How to Stabilize Financial Markets before EMU ?

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Abstract

A recent report from the Center for Economic Policy Research (CEPR) has advocated that fixed conversion rates for the start of EMU should be preannounced as soon as possible. The aim of this short paper is to focus on the potential dangers of such a decision. Indeed, the CEPR report propositions are based on the idea that, roughly speaking, fixing a conversion rate in advance should stabilize the exchange rate markets now. We show that this intuition is misleading: knowing the price of an asset at some point in the future has never meant that its current price has a low volatility. Moreover, we perform some simulations to evaluate how volatile exchange rate markets should have been for the past few months if such a rule had been announced. Finally, we provide a constructive proof of what a stabilizing rule should look like.

1 Introduction

Various rules have been proposed to set the conversion rates of the currencies of the future European Monetary Union (EMU). Indeed nothing in the Maastricht Treaty prevents governments from announcing some rules well in advance of the scheduled time (i.e., 1 January 1999). For example, M. Lamfalussy, the former President of the European Monetary Institute has proposed to set these rates as the average market rates over a given period. We don't want to discuss here the advantages and the drawbacks of all these rules since many things have already been said elsewhere (see De Grauwe (1996), Frachot (1997a, 1997b), CEPR (1997)). We prefer to focus on the rule which is the most likely.

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This rule is also the simplest one: it consists in preannouncing the conversion rates in advance of 1 January 1999. For example, the conversion rate between the french franc and the deutsche mark would be set at some predefined level[1]. Why such a rule ? The CEPR authors are legitimatly convinced that the run-up to EMU should be seriously endangered if a currency crisis ever happened during the next few months. As a matter of fact, such a currency crisis is not impossible regarding the amount of misunderstandings between governments on what the EMU should look like. As a result, a well-designed rule is crucially needed to remove as much volatility as possible from the currency markets. In the CEPR authors minds, the preannouncement of fixed conversion rates is the best way to stabilize these markets from now until the end. Moreover, they claim that this rule should be announced as soon as possible, that is before memberships decision (i.e., May 1998).

Their basic intuition[2] is the following : when the price of an asset is known at some point in the future (i.e., the FRF/DEM at 1 January 1999 is kown now) then its current price must have a low volatility. In other words, the certainty brought by this future fixed point would stabilize the whole path until this point. In fact, this intuition is simply incorrect. There are many examples which contradict this intuition. Let us consider the fixed income markets and take a classic bond. Imagine a 20 year 10 % bond whose facial value is 1000 FRF: all the cash flows generated by this particular asset are perfectly known at the origination of the asset and remain so until its very end. In particular, this asset entitled its holder to receive 100 FRF every year and 1100 FRF at the last year and all these cash-flows are known 20 years in advance. In short, the price of this asset is perfectly known for some point in the future. Moreover, in the case of government bonds, there is generally absolutely no credibility concerns about whether the bond will be reimbursed on time or whether the scheduled time will be postponed. Does it mean that bond markets experience low volatilities ? Are these volatilities considerably lower than in the share markets ? Even a two-year bond (which is the analogous of our EMU problem) has a significant volatility.

As a matter of fact, there are two sources of volatility in the exchange rate movements. The first one is not specific to the EMU process. We shall call it *intrinsic volatility.* This volatility comes directly from the market and simply reflects the shifts of (market) expectation regarding futures interest

[1] As discussed in the cited papers, rules can be defined on the *bilateral* conversion rates but not on the conversion rate against the euro.

[2] This is essentially an intuition since the CEPR authors do not provide any rigourous proof of what they claim.

rates. Inversely, the second source of volatility is linked to the EMU process itself. Indeed, there is some uncertainty about who will belong to the first wave of the EMU and when it will eventually take place. The resulting volatility of the exchange rate depends on how the conversion rule which will be announced, manages these two sources of volatility.

From our former counter-example (i.e., the bond market example), the CEPR authors rule is unlikely to be the one which minimizes the intrinsic volatility. More generally, the aim of this paper is to show that this rule is not the suitable rule to manage efficiently the two sources of volatility especially if it is announced before memberships decision. Moreover we give a rigourous proof of which rule should be announced to financial markets in order to minimize the volatility of the exchange rates from now until the end, even though the EMU is postponed.

