The Dornbusch Model with Chaos and Foreign Exchange Intervention

Sergio Da Silva*

1. Introduction

A possible explanation for exchange rate movements away from the level consistent with macro fundamentals is the existence of speculation. Expectations of market participants who use supposedly recurring patterns in graphs to make forecasts might be destabilizing. Such 'technical' or 'chart' analyses might also be a source of nonlinearity leading to chaos in the Dornbusch (1976) model, as shown by De Grauwe and Dewachter (1992) and De Grauwe, Dewachter, and Embrechts ((1993), Chapter 5), henceforth DD and DDE respectively. These chaotic models can mimic the alleged random walk pattern of actual exchange rates despite the fact that the 'stochastic' behavior is produced by deterministic solutions.

The model presented here belongs to the same line of research. It is able to conciliate the two apparently divergent pieces of evidence that the nominal exchange rate appears to follow a random walk although it also seems to be explained by fundamentals. A martingale process (i.e. a random walk with heteroskedasticity) may be a solution of a chaotic version of the Dornbusch model in which fundamentals still matter.

In De Grauwe and Vansanten (1990) intervention could stabilize a chaotic exchange rate within a framework that was the forerunner of the models of DD and DDE. However, nonlinearities were introduced in the De Grauwe-Vansanten model by assuming the existence of a J-curve. Such an assumption was dropped by DD and DDE.

This paper further generalizes the extension of the Dornbusch model accomplished by the DD and DDE models by rescuing the point made in the article of De Grauwe and Vansanten. Here a major novelty is the introduction of a policy rule linking the nominal exchange rate and the nominal money supply. The aim is to show that massive interventions can remove chaos from the foreign exchange market in the DD and DDE models.

Section 2 sets up the model, whose simulated solutions are presented in Section 3; the results are then contrasted with selected stylized facts and previous work (Section 4); and Section 5 concludes. Formal tests for chaos are presented in an appendix.

2. The Model

2.1. Building Blocks

The model is made up of equations (1)—(8) displayed in Table 1.

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Table 1. **The Extended Nonlinear Dornbusch Model with Speculative Dynamics and Foreign Exchange Intervention**

(1) $$S_t^* = \frac{P_t^*}{P_t^{f*}}$$

(2) $$\frac{P_t}{P_{t-1}} = \left(\frac{S_t}{S_t^*}\right)^{\chi}$$

(3) $$\left(\frac{\overline{M}_t}{P_t}\right)\left(\frac{S_t}{\overline{S}_t}\right)^{\varphi} = \frac{Y_t^{\delta}}{(1+i_t)^{\theta}}$$

(4) $$\frac{S_{t+1}^e}{S_t} = \frac{1+i_t}{1+i_t^f}$$

(5) $$\frac{S_{t+1}^e}{S_{t-1}} = \left(\frac{{}_C S_{t+1}^e}{S_{t-1}}\right)^{C_t}\left(\frac{{}_F S_{t+1}^e}{S_{t-1}}\right)^{1-C_t}$$

(6) $$\frac{{}_C S_{t+1}^e}{S_{t-1}} = \left[\left(\frac{S_{t-1}}{S_{t-2}}\right)\right]^{\nu}$$

(7) $$\frac{{}_F S_{t+1}^e}{S_{t-1}} = \left(\frac{S_{t-1}^*}{S_{t-1}}\right)^{\lambda}$$

(8) $$C_t = \frac{1}{1+\iota(S_{t-1}-S_{t-1}^*)^2}$$

Variables and parameter χ are defined as follows. Variable S_t is the nominal exchange rate (the price of the foreign currency in units of domestic currency) at time period t; S^* is the equilibrium nominal exchange rate; P_t is the domestic price level at t; P_{t-1} is the domestic price level at $t - 1$; P^* is the steady state value of the domestic price level; and P^{f*} is the steady state value of the foreign price level. Parameter $\chi \in (0, \infty)$ measures the actual speed of adjustment in the goods market and is seen as a proxy for the degree of domestic price level flexibility, as explained below. Equation (1) gives the long run equilibrium condition, which is defined as a situation in which purchasing power parity (PPP) holds. Since PPP is one of the long run properties of the Dornbusch model, equation (1) states that explicitly. Equation (1) is employed by both the DD and DDE models.

Equation (2) is a substitute for the Phillips curve in describing the short run price dynamics; it links domestic price level changes and nominal exchange rate deviations from equilibrium. Since $\chi > 0$, (2) states that whenever the nominal exchange rate S_t exceeds its PPP value S^* the domestic price level increases, i.e. $P_t > P_{t-1}$. So whenever the currency is undervalued an excess demand in the goods market follows, causing the domestic price level to increase (and vice versa). Equation (2) is part of the DD model. Since parameter χ measures the speed of adjustment in the goods market, the value of χ is interpreted as a proxy for the degree of domestic price level flexibility. Price rigidity occurs at the borderline case where $\chi \to 0$, whereas full price flexibility is represented by $\chi \to \infty$.

Money market equilibrium is given by equation (3). Variable Y_t is the domestic real income at time period t, which equals the exogenous level of domestic output by assumption; i_t is the domestic nominal interest rate at t; $\overline{M}_t$ is the central bank target to the domestic nominal money supply at t; and $\overline{S}_t$ is the nominal exchange rate target at t.

Parameters $\delta \in (0, \infty)$ and $\theta \in (0, \infty)$ are proxies for the income elasticity of money demand and the absolute value of the interest elasticity of money demand respectively. Parameter δ will not appear in the solution to this model (equation (14)). So the results presented in Section 3 should hold regardless of the value for the income elasticity of money demand. The central bank parameter φ captures the degree of official intervention in the foreign exchange market.

A major novelty in the model in Table 1 lies in equation (3); thus, it deserves a more detailed rationale. That equation is a standard LM such as $M_t/P_t = Y_t^{\delta}/(1 + i_t)^{\theta}$ (where M_t is the domestic nominal money supply at time period t) to which more structure is given by the introduction of the following policy rule:

(9) $$\frac{M_t}{\overline{M}_t} = \left(\frac{S_t}{\overline{S}_t}\right)^{\varphi}$$

where parameter φ is zero under free float and approaches either plus or minus infinity to a fixed exchange rate; leaning-against-the-wind intervention is represented by $\varphi \in (-\infty, 0)$, whereas leaning into the wind is given by $\varphi \in (0, \infty)$. Policy rule (9) was first suggested by Marston ((1985), p. 910) (see also Obstfeld and Rogoff (1996), p. 632).

The economy is under free float when $\varphi = 0$ because in such a situation the central bank focuses exclusively on the target to the domestic nominal money supply abstaining from any intervention in the foreign exchange market (i.e. $M_t = \overline{M}_t$ in (9)). When $M_t = \overline{M}_t$ and $\varphi = 0$, (3) collapses to the standard LM above, which makes up most versions of the Dornbusch model, including the DD and DDE models. Accordingly, it might be argued that the Dornbusch model implicitly presupposes free float. The DD and DDE models thus turn out to be particular cases of the model in Table 1 in that these implicitly assume $\varphi = 0$. The fixed exchange rate regime holds when $\varphi \rightarrow \pm\infty$ because in such a situation the authorities focus exclusively on the nominal exchange rate target without concern for the domestic nominal money supply (i.e. $S_t = \overline{S}_t$ in (9)). Since credibility issues are not discussed, $\varphi \rightarrow \pm\infty$ is a fixed exchange rate regime with perfect credibility.

