Fear of Floating: An optimal discretionary monetary policy analysis

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First Draft: October 2004
This Draft: September 2005

Abstract

This paper explores the idea that "Fear of Floating" and accompanying pro-cyclical interest rate policies observed in the case of some emerging market economies may be justified as an optimal discretionary monetary policy response to shocks. The paper also examines how the differences in monetary policies may lead to different degrees of this fear.

These questions are addressed with a small open economy, new-Keynesian model with endogenous capital accumulation and sticky prices. The economy consists of two sectors- traded and non-traded. International credit markets are assumed to be imperfect, so that only the traded sector enjoys the ability to borrow internationally in foreign currency. The firms in the traded sector could potentially hold a large proportion of their debt in foreign currency, while the liabilities of the non-traded sector firms are entirely denominated in the domestic currency. Domestic exchange rate volatility adversely affects the balance sheets of the traded sector firms, while interest rate volatility creates problems for the firms in the non-traded sector. In such a situation, the monetary authorities face a dilemma when reacting to shocks. The numerical solution of the model indicates that the central bank's reaction to shocks depends not only on the net effect of exchange rate movements on output gap and inflation, but also on the relative weight the central bank allocates to stabilizing output in the traded sector as against the non-traded sector. A central bank that assigns relatively higher importance to output stability in the traded goods sector also displays greater aversion for exchange rate volatility.

* I am grateful to Carl Walsh, Joshua Aizenman and Michael Hutchison for valuable suggestions and comments. All remaining errors are mine. I also wish to thank the participants of the brown bag seminar at University of California, Santa Cruz.

1. Introduction

The term “Fear of floating” indicates the behavior of several emerging market economics that have officially adopted flexible exchange rate regimes but seem reluctant to allow their currencies to move freely, in response to shocks.[1] Countries that exhibit fear of floating seem to use both reserves as well as interest rates as tools to heavily manage their exchange rates. They allow higher volatilities in reserves and interest rates (real and nominal) while keeping their exchange rate volatility low, as compared to other floaters like the United States and Japan. In trying to keep exchange rate volatility low, these countries sometimes seem to engage in pro-cyclical interest rate policies.[2]

This paper explores the idea that fear of floating and the accompanying pro-cyclical interest rate movements observed in several emerging market economies may be an outcome of an optimal discretionary monetary policy. In an economy made up of sectors that are characterized by differing degrees of openness, exchange rate movements will have differential qualitative and quantitative effects on output volatilities across the sectors. Monetary authorities need not view all sectors equally, and may have a relatively greater preference for output stability in some of the sectors. A relatively stronger preference for output stability in the more open sectors may lead to a fear of floating type behavior. Such a monetary policy can also explain the strikingly different degrees of fear that have been observed across emerging market countries. The degree of the fear is directly proportional to the relative preference of the monetary authorities for the sectors in the economy that are more open than others.

The above idea is analyzed in this paper with the help of a new-Keynesian macro model developed in Walsh (2003) and Woodford (2003). The economy under consideration is a two-sector, small open economy with endogenous capital accumulation, Bernanke-Gertler type credit constraints and nominal rigidities introduced through an assumption of price stickiness. There are three types of agents; households, firms, and a central bank.

[1] This term was first coined by Calvo G. and C. Reinhart (2000a)

[2] See Calvo G. and C. Reinhart (2000a, 2000b).

The two sectors are a traded goods sector and a non-traded sector. The traded sector produces goods for domestic consumption and exports. Firms in the traded sector enjoy the ability to borrow internationally in foreign currency. They could potentially hold a large proportion of their debt in foreign currency. The non-traded sector on the other hand produces goods for domestic consumption. Liabilities of the firms in the non-traded sector are always entirely denominated in the domestic currency.

The model is solved numerically, and is calibrated in order to examine the reaction of monetary authorities when the economy is subjected to both real (aggregate demand) and nominal (inflation and foreign interest rate) shocks.

The calibration results indicate that we should observe relatively lower exchange rate volatility and higher interest rate volatility, when the central bank assigns relatively greater importance to output stability in the traded sector. The degree of fear of floating indicated by the variance of the exchange rate also increases when the weight that the central bank assigns to output volatility in the traded sector increases.

The arguments that have been made so far in the literature to explain fear of floating are essentially similar to those for fixed and intermediate regimes. Calvo and Reinhart (2001a) have suggested a lack of credibility on the part of the monetary authorities as the primary reason. Hausman, Panizza and Stein (2001) have also suggested adverse effects of exchange rate movements on domestic price level, balance sheets and competitiveness as other possible reasons.[3]

Cespedes, Chang and Velasco (2001, 2002) have further examined whether a limited access to international capital markets and an inability of agents in emerging markets to borrow internationally in domestic currency leads to a fear of floating result. They examine the implications of optimal monetary policy with and without commitment, under a flexible exchange rate regime, with balance sheet effects and Calvo-type overlapping wage contracts and compare it to those of fixed exchange rates. In the first paper the authors show that flexible exchange rate regimes dominate fixed rate regimes

[3] See Hausman, Pannizza, Stein (2001)

even in the presence of dollar liabilities and Bernanke-Gertler type balance sheet effects. This is because real devaluation occurs in both regimes as a result of balance sheet effects, when there is a negative external shock. In flexible rate regimes, it occurs as a result of real depreciation and in the fixed case it will occurs via a domestic deflation. As a result there is a contraction of output under both the regimes. This contraction is greater in the fixed rate case because of the presence of nominal wage rigidities.

In the second paper, the authors study the determination of optimal monetary and exchange rate policy with and without commitment in an economy with balance sheet effects and Calvo-type wage setting. They compare the results to that of a completely fixed exchange rate regime. The idea is that fixed rates are a commitment device and may hence prove to be superior to flexible rates in case of discretionary monetary policy. They show that discretionary inflation targeting policies under flexible exchange rate regimes still yields higher welfare than a policy of strictly fixed rate regimes. Further the authors observe that the nominal interest rate is higher and more volatile in a flexible rate regime under discretion and flexible inflation targeting when there is an adverse shock, as compared to a fixed rate regime. However they rule out the argument that this is due to a fear of floating, since inflation is also higher (hence real interest rate may not be pro-cyclical) and the high interest rate tends to fall immediately after the initial rise.

Glick (2000) argues that devaluations can be contractionary in some countries, if they are associated with a reduction in aggregate demand, as a result of a reduction in real income or wealth. At the same time devaluation can lead to a reduction in aggregate supply, as a result of an increase in the cost of imported inputs or capital. In the financial sector the debt-servicing burden increases, and access to international credit markets is adversely affected ("sudden-stops" problem). All these factors contribute to a downturn. The paper further notes that trade of emerging market economies, which is mostly comprised of primary and manufactured commodities is adversely affected by exchange rate volatility. This is because the exposure to exchange risk is higher when the exchange rate is more volatile.

