

Valuing Volatility Spillovers

George Milunovich

Division of Economic and Financial Studies

Macquarie University

Sydney

gmilunov@efs.mq.edu.au

Susan Thorp

School of Finance and Economics

University of Technology

Sydney

susan.thorp@uts.edu.au

May 2005

Abstract

We measure the reduction in realized portfolio risk that can be achieved by allowing for volatility spillover in forecasts of equity covariance. The conditional second moment matrix of equity returns for pairs of major European equity markets is estimated via two asymmetric dynamic conditional correlation models (A-DCC): the unrestricted model includes volatility spillover effects and the restricted model does not. Data are daily returns on the London, Frankfurt and Paris equity market price indices synchronized at London 16:00 time. Covariance forecasts from the restricted and unrestricted models are combined with assumed expected returns to compute efficient three-asset portfolios (two equity indices and the risk-free asset). The impact of expected return choice on out-of-sample portfolio efficiency is minimized via the polar co-ordinates method of Engel and Colacito (2004), which allows expected equity returns to span all relatives. Out-of-sample realized portfolio returns and variances from efficient portfolios are computed and tested. Allowing for volatility spillover effects produces small, statistically significant reductions in portfolio risk. Portfolio standard deviations for the unrestricted model are at most one per cent smaller than standard deviations for restricted models. Significant risk reductions persist across daily, weekly, and monthly rebalancing horizons. Tests for second degree stochastic dominance indicate that realized returns from portfolios based on the volatility spillover model would be preferred by risk averse agents.

1. Introduction

A key ingredient in successful portfolio selection is an accurate prediction of covariance between asset returns. Better forecasts of second moments mean lower portfolio volatility, which benefits investors. However volatility patterns in financial time series are complex, and forecasters face the challenge of finding parsimonious, positive definite and stationary models of time-varying covariances, while still accounting for the salient features of the data.

Empirical studies of time-varying second moments are plentiful but fewer studies actually measure how much investors might profit from improved predictions. Whether realized portfolio efficiency is improved by a new approach to covariance forecasting seems an obvious question, and also suggests a method of forecast evaluation. Consequently, the aim of this study is to incorporate recent advances in volatility modelling into simple portfolios, and quantify how much benefit flows to investors.

We focus on volatility spillover, that is the transmission of turbulence from market to market. Volatility spillover occurs when changes in price volatility in one market produce a lagged impact on volatility in other markets, over and above local effects. Such patterns appear to be widespread in financial markets. There is evidence for spillovers between equity markets (see for example Hamao, Masulis and Ng 1990 and Lin, Engle and Ito 1994), bond markets (Christiansen 2003), futures contracts (Abrhyanekar 1995, Pan and Hsueh 1998), exchange rates (Engle, Ito and Lin 1990, Baillie and Bollerslev 1990), equities and exchange rates (Apergis and Rezitis 2001), various industries (Kaltenhauser 2002), size-sorted portfolios (Conrad, Gultekin and Kaul 1991), commodities (Apergis and Rezitis 2003), and swaps (Eom, Subrahmanyam and Uno 2002). Despite the interest that investors might have in these pervasive spillover effects, we are not aware of any study that explicitly measures their importance for efficient asset allocation. It therefore is natural to ask whether including spillover effects in covariance forecasts will generate significantly lower realized portfolio variance.

An important first step towards answering this question is to construct covariance models which comprehensively capture the data while isolating the impact of volatility

spillover. In this study, investors hold mean-variance portfolios allocated among the risk-free asset and equities in two major European stock markets.¹ Consequently, portfolio construction depends on forecasts of the bivariate conditional covariance matrix of stock market returns. To isolate the impact of volatility spillover on portfolio efficiency, we estimate nested forecasting models of returns volatility via (scalar) versions of the Asymmetric Dynamic Conditional Correlation (A-DCC) model (Cappiello, Engle and Sheppard 2004) over the first part of the data. The benchmark (restricted) model thus captures time-varying volatility and correlation, including asymmetric effects, but omits volatility spillover terms, which are then added to the unrestricted model.²

The second step is to make one-, five-, and twenty-step-ahead forecasts of conditional covariances over remaining data and calculate optimal portfolio weights at each forecast. Mean-variance portfolio weight calculations depend on expected returns as well as expected covariances, and it is well known that out-of-sample portfolio performance is often degraded by a poor choice of expected returns. A new approach, developed by Engel and Colacito (2004), offers a method for minimizing the impact of expected return choice on out-of-sample portfolio efficiency. In a two-asset portfolio, relative, rather than absolute, returns matter to optimal portfolio weighting, thus by computing weights for all possible returns ratios, one can identify the effects of covariance forecasting separately from returns forecasting. We employ the Engle and Colacito approach in order to better isolate volatility spillover effects from the influence of expected returns.

Finally, from optimal weights we compute realized portfolio returns and variances, and then test any advantages of the volatility spillover formulation over the benchmark. Section 5 below reports standard deviations of optimal portfolio returns, Diebold and Mariano (1995) tests of forecasting performance, and gives evidence of significant second degree stochastic dominance among portfolios via a time-series adaptation of the Barrett

¹Equity returns are proxied by the daily change in the FTSE 100 (London), DAX 30 (Frankfurt) and CAC 40 (Paris) price indices, in US dollars (USD). All price indices are synchronized at London 16:00 time and the estimation sample runs from 2 June 1992 to 28 December 2001, with remaining observations saved for forecasting. A fuller description is given in section 5.

²Volatility asymmetry was first introduced to the financial literature by Black (1976), and has since become a well-documented feature of volatility patterns hence a failure to account for asymmetries may result in distorted estimates of volatility spillover. See, for example, Nelson (1991), Koutmos (1992), and Poon and Taylor, (1992).

and Donald (2003) tests.

To summarize results, estimation of the benchmark and alternative A-DCC models indicates significant volatility spillover effects from Paris and Frankfurt to London, and from Frankfurt to Paris. Parameter estimates from London to the other markets are positive, but have large standard errors. Tests of realized portfolio returns show that accounting for volatility spillover makes a small but significant difference to portfolio efficiency, generating less portfolio risk for a given return than the benchmark model. The efficiency gains arising from modelling volatility spillover range from a 0.2 to 1 per cent reduction in portfolio standard deviations. In terms of a portfolio returning 10 per cent per year, this represents a risk-adjusted improvement of at most 0.1 per cent. However tests of forecasting performance confirm that risk reductions are statistically significant at all forecasting horizons. In addition, stochastic dominance tests point to significant improvements in investor utility arising from volatility spillover predictions for investors in two of the three possible equity pairings.

Overall, the impact of volatility spillover may not be large by commercial standards, but it is statistically significant, and since including volatility spillover effects in the portfolio selection process does not incur any additional transactions costs, even small gains represent improvement to investors.

The next section (Section 2) reviews some of the relevant features of volatility spillover literature. The benchmark and alternative models and estimation method are described in Section 3. Portfolio construction is developed in Section 4. Section 5 presents an outline of the data and estimated parameters, followed by tests comparing the performance of portfolios constructed from the benchmark and volatility spillover models. Section 6 concludes.

2. Literature Review

Interest in volatility spillovers across international equity markets intensified after the October 19, 1987 stock market crash when a sharp drop in the US equity markets ap-

peared to have a widespread ‘domino effect’ across international markets. In an attempt to explain this, King and Wadhvani (1990) put forward a ‘market contagion’ hypothesis, arguing that stock price turbulence in one country is partly driven by turbulence in other countries, beyond the influence of ‘fundamentals’. Identifying and testing the transmission of turbulence between markets has been the focus of the volatility spillover literature.

Early studies on volatility spillovers typically focus on developed country equity markets, and the transmission of volatility from larger to smaller country markets in particular. For example, unidirectional volatility spillovers from US markets to the UK and Japan, and the UK to Japan, are found by Hamao, Masulis and Ng (1990), while Theodossiou and Lee (1993) argue for additional transmissions from the US market to Canada and Germany.

Further, the large-small country effect appears to be mirrored *within* equity markets on a firm-size level. Studies document volatility spillover from large to small firms (Conrad, Gultekin and Kaul 1991, and Reyes 2001), although bad news may cause spillover in the reverse direction as well (Pardo and Torro 2003).