Leaning against the wind is the intervention operation that attempts to move the exchange rate in the opposite direction from its current trend. Leaning into the wind is the operation that is motivated by the central bank's desire to support the current exchange rate trend. Here both leaning-against-the-wind and leaning-into-the-wind interventions are carried out by changes in $\overline{M}_t$. Whether such changes are sterilized is not discussed.

If $S_t > \overline{S}_t$ ($S_t < \overline{S}_t$) for any reason, the aim of leaning against the wind is thus to reduce (increase) the current nominal exchange rate S_t; that can be achieved by reducing (increasing) $\overline{M}_t$ in (9) when $\varphi < 0$. By contrast, since leaning into the wind signifies supporting the current exchange rate trend, if $S_t > \overline{S}_t$ ($S_t < \overline{S}_t$) such an intervention operation means increasing (reducing) $\overline{M}_t$ in (9) when $\varphi > 0$. As will be seen in Section 3, the degree of such interventions also matters for a successful stabilization of chaotic nominal exchange rates.

Equation (4) is the uncovered interest rate parity (UIP) hypothesis. Variable S^{e}_{t+1} is the forecast made at time period t for the nominal exchange rate at $t + 1$; and i^f_t is the foreign nominal interest rate at t. Both i_t and i^f_t are the nominal interest rates available on similar domestic and foreign securities respectively, with the same periods to maturity. UIP states that the expected foreign exchange gain from holding one currency rather than another (the expected nominal exchange rate change) must be just offset by the opportunity cost of holding funds in this currency rather than the other (the nominal interest rate differential). UIP is a basic ingredient in even the simplest versions of the Dornbusch model and is also present in both the DD and DDE models.

Equations (5)—(8) describe the speculative dynamics of the model by introducing chartist behavior among speculators. In equations (5)—(8), speculators are assumed to take positions in the market at time period t based on the forecasts they have made for $t + 1$, and these forecasts were made by them using information available at $t - 1$. That is the reason why S_{t-1} appears rather than S_t in these equations. Since S_t is the solution obtained when speculators have taken their market positions, S_t is not observable by these agents at the moment they make their forecasts.

Equation (5) splits expectations between two components—the expectations based on charts ${}_C S^e_{t+1}$, and the expectations based on the fundamentals of the model ${}_F S^e_{t+1}$. Variable S_{t-1} is the nominal exchange rate at time period $t - 1$, and $C_t \in (0, 1)$ is the weight given to charting at t. If $C_t \in (0, 0.5)$ then there is less charting than forecasts based on fundamentals. If $C_t = 0.5$ then half of the speculators are involved in charting and the other half are making forecasts based on fundamentals. If $C_t \in (0.5, 1)$ then expectations are dominated by chartists. Equation (5) appears in the DDE model too.

The expectation rule for the forecasts based on charts is given by (6). Variables S_{t-2} and S_{t-3} are the nominal exchange rates at time periods $t - 2$ and $t - 3$ respectively; and parameter $\nu \in (0, \infty)$ is the degree of past extrapolation used in technical analysis. Since $\nu > 0$, the greater ν, the more the past will be extrapolated into the future in exchange rate forecasts, and chartists will expect the nominal exchange rate at time period $t + 1$ to be less than the nominal exchange rate prevailing at $t - 1$. Rule (6) is employed by DDE (Chapter 3, p. 80) in a simple chaotic model without money. Here it is used in the context of the Dornbusch monetary model. LeBaron (1996) empirically demonstrates significant forecastability from a simple moving average trading rule (similar to (6)) for series of the US dollar against the mark and the yen that uses both weekly and daily data. Equation (6) may seem a little odd at first sight, but the further discussion presented below will help to clarify it.

The rationale to (6) is the following. Speculators expect an increase in the nominal exchange rate whenever a short run moving average of past exchange rates S^S crosses a long run moving average of past exchange rates S^L from below (Figure 1). In such an event a buy order of the foreign currency is given by them. By contrast, they expect a decline of the nominal exchange rate whenever S^S crosses S^L from above. In the latter case speculators order a selling of the foreign currency. This can be postulated as

$$\text{(10)} \qquad \frac{{}_C S^e_{t+1}}{S_{t-1}} = \left(\frac{S^S_t}{S^L_t} \right)^{2\nu}$$

Equation (10) states that since $\nu > 0$, whenever $S^S_t > S^L_t$ ($S^S_t < S^L_t$) chartists expect an increase (fall) of the nominal exchange rate relative to the most recently observed value S_{t-1}.

By assumption, the short run moving average S^S_t is based on a one period change, i.e.

(11)
$$S^S_t = \frac{S_{t-1}}{S_{t-2}}$$

and the long run moving average S^L_t is based on a two period change, i.e.

(12)
$$S^L_t = \left(\frac{S_{t-1}}{S_{t-2}}\right)^{\frac{1}{2}}\left(\frac{S_{t-2}}{S_{t-3}}\right)^{\frac{1}{2}}$$

Rule (6) can be obtained by plugging (11) and (12) into (10).

While making forecasts based on the fundamentals of the model, speculators are assumed to use the rule given by (7). Variable S^*_{t-1} is the equilibrium nominal exchange rate at time period $t - 1$, and parameter $\lambda \in (0, \infty)$ is the expected speed of return of the current nominal exchange rate toward its equilibrium value. According to (7), whenever fundamentalists observe a market rate above (below) the PPP value, they will expect it to decline (increase) in the future. Since $\lambda > 0$, the greater λ, the higher the expected speed of return toward the fundamental rate. The greater λ, the faster fundamentalists will expect the nominal exchange rate to increase (fall) toward its equilibrium value if $S_{t-1} < S^*_{t-1}$ $(S_{t-1} > S^*_{t-1})$. Values of λ greater than one mean that fundamentalists expect some sort of overshooting. Equation (7) is also employed in the DDE model.

The weight of charting is endogenized by (8). The amount of technical analysis used by speculators is made dependent on the size of the deviation of the current nominal exchange rate from its equilibrium (fundamental) value. Equation (8) states that if $(S_{t-1} - S^*_{t-1})^2 \to \infty$ then $C_t \to 0$, i.e. whenever deviations from PPP increase, the expectations based on charts will be reduced. If $(S_{t-1} - S^*_{t-1})^2 \to 0$ then $C_t \to 1$, which means that whenever deviations from PPP tend to be eliminated, charting will grow in importance among speculators. Parameter $\iota \in (0, \infty)$ is the speed at which forecasts based on charts switch to those based on fundamentals. The higher ι, the faster chartist activity will decrease (and vice versa). The same weighting function (8) is found in the DDE model. In accordance with (8), LeBaron ((1994), p. 400) points out that predictability appears to be higher during periods of lower volatility, a phenomenon used by chartists to achieve some small improvements in forecasts. This completes the description of the model.

2.2. Solution

We can proceed toward the solution to the model. The eight endogenous variables are: S_t, P_t, i_t, S^*_t, S^e_{t+1}, ${}_CS^e_{t+1}$, ${}_FS^e_{t+1}$, and C_t. The model is recursive in that the block made up of equations (5)—(8) runs first.

An additional assumption beforehand helps to simplify matters. The rate at which speculators expect the nominal exchange rate to return toward its fundamental value is assumed to be the same as the speed at which prices in the goods market actually adjust, i.e.