The remainder of the paper is organized as follows. In section 2, we build a market expectation for the time to EMU and then we can compute what the exchange rates would have been for the past few months if the discussed rule had been preannounced a few months ago. The use of the expected time to EMU allows to take account explicitly the second source of volatility. In section 3, we derive what the "most stabilizing" rule should look like.

2 Exchange rate volatility and the Preannouncement of conversion rates

Fundamentally, preannouncing the conversion rates for the start of the EMU is equivalent to the fixing of the forward exchange rate for this particular maturity. As we mentionned above, it dose not mean that the current exchange rate has a low volatility. Moreover, we can't prevent financial markets from considering this maturity as uncertain. Though the scheduled time is 1 January 1999, financial markets may think differently and consider a postponement as a non zero probability event. Clearly, by a classic covered interest rate parity relation, it is obvious that any uncertainty on the maturity of the forward rate will be transfered to the current exchange rate and/or the current yield curves.

This is the point we want to illustrate. So we first build an expected time to EMU. Then we plug it in a covered-interest-rate-like relation to derive what the exchange rate should have been for the past few months if such a preannouncement had been released.

2.1 An expected time to EMU

Our computations are very simple. They are based on the ECU market and on the fact that two different versions of the ECU (i.e., the basket and the private ECU) co-exist (see Frachot (1997c) for a detailed analysis). The basic intuition is the following. We suppose that financial markets are uncertain about the precise date when the monetary union will eventually take place. We assume however that they are perfectly convinced that the private and official ECU will be set at par the day EMU starts. Then any deviation between the two ECU reveals some information about the time to EMU.

There are strong reasons why we can assume that financial markets believe that this parity rule won't be violated. Indeed the Maastricht Treaty states that the external value of the ECU (for example against Dollar) shall not change at the moment the single currency, the Euro, is introduced. Furthermore, at the Madrid summit, it was decided that the ECU would be converted into Euro at a rate of one for one at the start of EMU. Combining these two statements with the fact that all ECUs are refered as basket ECUs, will constrain the official and private ECU to be traded at par at the start of EMU.

What are the consequences of this one-for-one rule for the current exchange rates and yield curves ? Let us denote T the (uncertain) time when EMU takes place and $S_{\$\,/\,i}$ the Dollar price of one unit of currency i. The one-for-one rule imposes that:

$$S_{\$\,/\,private\,ECU}(T) = S_{\$\,/\,basket\,ECU}(T) = S_{\$\,/\,EURO}(T).$$

Since the basket ECU refers to a basket of currencies, we thus obtain:

$$\begin{aligned} S_{\$\,/\,private\,ECU}(T) &= S_{\$\,/\,basket\,ECU}(T) \\ &= \sum_i \omega_i S_{\$\,/\,i}(T) \end{aligned}$$

where i belongs to the set of currencies of the basket and ω_i is the amount of currency i in the basket.

We can now "discount" this equation at time t ($t < T$) by considering a trader who can invest either in the private ECU or in the currencies of the basket. Let us denote $r_i(t,T)$ (respectively $r_{ECU}(t,T)$) the zero-coupon yield for currency i (resp. private ECU) and for maturity date T. Investing $S_{\$\,/\,i}\ (t)/[1+\ r_i(t,T)]^{T-t}$ Dollars at time t ensures that 1 unit of currency i is obtained at time T. Consequently, if markets remove efficiently any free lunch then the following relation should hold:

$$\frac{S_{\$\,/\,private\,ECU}(t)}{[1+r_{ECU}(t,T)]^{T-t}} = \sum_i \omega_i \frac{S_{\$\,/\,i}(t)}{[1+r_i(t,T)]^{T-t}}.$$

Solving this equation in T gives an expected time to EMU[3]. The empirical implementation of this expected time is detailed in Frachot (1997c). Figure 1 illustrates the value of this expected time for the past few months. We see that this time to EMU is generally close to 1 January 1999 but sometimes diverges significantly. For example, the discussions between France and Germany during the Amsterdam summit have generated strong divergence of this time indicator. As this indicator simply reveals the way financial markets hedge their private ECUs with the currencies of the basket (under the one-for-one constraint), this divergence means that they have weighed more heavily the probability of a postponement. As a consequence, our expected time to EMU reflects the fact that financial markets have rebalanced their portfolios accordingly.

2.2 What would exchange rates have been ?

The expected time to EMU can now be plugged in a usual covered interest rate parity in order to evaluate its impact on current exchange rate and yield curves. In this way, we fundamentally focus on the second source of uncertainty, that is the uncertainty about the time to EMU.