More recent studies investigate spillover effects between developed and emerging markets, and among emerging markets themselves. A typical finding (see, for example, Wei et al, 1995) is that volatility transmits from developed to emerging markets, and that the smaller, less developed markets are likely to be more sensitive to transmitted shocks.

Geographic locality, regardless of market size, is also likely to be a factor in volatility spillover. Bekaert and Harvey (1997) are able to distinguish between local and global shocks, studying volatility spillovers across emerging stock markets. Regional factors are important for Pacific Basin markets, over and above the world-market effects of spillovers from the US (Ng 2000). In a related study, Miyakoshi (2003) goes further, arguing that regional effects are stronger than world market influence for markets in the Asian region.

Europe represents a particularly interesting geographic area for volatility spillover studies since it encompasses a number of developed markets with common economic and financial features, and overlapping trading hours. Thirteen European markets and the

US are studied by Baele (2003), who decomposes volatility spillovers into country specific, regional and world shocks. (The model also allows for regime switches in the spillover effects.) Both regional and world effects are reported as significant. Further, spillovers appear to have intensified over the 1980s and 1990s, with a more pronounced rise among European Union (EU) markets. In a related study, Billio and Pelizzon (2003) find that volatility spillovers to most European stock markets from both the world index and the German index have increased since the European Monetary Union (EMU) came into effect.

The importance of regional spillovers for Europe is not restricted to equity markets. Testing for volatility spillover effects in European bond markets, Christiansen (2003) finds evidence of spillover from both the US and Europe to individual country's bond markets. The European volatility spillover effects are stronger than the US volatility spillovers in bond markets as in equity markets.

An important methodological issue for transmission studies is whether volatility spillovers can be identified separately from lags in information transfer due to non-overlapping trading hours between markets. For example, in the foreign exchange market Engle, Ito and Lin (1990) investigate volatility spillovers across Tokyo and New York for the Yen/USD exchange rate. Since these two markets trade a common security, but operate in different time zones, the authors argue for a 'Meteor Shower' effect, whereby surprises in one market while the other is closed show up as soon as the second market opens. In addition, by studying open-to-close against close-to-open equity returns, Lin, Engle and Ito (1994) find that shocks to New York daytime equity returns are correlated with overnight Tokyo returns and vice versa. In the latter case they conclude that information revealed during the trading hours of one market has a simultaneous impact on the returns of the other market. Thus these two studies exemplify the need distinguish between contemporaneous shocks that appear lagged because of staggered trading hours, and real-time lead-lag effects between security markets (Martens and Poon 2001).

To summarize, existing empirical research provides ample evidence of volatility spillovers both across and within equity markets. Our choice of markets (London, Frankfurt and

Paris) facilitates investigation of larger-smaller market effects,³ and the interesting intra-regional influences which appear to be strengthening in Europe. In addition, we restrict the study to synchronous price observations, avoiding the confusion which can arise from trading lags. As well as estimating volatility spillover effects in the A-DCC framework, the remainder of this paper addresses the issue of how important these volatility spillovers are for mean-variance investors. The next section describes the benchmark (no volatility spillover) and alternative (volatility spillover) models used for forecasting volatilities and computing optimal portfolios.

3. Model Specification and Estimation

To effectively identify volatility spillover effects, other features of time-varying second moments should be well modelled in both the restricted and unrestricted models, since failure to properly capture other features may lead to biases in estimated coefficients and poor forecasts. The bivariate M-GARCH models set out in this section capture time-varying volatility and asymmetric effects while also allowing correlations between security returns to vary over time. At the same time, the tendency to long-run stationarity is captured via variance targeting. Models are built over pairs of equity market returns series, London-Frankfurt, London-Paris, and Frankfurt-Paris.

Initially, two variance equations are formulated for each market return series in Glosten Jagannathan and Runkle (GJR) (1,1,1) form, one with, and one without, volatility spillover effects. Hence the variance equations are similar to GARCH (1,1) processes with the addition of an asymmetry term. (Asymmetry refers to the observed tendency of financial markets to respond more to negative than positive price shocks, with volatility more likely to increase in the face of bad news.) Estimates of conditional standard deviations generated by these variance equations are then used to standardize the (demeaned) data and then to estimate conditional correlation matrices,⁴ as scalar asymmetric generalizations of the Dynamic Conditional Correlation (DCC) model of Engle (2002). DCC

³The percentages of the world stock market capitalization attributed to the UK, France and Germany as of January 1998 were 10.5%, 3.8% and 4.7% respectively, MSCI (1998).

⁴As in the two-step method of Engle and Sheppard (2001).

models allow time-varying conditional correlations. The Asymmetric-DCC formulation (described and applied recently in Cappiello Engle and Sheppard, 2004) goes a step further by not only capturing symmetric responses of conditional correlation to volatility shocks, but also increases in conditional correlations during periods of negative returns. Finally variances and correlation estimates are combined to compute a multi-variate conditional covariance matrix.

3.1 Model

Consider a vector of returns for two equity markets, $\mathbf{r}_t = [r_{1t} \ r_{2t}]'$ such that

$$\mathbf{r}_t = \mathbf{c} + \mathbf{u}_t \quad (1)$$

$$\mathbf{u}_t = \mathbf{D}_t \boldsymbol{\varepsilon}_t, \quad (2)$$

where \mathbf{c} is the unconditional mean vector of \mathbf{r}_t , \mathbf{D}_t contains conditional standard deviations on the main diagonal and zeros elsewhere, $\boldsymbol{\varepsilon}_t$ are the innovations standardized by their conditional standard deviations, and Ψ_{t-1} represents the conditioning information set at time t such that

$$\boldsymbol{\varepsilon}_t | \Psi_{t-1} \sim (0, \mathbf{R}_t). \quad (3)$$

Observe that the conditional correlation matrix of the standardized innovations is $E_{t-1}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \mathbf{R}_t$.

The conditional covariance matrix for the returns vector \mathbf{r}_t can therefore be specified as

$$\begin{aligned} \text{Var}(\mathbf{r}_t | \Psi_{t-1}) &= \text{Var}_{t-1}(\mathbf{r}_t) = E_{t-1} [(\mathbf{r}_t - \mathbf{c})(\mathbf{r}_t - \mathbf{c})'] \\ &= E_{t-1} [\mathbf{D}_t \boldsymbol{\varepsilon}_t (\mathbf{D}_t \boldsymbol{\varepsilon}_t)'] \\ &= E_{t-1} [\mathbf{D}_t \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \mathbf{D}_t], \end{aligned}$$

and since D_t is a function only of information at $t - 1$, one can write the conditional

covariance matrix as

$$\begin{aligned}\mathbf{H}_t &\equiv \text{Var}_{t-1}(\mathbf{r}_t) \\ &= \mathbf{D}_t E_{t-1}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') \mathbf{D}_t\end{aligned}\tag{4}$$

$$= \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t.\tag{5}$$

With this structure in mind we turn to the elements of the \mathbf{D}_t matrix, the conditional standard deviations, where

$$\mathbf{D}_t = \begin{bmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{bmatrix}.\tag{6}$$

As outlined above, we use two different specifications of conditional variances to capture the effects of asymmetric dynamics and volatility spillover separately:

1. Asymmetric GJR(1,1,1):

$$h_{ii,t} = \omega + (\alpha + \delta I_{t-1}) u_{ii,t-1}^2 + \beta h_{ii,t-1}\tag{7}$$

$$\text{where } I_t = \begin{cases} 1 & | \ u_t < 0 \\ 0 & | \ u_t \geq 0 \end{cases}.$$

2. Asymmetric GJR(1,1,1) with volatility spillover:

$$h_{ii,t} = \omega + (\alpha + \delta I_{t-1}) u_{ii,t-1}^2 + \beta h_{ii,t-1} + \gamma u_{jj,t-1}^2\tag{8}$$

$$\text{where } I_t = \begin{cases} 1 & | \ u_t < 0 \\ 0 & | \ u_t \geq 0 \end{cases} \text{ and } ii \neq jj.^5$$

Next we model the conditional correlation matrix \mathbf{R}_t following Cappiello, Engle and Sheppard (2004). From (1) and (2) above, one can see that the standardized residuals

⁵Engle (2002) shows that a Bollerslev-Wooldridge (1992) covariance matrix gives consistent standard errors for the estimates.