(13)
$$\lambda = \chi$$

One known property of the Dornbusch model is that after a possible overshooting of the nominal exchange rate in the impact period, it asymptotically moves back toward its equilibrium value at the same pace as the domestic price level movement (Dornbusch (1976), p. 1165). Therefore it is not unreasonable to think that such piece of information is taken into account by fundamentalists. Assumption (13) is also made by DD and DDE.

Substituting (13) in (7) and then plugging the resulting equation together with (6) and (8) into (5) yields an expression for S^e_{t+1}. Next, inserting (1) into (2) obtains an expression for P_t; and inserting the expression for P_t into (3) gives an expression for $1 + i_t$. Then, substituting the latter expression into (4) produces an expression for S_t. Without loss of generality we consider the exogenous (fundamental) variables constant and normalized to unity (and $i^f_t = 0$). Considering this assumption in the expression obtained earlier for S_t it becomes apparent that it depends only on a term for S^e_{t+1}. That assumption also implies $S^*_t = S^*_{t-1} = 1$ so that PPP holds in equilibrium. After inserting this result into the expression for S^e_{t+1} (and substituting it in the expression for S_t) we obtain the solution to the model for the nominal exchange rate given by the following weighted geometric moving average:

(14) $$S_t = S_{t-1}^{f_1} S_{t-2}^{f_2} S_{t-3}^{f_3}$$

where

(15) $$f_1 \equiv \frac{\theta[1 + \nu + \iota(1 - \chi)(S_{t-1} - 1)^2]}{(\theta + \chi - \varphi)[1 + \iota(S_{t-1} - 1)^2]}$$

(16) $$f_2 \equiv \frac{-2\theta\nu}{(\theta + \chi - \varphi)[1 + \iota(S_{t-1} - 1)^2]}$$

and

(17) $$f_3 \equiv \frac{\theta\nu}{(\theta + \chi - \varphi)[1 + \iota(S_{t-1} - 1)^2]}$$

Expression (14) is a nonlinear difference equation to which an analytical solution is not available. It needs to be solved numerically. To do that initial conditions (values for S_{t-3}, S_{t-2}, and S_{t-1}) are required. Equation (14) has as many solutions as there are parameter combinations. In the intertemporal equilibrium given by $S^*_t = S_{t-1} = S_{t-2} = S_{t-3} = 1$, variable S_t also equals one in (14) regardless of parameter values. Due to that independence the characteristics of solution to the model can be evaluated in the neighborhood of $(1, 1, 1)$.

To calibrate the model, the nominal exchange rate is assumed to be in equilibrium at the starting period, i.e. $S_{t-3} = 1$. Then in the two subsequent periods there occur small deviations from that equilibrium. As in DDE, here it is assumed that $S_{t-2} = 0.99$ and $S_{t-1} = 1.02$. As will be seen in the next section, the above set of initial conditions suffices to generate very complex dynamics in the Dornbusch model. The rich variety of solutions ranges from stability and cycles to chaos (accompanied or not by crashes) and instability. This has been shown by DD and DDE already. Here it is shown further that massive interventions in the foreign exchange market are able to collapse chaotic, cyclical, and unstable motions to stable ones.

3. Simulation Results

3.1. Methodology

This section presents the numerical solutions to (14) in the (ν, φ), (ι, φ), (χ, φ), and (θ, φ) spaces. The simulation results are shown in Tables 2—5, where the private parameters ν, ι, χ, and θ respectively are combined with the policy parameter φ.

In Table 2 the degree of past extrapolation into the future taking place in forecasts based on charts ν is combined with the type of foreign exchange intervention φ. The other three parameters ι, χ, and θ are fixed. The speed at which forecasts based on charts switch to those based on fundamentals is chosen to be $\iota = 10^4$ in Table 2, the same benchmark value used by DDE. Other values for ι are considered in Table 3. The actual speed of adjustment of the goods price χ—which is attached in (13) to the speed of exchange rate return toward the fundamental value λ—is assumed to be $\chi = \lambda = 0.45$, as in DD. Considering the range of possible values of $\chi \in (0, \infty)$, a strong price stickiness is assumed in Table 2. This assumption is relaxed in Table 4, however, where χ is allowed to vary. The proxy for the absolute value of the interest elasticity of money demand is picked as $\theta = 0.95$. Nevertheless, θ is allowed to vary between its theoretical range $\theta \in (0, \infty)$ in Table 5.

The conclusions about the nature of the solutions displayed in Tables 2—5 were reached after checking for the first 10000 data points, each one with eight decimal places. A cycle of periodicity above 10000 was considered as chaos for practical purposes.

The DD and DDE models may be thought of as being represented by the column for $\varphi = 0$ in Tables 2—5 because these models implicitly assume free float, as argued in Section 2. The information presented in the $\varphi = 0$ columns reveals that this model is able to replicate the same rich variety of solutions to the DD and DDE models. The pictures in Figure 2 display some selected chaotic solutions to our model.

3.2. Charting versus Intervention

As far as Table 2 is concerned, at the borderline case in which $\nu \to 0$ the model is stable for all φ values, except $\varphi = 1$. On the other hand, the very high degree of past extrapolation in chartist activity represented by $\nu = 10^5$ makes the model unstable with free float ($\varphi = 0$). Chaos mostly accompanied by crashes may also occur under free float. A currency crash is roughly defined in this model as a situation in which the nominal exchange rate suddenly depreciates by more than two digits.

Looking at both the first and the last columns in Table 2 one can see that the model is, in most cases, stable in the presence of massive foreign exchange intervention when φ is very large (an exception occurs for $\nu = 10^5$).

Table 2 also shows that chaotic solutions may emerge out of free float for small amounts of intervention in the foreign exchange market. Cases of chaos mostly accompanied by crashes for different degrees of past extrapolation in charting may appear with leaning-against-the-wind ($\varphi < 0$) and leaning-into-the-wind ($\varphi > 0$) interventions. The more chartists extrapolate the past into the future (the greater ν), the larger the variance of chaotic nominal exchange rate movements. Simple chaotic series become accompanied by slight crashes which turn violent soon after. We thus conclude that the more chartists extrapolate the past in forecasts, the greater the crashes. It might be noted that currency crashes always come to an end in this chaotic model due to the presence of centripetal forces operating for large deviations of the nominal exchange rate from equilibrium.

Figure 3 provides an illustration of the property of extreme sensitivity to initial conditions with past extrapolation in charting ($\nu = 15$) and free float ($\varphi = 0$). An exogenous shock of 1% is introduced at time period 9950; as a result, the new (dotted) series follows an entirely different trajectory. This is the famous 'butterfly effect' of the chaos literature. Massive intervention is able to stabilize the same series, however. Figure 4 displays the same degree of past extrapolation into the future by chartists as the one considered in Figure 3 (i.e. $\nu = 15$) being neutralized by massive leaning-into-the-wind intervention ($\varphi = 10^4$).

The most striking discovery obtained in the simulations presented in Tables 2—5 is the clear pattern emerging as far as stability is concerned. As a rule, stable solutions can be recognized in both the left and right hand sides in Tables 2—5. An exception occurs for the top of Table 5, as discussed below. This reveals that massive central bank intervention has the ability to reverse chaotic, cyclical, and unstable series to stable ones. In particular, the 'steps' of stable solutions shown in Table 2 indicate that the higher the past extrapolation by chartists, the larger must be the volume of intervention aiming at stabilizing the nominal exchange rate.