Let us consider i and j two currencies and let us denote $S^*_{i/j}$ the preannounced conversion rate between i and j. Let us also assume that preannouncement took place on the begining of March 1997 (simply because our data are available only from this date). Covered interest rate parity implies that, for any date $t > March$ 1997:

$$\frac{S^*_{i/j}}{S_{i/j}(t)} = \frac{B_j(t,T)}{B_i(t,T)}. \qquad (1)$$

[3] This relation can be rearranged as:

$$\frac{S_{\$\,/\,private\,ECU}(t)}{S_{\$\,/\,basket\,ECU}(t)} = \sum_i p_i(t) \left[\frac{1+r_{ECU}(t,T)}{1+r_i(t,T)}\right]^{T-t}$$

where $p_i(t)$ is the weight of currency i in the basket:

$$p_i(t) = \frac{\omega_i S_{\$\,/\,i}(t)}{S_{\$\,/\,basket\,ECU}(t)}.$$

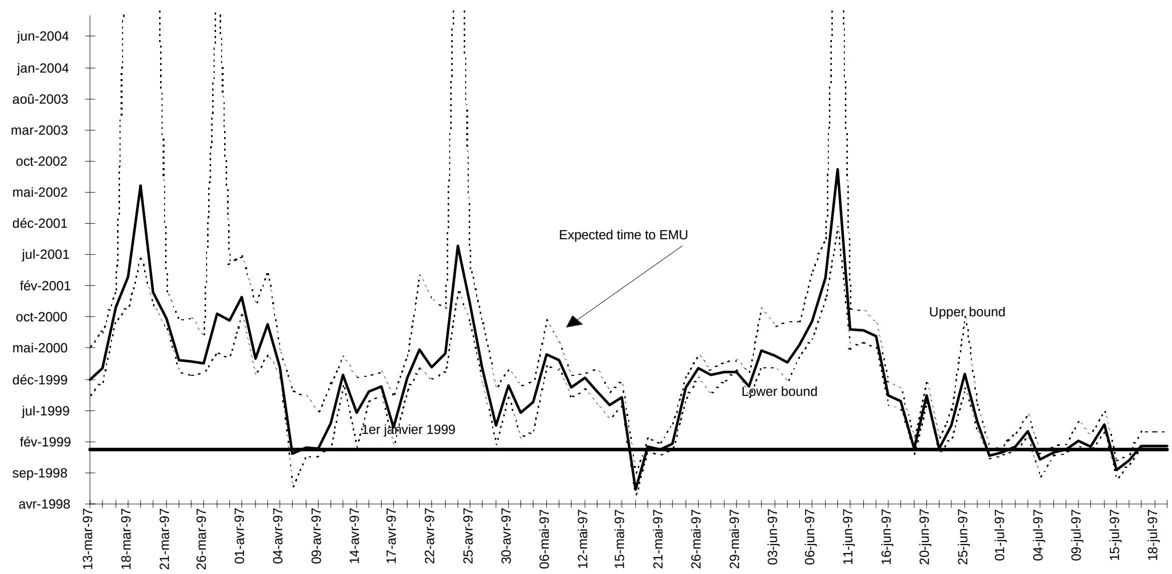

Figure 1: Expected time to EMU

Let us recall that a covered interest rate parity requires very few assumptions. It only necessitates that any arbitrage opportunities are efficiently removed by the markets. In particular, we make no assumptions regarding the risk neutrality or the utility preferences of the traders. However, we use this no arbitrage constraint in a non conventional way since T is an expectation and not a fixed, non random date.

The use of equation (1) is now rather trivial provided we say which of the exchange rate or the yield curves will adjust to the variations of T. Clearly, it is an open question. Some authors (Brookes (1996)) have argued that the exchange rate is more likely to adjust than the yield curves because yield curves are partially control by Central Banks. We shall adopt this point of view although we believe that the adjustement will be more likely divided between the two. Consequently, the "current" exchange rate obtained under the fixed conversion rate rule is computed as follows:

$$S_{i/j}(t) = S^*_{i/j} \frac{B_i(t,T)}{B_{ij}(t,T)}$$

with the estimated T given previously[4]. Figure 2 shows the resulting exchange rate path for the Deutsche Mark and the Italian Lira. In order to compare, we have also plotted the true exchange rate path observed during this period. The least we can say is that this conversion rule has no stabilizing properties. In comparaison, we give in figure 3 the same exchange rate path for the FRF/DEM parity.