can be calculated as

$$\mathbf{D}_t^{-1} \mathbf{u}_t = \boldsymbol{\varepsilon}_t, \quad (9)$$

where the elements of \mathbf{D}_t^{-1} have been derived from estimated equations for each of the formulations for $h_{ii,t}$ above. By using these standardized residuals we are able to estimate a conditional correlation matrix of the form:

$$\begin{aligned} \mathbf{R}_t &= \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1} \\ \mathbf{Q}_t &= \bar{\mathbf{Q}}(1 - \phi - \eta) - \varphi \bar{\mathbf{m}} + \phi \boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}'_{t-1} + \varphi \mathbf{m}_{t-1} \mathbf{m}'_{t-1} + \eta \mathbf{Q}_{t-1} \end{aligned} \quad (10)$$

where ϕ , φ and η are scalar parameters. $\mathbf{Q}_t^* = [q_{ii,t}^*] = [\sqrt{q_{ii,t}}]$ is a diagonal matrix with the square root of the i^{th} diagonal element of \mathbf{Q}_t on its i^{th} diagonal position. The vector $\mathbf{m}_t = I[\boldsymbol{\varepsilon}_t < 0] \circ \boldsymbol{\varepsilon}_t$ (where \circ is the Hadamard product) isolates observations where standardized residuals are negative. Notice that \mathbf{Q}_t resembles a GJR(1,1,1) process in the standardized volatilities. Finally, we implement variance targeting, where $\bar{\mathbf{Q}} = \frac{1}{T} \sum \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t$ and $\bar{\mathbf{m}} = \frac{1}{T} \sum \mathbf{m}_t \mathbf{m}'_t$ to enforce stationarity.

Combining estimates for (6) and (10) results in a conditional covariance matrix for the returns vector \mathbf{r}_t which can be used, along with a vector of expected returns, to predict optimal portfolio weights t periods ahead:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t. \quad (11)$$

3.2 Estimation Method

The model is estimated in two steps following Engle (2002). Assuming that the standardized residuals $\boldsymbol{\varepsilon}_t$ are conditionally normally distributed so that $\boldsymbol{\varepsilon}_t | \boldsymbol{\Psi}_{t-1} \sim N(0, \mathbf{R}_t)$, the log likelihood function for the vector of returns \mathbf{r}_t , can be expressed as

$$\mathcal{L} = -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + \log |H_t| + \mathbf{u}'_t H_t^{-1} \mathbf{u}_t). \quad (12)$$

Now let the mean parameters, \mathbf{c} , and the univariate GARCH parameters in \mathbf{D}_t be

represented by ψ , and the conditional correlation parameters in \mathbf{R}_t by ζ . The log likelihood can be written as the sum of a volatility part and a correlation part:

$$\mathcal{L}(\psi, \zeta) = \mathcal{L}_V(\psi) + \mathcal{L}_C(\zeta|\psi), \quad (13)$$

where the volatility term is

$$\mathcal{L}_V(\psi) = -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + 2 \log |\mathbf{D}_t| + \mathbf{u}'_t \mathbf{D}_t^{-2} \mathbf{u}_t), \quad (14)$$

and the correlation component is

$$\mathcal{L}_C(\zeta|\psi) = -\frac{1}{2} \sum_{t=1}^T (-\boldsymbol{\varepsilon}'_t \boldsymbol{\varepsilon}_t + \log |\mathbf{R}_t| + \boldsymbol{\varepsilon}'_t \mathbf{R}_t^{-1} \boldsymbol{\varepsilon}_t). \quad (15)$$

The procedure is further simplified by recognizing that the volatility part of the log likelihood is just the sum of the individual univariate GARCH likelihoods:

$$\mathcal{L}_V(\psi) = -\frac{1}{2} \sum_{t=1}^T \sum_{i=1}^n \left(\log(2\pi) + \log(h_{i,t}) + \frac{u_{it}^2}{h_{i,t}} \right).$$

The two-step estimation method involves maximizing each univariate GARCH term separately, standardizing the returns by estimated standard deviations and then jointly estimating elements of \mathbf{R}_t by maximizing the correlation component of the log likelihood $\mathcal{L}_C(\psi, \zeta)$. We maximize log likelihoods numerically using the Max SQP procedure in OX 3.4. This procedure implements a sequential quadratic programming technique to maximise a non-linear function subject to non-linear constraints.⁶

Although the assumption of normality in $\boldsymbol{\varepsilon}_t$ is convenient for estimation, it is not necessary for consistency, since quasi-maximum likelihood arguments apply as long as the conditional mean and variance equations are correctly specified (Hamilton, 1994, p.126). However the standard errors need to be adjusted according to the method described for the univariate GARCH volatility equations. Standard errors for the correlation parameters require a more complicated process explained in Engle (2002).

⁶See OX documentation for more information.

4. Portfolio Allocation

This study aims to measure the value of information on volatility spillover using simple mean-variance portfolios. The individual variance formulations described by equations (7) and (8), in combination with the A-DCC correlation estimates, generate two sets of conditional covariance matrices for each pair of market returns, $\{H_t^i\}_{i=1}^2$, where model $i = 2$ includes volatility spillover effects and model $i = 1$ does not. While these alternative characterizations of volatility dynamics may be interesting in themselves, the economic value of any covariance forecast ultimately shows up in better investment outcomes.

In the past, portfolio-performance-based tests have been constrained by the need to simultaneously choose expected returns and variances, so that researchers have been unable to isolate the impact of covariance prediction from mean prediction. However recent work by Engle and Colacito (2004) outlines a method for fixing a range of assumed returns (for a two-asset portfolio) which can isolate the value of covariance prediction from return prediction. By applying their method to create portfolios from the A-DCC models, we can test for the impact of volatility spillover on portfolio efficiency without jointly testing a hypothesis about expected returns.

This section outlines the portfolio allocation problem and the method for fixing portfolio returns.

4.1 Minimum Variance Portfolios

The key feature of the original Markowitz (1959) model is a recognition that covariance between security returns can be exploited to optimally reduce portfolio risk. The (myopic) investor will choose portfolio weightings for each asset to minimize variance subject to a required return.

$$\min_{w_t} \mathbf{w}_t' \mathbf{H}_t \mathbf{w}_t \tag{16}$$

$$s.t. \quad \mathbf{w}_t' \boldsymbol{\mu} = \mu_o \tag{17}$$

which produces an optimal weighting vector of the form:

$$\mathbf{w}_t = \frac{\mathbf{H}_t^{-1}\boldsymbol{\mu}}{\boldsymbol{\mu}'\mathbf{H}_t^{-1}\boldsymbol{\mu}}\mu_o, \quad (18)$$

where $\boldsymbol{\mu}$ is an assumed vector of expected returns to be described below, and μ_o is the required rate of return to the portfolio, here set to unity. \mathbf{H}_t is the expected (forecasted) covariance matrix of returns. We forecast \mathbf{H}_t and rebalance the portfolio on daily, weekly (5 days) and monthly (20 days) basis, using the A-DCC models described above, testing to see if the impact of volatility spillover tapers off over longer rebalancing horizons.

Notice that (following Engle and Colacito 2004) we do not impose full investment or short-sales constraints. Omitting these constraints implies, firstly, that any wealth not accounted for by \mathbf{w}_t will be invested in the risk-free (assumed zero return) asset, and, secondly, that the weight vector may include negative values. A useful feature of this choice of optimal weighting vector is that the required return is held fixed in every portfolio, allowing direct comparison between alternative covariance predictions. Consider the contrast between (18) and $\mathbf{w}_t^T = \frac{\mathbf{H}_t^{-1}\boldsymbol{\mu}}{\boldsymbol{\mu}'\mathbf{H}_t^{-1}\boldsymbol{\mu}}$, the ‘tangency’ portfolio. The tangency portfolio is fully invested in risky assets, but does not hold required return constant, so that comparisons between tangency portfolios depend on return to risk ratios. By contrast, the weight vector in (18) allows us to compare portfolio standard deviations without comparing returns.

It can be shown that, for a given required rate of return, the portfolio with the smallest realized standard deviation will be the portfolio constructed from the most accurate covariance forecast (Engle and Colacito 2004). So that if σ^* is the portfolio standard deviation achieved using the true covariance matrix, and $\hat{\sigma}$ is the standard deviation from an inefficiently estimated covariance matrix, then σ^* will be less than for $\hat{\sigma}$, such that

$$\frac{\sigma^*}{\mu_o} < \frac{\hat{\sigma}}{\mu_o}. \quad (19)$$

Consequently, if including volatility spillover effects improves conditional covariance forecasts then portfolios constructed from the better forecasts will have lower realized stan-

dard deviations.