In a recent paper, Devereux and Lane (2003) develop a two-sector model similar to the one developed in this paper, in order to evaluate alternative monetary policy rules for emerging market economies in the presence of financial friction in capital formation and delayed exchange rate pass through. The authors show that there exists a trade-off between output stability and inflation stability when the pass-though from exchange rate to import prices is very high.

The model developed in this paper is similar in flavor to Cespedes, Chang and Velasco (2001a, 2001b), Gertler, Gilchrist and Natallucci (2001) and Devereux and Lane(2003). The key difference is the assumption that only the traded goods sector in the economy has access to international capital markets. Moreover, the monetary authorities are assumed to have different preferences regarding output stability in the different sectors. A fear of floating is found to be an outcome of a discretionary optimal monetary policy if the monetary authorities care a lot about stability of output in the traded-goods sector.

This chapter is divided into eight sections. The model is developed in the following section 2. Section 3 derives the key equations characterizing the underlying dynamics of the economy. Section 4 discusses the setup of the numerical analysis. Sections 4 and 5 present the results. Section 8 summarizes the conclusions.

2. The Model

We consider a small open economy with two sectors; the traded goods sector (T) and the non-traded goods sector (N). Both the goods are produced with constant returns to scale technology, by a number of monopolistically competitive firms in each of the two sectors. There are two factors of production -- labor (L) and capital (K). Capital depreciates over time at the rate δ.

Labor used in the production of the two sectors is supplied by households, made up of individuals. All individuals are alike and supply labor to both the sectors. We assume however, that labor is perfectly substitutable between the traded and non-traded sectors, and as a result the individuals receive nominal wages W in return for their labor effort.

We further assume that nominal wages are free to adjust every period, so that workers are always on their labor supply curve. In addition to working, the households consume and save. They consume a composite of the traded (C^T) and non-traded goods (C^N). The traded good (C^H) is a composite of a domestically produced traded good (C^H), and one that is imported from abroad (C^F).

Neither the households, nor the firms in the non-traded sector, have access to the international credit markets and have to therefore borrow entirely from the domestic credit markets. Only the firms producing traded goods have access to the international capital markets. These firms can therefore borrow in both the domestic as well as the foreign currency.

The capital is financed through the net worth left over from the previous period and by borrowing. In both the sectors, firms face Bernanke-Gertler type domestic credit constraints, so that they can borrow only a certain proportion of their net wealth. Following Bernanke, Gertler and Gilchrist (2000) the rate of borrowing is assumed to be subjected to a risk premium, which is an increasing function of the leverage ratio (ratio of debt over net worth). The risk premiums may differ for borrowing in domestic versus foreign currency.

Nominal rigidities are introduced in the model through the price setting behavior of the firms. Domestic firms set prices, one period in advance. We assume Calvo type price setting behavior so that the firms in the traded and non-traded sectors face the probability ($1-\omega^H$) and ($1-\omega^N$) respectively in every period of altering their price, no matter how long their price has been fixed in the past periods.

Finally, there is a central bank that minimizes the loss arising out of inflation and output volatility in every period. We assume that the Central Bank cares about fluctuations in output in both the sectors as well as inflation. However, the central bank may not attach equal weights to stabilizing output in the traded sector as against the non-traded sector. A relatively higher weight on the traded sector output gap implies a higher preference for output stability in this sector.

The timing is standard. Firms make production and pricing decisions and households make consumption and labor supply decisions, at the start of a period. The economy is hit by a shock (real or nominal) in period t. The central bank observes the shock before making its policy decisions. The policy instrument used by the central bank in the decision is the domestic interest rate. It reacts to the shock by setting the interest rate.

The model investigates whether the fear of floating behavior may arise from a time-consistent discretionary policy of the central bank in such an economy.

2.1 Households

2.1.1 Preferences

The households consume a composite of traded and non-traded goods. The composite of a representative household's consumption goods $\left(C_t\right)$ is a CES index that gives the relative preference between traded $\left(C_t^T\right)$ and non-traded consumption goods $\left(C_t^N\right)$.

(1) $$C_t = \frac{C_t^{T^{\gamma}} C_t^{N^{(1-\gamma)}}}{(\gamma)^{(\gamma)} (1-\gamma)^{(1-\gamma)}}$$

The household consumes two types of traded goods – those that are produced at home $\left(C_t^H\right)$ and ones that are imported $\left(C_t^F\right)$. We write the composite of tradable goods as follows.

(2) $$C_t^T = \frac{C_t^{H^{\phi}} C_t^{F^{(1-\phi)}}}{(\phi)^{(\phi)} (1-\phi)^{(1-\phi)}}$$

The home manufactured traded goods are also exported.

Foreign consumption of domestically produced traded goods is

(3) $$C_t^{H^*} = \left(\frac{P_t^H}{S_t P_t^{F^*}} \right)^{-\varepsilon} Y_t^*$$

where P_t^H and P_t^F are the price indices for the domestically produced traded good and the imported goods respectively, S_t denoted the nominal exchange and Y_t^* is the income of the rest of the world.

Furthermore, in every period, each household supplies the labor required in the production process of both the traded and non-traded goods, in return for wages. Labor is perfectly substitutable across the two sectors, so that total labor effort in period t (L_t) takes the form;

(4) $$L_t = L_t^H + L_t^N$$

2.1.2 Relative Prices

Consumer Price Index (P) is the minimum expenditure $Z = p^T C^T + p^N C^N$ such that $C = F\left(C^T, C^N\right) = 1$, given p^T and p^N. We can therefore write the Consumer Price Index as follows.

(5) $$P_t = P_t^{T^\gamma} P_t^{N^{(1-\gamma)}}$$

Similarly, we can write the traded goods price index between the home produced and the imported traded goods.

(6) $$P_t^T = P_t^{H^\phi} P_t^{F^{(1-\phi)}}$$

We assume that the law of one price holds in the traded sector.

(7) $$P_t^F = S_t P_t^{F^*}$$

$$\Rightarrow \qquad P_t^F = S_t , \qquad \text{for } P_t^{F^*} = 1$$

We can rewrite equation (8) as:

(9) $$P_t^T = P_t^{H^\phi} S_t^{(1-\phi)}$$

Combining the equations (5) and (9) we can write the aggregate price index as;

(10) $$P_t = \left(P_t^{H^\phi} S_t^{(1-\phi)} \right)^\gamma P_t^{N(1-\gamma)}$$

2.1.3 Households' decision problem:

The representative household maximizes expected value of lifetime utility subject to its budget constraint. We assume that utility depends positively on consumption and real money balances, and negatively on total labor effort. The utility function of the household is assumed to take the following form.

(11) $$E_t \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \chi \ln \frac{M_t}{P_t} - \eta \frac{L_t^\kappa}{\kappa} \right]$$

C_t, M_t/P_t and L_t denote consumption, real money balances and total labor effort respectively.

