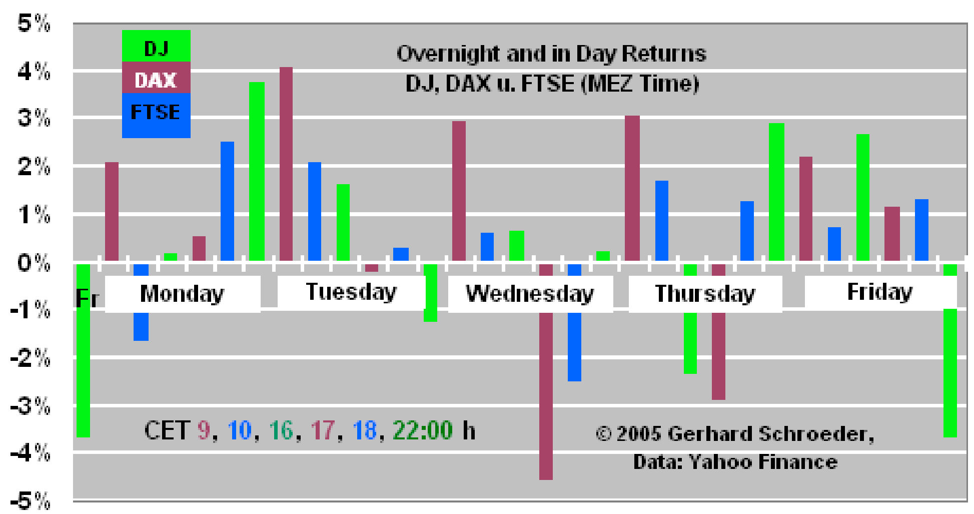
Figure 15: Intra Week Returns

DAX, DOW and FTSE	Gesamt		
90 - 02 CET:	FTSE	DOW	DAX
Monday	4,5%	7,0%	5,3%
Tuesday	-0,9%	1,9%	1,9%
Wednesday	0,5%	1,4%	-1,6%
Thursday	1,3%	0,6%	-1,8%
Friday	4,1%	-0,9%	1,7%
Day:	(t, t+1)	t	(t, t+1)

Data Commodity Systems, Inc. (CSI).via Yahoo © 2000-2002

How does it come that the weekdays fail so differently? Particularly Monday is unsatisfactory. There the DOW carries the largest upswing out (+ 4.6%). The DAX has here also its best value - however comes only on 1,2 per cent. Why does the Tuesday DAX and the FTSE show growing negative or weak impulses until Thursday? In addition further investigations are necessary. There also psychology comes into the play which is a far field.

Figure 16: Synoptic View of DJ, DAX and FTSE Intra Week



A more simplified interpretation might suggest: Buy DJ shares on Friday close (DAX and FTSE on Wednesday close) and sell DJ and FTSE on Monday close (DAX on Tuesday open).

F.III. Methodology and Explanations

To the tables: The performance were determined by comparing the open prices of the last 12 years in Frankfurt, London and Wallstreet with their previous close individually and cross over.

The past night to the Wallstreet was compared with the day of the Frankfurt stock exchange like the events of the day to the Wallstreet are confronted to the following night of DAX and FTSE respectively. Naturally that is expressive only conditionally, because the stock exchanges share some hours per day the same time. The open in Europe and the close in New York are settled independent from the "other side". The stock exchanges run parallel for four or five hours. For each weekday the performance was scaled considering 52 weeks (i.e. 52 Mondays, 52 Tuesdays etc.), in order to determine those on the day and the night resulting contingents at the annual return (approximately 260 stock exchange days).

The night performance is determined from the closing quotation and the opening rate on the next morning. Constantly quoted DJ open were available only starting from1998. The DAX and FTSE daily performance was compared with the preceding DJ overnight performance and the DJ in day performance respectively. The volatility is an iridescent term. It is - undisputed - a kind of indicator to reflect the "temperature" of the current markets. The meaning for the prediction of options and their underlyings has to be disputed.

G. References

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[12] Rainer Schöbel, Jianwei Zhu: „Stochastic Volatility with an Ornstein-Uhlenbeck [OU] Process: An Extension", European Finance Review 1999 (submitted)

[13] Zhou, Chunseng "A Jump-Diffusion Approach to Modeling Credit Risk and Valuing Defaultable Securities." Board of Governors of the Federal Reserve System Working Paper No. 97-15. 1997

[1] An easy introduction into the mathematics of advanced models is published by Baxter, Rennie (1998) and (similar, however less easy) by Bingham, Kiesel (1998)

[2] Aït-Sahalia (1998 et al.) is an excemption, who provides his detailed sample data (daily S&P Futures during 1993) under his home page.

[3] It is not suffiecient if the the computation is offered as an options calculator, as a „Black Box" only. This would make the computation of large samples practically impossible and it is not sufficient from the scientific point of view.

[4] Nandi (1988) provides an overview of models, see also Zhuo (1997).

[5] In cases diagrams had to be changes the DEM was replaced by Euro.

[6] Duffie, Pan, and Singleton (1999) use in their model combined jumps of the volatility and of the returns varying in intensity and time.

[7] [1c] Without constant C3 the formula isn't necessarily hyperbolic. One could find the formula just by chance. However, when replacing the percent scale by a scale x/mean(x) -1 the relation to hyperbolic curves would become evident again and a constant would be part of the root expression.

[8] The phenomenon was decribed first by Mandelbroijt 1963 (S. 418) - relatively vague - : „large changes tend to be followed by large changes - of either sign - and small changes ... by small changes". It could be applied as a rule for absolute returns instead of volatility as well.

[9] Mean Reversion is discussed by Schöbel, Zhu 1999 S. 7

[10] The Deutsche Mark (DEM) was still valid during research. For comparisons with the Euro please use 1 Euro = 1.95583 DEM. Properties like the Euro volatility didn't change despite some expectations that the Euro volatility would

[11] Rady (1995) included a bandwidth limitation thus allowing a generalization of the Black and Scholes model. The underlying limits of the B&S model are simply 0 and ∞, in Rady's model given values in beween, sign bias effects included. However, 0 and ∞ are not values allowed, once a lognormal distribution is assumed. The proof in section BIII, extrema, is similar for the hyperbolic distribution as well as for the lognormal one.

[12] The Clustering rule should be seen complementary to the Mean Reversion rule since once the volatile returns are consumed Clustering results in an smoothing effect also.

[13] The Brownian Motion is well described along with the model of B&S. However the details of stock exchange procedures and OTC are not covered. An exemption is Aït-Sahalia (1998).

[14] The Brownian Motion is three dimensional. Considering the room between the preparation glasses as a plane it is two dimensional.

[15] NASDAQ's trading system Opti-Mark will allow to split large orders into different volumes at different prices to make them less visible. (FAZ, 1.10.1999, P. 25)

[16] ODE / PDE: Abbreviation for ordinary (gewöhnliche) / partial (partielle) differential equations.

[17] It is questionnable to compute a daily volatility deviating from the mean (or expected value) when the formula is allowed for stable volatility only. Clustering is the only rational for doing so but it happens during 67 weeks (out of 522) only.

[18] Gerhard Schroeder Empirical Contributions to Optionpricing analyzing Black and Scholes and other Models, 2005, p. 17 -18, provided by Economics Working Paper Archive EconWPA

[19] A. Pitts, Warwick Business School, did a survey in 1999 including 100 directors of major banks in the UK: Only four of them heard of applied mathematic terms like „real-option analysis" or CAPM etc.(The Economist, Aug 14, 1999, p. 68)

[20] Pls see note 16.