4.2 Expected Returns

Performance tests of mean-variance portfolios are typically joint tests of both expected returns and variances,⁷ but our aim is to test the effects of volatility spillover in isolation from choices of expected returns. To get around the problem of jointly testing a specific return prediction, we calculate portfolios for each A-DCC model over a complete range of expected returns pairs. In a two-asset portfolio what matters to allocation is the relative size of the elements of the expected returns vector. In fact, one can span all possible relatives by choosing pairs of expected returns $\boldsymbol{\mu} = (\sin \frac{\pi j}{20}, \cos \frac{\pi j}{20})$, where $j \in \{0, \dots, 10\}$. The resulting values (listed in Table 1) range from zero to one for each asset, including a mid-point where the expected return of assets are equal. The next step is to compute optimal portfolio weights for each of these eleven expected return pairs $\{\boldsymbol{\mu}_t^k\}_{k=1}^{11}$ in combination with forecast covariance matrices, $\{H_t^i\}_{i=1}^2$. If one conditional covariance model performs better for all eleven expected returns relatives, we can be confident that it is a better model for any choice of return.

⁷Earliest studies of mean-variance portfolio allocations use historical sample means as expected returns, despite the fact that, for most financial data, they are measured with low precision. The finely-tuned optimisation process implied by (16) and (17) is, however, sensitive to small changes in input vectors, and will amplify any measurement errors in predicted returns. Common responses to this problem include either imposing *ad hoc* constraints on the weights vector, or Bayesian adjustment of the means and/or covariances according to a plausible prior (See Jorion 1985 and Connor 1997 for examples). Adjustments which moderate the differences between individual asset returns tend to improve the out-of-sample performance of optimized portfolios.

Table 1: Pairs of Expected Returns

j	$\mu(1)$	$\mu(2)$	θ
0	0.000	1.000	0
1	0.156	0.988	0.1
2	0.309	0.951	0.2
3	0.454	0.891	0.3
4	0.588	0.809	0.4
5	0.707	0.707	0.5
6	0.809	0.588	0.6
7	0.891	0.454	0.7
8	0.951	0.309	0.8
9	0.988	0.156	0.9
10	1.000	0.000	1

Nevertheless, a single summary measure is always useful. We create a single point estimate of portfolio risk across all returns by combining these eleven different standard deviations using the empirical Bayesian approach set out in Engle and Colacito (2004). Briefly, non-overlapping sample means (using 40 observations) $\{\bar{\mu}_1^l, \bar{\mu}_2^l\}_{l=1}^L$, are calculated from the sample data for each market pairing. Any mean pair where either value is negative is dropped, leaving a subset of size $d = 1, \dots, D$. From this sample we back out D values of $\theta_d = \frac{2}{\pi} a \cos\left(\frac{\bar{\mu}_{2,d}}{\sqrt{\bar{\mu}_{2,d}^2 + \bar{\mu}_{1,d}^2}}\right)$ and use these values of θ to calculate maximum likelihood parameters of the Beta distribution \hat{a} and \hat{b} . Finally, the empirical probability of each pair of the eleven polar co-ordinate returns $\boldsymbol{\mu}^k = \left(\sin \frac{\pi j}{20}, \cos \frac{\pi j}{20}\right)$ can be inferred by computing the value

$$\Pr(\theta = \theta_j) = \frac{1}{\Upsilon} \frac{\theta_j^{\hat{a}-1} (1 - \theta_j)^{\hat{b}-1}}{\int_0^1 t^{\hat{a}-1} (1-t)^{\hat{b}-1} dt} \quad (20)$$

(where $\frac{1}{\Upsilon}$ is a normalizing constant) for each pair of markets.

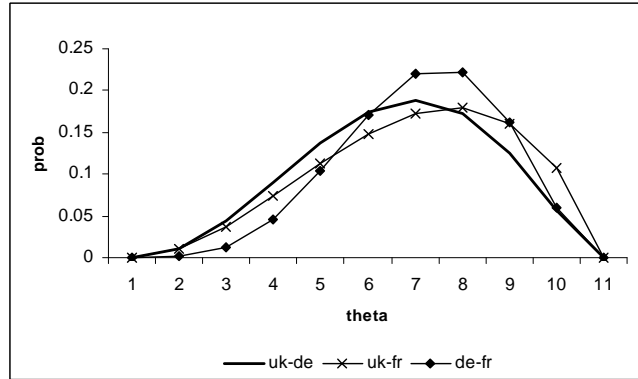
Probability density functions for θ computed from this procedure are graphed in Figure 1, with all showing some skewness. Skewness in the distribution for the London-Paris distribution, for examples, indicates that returns are likely to be higher in London than in Paris. A similar observation applies to Frankfurt and Paris, with the London-

⁸Recall that

$$\int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.$$

Frankfurt pair likely to be more equal. In each case, a most probable value for θ , and hence most probable expected returns relative is given by the maximum of the density function. However, all but the most extreme values of θ have some weight in the density, and focusing on the most likely value may be misleading.

Figure 1: Probability Density Functions



Notes: Empirical Bayesian estimate of the probability of assumed expected returns such that each pair is $\sin(\frac{\pi}{2}\theta)$ and $\cos(\frac{\pi}{2}\theta)$. Numerical values are listed in Table 1.

5. Empirical Results

In this section we report the main empirical results: firstly the data and model estimation results, then realized standard deviations for optimal portfolios over each market pairing and specification, forecasting one, five, and 20 days ahead. We go on to compare the relative efficiency of the volatility spillover model for each rebalancing horizon by calculating portfolio standard deviations and testing for improvement using Diebold and Mariano (1995) tests. Finally we extract portfolio returns for the one step ahead forecasts and search for second degree stochastic dominance relations of the volatility spillover specification against the benchmark.

5.1 Data and Estimation

The data are daily returns computed from three major European stock market price indices: FTSE 100 for London; DAX 30 from Frankfurt; and CAC 40 from Paris. Returns are calculated as log differences and do not include dividends. Trading hours for the Lon-

don, Frankfurt and Paris stock exchanges overlap imperfectly, so to ensure synchronous prices we take index values at London 16:00 time.⁹ The sample runs from 2 June 1992 to 4 February 2005.

The importance of synchronous data for studies of daily conditional correlation and volatility spillover was pointed out by Martens and Poon (2001). Substantial misestimation of returns correlation and spillovers can result from a failure to account for timing differences at the daily level. Martens and Poon (2001) show that correlations will be under-estimated, and estimated spillover patterns changed, if non-synchronous daily data are used in correlation models. By synchronizing prices we ensure that estimated spillovers and correlations more accurately expose real-time interactions, rather than representing lags in information flows, misalignments in trading, or mismatched data collection.

In addition we assume that our investor uses a single currency to value portfolio returns and take all values in US dollars. No currency hedging is implemented.

Key features of the data sample are reported in Table 2. Average returns are highest for the DAX 30 index, which also displays the largest standard deviation and degree of skewness. The FTSE 100 has annualized returns around two per cent lower than the DAX 30 and the least variance of the three markets. All three daily returns series show considerable non-normality manifested in negative skewness and excess kurtosis. Average skewness is -0.11, and kurtosis, 5.35.

⁹Datastream supplies London 16:00 data for a group of major markets. Codes for the series described here are FOOTC16(PI) , DAXIN16(PI), and CAC4016(PI).

Table 2: Summary Statistics- Daily Stock Index Returns, % p.a.

	FTSE 100	DAX 30	CAC 40
Mean	5.10	7.64	5.92
Median	6.71	21.25	12.28
Std. Dev.	18.35	25.23	23.02
Skewness	-0.01	-0.23	-0.08
Kurtosis	5.42	5.70	4.92
Jarque-Bera	807.99	1031.18	512.68
Observations	3310	3310	3310

Notes: Daily returns calculated from price indices synchronized at London 16:00 time, 1 June 1992 to 4 February 2005. All indices are in USD, unhedged. Data supplied by Datastream.

Correlations shown in Table 3 are above 0.65, with the greatest correlation between the two continental markets, Frankfurt and Paris, at 0.77.