Obviously this graph is just an illustration as the volatility is more likely to be split between interest rate and exchange rate movements. Moreover, facing to such movements, Central Banks would certainly intervene. However it is not clear whether they should raise or lower their official rates as the exchange rate movements illustrated in figure 2 do not follow a clear trend: the exchange rate depreciates or appreciates alternatively depending on whether financial markets are convinced or not about the 1 January 1999. In this respect, exchange rates movements seem higly uncontrolable.

Furthermore, in the particular case of Italy, we do not take into account that T is probably not the appropriate time since there is another source of uncertainty due to membership. Indeed, Italy has a lower probability of belonging to the first wave of the EMU than France or Germany. As a result, the time T to be used should certainly be much more volatile. The corresponding exchange rate computed with our methodology would be even more volatile.

[4] Zero coupon yield curves are drawn from Reuter (see Frachot (1997c) for more details on the implementation).

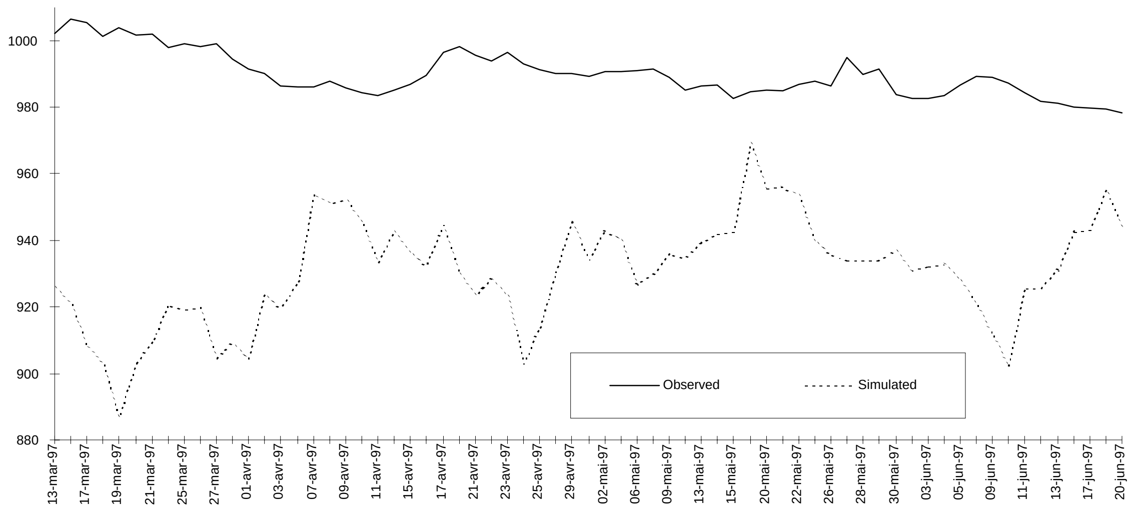

Figure 2: Simulated exchange rate (ITL/DEM) under preannouncement

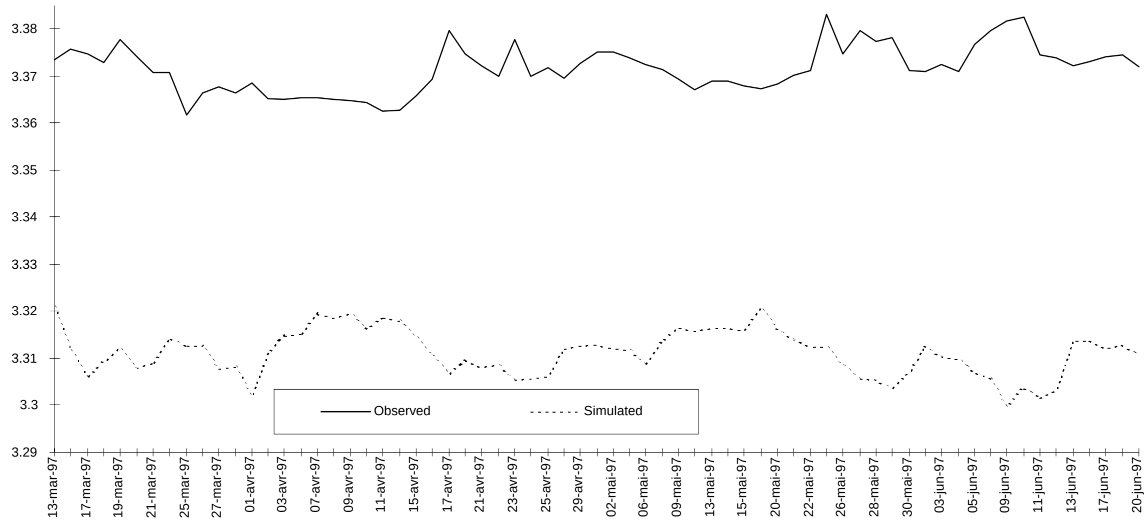

Figure 3: Simulated exchange rate (FRF/DEM) under preannouncement

As a consequence, if the fixed conversion rate rule was adopted, it should be announced at the same time as memberships decision (i.e. in May 1998) and not before, precisely because the most stabilizing rule is *not* this one. If it was announced before May 1998, the Italian Lira would suffer from excessive volatility and, in some sense, would pay for the uncertainty (imposed by other countries) concerning its membership. Announcing the rule after May 1998 would certainly be safer. However, as we already mentionned about the "bond market" comparison, the volatility of the exchange rate has no reason to be zero.

Finally, the question is whether one can build a stabilizing rule. The answer is yes: it suffices to use the fixed conversion rate rule with a small correction related to the interest rate spreads between countries. In the case of the core EMU (i.e. France, Germany, Netherlands, Belgium...), this correction is rather innocuous but, for Italy, this rule would be much more favorable as it would remove a large part of volatility.

3 An Optimal Rule

3.1 Some Intuition

The fixed conversion rule doesn't eliminate the intrinsic volatility of the exchange rate. Moreover, when the scheduled time (i.e., 1 January 1999) is not fully credible, then the rule increases this volatility and makes the exchange rate unstable. As a matter of fact, the underlying idea of the CEPR report is that one should find a stabilizing rule, that is a rule which minimizes exchange rate movements from now until the end, even if the "end" doesn't correspond to 1 January 1999.