The representative households get wages for the hours of labor supplied to the traded and non-traded sector firms, W_t, domestic money M_t, and repayment of last period's domestic debt $(1+i_{t-1})B_t$, where i_t is the domestic nominal interest rate and B_t is the outstanding amount of domestic currency debt. The households also get dividends Π_t from the shares in the traded and non-traded sector funds. Households only have access to the domestic credit markets and therefore can only borrow in the domestic currency.

The household's period budget constraint in nominal terms can be written as follows.

(12) $$B_{t+1} + M_t + P_t C_t = W_t\left(L_t^H + L_t^N\right) + \left(1 + i_{t-1}\right) B_t + M_{t-1} + \Pi_t$$

The household maximizes (11) subject to equations (1), (2), (4), and the period budget constraint (12).

The optimality conditions for consumption and saving are derived from the household's utility maximization problem.

Consumption allocation between traded and non-traded goods is

(13) $$\frac{C_t^T}{C_t^N} = \frac{\gamma}{1-\gamma}\left(\frac{P_t^N}{P_t^T}\right)$$

Consumption allocation between imported traded goods and those domestically produced is

(14) $$\frac{C_t^H}{C_t^F} = \frac{\phi}{1-\phi}\left(\frac{s_t}{P_t^H}\right)$$

Optimality condition for savings is

(15) $$\frac{1}{C_t} = \beta E_t\left\{\frac{1}{C_{t+1}}\left(1 + i_t\right)\frac{P_t}{P_{t+1}}\right\}$$

where, P_t denotes the price of the composite good.

Allocation of labor in the economy is given by;

(16) $$\eta\left(L_t^H + L_t^N\right)^{\kappa-1} C_t = \frac{W_t}{P_t}$$

Money demand equation;

(17) $$\frac{\chi}{M_t/P_t} = \frac{1}{C_t} - E_t\left[\beta\frac{1}{C_{t+1}}\frac{P_t}{P_{t+1}}\right]$$

2.2 Firms

2.2.1 Domestic Production

Production is carried out by monopolistically competitive firms, in each of the two sectors; H and N. Firms in both the sectors, produce goods with the help of identical Cobb-Douglas production technologies.

Let Y^J, K^J and L^J denote capital and labor in sector J. A^J is the technology coefficient in each sector J, where J = H, N.

Output of a single firm j in the traded goods sector is

(18) $$Y_{jt}^{H} = e^{z} K_{jt}^{H\alpha} L_{jt}^{H(1-\alpha)} \qquad 0 < \alpha < 1$$

Output of a single firm I in the non-traded sector is

(19) $$Y_{it}^{N} = e^{z'} K_{it}^{Na} L_{it}^{N(1-a)} \qquad 0 < a < 1$$

z and z′ are real mean zero and i.i.d shocks to the economy.

The total output in each of the sectors is a CES composite of the output produced by each individual monopolistically competitive firm in each of the sectors. Therefore for each sector *J* the output and the corresponding composite price index in each sector is;

(20) $$Y_t^J = \left[\int_0^1 Y_t^J(z)^{\frac{\varphi-1}{\varphi}} dz \right]^{\frac{\varphi}{\varphi-1}}$$

and,

(21) $$P_t^J = \left[\int_0^1 P_t^J(z)^{1-\varphi} dz \right]^{\frac{1}{\varphi-1}}$$

Demand for the final output of the traded sector firms comes from households, which consume the goods, the rest of the world in the form of exports and the traded sector firms for next period investment. The demand for the output of the traded goods produced by a single firm in the sector (as derived in the appendix C) is;

(22) $$Y_t^H(z) = \left(\frac{p_t^H(z)}{P_t^H}\right)^{-\varphi} Y_t^H$$

φ is the price elasticity of demand for the output of the firm's output, and as $\varphi \to \infty$, individual firms have less market power as their goods become closer substitutes.

Similarly, the demand for the output of the non-traded goods produced by a single firm in the sector is;

(23) $$Y_t^N(z) = \left(\frac{p_t^N(z)}{P_t^N}\right)^{-\varphi} Y_t^N$$

The standard cost minimization problem that a firm *j* in traded sector faces is;

(24) $$\min_{L_{jt}^H,\, K_{jt}^H} R_t K_{jt}^H + \frac{W_t}{P_t} L_{jt}^H - MC_t^H \left(Y_{jt}^H - e^z K_{jt}^{H^\alpha} L_{jt}^{H^{(1-\alpha)}}\right)$$

Similarly, a firm *i* in the non-traded sector solves the following cost function.

(25) $$\min_{L_{it}^N,\, K_{it}^N} R_t K_{it}^N + \frac{W_t}{P_t} L_{it}^N - MC_t^N \left(Y_{it}^N - e^{z'} K_{it}^{N^a} L_{it}^{N^{(1-a)}}\right)$$

where, MC_t^H and MC_t^N denote the real marginal costs for the firms in the traded and non-traded sectors respectively.

The real return to capital is the marginal product of capital in the two sectors;

(26) $$R_t^H = MC_t^H \alpha \frac{Y_{jt}^H}{K_{jt}^H} - \delta$$

$$(27) \qquad R_t^N = MC_t^N a \frac{Y_{jt}^N}{K_{jt}^N} - \delta$$

here, δ is the depreciation rate.

Labor demand functions in the two sectors can be derived as;

$$(28) \qquad L_{jt}^H = MC_t^H (1-\alpha) \frac{P_t Y_{jt}^H}{W_t}$$

$$(29) \qquad L_{it}^N = MC_t^N (1-a) \frac{P_t Y_{it}^N}{W_t}$$

2.2.3 Price Setting:

All domestic firms set prices in advance. We assume Calvo type price setting behavior so that the traded and non-traded sector firms face a probability (1-ω^H) and (1-ω^N) probabilities respectively, in every period, of altering their price no matter how long their price has been fixed.

For a single firm in sector J (= H, N), the pricing decision involves picking a price $\tilde{p}_t^J(z)$, to maximize the expected discounted value of current and future profits. The profits of the firm in some future period $t + i$ are affected by the price chosen in the earlier period t only if it has not had a chance to readjust its price, and the probability of such a situation is $\left(\omega^J\right)^i$. The profit maximization problem implies that the firm will choose its price in period t so as to equalize the present discounted value of marginal revenue with the discounted value of marginal cost. (See Appendix B for the solution).

The price chosen by a firm in the traded sector firms that get to set a new price in period t can be written as follow.