3.3. Change of Forecast Rule and Intervention

The speed at which forecasts based on charts switch to those based on fundamentals is now allowed to vary along with the policy parameter in Table 3, where the solutions to the model in the (ι, φ) space are presented. The other fixed parameter values are $\nu = 500$, $\chi = \lambda = 0.45$, and $\theta = 0.95$. Thus the row for $\iota = 10^4$ in Table 3 matches with the row for $\nu = 500$ in Table 2 (and with the row for $\chi = 0.45$ in Table 4, and the row for $\theta = 0.95$ in Table 5).

DDE (p. 109 n3) report that the size of parameter ι does not affect their results in that if a large ι produces chaos then a smaller ι also does. The results displayed in Table 3 do confirm that if a large ι generates chaos then a smaller value for this parameter also does. Looking at the column for $\varphi = 0$ one can note the presence of chaos for both high and low speeds at which chartist activity takes place. The discovery that massive interventions in the foreign exchange market are capable of stabilizing chaotic, cyclical, and unstable nominal exchange rates is also robust regarding the results in Table 3, as can be seen in the columns on both the left and right hand sides.

3.4. Price Flexibility and Intervention

Table 4 displays the solutions to (14) focusing on the relationship between goods price flexibility (parameter χ) and central bank intervention (parameter φ). Since the assumption of fixed prices is ad hoc in the context of the Dornbusch model, it is reasonable to relax it allowing for increasing goods price flexibility. As a result, nominal exchange rate variability no longer necessarily means real volatility.

The solutions to the model in the (χ, φ) space are obtained regarding $\nu = 500$ as given, and the other parameters and initial conditions are the same as in Table 2, namely $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\iota = 10^4$, and $\theta = 0.95$. The same findings shown in Tables 2 and 3 can be seen in Table 4, i.e. massive foreign exchange intervention has the ability to stabilize the nominal exchange rate. This can be checked in both the first three columns on the left hand side and the last two columns on the right hand side.

3.5. Money Demand and Intervention

The interest elasticity of money demand is now related to the policy parameter in Table 5, which shows the solutions to the model in the (θ, φ) space. The fixed parameters are $\nu = 500$, $\iota = 10^4$, and $\chi = 0.45$, and the initial values for the nominal exchange rate are the same as those assumed so far.

If the interest elasticity of money demand approaches minus infinity the economy is in the so-called liquidity trap. Since θ refers to the absolute value of the interest elasticity, the liquidity trap is defined in this model as $\theta \to \infty$. As long as figure 10^5 is considered as a proxy to plus infinity, the first row at the top of Table 5 displays the liquidity trap situation.

Two major patterns can be recognized in Table 5. The first row at the bottom for $\theta \to 0$ shows that the system is stable regardless of the type of intervention if the demand for money is not influenced by the nominal interest rate. Taking the $\varphi = 0$ column as a benchmark, it can be noted—from the row for $\theta = 0.1$ to around the row for $\theta = 10$—that chaotic, cyclical, and unstable series can be stabilized by massive interventions. Thus, these sensible values for the interest elasticity of money demand reproduce our findings shown in Tables 2—4 concerning the stabilizing power of intervention. However, commencing at the row for $\theta = 10^2$ until very high interest elasticities of money demand, it can be realized that intervention plays no role. This result is not unexpected. Actually, it is in line with the textbook wisdom that monetary policy becomes powerless under the liquidity trap.

4. Contrast with Stylized Facts and Previous Work

The above findings are now contrasted with some stylized facts and previous results in literature.

Most chaotic series displayed in Tables 2—5 show that chaos is accompanied by crashes, which means that not only does the nominal exchange rate exhibit a random-like behavior but also heteroskedasticity is present (e.g. panel *b* in Figure 2). Therefore, the stylized fact that a martingale process is more likely to describe nominal exchange rate behavior is replicated in chaotic models.

Some studies adopt the modeling strategy of reducing all structure of a model to only one single variable. This is intended to focus analysis on the effect of 'news', i.e. unexpected changes in the nominal exchange rate that result from changes in the fundamentals that come as a surprise. The news approach thus relies on the existence of an unexpected shock underlying any one nominal exchange rate movement. However, only a small proportion of spot movements of the nominal exchange rate seems to be caused by news (Goodhart (1990)). As in DD and DDE, the results presented in Section 3 are consistent with the fact that large variations in the nominal exchange rate may occur without it being possible to identify the cause in any shock. Violent currency crashes may emerge in chaotic series with no change in the exogenous variables of the model. Crashes are caused by dynamic chaos without random external influences. Hence an advantage of chaotic models is not to rely on random shocks to explain nominal exchange rate swings.

For a given foreign price level, since the domestic price level is rigid in the impact period, the real exchange rate follows the nominal rate in the Dornbusch model. This feature together with the circumstance that free float is implicit in that model makes it consistent with the stylized fact that the volatility of the real exchange rate is much higher under flexible exchange rates than under fixed rates. It has been suggested that a factor explaining the bad empirical performance of the Dornbusch model is that actual data after the Bretton Woods era are managed floating data rather than the pure float data which the model addresses (e.g. Gartner (1993), p. 196). The chaotic model presented here is consistent with the stylized fact that real exchange rates are more volatile under free float, as can be appreciated in Tables 2, 3, and 5.

The model presented here also makes a case for the importance of macromodels—in which fundamentals play a role—to explain nominal exchange rate behavior. This is thus in line with attempts to revive explanations based on fundamentals to beat the simple random walk model (e.g. Mark (1995)). Here fundamentals matter because the interaction between speculative private behavior and foreign exchange intervention can give rise to chaos and thus mimic a random walk. Fundalmentals also matter because massive foreign exchange interventions are able to stabilize the chaotic motions and thus influence the nominal exchange rate.

There is a piece of indirect evidence addressing the implication that intervention may stabilize a chaotic nominal exchange rate. Dominguez (1993), for instance, presents evidence that foreign exchange intervention actually reduces the volatility of the nominal exchange rate. There is an apparent puzzling piece of evidence too. Using both weekly and daily data of foreign exchange intervention and foreign exchange series of the US dollar against the mark and the yen, LeBaron (1996) shows that after removing periods in which the Federal Reserve is active, the ability to predict future exchange rates coming from technical trading rules is dramatically reduced. This suggests that central bank intervention may introduce noticeable trends into the evolution of the nominal exchange rate and thus create profit opportunities coming from speculation against the central bank. Taylor (1982) and Leahy (1995) find evidence that central banks make money on their foreign exchange intervention operations; and Silber (1994) presents evidence in a cross sectional context that technical rules have value whenever governments are present as major players. Szpiro (1994) even argues that an intervening central bank may induce chaos in the nominal exchange rate. These results suggest that the more central banks intervene, the more they give incentives to chartists to enter the market, thereby increasing the chance of chaos.

However, our model shows that the more central banks intervene, the less the likelihood of chaos. Most precisely, *massive* intervention can remove chaos from the foreign exchange market. Nevertheless the 'puzzling' effect of intervention generating chaos also appears in the results displayed in Tables 2—5, where *low* volumes of foreign exchange intervention can also lead to chaos. Here low amount of intervention can be interpreted as either intervention toward profitability on the part of the central bank or noncredible attempts at stabilization of the nominal exchange rate.