Actually, this goal can be achieved through a simple rule which ensures that the instantaneous volatility of exchange rates is zero along the whole path to EMU *without adding any constraint on monetary policies* and *even though EMU would have to be delayed.* It means that exchange rates may move from one day to another since Central Banks remain free, under the rule, to influence the interest rate spread through their monetary policies. But the rule ensures that, whatever monetary policies are, the exchange rate has no intrinsic volatility (i.e., the volatility provided by the market). As a result, Central Banks have a full control of the exchange rate through their central interest rates. Contrary to the previous rules, the market becomes unable to add volatility to the exchange rate and then the only source of variation of the exchange rate is directly due to variation of the Central Bank interest rates.

This rule is precisely not the rule advocated by the CEPR authors. As a matter of fact, the rule which achieves the best "certainty" (i.e., no volatility along the path) is *not* the rule which would set in advance some conversion rates (i.e., the "pre-announcement of fixed conversion rates" rule promoted by the CEPR authors). In order to have the most important certainty along the path, we shall show that one must keep some randomness on the conversion rates until the very end of the process.

The intuition of the rule is as follows. If we want almost no uncertainty in the exchange rates process, we mean that, in some sense, we want the EMU to be partially achieved *before* the scheduled time (1 January 1999). What does "the EMU is achieved" mean ? It simply means that the French Franc and the Deutsche Mark (say) are considered by financial markets as perfect substitutes. So the rule must guarantee that any investment in French Francs must have the same return as an investment in Deutsche Marks (converted in French Francs on 1 January 1999). The solution is to announce that the French Franc/Deutsche Mark parity will be equal to a fixed rate (for example, 1 DEM = 3.35 FRF/DEM) *modified* for the spreads between the day-to-day interest rates of the two currencies. If this spread is equal to zero along the path to 1 January 1999 then the conversion rate will be exactly the fixed rate (here 3.35 FRF/DEM). Otherwise, the rule garantees to the markets that, if a currency has benefitted from a higher interest rate before 1 January 1999 then this higher return will be offset by a corresponding depreciation of the currency at that time. In this way, the two following investments will appear as fully equivalent:

- invest 1 FRF at the (french) day-to-day interest rate;
- convert 1 FRF in DEM, invest these DEM in the (german) day-to-day rate, convert them back in FRF on 31 December 1998[5].

This implies that the two currencies will be treated as (almost) perfect substitutes by financial markets[6].