(30) $$\tilde{p}_t^H(z) = \frac{\varphi}{1-\varphi} \frac{E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_t}{C_{t+i}} \right) MC_{t+i}^H \left(\frac{1}{P_{t+i}} \right)^{-\varphi} Y_{t+i}^H}{E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_t}{C_{t+i}} \right) \left(\frac{1}{P_{t+i}} \right)^{(1-\varphi)} Y_{t+i}^H}$$

The price set by traded sector firms in period t will therefore depend on the present and expected future aggregate price level, which is defined by equation (5). The aggregate price index of the traded sector goods is the average of the price charged by the fraction $\left(1-\omega^H\right)$ of firms that set their prices in period t and the average of the prices charged by the remaining firms. Since the firms that are able to revise their prices in period t are chosen randomly, the average price of the firms that do not adjust in period t is just the average price in period t-1.

Thus the average price of the traded good follows;

(31) $$P_t^{H^{(1-\varphi)}} = (1-\omega^H)\tilde{p}_t^{H^{(1-\varphi)}} + \omega^H P_{t-1}^{H^{(1-\varphi)}}$$

We can derive corresponding expressions for pricing decisions of the non-traded sector firms. The price chosen by non-traded sector firms that get to set a new price in period t satisfies the following equation.

(32) $$\tilde{p}_t^N(z) = \frac{\varphi}{1-\varphi} \frac{E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_t}{C_{t+i}} \right) MC_{t+i}^N \left(\frac{1}{P_{t+i}^N} \right)^{\varphi} Y_{t+i}^N}{E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_t}{C_{t+i}} \right) \left(\frac{1}{P_{t+i}^N} \right)^{1-\varphi} Y_{t+i}^N}$$

The average price of the non-traded good follows;

(33) $$P_t^{N^{(1-\varphi)}} = (1-\omega^N)\tilde{p}_t^{N^{(1-\varphi)}} + \omega^N P_{t-1}^{N^{(1-\varphi)}}$$

As stated earlier, we assume law of one price in the imported goods. Therefore, the price of imports is simply $P_t^F = S_t P_t^{F^*} = S_t$.

2.3.1 Financing of capital:

Because of informational asymmetries, all the borrowing by domestic firms is subject to Bernanke-Gertler type domestic credit constraints that stem from agency costs. Further, international credit markets are assumed to be segmented, so that only the traded goods sector has access to it. Firms in the non-traded sector can borrow only from the domestic credit market in domestic currency, while the traded sector firms can borrow from both the domestic and international credit markets, in domestic and international currencies respectively.

Moreover, all domestic borrowing is done from domestic savings so that the firms' domestic currency denominated debt equals the saving of households in every period t.

(34) $$D_t^H + D_t^N = B_t$$

We denote domestic currency debt of the traded and non-traded sector firms as D_t^H and D_t^N. The debt of the traded sector firms, which is denominated in foreign currency, is denoted as $D_t^{H^*}$.[4]

2.3.2 Borrowing Decisions:

The firms' demand for capital will depend on the expected marginal cost of borrowing and the expected marginal return on investment. A firm in the non-traded sector will acquire capital till the expected net real return on capital is equal to the expected real rate of interest on domestic currency debt. This is denoted by the following equation (36). A firm in the non-traded sector can borrow at the rate $(1+i_t)(1+\eta_t)$ where i_t is the nominal interest rate and η_t is the risk premium, defined as a function of the leverage ratio.

[4] Both terms D_t^H and $D_t^{H^*}$ are in terms of domestic currency.

(35) $$\eta_t = \eta\left(\frac{D_t^N}{N_t^N}\right), \qquad \eta' > 0,\ \eta(0) = 0,\ \eta(\infty) = \infty$$

(36) $$E_t\left(1 + r_{t+1}^k\right) = E_t\left\{(1 + i_t)\left(1 + \eta_t\right)\frac{P_t}{P_{t+1}}\right\}$$

Likewise, the firms in the traded sector have access to the domestic as well as the international credit markets. They can borrow in foreign currency in the international credit markets at the rate $\left(1 + i_t^*\right)\left(1 + \eta_t^*\right)$ where, i_t^* is the nominal world interest rate and η_t^* is the risk premium, which is a function of the leverage ratio.

(37) $$\eta_t^* = \eta^*\left(\frac{D_t^H + D_t^{H^*}}{N_t^H}\right), \qquad \eta^{*\prime} > 0,\ \eta^*(0) = 0,\ \eta^*(\infty) = \infty$$

They can borrow domestic currency at the domestic interest rate $(1 + i_t)\left(1 + \eta_t\right)$. Just as in the case of the non-traded sector, the firms in the traded sector will borrow till the expected cost debt - at the margins, is equal to the expected rate of return on capital.

We can consider three situations here. In the first situation, the traded sector firms will borrow entirely from the domestic credit market. As long as the expected cost of domestic currency debt is initially less than that of the foreign currency debt and the following condition is satisfied, the firms will never borrow from abroad.

(38) $$E_t\left(1 + r_{t+1}^k\right) = E_t\left(1 + i_t\right)\left(1 + \eta_t\right) < E_t\left\{\left(1 + i_t^*\right)\left(1 + \eta_t^*\right)\frac{S_{t+1}}{S_t}\right\}$$

In the second situation the traded sector firms may borrow from both the markets. This will be the case if the following equation is satisfied at the margins.

(39) $$E_t\left(1 + r_{t+1}^k\right) = E_t\left(1 + i_t\right)\left(1 + \eta_t\right) = E_t\left\{\left(1 + i_t^*\right)\left(1 + \eta_t^*\right)\frac{S_{t+1}}{S_t}\right\}$$

In this case, the firms will be indifferent between borrowing domestically and borrowing in foreign currency.

Lastly, the firms will borrow in the foreign currency only if the following equation (40) is satisfied at the margins.

(40) $$E_t\left(1+r_{t+1}^k\right)=E_t\left\{\left(1+i_t^*\right)\left(1+\eta_t^*\right)\frac{S_{t+1}}{S_t}\right\}<E_t\left(1+i_t\right)\left(1+\eta_t\right)$$

In this paper we focus on the initial situation given by equation (39), where the expected cost of borrowing for the traded sector firms is the same for domestic and foreign currency, so that they firms are indifferent between the two.

2.4 Profit maximization

Firms in both sectors maximize profits.

Profits in the traded goods sector in period t:

(41)

$$V_t^H=Q_t^{kh}K_t^H-\left[D_t^{H^*}\left(1+i_{t-1}^*\right)\left(1+\eta_{t-1}^*\right)\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}+D_t^H\left(1+i_{t-1}\right)\left(1+\eta_{t-1}\right)\frac{P_{t-1}}{P_t}\right]$$

Profits in non-traded goods sector:

(42) $$V_t^N=Q_t^{kn}K_t^N-D_t^N\left(1+i_{t-1}\right)\left(1+\eta_{t-1}\right)\frac{P_{t-1}}{P_t}$$

Q^{kh} and Q^{kn} denote ex-post real rate of return on capital. According to equations (41) and (42), value of period t profits in each sector in real terms is equal to the difference between ex-post gross real value of the return capital invested, and the ex-post real cost of debt.