Elsewhere (Da Silva (2001)), I show that chaos is possible in a sticky price model with microfoundations, where the model of Obstfeld and Rogoff ((1995), (1996)) is extended to encompass the speculative dynamics and the modeling of foreign exchange intervention discussed in this paper. In such a model, chaos is also possible under a nonzero amount of foreign exchange intervention.

5. Conclusion

This paper examines whether a result obtained in De Grauwe and Dewachter (1992) and De Grauwe, Dewachter, and Embrechts ((1993), Chapter 5), that chaos can be generated in a Dornbusch style model if some traders are chartists, will continue to hold when central banks engage in foreign exchange intervention. The answer seems to be a qualified no, that is, generally massive interventions are able to stabilize the chaos.

Central bank intervention is introduced to represent anti-chartist behavior through a rule connecting the nominal exchange rate and the nominal money supply. The analysis of both leaning-against-the-wind and leaning-into-the-wind interventions becomes possible and the Dornbusch model is reduced to the particular case of free float. Chaotic, cyclical, and unstable nominal exchange rate series under free float and low amount of intervention are shown to collapse to stable ones as long as massive central bank interventions are carried out.

The results in this paper show that past extrapolation into the future in chartists' forecasts can produce very complex dynamics in the Dornbusch model. The possible solutions to the model range from stability and cycles of different periodicities to chaos (with or without crashes) and instability. Under both free float and low volumes of intervention, the more chartists extrapolate the past into the future, the larger the variance of the chaotic nominal exchange rate, and the greater the currency crashes. The higher the past extrapolation by chartists, the larger must be intervention to stabilize the nominal exchange rate. Also, the emergence of chaos does not show dependence on the speed at which forecasts based on charts switch to those based on fundamentals.

Massive interventions can also remove chaotic, cyclical, and destabilizing movements even if the assumption that prices are sticky is relaxed. Chaos, crashes, cycles, and instability emerge with both free float and low volumes of intervention, when the interest elasticity of money demand assumes sensible values. Massive interventions can again produce stability in such a scenario. However, intervention plays no role in the liquidity trap.

This study shows consistency with a number of stylized facts and previous results in the literature on exchange rates and foreign exchange intervention. Most remarkably, the model presented here is able to conciliate the two apparently divergent pieces of evidence that the nominal exchange rate appears to follow a martingale process although it also seems to be explained by fundamentals. The random-like behavior of the nominal exchange rate in which crashes crop up is generated by deterministic solutions to the model, and since massive foreign exchange interventions are able to stabilize the chaotic motions, fundamentals—most precisely, exchange rate policy—also influence the nominal exchange rate.

Appendix

The decision that a given solution to the model is chaotic is made in Tables 2—5 on the grounds that no data point repeats itself in the range of 10000 periods. A problem with simulations is that there is no formal guarantee that a reached conclusion still applies for the simulation range plus one. For that reason, formal tests for chaos are carried out in this Appendix for the obtained chaotic solution with parameters $\nu = 15$, $\varphi = 0$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, and initial values 1.00000000, 0.99000000, and 1.02000000. A data record of 15000 points is taken, and the first 100 values are skipped to allow for the nominal exchange rate to settle into its final behavior. The program employed for data analysis was Chaos Data Analyzer: The Professional Version 2.1® by Sprott (1995). The pictures in Figure 5 were obtained using such a software. Information regarding description of statistics as well as suggestions for analysis strategy are given by Sprott and Rowlands (1995).

Panel *a* in Figure 5 shows that a discernible structure emerges in a three-dimensional embedding plotting, i.e. there is a 'strange attractor'. Strange attractors are suggestive pictures that can be plotted from chaotic series showing some order in fake randomness. The shape of the attractor is very similar to that in DD (p. 39), which is generated by a plotting of 6000 observations in a two-dimensional diagram. It also retains a certain resemblance to the two-dimensional phase diagram shown in DDE (p. 136). DD (p. 39) display a blow-up showing that no datapoint repeats itself in this attractor. Since the data are aperiodic but not random, they are chaotic. Indeed, chaos is defined as apparently stochastic behavior occurring in deterministic systems (Stewart (1997), p. 12).

A calculated Hurst exponent of about 0.99 indicates that the data are highly 'persistent', i.e. past trends persist into the future. The Hurst exponent gives a measure of the extent to which the data can be represented by a random walk (or a fractional Brownian motion). White (uncorrelated) noise has a Hurst exponent of 0.5. Indeed, an IFS clumpiness test (panel *b* in Figure 5) shows that the data cannot be represented by a random walk. In a picture displaying the IFS clumpiness test, white noise fills it uniformly whereas chaos or correlated noise generates localized clumps. The data are not periodic either, because the calculated LZ complexity is around 0.6. Relative LZ complexity gives a measure of the algorithmic complexity of a time series. Maximal complexity (randomness) has a value of 1.0, whereas perfect predictability (cycles) has a value of 0.

An attractor can be quantified by measures of its dimension and its Lyapunov exponents. The dimension evaluates the complexity of the attractor, whereas the Lyapunov exponent measures the sensitivity to initial conditions, i.e. the famous 'butterfly effect' of chaotic series. Capacity dimension and correlation dimension are major measures of dimension of a chaotic attractor. Values greater than about 5 for these measures give an indication of randomness, whereas values less than 5 provide further evidence of chaos. Extreme sensitivity to tiny changes in initial conditions and therefore evidence of chaos is obtained as long as the largest Lyapunov exponent is positive. A zero exponent occurs near a bifurcation; periodicity is associated with a negative Lyapunov exponent; and white (uncorrelated) noise is related to an exponent approaching infinity.

The correlation dimension calculated from the data is about 1.7. This gives evidence of chaos. Panel *c* in Figure 5 shows a saturation in the calculated correlation dimension as the embedding dimension is increased. Such a well-defined plateau indicates an appropriate embedding dimension in which to reconstruct the attractor. The picture suggests the proper embedding as given by 3. Thus, panel *c* provides an indication of low-dimensional chaos, albeit some quasi-periodic data may also exhibit a plateau.

The capacity dimension calculated from the data is about 1.9. So the calculated capacity dimension is compatible with the presence of chaos in the data.

The largest Lyapunov exponent calculated from the data is about 0.3, whereas the largest Lyapunov exponent to the base *e* is about 0.2. These calculations considered 3 time steps and the proper embedding dimension as given by 3. Such positive values for the Lyapunov exponents give further evidence of chaos.

To test whether the evidence of hidden determinism in the data is robust, it is prudent to repeat the calculations of the quantitative measures of the attractor using surrogate data that resemble the original data but with the determinism removed. Robustness implies that analysis of the surrogate data should provide values that are statistically distinct from those calculated from the original data. As observed, "this test is a very important one and is rarely included in papers claiming observation of low-dimensional chaos in experimental data" (Sprott and Rowlands (1995), p. 15). The most useful method is to Fourier-transform the data, randomize the phases, and then inverse Fourier-transform the result to get a new time series with the same spectral properties as the original but lacking determinism; this implies a different probability distribution. After employing such a method, a bell-shaped distribution (indicating lack of determinism) was indeed obtained.