Table 3: Sample Correlations - Daily Stock Index Returns

	FTSE 100	DAX 30	CAC 40
FTSE 100	1.00		
DAX 30	0.66	1.00	
CAC 40	0.69	0.77	1.00

A graph of the daily returns in Figure 2 clearly shows clusters of volatility, where groups of large or small changes persist for a number of periods. More frequent periods of turbulence are evident since 1998 and volatility patterns are clearly related, as might be expected among such closely-aligned equity markets.

Figure 2: Daily Stock Index Returns, 2 June 1992 – 4 February 2005.

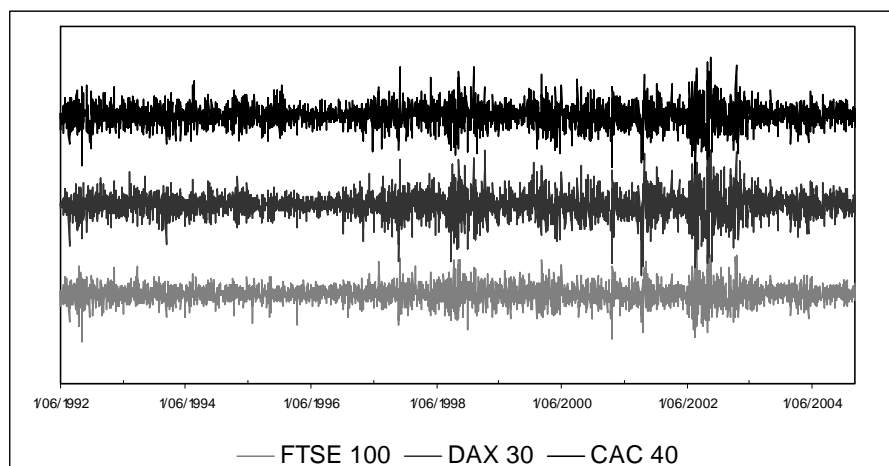


Table 4 presents autocorrelation functions of squared returns series. One can see that all three squared series are strongly autocorrelated with statistically significant Q statistics, calculated up to the fifth lag. Dependence in squared residuals is indicative of autocorrelated volatilities and lends support to our earlier remarks about volatility clustering.

Table 4: Autocorrelation Functions of Squared Daily Returns

	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$Q(5)$
Sqr(FTSE 100)	0.157	0.286	0.248	0.160	0.249	847.67
Sqr(CAC 40)	0.151	0.230	0.146	0.175	0.128	478.33
Sqr(DAX 30)	0.164	0.241	0.195	0.174	0.158	589.19

Daily returns to the London, Frankfurt and Paris equity markets are highly correlated and non-normal, exhibiting time-varying and inter-related volatility patterns.

5.2 Estimated Parameters

Table 5 reports estimates for a total of six bivariate A-DCC models: for each of the three pairs of returns series (London-Frankfurt, London-Paris and Frankfurt-Paris) we compute a benchmark without volatility spillover and an alternative with volatility spillover. The models were estimated using the first 2500 observations of the 3310 size sample, leaving

the remaining 810 observations for testing. The estimation period runs from 2 June 1992 to 28 December 2001, and predictive power for portfolio formation is tested over the three years from 2002.

Parameter estimates and standard errors for the variance equations are reported in the top portion of Table 5, and estimates of the parameters of the correlation matrices in the lower portion.

Table 5: Parameter Estimates, A-DCC Models.

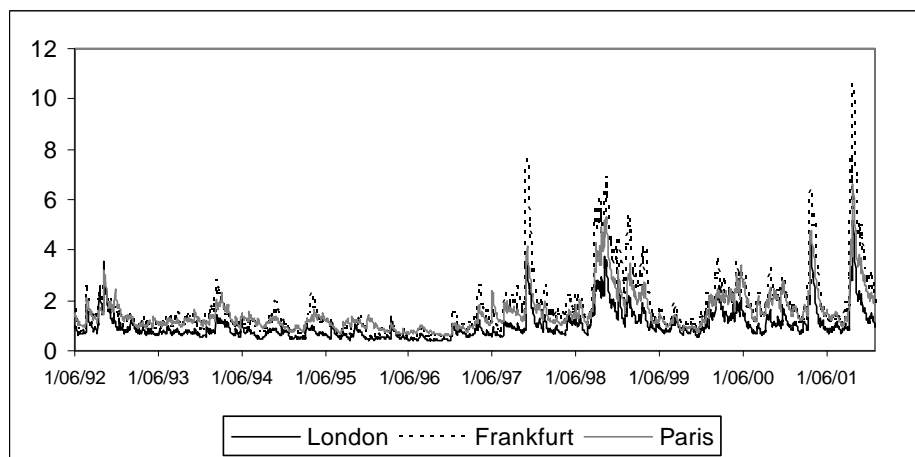
Parameter	London-Frankfurt				London-Paris				Frankfurt-Paris			
	GJR (1,1,1)		GJR (1,1,1) Volatility spillover		GJR (1,1,1)		GJR (1,1,1) Volatility spillover		GJR (1,1,1)		GJR (1,1,1) Volatility spillover	
	UK	DE	UK	DE	UK	FR	UK	FR	DE	FR	DE	FR
ω	0.0261 (0.0141)	0.0256 (0.0125)	0.0272 (0.0125)	0.0253 (0.0146)	0.0261 (0.0141)	0.0243 (0.0156)	0.0228 (0.0116)	0.0263 (0.0236)	0.0256 (0.0125)	0.0243 (0.0156)	0.0256 (0.0138)	0.0286 (0.0179)
α	0.0196 (0.0136)	0.0466 (0.0148)	0.0001 (0.0155)	0.0476 (0.0163)	0.0196 (0.0136)	0.0240 (0.0083)	0.0047 (0.0126)	0.0209 (0.0109)	0.0466 (0.0148)	0.0240 (0.0083)	0.0466 (0.0169)	0.0032 (0.0092)
β	0.9237 (0.0263)	0.9189 (0.0188)	0.9082 (0.0292)	0.9158 (0.0206)	0.9237 (0.0263)	0.9409 (0.0188)	0.9163 (0.0257)	0.9366 (0.0278)	0.9189 (0.0188)	0.9409 (0.0188)	0.9189 (0.0189)	0.9328 (0.0226)
δ	0.0567 (0.0118)	0.0348 (0.0183)	0.0644 (0.0208)	0.0292 (0.0184)	0.0567 (0.0182)	0.0351 (0.0148)	0.0643 (0.0197)	0.0311 (0.0145)	0.0348 (0.0183)	0.0351 (0.0148)	0.0344 (0.0183)	0.0442 (0.0151)
γ			0.0186 (0.0076)	0.0074 (0.0184)			0.0147 (0.0083)	0.0114 (0.0105)			0.0001 (0.0092)	0.0198 (0.0088)
ϕ	0.0263		0.0255		0.0272		0.0270		0.0317		0.0336	
η	0.9360		0.9363		0.9407		0.9417		0.9575		0.9552	
φ	0.0178		0.0204		0.0060		0.0065		0.0192		0.0201	

Notes: Standard errors in brackets. Estimated over 2500 daily returns, sampling 2/6/1992 – 28/12/2001.

All parameters have the (expected) positive sign. High levels of volatility persistence are evident in all models with parameters on lagged variables summing to just below one. Estimates from the benchmark model (GJR (1,1,1)) show asymmetry effects (δ) in London and Paris but not Frankfurt. Furthermore, the asymmetric effect is strongest for the UK market, dominating the symmetric volatility shock component. In terms of volatility spillover (γ), we find significant transmission from Frankfurt and Paris to London, and from Frankfurt to Paris, so we observe that Frankfurt is unaffected by lagged news shocks from the other markets. Although all volatility spillover coefficients are small in magnitude, Frankfurt to Paris shocks are strongest. Estimates of volatility spillover effects from London to the continental markets are positive, but smaller and poorly

estimated. Graphs of estimated conditional variance series for the volatility spillover model are presented in Figure 3.

Figure 3: Daily Conditional Variances, 2 June 1992 – 28 December 2001.