Capital stock in the traded good sector can therefore be written as follows. The value of the capital stock in the traded sector in period t is equal to the sum of the total net worth of traded sector firms and the borrowing that the firms undertakes in that period

(43) $$Q_t^{hk} K_t^H = N_t^H + D_t^H + D_t^{H^*}$$

Similarly, capital stock in the non-traded sector in period t is equal to the sum of net worth in period t and the borrowing that the non-traded sector firms undertakes in period t.

(44) $$Q_t^{nk} K_t^N = N_t^N + D_t^N$$

The net worth in sector J (= H, N) is just whatever is left over from the previous period profit after the households are paid off.

(45) $$N_{t+1}^J = V_t^J - \Pi_t^J ,$$

2.5 Resource Constraints

The economy wide resource constraints in the traded goods sector is

(46) $$Y_t^H = C_t^H + C_t^{H^*} + I_t^H$$

The economy wide resource constraint in the non-traded goods sector is

(47) $$Y_t^N = C_t^N + I_t^N$$

Following Woodford (2003), convex adjustment costs are assumed for investment in each sector, such that for any firm in sector J

(48) $$I_t^J (i) = I_t^J \left(\frac{K_{t+1}^J (i)}{K_t^J (i)} \right) K_t^J (i)$$

The function I(.) is assumed to be increasing and convex. I(1) = δ, I'(1) =1 and I''(1) =ε near a zero growth rate of capital stock. Parameters ε>0 and 0<δ <1, where δ is the rate of depreciation of capital and ε indicates the adjustment costs. In order to keep the analysis simple we assume that the rate of depreciation and the degree of adjustment costs ε are the same in both sectors.

Capital evolves such that

(49) $$K_{t+1}^{H} = I_t^{H} + (1-\delta)K_t^{H}$$

(50) $$K_{t+1}^{N} = I_t^{N} + (1-\delta)K_t^{N}$$

where, I_t^H and I_t^N denotes the investment in the traded and the non-traded sectors respectively, in period t.

2.6 Central bank

Given the forward-looking nature of the model it is reasonable to assume that the central bank cannot commit and follows a discretionary monetary policy.

We consider an economy that is in steady state equilibrium in period t-1. Firms set their prices and households make consumption decisions for period t at the beginning of the period. At the same time the firms in both the sectors make investment and financing decisions for period t. There is shock to the economy (nominal or real) in period t after the households' and firms' decisions have been made. The monetary authorities decide on the period t monetary policy after observing the shock.

Exchange rate and interest rate movements affect the two sectors in the economy differently. The traded sector, which produces goods for export and holds liabilities denominated in foreign currency, is relatively more affected by exchange rate movements, than the non-traded sector. Domestic interest rate volatility is of concern to both the sectors, but more so to the non-traded sector, which is assumed to have all its liabilities in domestic currency. In such an economy, the inter-temporal loss function of the central bank can be written as follows.

(51) $$\Gamma_t = E_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda_X \left(\theta_H \left(\hat{X}_{t+i}^H \right) + \theta_N \left(\hat{X}_{t+i}^N \right) \right)^2 \right)$$

Where, π is the economy wide inflation rate, $\hat{X}_t^H = \hat{Y}_t^H - \hat{Y}_t^{H^{Flex}}$ and $\hat{X}_t^N = \hat{Y}_t^N - \hat{Y}_t^{N^{Flex}}$ are the output gaps in the traded and non-traded sectors, respectively. Here, $Y_t^{H^{Flex}}$ and $Y_t^{N^{Flex}}$ are defined as flexible price equilibrium outputs in the traded and non-traded sectors. Thus output gaps are defined as log deviation of traded sector and non-traded sector outputs from the flexible price levels.

The parameters λ_X denotes the weight that the central bank assigns to overall output stability. The importance that the central bank assigns to relative output stability in the traded and the non-traded sectors is given by the weights θ_H and θ_N respectively. A relatively higher θ_H would imply that the central bank cares more about output stability in the traded sector. The parameter on inflation is normalized to 1. A higher value of a parameter indicates greater desire of the central bank to stabilize the deviations of that variable from its desired value.

3. Inflation, output gap and investment dynamics

Following Woodford (2004), the aggregate supply and aggregate demand relations can be solved in terms of "gap" variables. The output gaps and capital gaps in the sectors j = H, N, are defined as $\hat{X}_t^J \equiv \hat{Y}_t^J - \hat{Y}_t^{J^n}$ and $\tilde{K}_{t+1}^J \equiv \hat{K}_{t+1}^J - \hat{K}_{t+1}^{J^n}$ respectively. The natural rate of output $\hat{Y}_t^{J^n}$ and the natural rate of capital $\hat{K}_{t+1}^{J^n}$ are flexible price equilibrium levels of output and capital respectively, given the actual level of capital stock $\hat{K}_t^J$.

The aggregate demand equation for the traded sector is derived (in appendix E) as;

(52)

$$\hat{X}_t^H = E_t \hat{X}_{t+1}^H - \frac{I^H}{Y^H}\left(E_t \tilde{I}_{t+1}^H - \tilde{I}_t^H\right) + \frac{C^H}{Y^H}(\zeta+1) E_t \hat{\pi}_{t+1}^H - \left(\frac{C^H}{Y^H}\zeta\right) E_t \hat{\pi}_{t+1}^F - \frac{C^H}{Y^H}\left(\tilde{i}\right)_t$$

The aggregate demand equation for the non-traded sector is similarly derived (in Appendix D) as;

(53)

$$\hat{X}_t^N = E_t \hat{X}_{t+1}^N + \left[\frac{\left(\frac{1}{\beta}-(1-\delta)\right)a-\delta}{\left(\frac{1}{\beta}-(1-\delta)\right)a}\right] E_t \hat{\pi}_{t+1}^N$$
$$-\left(\frac{\delta}{\left(\frac{1}{\beta}-(1-\delta)\right)a}\right)\left(E_t \tilde{I}_{t+1}^N - \tilde{I}_t^N\right) - \left(\frac{\left(\frac{1}{\beta}-(1-\delta)\right)a-\delta}{\left(\frac{1}{\beta}-(1-\delta)\right)a}\right)\tilde{i}_t$$

Following Woodford (2003) he joint dynamics of output and capital in the traded and non-traded sectors can be derived as:

(54) $\left(1-\beta(1-\delta)\right)\left(E_t mc_{t+1}^H + E_t y_{t+1}^H\right) + \beta\varepsilon_\psi\left(E_t k_{t+2}^H - k_{t+1}^H\right) = \varepsilon_\psi\left(k_{t+1}^H - k_t^H\right) + E_t\left(i_t - \pi_{t+1}\right)$

(55) $\left(1-\beta(1-\delta)\right)\left(E_t mc_{t+1}^N + E_t y_{t+1}^N\right) + \beta\varepsilon_\psi\left(E_t k_{t+2}^N - k_{t+1}^N\right) = \varepsilon_\psi\left(k_{t+1}^N - k_t^N\right) + E_t\left(i_t - \pi_{t+1}\right)$

Economy wide inflation is a composite of inflation in the traded goods prices and the non-traded goods prices. Thus economy wide inflation can be expressed as follows.