The major quantitative measures of the surrogate data were also different from those of the original data. The largest Lyapunov exponent and the largest Lyapunov exponent to the base *e* were 0.847 ± 0.011 and 0.587 ± 0.008 respectively. The correlation dimension was 4.505 ± 0.089, and the calculated capacity dimension was 2.238 ± 0.128. To know whether this difference is statistically significant, one should ideally generate many surrogate data sets and see whether the results from the original data lie within the range of values corresponding to the surrogates. If they do, then the difference is not statistically significant and the original data are indistinguishable from correlated noise. By repeating the above test twice, the calculated Lyapunov exponents were found to be greater than the values shown above. Another surrogate data set was generated by simply shuffling the original data values, as one shuffles a deck of cards. The calculated Lyapunov exponents were still greater than the ones obtained from the original data. Thus, the conclusion that the data obtained from the Dornbusch model are chaotic and distinguishable from correlated noise seems to be robust.

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Table 2. **Solutions to the Model in the (ν, φ) Space: Degree of Past Extrapolation in Charting versus Foreign Exchange Intervention**

ν φ	-10^5	-10^4	-10^3	-10^2	-10	-1	-0.5	0	0.5	1	10	10^2	10^3	10^4	10^5
10^5	C4	C4	CH*	C8	U	CH*	U	U	U	U	U	U	C8	C2	U
10^4	ST	C4	C4	C4	C4	U	U	U	U	U	C2	C2	C2	C2	ST
10^3	ST	ST	C8	C4	U	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
500	ST	ST	ST	CH*	U	CH	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
250	ST	ST	ST	C4	U	CH*	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
225	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
200	ST	ST	ST	C4	CH*	CH	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
175	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
150	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
125	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
10^2	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
75	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
50	ST	ST	ST	ST	CH	C8	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
25	ST	ST	ST	ST	C4	CH*	CH*	CH*	CH	U	C2	ST	ST	ST	ST
15	ST	ST	ST	ST	C4	CH	CH	CH	CH	U	C2	ST	ST	ST	ST
10	ST	ST	ST	ST	C4	CH	CH	CH	CH	U	C2	ST	ST	ST	ST
5	ST	ST	ST	ST	ST	C4	C4	CH	CH	U	C2	ST	ST	ST	ST
0	ST	ST	ST	ST	ST	ST	ST	ST	ST	U	ST	ST	ST	ST	ST

ST = stable solution; Ci = cycle of periodicity i; CH = chaotic solution (* denotes occurrence of crashes); U = unstable solution.

The greater ν, the more the past is extrapolated into the future in nominal exchange rate forecasts.
$\varphi = 0$ represents free float; intervention increases as one moves to both the left and right hand sides of the $\varphi = 0$ column.
$S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$.

Table 3. **Solutions to the Model in the (ι, φ) Space: Speed at which Forecasts Based on Charts Switch to those Based on Fundamentals versus Foreign Exchange Intervention**

ι φ	-10^5	-10^4	-10^3	-10^2	-10	-1	-0.5	0	0.5	1	10	10^2	10^3	10^4	10^5
10^5	ST	ST	ST	C4	U	CH*	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
10^4	ST	ST	ST	CH*	U	CH	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
10^3	ST	ST	ST	C4	C4	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
500	ST	ST	ST	C4	C4	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
300	ST	ST	ST	C4	C4	U	U	U	CH*	U	C4	C2	C2	ST	ST
275	ST	ST	ST	C4	C4	U	U	CH*	CH*	U	C6	C2	C2	ST	ST
250	ST	ST	ST	C4	U	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
225	ST	ST	ST	C4	U	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
200	ST	ST	ST	C4	C4	U	U	U	CH*	U	C2	C2	C2	ST	ST
10^2	ST	ST	ST	C4	U	U	U	CH*	CH*	U	U	C2	C2	ST	ST
75	ST	ST	ST	C16	U	U	U	CH*	CH*	U	U	C2	C2	ST	ST
50	ST	ST	ST	C4	U	U	U	CH*	CH*	U	U	C2	C2	ST	ST
25	ST	ST	ST	C4	U	CH*	U	CH*	U	U	U	C2	C2	ST	ST
20	ST	ST	ST	C4	U	CH*	U	CH*	CH*	U	U	C2	C2	ST	ST
10	ST	ST	ST	U	CH	CH*	CH*	CH*	CH*	U	U	U	C2	ST	ST
5	ST	ST	ST	U	CH	CH*	CH*	CH*	CH*	U	U	U	C2	ST	ST
1	ST	ST	ST	CH*	CH*	U	U	CH*	U	U	U	U	C2	ST	ST
0	ST	ST	ST	CH*	U	U	U	U	U	U	CH*	CH*	U	ST	ST

ST = stable solution; Ci = cycle of periodicity i; CH = chaotic solution (* denotes occurrence of crashes); U = unstable solution.

The greater ι, the faster chartist activity decreases.
$\varphi = 0$ represents free float; intervention increases as one moves to both the left and right hand sides of the $\varphi = 0$ column.
$S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\nu = 500$, $\chi = \lambda = 0.45$, $\theta = 0.95$.

Table 4. **Solutions to the Model in the (χ, φ) Space: Price Flexibility versus Foreign Exchange Intervention**

χ \ φ	-10^5	-10^4	-10^3	-10^2	-10	-1	-0.5	0	0.5	1	10	10^2	10^3	10^4	10^5
10^5	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST
10^4	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST
10^3	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	C3	ST	ST	ST
900	ST	ST	ST	ST	C3	C3	C3	C3	C3	C3	C3	C6	U	ST	ST
800	ST	ST	ST	C10	C6	C6	C6	C6	C6	C6	C6	C6	U	ST	ST
700	ST	ST	ST	C4	C12	C12	C12	C12	C12	C12	C12	U	U	ST	ST
600	ST	ST	ST	C12	CH	CH	CH	CH	CH	CH	CH	U	U	ST	ST
500	ST	ST	ST	CH	CH	CH	CH	C24	C24	C12	C6	U	CH	ST	ST
250	ST	ST	ST	CH	C3	C3	C3	C3	CH	CH	CH	CH*	C2	ST	ST
200	ST	ST	ST	CH	C12	CH	CH	CH	CH	CH	CH	U	C2	ST	ST
10^2	ST	ST	ST	CH	C6	CH	CH	CH	CH	CH	U	CH*	C2	ST	ST
50	ST	ST	ST	CH	CH*	CH*	CH*	CH*	CH*	CH*	U	CH	C2	ST	ST
25	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	CH*	U	CH	C2	ST	ST
10	ST	ST	ST	C4	CH*	CH*	CH*	C6	C3	CH*	CH*	CH	C2	ST	ST
5	ST	ST	ST	C4	C4	CH*	CH*	CH*	CH*	C3	C2	CH	C2	ST	ST
1	ST	ST	ST	CH*	CH*	CH*	CH*	CH	CH*	CH*	C8	C2	C2	ST	ST
.45	ST	ST	ST	CH*	U	CH	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
0	ST	ST	ST	C16	CH	CH*	CH*	ST	U	CH*	C2	C2	C2	ST	ST

ST = stable solution; Ci = cycle of periodicity i; CH = chaotic solution (* denotes occurrence of crashes); U = unstable solution.

$\chi = 0$ means price rigidity; $\chi \to \infty$ signifies full price flexibility.
$\varphi = 0$ represents free float; intervention increases as one moves to both the left and right hand sides of the $\varphi = 0$ column.
$S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\nu = 500$, $\iota = 10^4$, $\theta = 0.95$.