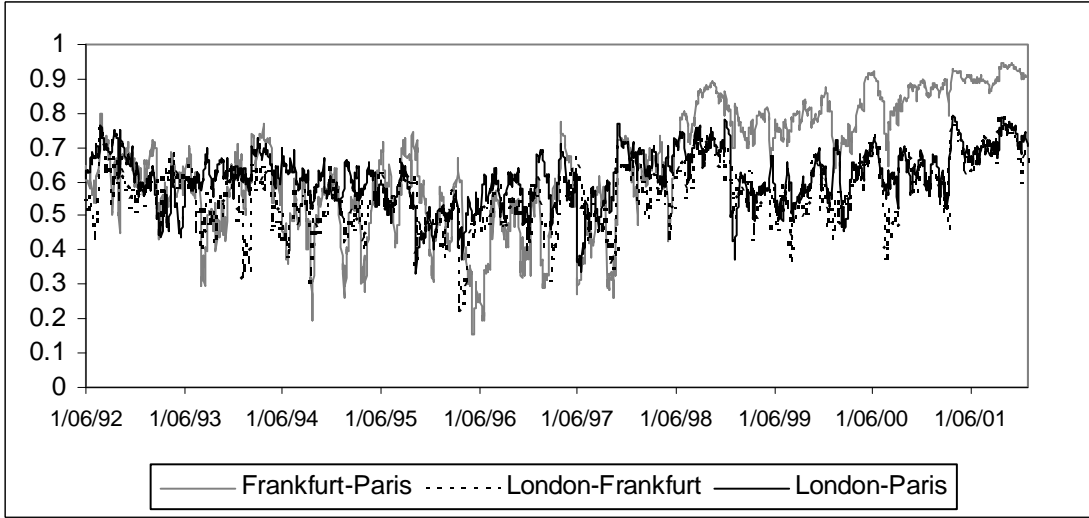
Notes: Conditional variances generated from Model 2, GJR(1,1,1) A-DCC.

Conditional variances confirm earlier observations (Figure 2) that the three markets have become increasingly volatile since early 1997, possibly in connection with the beginning of the Asian crisis. The German market shows the most, and the UK market, the least, volatility over the whole sample.¹⁰

Conditional correlation parameter estimates (ϕ, η, φ) for the benchmark and alternative models differ only slightly. This result should help us isolate the effects of volatility spillovers on the portfolio selection process. The Frankfurt-Paris combination displays the most persistence in conditional correlations, confirming our earlier observation that unconditional correlation is highest for this market pair. Asymmetric effects in conditional correlations are smaller than their symmetric counterparts in all three combinations, with the London-Frankfurt pair exhibiting the largest asymmetric effect and London-Paris the smallest. Figure 4 below graphs the three estimated conditional correlation series.

¹⁰We note that daily returns to the DAX 30 have the largest unconditional variance of the three indices.

Figure 4: Daily Conditional Correlations, 2 June 1992 – 28 December 2001.



Daily conditional correlations generated from Model 2, GJR (1,1,1) A-DCC.

In preparation for computing optimal portfolios, these sub-sample models were forecast forward over the remaining data. Predictions of covariances were made at one-step, five-step and 20-step horizons to line up with daily, weekly and monthly forecasts. This generates time series of two covariance matrices for each pair of markets $\{H_t^j\}_{j=1}^2$ at the three forecasting horizons. In the next section we apply these predictions in computing optimal portfolio weights and use a number of performance measures to compare the volatility spillover formulation with the benchmark.

5.3 Portfolio Standard Deviations

Optimal portfolio weights $\{\mathbf{w}_t^{i,k}\}$, are based on predicted covariances $\{H_t^i\}_{i=1}^2$ from the benchmark and volatility spillover models and computed as the solution to the optimization problem set out in (16) and (17) for two equity markets and the risk-free asset. Portfolio returns at each forecasting horizon can be simulated using the remaining (810) observations of the data set, where realized portfolio return is:

$$\boldsymbol{\pi}_t^{i,k} = \mathbf{w}_t^{i,k'} \mathbf{r}_t. \quad (21)$$

where $i = 1, 2$ corresponds to the benchmark and alternative portfolios and k indicates the vector of expected returns.

As outlined in Section 4, we expect the more efficient covariance model to produce a lower portfolio risk for the any specified required return. (Here, $\mu_o = 1$.) Standard deviations for the benchmark and volatility spillover models are set out in Table 6 for London-Frankfurt, Table 7 for London-Paris and Table 8 for Frankfurt and Paris. To make comparison easier, we re-weight standard deviations so that the smallest standard deviation at each value of k is set to 100. Less efficient forecasts generate a higher portfolio standard deviation and hence a value greater than 100. The final row in each table reports a weighted average of the values in the column, where the weights are derived from the relevant Bayesian distribution for theta. (See Figure 1.) On a weighted average basis, the volatility spillover model performs better than the benchmark at every forecast horizon, and for all market pairs.

Table 6: Portfolio Standard Deviations, London - Frankfurt

j	One-step-ahead forecasts		Five-steps-ahead forecasts		20-steps-ahead forecasts	
	GJR(1,1,1)	GJR(1,1,1) VOLATILITY SPILLOVER	GJR(1,1,1)	GJR(1,1,1) VOLATILITY SPILLOVER	GJR(1,1)	GJR(1,1,1) VOLATILITY SPILLOVER
0	100.00	100.26	101.53	100.00	100.00	100.15
1	100.00	100.29	100.62	100.00	100.00	101.15
2	100.50	100.00	100.00	100.26	100.04	100.00
3	100.80	100.00	101.37	100.00	100.32	100.00
4	100.10	100.00	100.29	100.00	100.19	100.00
5	100.01	100.00	100.53	100.00	100.70	100.00
6	100.15	100.00	100.56	100.00	100.02	100.00
7	100.31	100.00	100.34	100.00	100.39	100.00
8	100.55	100.00	100.43	100.00	100.89	100.00
9	100.79	100.00	101.67	100.00	100.20	100.00
10	100.99	100.00	101.98	100.00	100.33	100.00
	100.28	100.00	100.98	100.00	100.57	100.00

Notes: Smallest portfolio standard deviation for each pair of expected returns is scaled to 100.

Values over 100 represent proportional increases in standard deviations. The final row is a weighted average of the preceding rows where weights are the Bayesian probabilities reported in Figure 1.

Table 7: Portfolio Standard Deviations, London - Paris

	One-step-ahead forecasts		Five-steps-ahead forecasts		20-steps-ahead forecasts	
j	GJR(1,1,1)	GJR(1,1,1) VOLATILITY SPILLOVER	GJR(1,1,1)	GJR (1,1,1) VOLATILITY SPILLOVER	GJR(1,1,1)	GJR(1,1,1) VOLATILITY SPILLOVER
0	100.09	100.00	105.54	100.00	103.08	100.00
1	100.00	100.07	103.49	100.00	101.89	100.00
2	100.21	100.00	100.03	100.00	100.06	100.00
3	100.74	100.00	100.00	101.97	100.00	100.77
4	100.44	100.00	100.19	100.00	100.41	100.00
5	100.18	100.00	100.60	100.00	101.28	100.00
6	100.34	100.00	100.09	100.00	101.14	100.00
7	100.41	100.00	100.00	100.01	100.40	100.00
8	100.54	100.00	100.00	102.01	100.00	100.57
9	100.74	100.00	100.00	103.27	100.00	101.40
10	100.94	100.00	100.00	103.75	100.00	101.93
	100.41	100.00	100.35	100.00	100.36	100.00

Table 8: Portfolio Standard Deviations, Frankfurt-Paris

	One-step-ahead forecasts		Five-steps-ahead forecasts		20-steps-ahead forecasts	
j	GJR(1,1,1)	GJR(1,1,1) VOLATILITY SPILLOVER	GJR(1,1,1)	GJR(1,1,1) VOLATILITY SPILLOVER	GJR(1,1,1)	GJR(1,1,1) VOLATILITY SPILLOVER
0	100.60	100.00	100.98	100.00	101.91	100.00
1	100.59	100.00	100.95	100.00	101.89	100.00
2	100.52	100.00	100.83	100.00	101.69	100.00
3	100.34	100.00	100.54	100.00	101.08	100.00
4	100.00	100.08	100.01	100.26	100.09	100.00
5	100.00	100.12	100.14	100.00	101.78	100.00
6	100.87	100.00	100.72	100.00	100.32	100.00
7	100.03	100.00	100.00	100.16	100.00	100.23
8	100.22	100.00	100.00	100.23	100.00	100.09
9	100.36	100.00	100.01	100.00	100.28	100.00
10	100.43	100.00	100.23	100.00	100.61	100.00
	100.27	100.00	100.21	100.00	100.42	100.00