Moreover, the French Franc and the Deutsche Mark remain substituable until the start of EMU *whenever it actually takes place.* Indeed, the previous

[5] Of course, the same equivalence holds for the symmetric investment of 1 DEM.

[6] We can carry on the "bond market" comparison. As mentioned in the introduction, a traditional bond has some intrinsic volatility although its future cash-flows are fixed and perfectly known. Our optimal rule can be understood through the floating rate note market where bonds generate interest rate related cash flows. Standard finance theory shows that a pure floating rate note is traded at par. So its price does not vary over time and has no volatility. Fundamentally, our optimal rule relies on exactly the same intuition.

intuition is totally independent of the actual starting time. EMU could be delayed and would not endanger exchange rate markets provided that they are convinced that conversion rates will be finally fixed through this mechanism. As a conclusion, the two sources of volatility are removed.

3.2 The Formal Rule

Appendix 1 and 2 propose two constructive proofs of the "most stabilizing" rule whose intuition is given above. The first one (i.e., appendix 1) is the standard proof. Alternatively, a second proof (i.e., appendix 2) is derived using the way of reasoning of financial engineers. As a matter of fact, it gives the shortest and most elegant proof.

Starting from the previous intuition, it is straightforward to write the formula which has to be announced. Ler $r_{FRF}(s)$ and $r_{DEM}(s)$ the day-to-day money rates for the French Franc and the Deutsche Mark. Let $S_{FRF/DEM}$ be a target for the conversion rate (e.g. $S_{FRF/DEM} = 3.35$): the fixed conversion rates rule of the previous section would consist in announcing that the conversion rate will actually be $S_{FRF/DEM}$. Here, we slightly modify the pre-announcement: at time t_0, it is announced that the final conversion rate will be equal to $S_{FRF/DEM}$ with an additional term to take into account that the two money rates might be different until the final term. So the equation giving $S_{FRF/DEM}(T)$ is the following:

$$S_{FRF/DEM}(T) = S_{FRF/DEM} \cdot \prod_{s=t_0}^{T-1} \frac{1 + r_{FRF}(s)/360}{1 + r_{DEM}(s)/360}. \qquad (2)$$

Before we investigate its properties, let us make a few remarks. First this formula captures the intuition provided in the previous subsection: if the French Franc has a better return than the Deutsche Mark (i.e., $r_{FRF} > r_{DEM}$) then this higher return will be exactly offset by a corresponding depreciation of the Frenc Franc against the Deutsche Mark. As a result, investing in the two currencies is equivalent. So any movement in the exchange rate is not due to shifts in market expectations but comes from change in monetary policies.

Secondly, whether T is equal to 1 January 1999 or not has no importance: the only thing needed to ensure the currency substituability is that the EMU will take place "some day" (i.e., $T < \infty$). Consequently, the two sources of volatility (i.e., the intrinsic one and the one related to the uncertainty of the time to EMU) are eliminated.

Aside these important features, the optimal rule has other interesting advantages which are described in the following subsection.

3.3 Some properties of the optimal rule

This optimal rule has many advantages compared with the other rules investigating in the previous sections. First, this rule doesn't lead to strong discontinuities the day it is publicly released. Applying the master formula (4), we obtain that, at time t_0, the equilibrium exchange rate is exactly $S_{FRF/DEM}$. As a result, the jump is equal to:

$$\Delta = S_{FRF/DEM} - S_{FRF/DEM}(t_0 - 1)$$

which *is independent of the current yield curves*. So the jump doesn't depend on which degree of convergence the yield curves have achieved. It only depends on the current exchange rate and the target $S_{FRF/DEM}$. Moreover the jump is likely to be quite small either because the target has been chosen such as Δ is zero (i.e., $S_{FRF/DEM}$ is taken not too far from the current exchange rate) or because the central parity of the ERM has been choosen. Indeed, as we mentionned earlier, the fixed conversion rate rule advocated by the Governor of the BoF (with central parities) is impossible at the moment since it could imply strong discontinuities due to insufficient convergence of the yield curves. With the optimal rule, central parities can be announced at any time without any significant jump of the current exchange rates. On the other hand, the effective exchange rate at the final term will be slightly different from the target since it will compound the remaining spread between day-to-day interest rates. However, as it will become clear in the sequel, Central Banks will have a strong incitation, under this rule, to make their official rates converge to one another because (among other reasons) the no-volatility property of the exchange rate doesn't necessitate to keep any risk premium. In any case, taking the central parities (i.e., small jump) or the current exchange rate (i.e., zero jump) would be rather equivalent at the moment since all currencies of the future EMU trade around their central rate.