Table 5. **Solutions to the Model in the $(\theta,\ \varphi)$ Space: Interest Elasticity of Money Demand (Absolute Value) versus Foreign Exchange Intervention**

θ φ	-10^5	-10^4	-10^3	-10^2	-10	-1	-0.5	0	0.5	1	10	10^2	10^3	10^4	10^5
10^5	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	U
10^4	U	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	U	C2
10^3	CH*	U	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	U	C2	C2
10^2	CH	C24	U	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	U	C2	C2	CH
50	ST	C4	U	U	CH*	CH*	CH*	CH*	CH*	CH*	CH*	C2	C2	C2	C2
10	ST	CH	CH*	U	CH*	CH*	CH*	CH*	CH*	CH*	U	C2	C2	CH	ST
1.5	ST	ST	C4	CH*	U	CH*	C4	CH*	CH*	CH*	C2	C2	C2	ST	ST
1	ST	ST	CH	CH*	U	CH*	CH*	CH	CH*	U	C2	C2	CH	ST	ST
.95	ST	ST	ST	CH*	U	CH	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
.8	ST	ST	ST	C68	U	CH*	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
.7	ST	ST	ST	C4	U	CH*	CH*	C4	CH*	U	C2	C2	C2	ST	ST
.6	ST	ST	ST	C4	U	C3	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
.5	ST	ST	ST	C4	U	C12	U	CH*	CH*	CH*	C2	C2	C2	ST	ST
.4	ST	ST	ST	C4	CH*	U	C3	CH*	CH*	CH*	C2	C2	ST	ST	ST
.3	ST	ST	ST	C4	CH*	U	U	CH	CH*	C2	C2	C2	ST	ST	ST
.2	ST	ST	ST	C4	CH*	U	U	C6	CH*	C2	C2	C2	ST	ST	ST
.1	ST	ST	ST	ST	CH*	U	U	U	U	C2	C2	C2	ST	ST	ST
0	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST

ST = stable solution; Ci = cycle of periodicity i; CH = chaotic solution (* denotes occurrence of crashes); U = unstable solution.

Reasonable values of θ fall into the interval between zero and one; very high values of θ give the situation of liquidity trap.
$\varphi = 0$ represents free float; intervention increases as one moves to both the left and right hand sides of the $\varphi = 0$ column.
$S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\nu = 500$, $\iota = 10^4$, $\chi = \lambda = 0.45$.

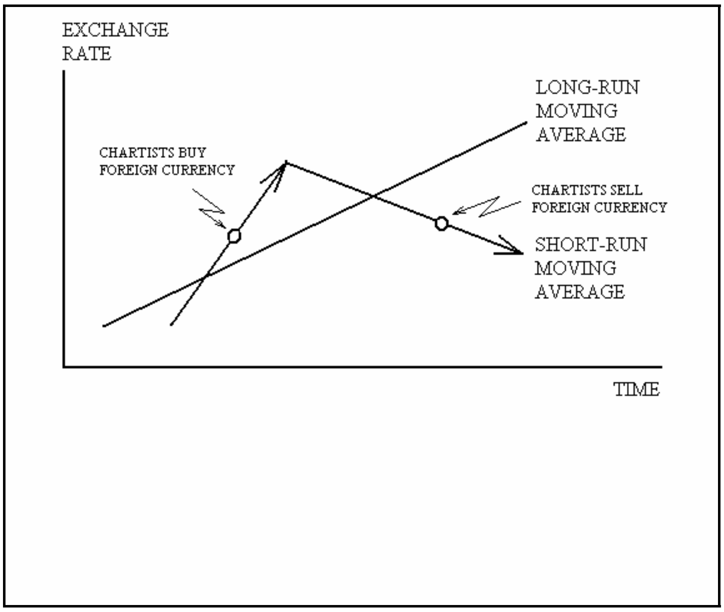


Figure 1. The Chart Used in the Model Forecasts.

Speculators expect an increase in the nominal exchange rate whenever a short run moving average of past exchange rates $S^S{}_t$ crosses a long run moving average of past exchange rates $S^L{}_t$ from below; in such an event they give a buy order for the foreign currency.
By contrast, they expect a decline of the nominal exchange rate whenever $S^S{}_t$ crosses $S^L{}_t$ from above; in the latter case speculators order a selling of the foreign currency.
Source: DDE (p. 73) with minor modifications.

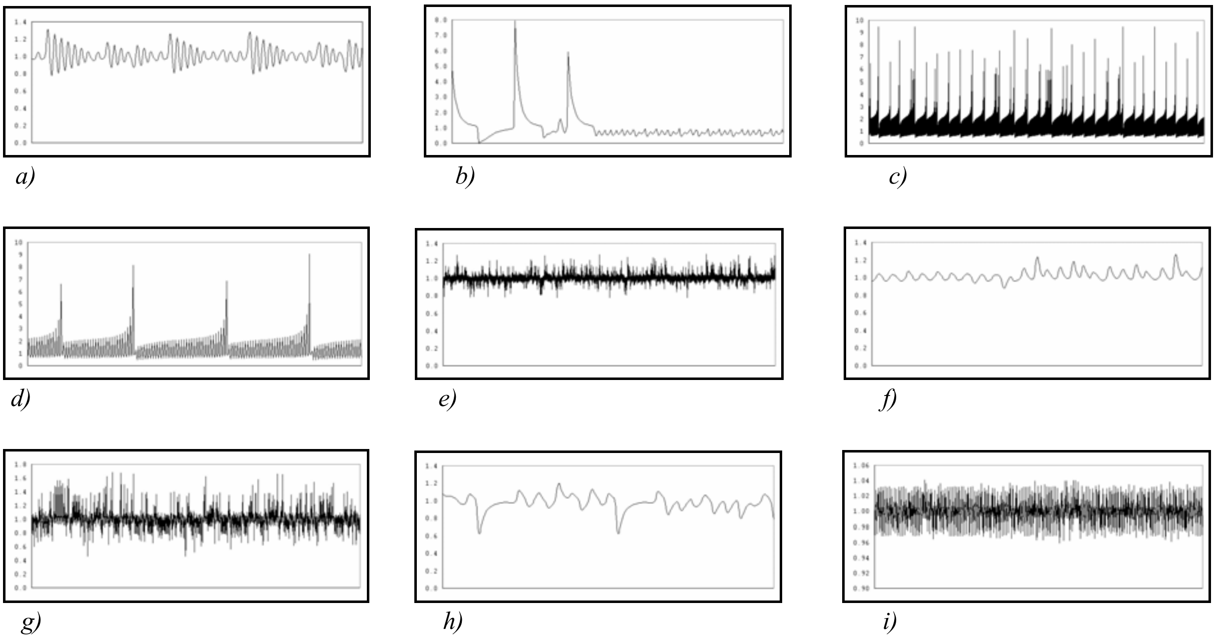


Figure 2. Display of Selected Chaotic Solutions.

a) $\varphi = 0$, $v = 500$, $\iota = 10^4$, $\chi = \lambda = 200$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: 9900—10000.

b) $\varphi = 1$, $v = 500$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 1.5$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: 9700—10000.

c) $\varphi = -10$, $v = 50$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: 6001—10000.

d) $\varphi = -10$, $v = 50$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range 9500—10000.

e) $\varphi = 0$, $v = 5$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: 6001—10000.

f) $\varphi = 0$, $v = 5$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: 9900—10000.

g) $\varphi = 0.5$, $v = 10$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: 6001—10000.

h) $\varphi = 0.5$, $v = 10$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: 9900—10000.

i) $\varphi = 10^3$, $v = 500$, $\iota = 10^4$, $\chi = \lambda = 500$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: 6001—10000.