In terms of economic value the relative efficiency gains are not large, but can be gotten without additional transactions costs or rebalancing costs. The greatest efficiency gain for the volatility spillover model on a weighted average basis is for the 5-step ahead forecast model for London-Frankfurt, where the benchmark model standard deviation is 100.98, meaning that neglecting volatility spillover effects increases portfolio risk by about one per cent of standard deviation. Or, in terms of risk-adjusted returns, if investors who allow for volatility spillover (σ^*) are receiving 10 per cent returns ($\mu^* = 10$), then investors who forecast using the benchmark ($\hat{\sigma}$) would need to get $\hat{\mu} = 10.098$ per cent returns to

equalize the return to risk ratio such that $\frac{\mu^*}{\sigma^*} = \frac{\hat{\mu}}{\hat{\sigma}}$. In other words, the efficiency gains to predicting covariance using the volatility spillover model represent risk-free return improvements around 10 basis points on a ten per cent return portfolio. Nevertheless these small efficiency improvements do not disappear at longer forecast horizons, as can be seen from weekly and monthly portfolio standard deviations.

5.4 Diebold-Mariano Tests

Tests of the statistical significance of the efficiency improvements attributable to modelling volatility spillover confirm the value of the alternative model over the benchmark. To implement a test of forecasting accuracy, we calculate a series of differences in portfolio variances, subtracting the volatility spillover portfolio variance from the benchmark portfolio variance so that

$$u_t^k = \left(\pi_t^{1,k}\right)^2 - \left(\pi_t^{2,k}\right)^2, \quad (22)$$

forming 11 series for each market pairing, $\{u_t^k\}_{k=1}^K$. Following Engle and Colacito (2004), we note that the null hypothesis in this test is that the mean of each u series is zero. We conduct a joint test of this null hypothesis using a GMM estimate of the parameter β from the regression $U_t = \beta u_t + \epsilon_t$. We use $k = 11$ moment conditions, one for each $\{u_t^k\}$, restricting the system to a single estimate of β . We report t -tests of the null hypothesis that $\beta = 0$, using the robust Newey-West standard errors from the GMM estimation. Results are given in Table 10 for each market pairing and forecast horizon. All reject the null hypothesis and confirm that portfolio variances are significantly lower (since $\beta > 0$ in every case) when volatility spillover is modelled in the conditional covariance matrix.

Table 10: Diebold Mariano Tests for Difference in Portfolio Variance

	1 step ahead	5 steps ahead	20 steps ahead
London-Frankfurt	0.0023 (2.2068)	0.1045 (5.1615)	0.2618 (10.0068)
London-Paris	0.0023 (2.2068)	0.0262 (3.6656)	0.0967 (3.7609)
Frankfurt-Paris	0.0042 (3.2277)	0.0079 (1.9606)	0.2293 (3.6127)

Notes: Table reports estimated values of β with t-statistics in brackets.

5.5 SD2 Tests

Here we implement tests for second-degree stochastic dominance to compare the performance of benchmark and alternative models. Our aim is to assess whether improvements measured by standard deviations are likely to matter to a risk averse investor.

Stochastic dominance tests offer a general, non-parametric addition to the set of performance tests. Specifically, following Barrett and Donald (2003), consider two samples of portfolio returns $\{Y_I\}_{I=1}^M$ and $\{X_I\}_{I=1}^M$ with cumulative distributions (CDFs) G and F . Second degree stochastic dominance (SD2) establishes the conditions under which any risk averse agent prefers one portfolio to another. Portfolio Y will be preferred to portfolio X by any agent whose utility over returns $U(r)$ obeys $U'(r) \geq 0$, $U''(r) \leq 0$ when $\int_0^r G(t)dt \leq \int_0^r F(t)dt$ for all r .

Barrett and Donald derive a Kolmogorov-Smirnov style test for stochastic dominance of any degree, evaluating the CDFs at all points in the support. This technique avoids the problem of choosing an arbitrary set of comparison points which can result in inconsistency.¹¹

The null hypothesis to be tested is that G (weakly) dominates F to the second degree, against the alternative that it does not. From random samples of equal size, the test statistic is given by:

$$\hat{S}_2 = \left(\frac{M}{2}\right)^{1/2} \sup_r (\mathcal{I}_2(r; \hat{G}_M) - \mathcal{I}_2(r; \hat{F}_M)), \quad (23)$$

¹¹To make the test tractable, each pairing of returns distributions was shifted to the right by the same fixed positive amount, sufficient to ensure a lower bound of zero for a support $r[0, \tilde{r}]$ where $\tilde{r} < \infty$.

where

$$\mathcal{I}_2(r; \hat{G}_M) = \frac{1}{M} \sum_{i=1}^M 1(Y_i \leq r)(r - Y_i), \quad \mathcal{I}_2(r; \hat{F}_M) = \frac{1}{M} \sum_{i=1}^M 1(X_i \leq r)(r - X_i),$$

and $1(\cdot)$ is the indicator function, returning the value 1 when $(X_i \leq r)$ and zero otherwise. Under the null hypothesis, the test statistic is no greater than zero. Bald comparisons between CDFs or their integrals are subject to non-trivial sampling error when the population density is unknown, so we need some approximation to the sampling distribution, here derived by block bootstrapping.

We follow Linton, Maasoumi and Whang (2002), and Lim, Maasoumi and Martin (2004), and adjust the bootstrapping method to keep underlying serial dependence intact. Block size is set at $B = 28$ where $B = \alpha\sqrt{T}$, α is a positive constant and T is sample size, here 810.¹² Each set of portfolio returns is divided into overlapping blocks of size B , then a random selection is made, choosing sufficient (contemporaneous) blocks to create a distribution of size T . Bootstrap samples are used to build an empirical distribution of the test statistic.

Test results are reported only for the 1-step ahead forecasts since 5 and 20 step forecasting generate samples too small for reliable testing. Of the k possible portfolios, we select realized returns for the portfolios where expected returns to the two markets were assumed to be equal, $\theta = 5$. Results in Table 11 show that the null hypothesis that the benchmark model dominates the volatility spillover model can be rejected for London-Paris and Frankfurt-Paris. In the London-Frankfurt case, no clear second degree dominance ordering can be identified. SD2 tests favour the volatility spillover model in two out of three cases.

¹²Before forming the blocks, the returns from each portfolio are weighted to adjust for the number of times they are sampled in the overlapping blocks. The weights follow the rule:

$$\omega_t = \begin{cases} t/B : t < B \\ 1 : B \leq t \leq T - B + 1 \\ (T - t + 1)/B : T - B + 2 \leq t \leq T \end{cases},$$

where ω_t is the weight and B is block size.

Table 11: Stochastic Dominance Relations, One-Step-Ahead Forecasts.

Market pairing	Null Hypothesis	
	Volatility Spillover dominates GARCH (1,1)	GARCH (1,1) dominates Volatility Spillover
London Frankfurt	0.82	0.11
London Paris	0.95	0.06*
Frankfurt Paris	0.72	0.03**

Notes: Bootstrapped P-values for tests of second degree stochastic dominance relations between pairs of portfolio returns based benchmark and volatility spillover models, assuming most probable expected returns. An asterisk indicates rejection at the 5 % (**) or 10 % (*) level when the reverse null is not rejected. Failure to reject both nulls is inconclusive. Grey shaded cells indicate pairings where the volatility spillover model is dominant.

6. Conclusions

Models of time-varying volatility have been introduced to the empirical finance literature over the past few decades with considerable success. While many of these models have been shown to be succinct descriptions of second moments, their economic value to investors has sometimes been glossed over. This study presents a valuation of one aspect of time-varying volatility, volatility spillover, from the perspective of an investor choosing a two-asset equity portfolio from among equity markets in London, Frankfurt and Paris.

By studying the conditional second moments of the London, Frankfurt and Paris equity markets in an A-DCC set-up, we isolate portfolio risk reductions that can be attributed to correct modelling of volatility spillovers between these markets. Significant spillovers are estimated from Paris and Frankfurt to London, and from Frankfurt to Paris. Frankfurt appears to be unaffected by lagged volatility from the other markets.