(56) $$\frac{P_t}{P_{t-1}} = \left(\frac{P_t^H}{P_{t-1}^H}\right)^{\phi\gamma}\left(\frac{S_t}{S_{t-1}}\right)^{(1-\phi)\gamma}\left(\frac{P_t^N}{P_{t-1}^N}\right)^{(1-\gamma)}$$

Taking linear approximation of the above equation around the steady state, we can write an expression for economy wide inflation as follows.

(57) $$\pi_t = \phi\gamma\pi_t^H + (1-\phi)\gamma\pi_t^F + (1-\gamma)\pi_t^N$$

The forward looking inflation adjustment equation for the traded sector is derived by approximating equations (30) and (31) around the steady state, where for the initial steady state, the rate of change in the traded goods price is zero. (See appendix D for the derivation)

(58) $$\pi_t^H = \varsigma^H mc_t^H + \beta E_t \pi_{t+1}^H$$

where, $\varsigma^H = \dfrac{(1-\omega^H)(1-\beta\omega^H)}{\omega^H}$, and mc_t^H is the log deviation of the real marginal cost from its steady state level.

Following the same procedure with equations (32) and (33) in the case of the non-traded sector, we can derive the inflation adjustment equation. (See appendix D)

(59) $$\pi_t^N = \varsigma^N mc_t^N + \beta E_t \pi_{t+1}^N$$

where, $\varsigma^N = \dfrac{(1-\omega^N)(1-\beta\omega^N)}{\omega^N}$, and mc_t^N is the log deviation of the real marginal cost from its steady state level.

Since we assume the law of one price in imported goods, we can write the inflation rate for the imported good is,

(60) $$\pi_t^F = s_t - s_{t-1}$$

Substituting equations (67), (68) and (69) into equation (66), we can express the economy wide inflation rate as;

(61) $$\pi_t = \phi\gamma\left(\varsigma^H mc_t^H + \beta E_t \pi_{1+t}^H\right) + (1-\phi)\gamma\left(\varsigma^N mc_t^N + \beta E_t \pi_{1+t}^N\right) + (1-\gamma)(s_t - s_{t-1}) + e_t$$

The deviation of the traded sector firms' real marginal cost from the steady state level itself can be written in terms of output gap and investment gap. (The following expressions (62) and (63) are derived in Appendix D).

$$mc_t^H = \left[\frac{1}{1-\alpha}\left(\left(\kappa-1\right)\frac{L^H}{L}+1\right)+\gamma\frac{Y^H}{C^H}-1\right]\hat{X}_t^H$$

(62)

$$+\left[\frac{1}{1-a}\left(\left(\kappa-1\right)\frac{L^N}{L}\right)+\left(1-\gamma\right)\frac{Y^N}{C^N}\right]\hat{X}_t^N-\gamma\left(\frac{I^H}{C^H}\right)\tilde{I}_t^H-(1-\gamma)\left(\frac{I^N}{C^N}\right)\tilde{I}_t^N$$

$$-\frac{1}{1-\alpha}\left(\left(\kappa-1\right)\frac{L^H}{L}+1\right)\alpha\tilde{K}_t^H-\frac{1}{1-a}\left(\left(\kappa-1\right)\frac{L^N}{L}\right)a\tilde{K}_t^N$$

$$-\left(\gamma\frac{C^{H^*}}{C^H}+\left(1-\phi\right)\right)\left(\hat{\pi}_t^F-\hat{\pi}_t^H\right)$$

Similarly for the non-traded sector firms, the deviation of the real marginal cost is expressed as;

$$mc_t^N = \left[\frac{1}{1-\alpha}\left(\left(\kappa-1\right)\frac{L^H}{L}+1\right)+\gamma\frac{Y^H}{C^H}\right]\hat{X}_t^H$$

(63)

$$+\left[\frac{1}{1-a}\left(\left(\kappa-1\right)\frac{L^N}{L}+1\right)+\left(1-\gamma\right)\frac{Y^N}{C^N}-1\right]\hat{X}_t^N$$

$$-\gamma\left(\frac{I^H}{C^H}\right)\tilde{I}_t^H-(1-\gamma)\left(\frac{I^N}{C^N}\right)\tilde{I}_t^N-\frac{1}{1-\alpha}\left(\left(\kappa-1\right)\frac{L^H}{L}\right)\alpha\tilde{K}_t^H$$

$$-\frac{1}{1-a}\left(\left(\kappa-1\right)\frac{L^N}{L}+1\right)a\tilde{K}_t^N-\left(\gamma\frac{C^{H^*}}{C^H}+\left(1-\phi\right)\right)\left(\hat{\pi}_t^F-\hat{\pi}_t^H\right)$$

Equations (57) to (63) describe the aggregate supply block. This set of equations along with the aggregate demand block (52) to (55), and the system of equation in Appendix 1, gives us the complete description of the economy in terms of deviations from steady state.

The purpose of this paper is to analyze how the central bank may react to nominal and real shocks under different policy assumptions. The distinctions between different policies of the central bank arise from the relative differences in the importance the central bank attaches to stability of output in the traded versus the non-traded sector.

A change in exchange rate affects the economy through three channels; inflation rate, terms of trade and the value of net worth in the traded sector (balance sheets of the firms). A change in interest rate affects the outputs in both the sectors since it changes the cost of borrowing in domestic currency.

The central bank reacts to a nominal shock, such as an unanticipated positive foreign interest rate shock by either increasing the domestic interest rate or by allowing the exchange rate to depreciate or both. The exact response of the central bank will depend on a number of factors such as terms of trade, foreign price elasticity of demand for exports, domestic price elasticity of demand for imports, the relative sizes of the two sectors, the proportion of the foreign currency denominated debt held by domestic agents, the relative weight the central bank attaches to price stability as against aggregate output stability and the relative importance the central bank attaches to output stability in the traded sector as against the non-traded sector.

The last factor i.e. the relative importance attached to stability of output in the traded sector may be very important for explaining why we see lower volatility of exchange rates in emerging market economies.

4. The Calibration Exercise

The model is solved using the calibration method by Soderlind (1999). The parameter values used in order to solve the model are taken from Natalucchi, Gertler and Gilchrist (2000) and Devereux and Lane (2003). The parameter values are fairly standard in this literature and are listed in a table 1.

The amount of labor employed in the two sectors is initially assumed to be equal. We also assume that the traded sector's initial debt in domestic currency and foreign currency is equal. The two sectors are assumed to be identical in all other respects.