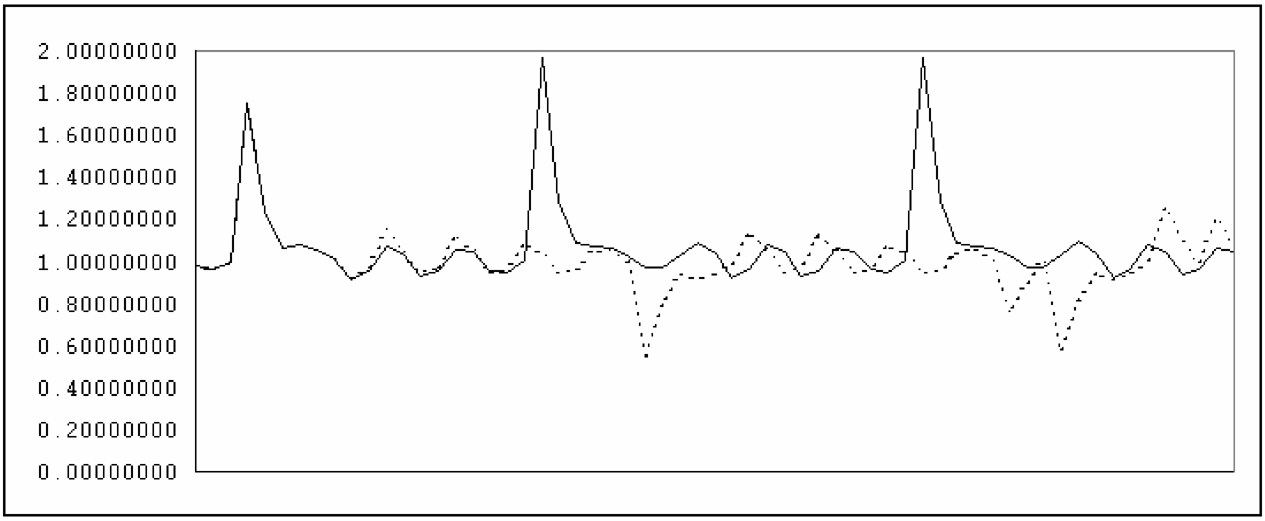


Figure 3. Extreme Sensitivity to Changes in Initial Conditions ('Butterfly Effect') with Past Extrapolation into the Future in Charting ($\nu = 15$) *and Free Float* ($\varphi = 0$).

Other values are: $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\iota = 10^4$, $\chi = \lambda = 0.45$, and $\theta = 0.95$. Range: 9940–10000. A shock (increase) of 1% is introduced at time period 9950; as a result, the new (dotted) series follows an entirely different trajectory, showing extreme sensitivity to changes in initial conditions.

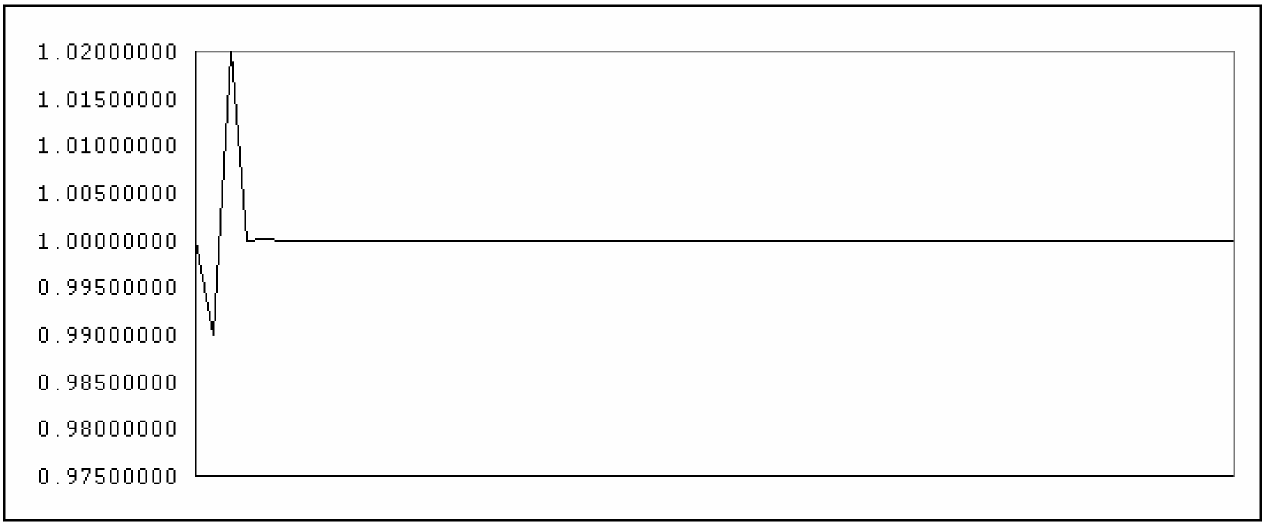
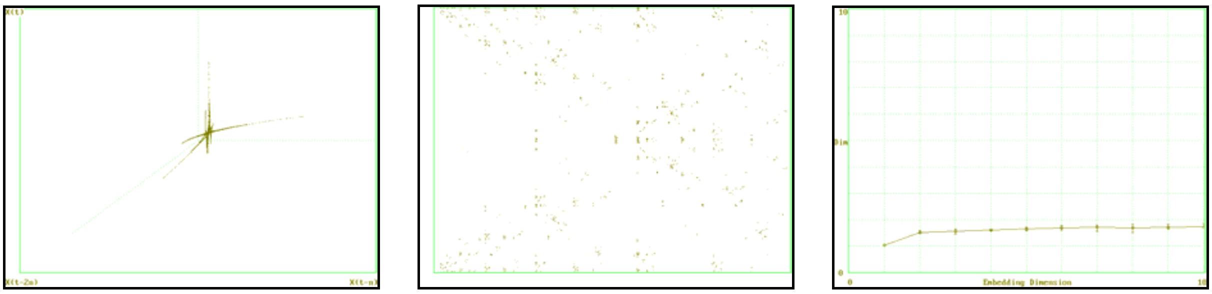


Figure 4. Massive Foreign Exchange Intervention Stabilizes Chaos and Currency Crashes in the Extended Dornbusch Model of the Foreign Exchange Market.

Other values are: $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\iota = 10^4$, $\chi = \lambda = 0.45$, and $\theta = 0.95$. This picture shows the first 60 data points for $\nu = 15$ and $\varphi = 10^4$.



a) Strange Attractor *b) IFS Clumpiness Test* *c) Correlation Dimension*

Figure 5. Chaotic Solution with Parameters $\nu = 15$, $\varphi = 0$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, *and Initial Values* $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, *and* $S_{t-1} = 1.02000000$.

a) This plot in a three-dimensional embedding reveals a clear structure in the data: a strange attractor can be recognized.
b) This picture shows localized clumps indicating chaos or correlated noise.
c) This picture shows a clear saturation in the calculated correlation dimension as the embedding dimension is increased; the well-defined plateau gives an indication of the proper embedding associated with chaos, although some quasi-periodic data may also exhibit such a property.