Secondly, there is a strong incitation for Central Banks to make their official rates converge to one another. We have just mentionned the fact that risk premia should vanished away due to the absence of exchange rate volatility. This is typically the case for France where the BoF states that it has to maintain a spread with Germany to keep the exchange rate under control[7]. It is worth noting that the exchange rates become under the full control of Central Banks since they completely depend on the day-to-day interest rates which are under the influence of the official rates. It may seem dangerous to give the Central Banks such a power but this situation is certainly better and safer than a control by financial markets. However, the optimal rule implies a different behavior of the Central Banks. A quick look of the master formula (4) shows that an increase of the french interest rate leads to a *depreciation* of the French Franc vis-à-vis of the Deutsch Mark instead of an appreciation[8]. This means that Central Banks are strongly incitated to converge. The proof is straightforward. Indeed, whatever the criteria focused by Central Banks are when they implement their monetary policies, we know that their reaction functions are more or less some weighted functions of inflation, activity and exchange rate volatility. Let us drop the exchange rate part since, under the optimal rule, exchange rate volatility has disappeared. There remain (domestic) inflation and activity. If a given Central Bank weighs more the activity variable, it should be incitated to lower significantly its interest rates (other foreign interest rates being equal). The consequence is a global appreciation of its currency. For this country, the gain of activity drawn from such a monetary policy will be diminished by the appreciation of its currency. Conversely, if the Central Bank puts more weigh on inflation, any tentative to rise its interest rates turns out to be less productive since the exchange rate depreciates and imported inflation increases. As a result, the optimal policy of Central Banks is to equalize their interest rates as quickly as possible. As soon as equalization is reached, exchange rates become perfect substitutes as no drift nor volatility remain any longer.

Thirdly, the whole yield curves will be pushed to converge to one another. As soon as currencies are perfect substitutes, the yield curves must mechanically equalized as well. Any remaining spread would mean that some arbitrage are profitable. However, as for any rule, a credibility spread might remain if there is a risk for the EMU to be postponed or for the rule to be abandoned. Obviously, this risk is not specific to the optimal rule but is shared by all other rules.

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[7] This spread is currently of 15 basis points.

[8] Interestingly, the optimal rule imposes that the Uncovered Interest Rate Parity Hypothesis strictlt holds (see Frachot (1996).

4 Concluding Remarks

The most likely rule for fixing conversion rates on 1 January 1999 is certainly the one advocated by the CEPR authors, that is the preannouncement of fixed conversion rates. However, there are no theoretical reasons which would prove that this rule is the most stabilizing one. Conversely, our theoretical discussions as well as our simulations show that this rule is higly sensitive to any postponement of the EMU process and to the memberships decision. Moreover, if this rule was announced before memberships decision, then the ITL/DEM exchange rate would certainly be higly volatile.

Moreover, we derive what the best rule should look like. This rule is closed to the previous one except that it takes into account the interest rate spreads between the currencies. Interestingly, this rule minimizes the volatility of the exchange rate from now until the end and is not sensitive to a postponement of the EMU nor to memberships decision.

Appendix 1

Here is the standard proof of the stabilizing feature of the optimal rule. Let us recall that, under this rule, the conversion rate is fixed by the following formula:

$$S_{i/j}(T-1) = S_{i/j} \ . \prod_{s=t_0}^{T-1} \frac{1 + r_i(s)/360}{1 + r_j(s)/360}$$

Proposition 1 *Under the optimal rule, the variance of the time-t exchange rate $S_{i/j}(t)$ conditionally to $t-1$ is exactly equal to zero:*

$$\forall\, t_0 < t \leq T,\ V_{t-1}\left(S_{i/j}(t)\right) = 0$$

Proof: The proof is rather straightforward and needs no assumptions except the no-arbitrage hypothesis and, as usual, perfect credibility of the rule. *However, whether EMU starts on time or not has no importance.*

As a first step, let us show how the time-$T-1$ exchange rate is determined. The two usual strategies give the following earnings:

- invest 1 unit of currency i at $T-1$ to obtain a time-T a flow of $1 + r_i(T-1)/360$;
- convert 1 unit of currency i in currency j (that is, $1/S_{i/j}(T-1)$ units of j), invest this amount of currency j and convert it back at time T. The time-T flow is:

$$\frac{S_{i/j}(T)}{S_{i/j}(T-1)} \ . \ (1 + r_j(T-1)/360).$$

These two strategies cost 1 unit of currency i at time $T-1$ and provide two *known* flows since, by formula (2), $S_{i/j}(T)$ is known at time $T-1$. So these two flows must be equal; otherwise, one could finance one investment by the other in order to obtain a strategy which would cost nothing and would give a positive gain with certainty. This would be a violation of the No-Arbitrage hypothesis. As a result, $S_{i/j}(T-1)$ is necessarily given by:

$$1 + r_i(T-1)/360 = \frac{S_{i/j}(T)}{S_{i/j}(T-1)} \,.\, (1 + r_j(T-1)/360).$$

Replacing $S_{i/j}(T)$ by its value yields:

$$S_{i/j}(T-1) = S_{i/j} \,.\, \prod_{s=t_0}^{T-2} \frac{1 + r_i(s)/360}{1 + r_j(s)/360} \tag{3}$$

which is exactly the same formula as in the definition rule (2) taken one step before. It is straightforward to understand that the same argument applies for the whole path between t_0 and T; so, we have proved the following lemma:

Lemma 2 (Master Formula) *Under the optimal rule, the time-t exchange rate is necessarily given by: (Master Formula)*

$$\forall\; t_0 < t \leq T, \quad S_{i/j}(t) = S_{i/j} \,.\, \prod_{s=t_0}^{t-1} \frac{1 + r_i(s)/360}{1 + r_j(s)/360} \tag{4}$$

Since the time-t exchange rate is perfectly known at time t, its variance conditional to $t-1$ is zero:

$$\forall\; t_0 < t \leq T, \;\; V_{t-1}\;(S_{i/j}(t)) = 0.$$

This ends the proof. ■

It is straightforward to show that our optimal rule is the *only* rule which removes all intrinsic volatility from the exchange rate process without imposing any constraint to monetary policies.

We have thus proved that, under the optimal rule, once traders know the current interest rates, they perfectly agree on what the next period exchange rate must be, whatever their expectations, their preferences or their risk aversions are. Any movement of the exchange rate results from a variation of the interest rate spread.

More importantly, the whole proof is independent of the time when the EMU starts. We only need that $T < \infty$, that is that the EMU will eventually take place "some day".

Appendix 2

We give a simple, finance-oriented proof of the stabilizing propety of our optimal rule. Under the no-arbitrage assumption, Harrison and Kreps (1979) and Harrison and Pliska (1981) have shown that there exists a risk-neutral probability measure under which discounted prices were martingale. In the particular case of the exchange rate market (see for example Amin and Jarrow (1992)), this fundamental proposition can be translated into the following equation:

$$\forall\, T > t,\; S_{i/j}(t) = E_t\left(S_{i/j}(T)\exp - \int_t^T (r_i(s) - r_j(s))\, ds\right) \tag{5}$$

where $r_i(s)$ (resp. $r_j(s)$) is the instantaneous spot rate for country i (resp. j) and where the expectation is taken under the risk-neutral probability. We see immediately that the best rule consists in announcing at time t_0 the following rule:

$$S_{i/j}(T) = S_{i/j}(t_0)\exp \int_{t_0}^T (r_i(s) - r_j(s))\, ds. \tag{6}$$

Indeed, if we plug equation (6) into equation (5), we obtain:

$$\forall\, t > t_0,\; S_{i/j}(t) = S_{i/j}(t_0)\exp \int_{t_0}^t (r_i(s) - r_j(s))\, ds \tag{7}$$

which exactly means that the exchange rate has no intrinsic volatility since now:

$$\frac{dS_{i/j}(t)}{S_{i/j}(t)} = (r_i(t) - r_j(t))\, dt.$$

Furthermore, equation (5) remains valid even if T is random. We do not want to enter a technical discussion on what random means in this context. In particular, T may be random for political reasons independently of market conditions. We know that, in this case, markets are incomplete and then equation (5) is satisfied for more than one risk neutral probability measures. Remarkably, our result still holds no matter there are several risk neutral probability measures: equations (5) and (6) imply formula (7) for all risk neutral probability measures.

As a result, our rule keeps its stabilizing effect even if the time to EMU is uncertain and is robust to any kind of postponement.

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