Although relatively small in magnitude, volatility spillover estimation improves the out-of-sample covariance forecasts and, consequently, portfolio performance. Standard

deviations of realized portfolio returns are lower for volatility spillover models, across all market choices and forecast horizons. Further, estimates of lower portfolio risk are confirmed by Diebold-Mariano tests, which show that reductions in portfolio risk in the volatility spillover model are statistically significant and do not disappear as the forecasting horizon increases from daily to monthly. In addition, for London-Paris and Frankfurt-Paris the distribution of realized portfolio returns from the volatility spillover model stochastically dominate returns from the benchmark model according to Barrett-Donald tests.

Failing to incorporate volatility spillover effects in variance equations makes portfolio standard deviations about one per cent higher. While such losses are not dramatic, they could be eliminated without incurring higher rebalancing costs and without additional portfolio risk.

References

- Abhyankar, A.H. (1995), 'Trading round-the-clock: Return, volatility and volume spillovers in the Eurodollar futures markets', *Pacific-Basin Finance Journal*, **3**, 75-92.
- Apergis, N., and Rezitis, A. (2001), 'Asymmetric cross-market volatility spillovers: evidence from daily data on equity and foreign exchange markets', *Manchester School*, **69**, 81-96.
- Apergis, N., and Rezitis, A. (2003), 'Agricultural price volatility spillover effects: the case of Greece', *European Review of Agricultural Economics*, **30**(3), 389-406.
- Baillie, R., and Bollerslev, T. (1990), 'A multivariate generalized ARCH approach to modeling risk premia in forward foreign exchange rate markets', *Journal of International Money and Finance*, **9**(3), 309-324.
- Baele, L. (2003), 'Volatility spillover effects in European equity markets', Ghent University Working Paper.
- Barrett, G.F., and Donald, S.G. (2003), 'Consistent tests for stochastic dominance', *Econometrica*, **71**(1), 71-104.
- Bekaert, G., and Harvey, C. (1997), 'Emerging equity market volatility', *Journal of Financial Economics*, **43**(1), 29-77.
- Billio M., and Pelizzon, L. (2003), 'Volatility and shocks spillover before and after EMU in European stock markets', *Journal of Multinational Financial Management*, **13** (4/5), 323-340.

- Black, F. (1976), ‘Studies of stock price volatility changes’, Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economical Statistics Section, 177-181.
- Bollerslev, T. (1986), ‘Generalized autoregressive conditional heteroskedasticity’, *Journal of Econometrics*, **31**(3), 307-327.
- Bollerslev, T. and Wooldridge, J.M. (1992), ‘Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances’, *Econometric Reviews*, **11**, 143-172.
- Cappiello, L., Engle, R.F., and Sheppard, K. (2004), ‘Asymmetric dynamics in the correlations of global equity and bond returns’, European Central Bank Working Paper No.204, European Central Bank, Frankfurt. URL: <http://www.ecb.int>.
- Christiansen C. (2003), ‘Volatility-spillover effects in European bond markets’, Working Paper Series No. 162, November 2003, Centre for Analytical Finance, Aarhus School of Business, University of Aarhus.
- Connor, G. (1997), ‘Sensible return forecasting for portfolio management’, *Financial Analysts Journal*, September/October, 44-51.
- Conrad, J., Gultekin, M., and Kaul, G. (1991), ‘Asymmetric predictability of conditional variances’, *Review of Financial Studies*, **4**, 597-622.
- Diebold, F.X., and Mariano, R.S. (1995), ‘Comparing predictive accuracy’, *Journal of Business and Economics Statistics*, **13**(3), 253-263.
- Engle, R.F. (2002), ‘Dynamic conditional correlation - A simple class of multivariate GARCH models’, *Journal of Business and Economic Statistics*, **20**(3), 339-350.
- Engle, R.F., and Colacito, R. (2004), ‘Testing and valuing dynamic correlations for asset allocations’, unpublished manuscript, New York University, NY.
- Engle, R.F., Ito, T., and Lin, W. (1990), ‘Meteor showers or heat waves? Heteroskedastic intra-daily volatility in the foreign exchange market’, *Econometrica*, **58**(3), 525-542.
- Engle, R.F. and Sheppard, K. (2001), ‘Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH,’ USC Working Paper No. 2001-15, University of California at San Diego.
- Eom, Y., Subrahmanyam, M., and Uno, J. (2002), ‘The transmission of swap spreads and volatilities in the international swap markets’, *Journal of Fixed Income*, **12**(1), 6-28.
- Glosten, L., Jagannathan, R., and Runkle, D. (1993), ‘On the relationship between the expected value and the volatility of the nominal excess return on stocks’, *Journal of Finance*, **48**(5), 1779-1801.
- Hamao, Y., Masulis, R., and Ng, V. (1990), ‘Correlations in price changes and volatility across international stock markets’, *Review of Financial Studies*, **3**, 281-307.
- Hamilton, J. D. (1994), *Time Series Analysis*, Princeton University Press, Princeton.

- Jorion, P. (1986), ‘Bayes-Stein estimation for portfolio analysis’, *Journal of Financial and Quantitative Analysis*, **21**(3), 279-305.
- Kaltenhauser, B. (2002), ‘Return and volatility spillovers to industry returns: does EMU play a role?’, Center for Financial Studies Working Paper No. 2002-05, an der Johann Wolfgang Goethe-Universität.
- King, M. and Wadhvani, S. (1990), ‘Transmission of volatility between stock markets’, *Review of Financial Studies*, **3**, 5-33.
- Koutmos, G. (1992), ‘Asymmetric volatility and risk trade off in foreign stock markets’, *Journal of Multinational Financial Management*, **2**, 27-42.
- Lim, G.C., Maasoumi, E., and Martin, V.L. (2004), ‘Discounting the equity premium puzzle’, unpublished manuscript, Economics Department, University of Melbourne, Victoria.
- Lin, W., Engle, R., and Ito, T. (1994), ‘Do bulls and bears move across borders? International transmission of stock returns and Volatility’, *Review of Financial Studies*, **7**, 507-38.
- Linton, O., Maasoumi, E., and Whang, Y-J. (2002), ‘Consistent tests for stochastic dominance: a subsampling approach’, Cowles Foundation Discussion Paper no. 1356, Cowles Foundation for Research in Economics, Yale University, New Haven.
- Markowitz, H. M. (1959), *Portfolio Selection: Efficient Diversification of Investments*, Yale University Press, New Haven.
- Martens, M., and Poon, S.-H. (2001), ‘Returns synchronization and daily correlation dynamics between international stock markets,’ *Journal of Banking and Finance*, **25**, 1805-1827.
- MSCI (1998), *Morgan Stanley Capital International Global Investment Monitor*, January 30.
- Miyakoshi, T. (2003), ‘Spillovers of stock return volatility to Asian equity markets from Japan and the US’, *International Financial Markets, Institutions & Money*, **13**, 383-399.
- Nelson, D. (1991), ‘Conditional heteroskedasticity in asset returns: A new approach’, *Econometrica*, **59**, 347–70.
- Ng, A. (2000), ‘Volatility spillover effects from Japan and the U.S. to the Pacific-Basin’, *Journal of International Money and Finance*, **19**, 207-233.
- Pan, M. and Hsueh, P. (1998), ‘Transmission of stock returns and volatility between the U.S. and Japan: evidence from stock index futures markets’, *Asia-Pacific Financial Markets*, **5**, 211-225.
- Pardo, A., and Torro, H., (2003), ‘Trading with asymmetric volatility spillovers’, Universitat de Valencia, working paper.
- Poon, S. and Taylor, S. (1992), ‘Stock returns and stock market volatility: An empirical study of the U.K. stock market’, *Journal of Banking and Finance*, **16**, 37-59.

Reyes, M.G. (2001), 'Asymmetric volatility spillover in the Tokyo Stock Exchange', *Journal of Economics and Finance*, **25**(2), 206-213.

Theodossiou, P., and Lee, U. (1993), 'Mean and volatility spillovers across major international stock markets: further empirical evidence', *Journal of Financial Research*, **16**, 337-350.

Wei, K.C.J., Liu, Y.J., Yang, C.C., and Chaung, G.S. (1995), 'Volatility and price change spillover effects across the developed and emerging markets', *Pacific-Basin Finance Journal*, **3**, 113-136.