Since, the traded sector initially holds half of its initial debt in domestic currency and the rest in foreign currency, the net worth in this sector is directly affected by exchange rate movements, the foreign interest rate, and domestic interest rate movements. Exchange rate movements also matter to this sector through the terms of trade channel. On the other hand, in case of the non-traded sector the domestic interest rate movements have a direct effect on the firms' borrowing decision.

We consider four types of shocks: an aggregate demand shock, an inflation shock, an exchange rate shock, and a foreign interest rate shock.

In order to keep the numerical analysis straightforward, it is assumed that the parameter denoting the weight assigned to the traded sector output gap in the quadratic loss function (51), takes values between zero and one, i.e. $0 < \theta_H < 1$ and $\theta_N = 1 - \theta_H$. We assign different values to the weight on aggregate output in the loss function, given by λ, from zero to 3. A weight of zero on this parameter implies that the central bank is a pure inflation targetter. The variances of exchange rate and interest rate are computed for different values of θ_H between zero and one, for every value of λ.

5. Central Bank Preferences and Relative Variance of Exchange and Interest rates

The purpose of this exercise to understand the relationship between relative volatilities of interest rate and exchange rate and the relative weights assigned by the central bank to the output volatilities in the two sectors, while holding the relative weight on inflation constant.

The results from the calibration analysis are summarized in tables 2-4 in the appendix. The results indicate that as the ratio of the weight on non-traded sector output gap to the weight on the traded sector output gap increases, the ratio of domestic interest rate variance to exchange rate variance decreases. This implies a negative relationship between the relative importance the central bank assigns to output stability in the non-traded sector and relative exchange rate volatility.

The figure 1 plots the ratio of the volatility of exchange rate to domestic interest rate as the relative weight on the traded sector output gap increases. This is plotted for different values of λ, which denotes the weight assigned to output stability at the aggregate level. A higher θ_H indicates that the central bank cares more about output stability in the traded sector relative to the non-traded sector. We see that for a given level of λ, as θ_H increases, the ratio of exchange rate volatility to interest rate volatility decreases. In other words, an increase in the importance given to stability of output in the traded sector relative to the non-traded sector, leads to lower flexibility of exchange rate. This indicates that what we observe as fear of floating may be a result of monetary authorities conducting policy geared towards protecting the traded sector from real and nominal shocks.

A change in λ, the weight assigned to overall output stability, changes the slope of the line plotting relative exchange rate volatility and θ_H. The explanation for this change in slope is as follows. At very low value of θ_H, i.e. if the central bank cares very little about output stabilization in the traded sector, exchange rate volatility primarily matters in terms of its effect on inflation. In this case, with a lower λ, and thereby relatively higher weight placed on price stabilization, we should see a relatively lower variance of exchange rate. As λ increases, with greater importance attached to overall output stability, the exchange rate would become relatively more volatile. On the other hand, if the monetary authorities attach a great deal of importance to output stability in the traded sector, i.e. θ_H is close to 1, a lower λ would mean higher relative exchange rate volatility, and vice versa. When $\frac{\theta_H}{\theta_N} = 0.5$, for any values of λ, the ratio of the variances of exchange rate and interest rate is 0.1268. This result is because of the way the model is set up. The ratio of the weights the central bank attaches to the traded and non-traded sector can be derived from the first order conditions from the central banks decision problem given by equation (51).

We can write these first order conditions as equations (64) and (65) below.

(64) $$\frac{\partial L}{\partial X_t^H} = \sum_i \beta^i \frac{\partial \pi_{t+i}}{\partial X_t^H} + \lambda\ \theta_H \left(\theta_H X_t^H + \theta_N X_t^N\right) = 0$$

(65) $$\frac{\partial L}{\partial X_t^N} = \sum_i \beta^{\,i} \frac{\partial \pi_{t+i}}{\partial X_t^N} + \lambda\ \theta_N \left(\theta_H X_t^H + \theta_N X_t^N\right) = 0$$

The ratio of the weights can then be solved by taking the second terms in the two equations to the right hand side, and dividing (1) by (2).

(66) $$\frac{\theta_H}{\theta_N} = \frac{\sum_i \beta^{\,i} \dfrac{\partial \pi_{t+i}}{\partial X_t^H}}{\sum_i \beta^{\,i} \dfrac{\partial \pi_{t+i}}{\partial X_t^N}} = \frac{\phi\ \gamma}{(1\text{-}\gamma)}$$

(67) $$\frac{\theta_H}{\theta_N} = \frac{\phi\ \gamma}{1-\gamma} = \frac{(0.5)(0.5)}{(0.5)} = 0.5$$

$$\therefore \theta_H = \left(1-\theta_N\right) = \frac{1}{3}$$

ϕ is the consumption share of the domestically produced traded goods in the composite of traded goods consumption, and $(1-\gamma)$ is the share of non-traded goods in the composite of total consumption as given by equations (1) and (2).

6. Sector Sizes and Relative Variance of Exchange and Interest rates

We would intuitively expect that, larger the relative size of the traded sector, lower will be the relative exchange rate volatility. This result can be seen in the figure 2. There is a negative relationship between the share of labor in the traded sector and relative exchange rate volatility.

The figure plots the ratios of the variances of exchange and interest rates against the labor share in the traded sector for different values of λ. These relationships are plotted under the assumption that the central bank assigns equal weights to traded and non-traded sector output gaps.

The figure 2 also shows that given the size of the traded sector, as weight allocated to aggregate output λ increases, the relative volatility of exchange rate decreases. This observation is again as expected in case of an economy in which the traded sector output volatility increases with an increase in relative exchange rate volatility and the non-traded sector output volatility increases with an increase in relative interest rate volatility.

7. Impulse response functions

Finally, the fear of floating result can be further observed from the impulse response functions. We see from figures 3-5 that the domestic interest rate reacts more strongly to a positive foreign interest rate shock when the relative weight on the output gap in the traded sector is larger. In other words, the central bank will react to the same foreign interest rate shock, differently depending on the relative importance allocated to the traded and non-traded sector. The central bank is less willing to allow the exchange rate to move in response to an external, nominal shock if it cares relatively more about output volatility in the traded goods sector.

8. Conclusion

The purpose of this research is to understand fear of floating from an optimal monetary policy perspective. The model developed in this paper shows that the fear of floating observed in the case of emerging market economies may result from an optimal discretionary monetary policy. It has been shown in the literature with similar models that domestic exchange rate volatility adversely affects the balance sheets of firms that have liabilities denominated in foreign currencies. In this model we make a distinction between such firms that belong to sectors of an economy, which are relatively more open and therefore have a greater access to foreign currency markets, and others that belong to non-traded goods sectors, and therefore only borrow domestically. In such an economy while exchange rate volatility may adversely affect firms in the more traded sectors, through balance sheets as well as trade and investment channels, domestic interest rate volatility creates problems for the non-traded sector firms. Therefore, the negative relationship between exchange rates and interest rates translates into real trade-offs. The central bank will be relatively more reluctant to allow the exchange rate to move freely in reaction to an external shock, if it attaches greater importance to the traded goods sector.

This model does not give any explanation for the source of the greater preference for the traded sector. It may arise from several factors such as long term growth strategies that are geared towards developing the manufacturing sector or simply from political economy considerations.

The model is solved numerically in order to understand how central bank preferences can affect exchange rate volatility relative to interest rate volatility. The results show that the relative exchange rate volatility decreases as the preference of the central bank for output stabilization in the traded sector relative to that in the non-traded sector increases. We also see that the relative size of the traded sector of an economy also matters. The relative exchange rate volatility decreases, as the traded sector becomes relatively bigger.

Appendix A

Linearized system of equations

(68) $$\hat{C}_t = \gamma \hat{C}_t^T + (1-\gamma)\hat{C}_t^N$$

(69) $$\hat{C}_t^T = \phi \hat{C}_t^H + (1-\phi)\hat{C}_t^F$$

(70) $$\hat{C}_t^T - \hat{C}_t^N = \hat{P}_t^N - \hat{P}_t^T$$

(71) $$\hat{C}_t^H - \hat{C}_t^F = s_t - \hat{P}_t^H$$

(72) $$\hat{P}_t = \gamma \hat{P}_t^T + (1-\gamma)\hat{P}_t^N$$

(73) $$\hat{P}_t^T = \phi \hat{P}_t^H + \left(1-\phi\right) s_t$$

(74) $$\hat{C}_t^{H^*} = \xi\left(s_t - \hat{P}_t^H\right)$$

(75) $$\hat{L}_t = \frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N$$

(76) $$\hat{W}_t - \hat{P}_t = \left(\kappa - 1\right)\hat{L}_t + \hat{C}_t$$

(77) $$E_t \hat{C}_{t+1} = \hat{C}_t + E_t\left(\hat{i}_t - \left(\hat{P}_{t+1} - \hat{P}_t\right)\right)$$

(78) $$\hat{Y}_t^H = \hat{z}_t^H + \alpha \hat{K}_t^H + (1-\alpha)\hat{L}_t^H$$

(79) $$\hat{Y}_t^N = \hat{z}_t^N + a\hat{K}_t^N + (1-a)\hat{L}_t^N$$

(80) $$\hat{z}_t^H = \hat{z}_{t-1}^H + \varepsilon_t^H$$

(81) $$\hat{z}_t^N = \hat{z}_{t-1}^N + \varepsilon_t^N$$

(82) $$\hat{R}_t^H = mc_t^H + \hat{Y}_t^H - \hat{K}_t^H$$

(83) $$\hat{R}_t^N = mc_t^N + \hat{Y}_t^N - \hat{K}_t^N$$

(84) $$\hat{W}_t - \hat{P}_t = mc_t^H + \hat{Y}_t^H - \hat{L}_t^H = mc_t^N + \hat{Y}_t^N - \hat{L}_t^N$$

(85) $$\overline{Y}^H \hat{Y}_t^H = \overline{C}_t^H \hat{C}_t^H + \overline{C}_t^{H^*} \hat{C}_t^{H^*} + \overline{I}_t^H \hat{I}_t^H$$

(86) $$\overline{Y}^N \hat{Y}_t^N = \overline{C}_t^N \hat{C}_t^N + \overline{I}_t^N \hat{I}_t^N$$

(87) $$\overline{K}^H \hat{K}_{t+1}^H = \overline{I}^H \hat{I}_t^H + (1-\delta)\overline{K}^H \hat{K}_t^H$$

(88) $$\overline{K}^N \hat{K}_{t+1}^N = \overline{I}^N \hat{I}_t^N + (1-\delta)\overline{K}^N \hat{K}_t^N$$

(89) $$\hat{S}_{t+1} - \hat{S}_t = E_t \hat{S}_{t+1} - \hat{S}_t + \hat{\phi}_t^s = \left(\hat{i}_t - \hat{i}_t^*\right) + \left(\hat{\eta}_t^H - \hat{\eta}_t^{H^*}\right)$$

(90) $$\hat{\eta}_t^H = \eta\left(\hat{d}_t^H + \hat{n}_t^H\right)$$

(91) $$\hat{\eta}_t^{H^*} = \eta^*\left(\hat{d}_t^H + \hat{n}_t^H\right)$$

(92) $$\hat{\eta}_t^N = \eta\left(\hat{d}_t^N + \hat{n}_t^N\right)$$

(93) $$\overline{K}^H \hat{K}_{t+1}^H = \overline{d}^H \hat{d}_{t+1}^H + \overline{n}^H \hat{n}_{t+1}^H$$

(94) $$\overline{K}^N \hat{K}_{t+1}^N = \overline{d}^N \hat{d}_{t+1}^N + \overline{n}^N \hat{n}_{t+1}^N$$

(95) $$\begin{aligned}\overline{n}^H \hat{n}_{t+1}^H = \overline{q}^H \overline{k}^H \left(\hat{q}_t^H + \hat{K}_t^H\right) - (1+\overline{i})(1+\overline{\eta}^H)\overline{d}^H \left(\hat{i}_t - \hat{\pi}_{t+1} + \hat{\eta}_t^H + \hat{d}_t^H\right) \\ - (1+\overline{i}^*)(1+\overline{\eta}^*)\overline{d}^{H^*} \left(\hat{i}_t^* - \hat{\pi}_{t+1} + \hat{\eta}_t^{H*} + \hat{s}_{t+1} - \hat{s}_t + \hat{d}_t^{H^*}\right)\end{aligned}$$

(96) $$\overline{n}^N \hat{n}_{t+1}^N = \overline{q}^N \overline{k}^N \left(\hat{q}_t^N + \hat{K}_t^N\right) - (1+\overline{i})(1+\overline{\eta}^N)\overline{d}^N \left(\hat{i}_t - \hat{\pi}_{t+1} + \hat{\eta}_t^N + \hat{d}_t^N\right)$$

Appendix B

Price setting in the two sectors:

For a single traded sector firm, the pricing decision problem involves picking a price $\tilde{p}_t^H(z)$ to maximize the expected discounted value of current and future profits. The profits of a firm in some future period $t + i$ are affected by the price chosen in the earlier period t only if it has not had a chance to readjust its price, and the probability of such a situation is $\left(\omega^H\right)^i$. The profit maximization problem implies that the firm will choose its price in period t so as to equalize the present discounted value of marginal revenue with the discounted value of marginal cost. The profit maximization problem of a firm producing the differentiated product z is:

Maximize

$$(97)\qquad E_t \sum_{i=0}^{\infty} \left(\omega^H\right)^i \left(\beta^i \frac{C_t}{C_{t+i}}\right) \left[\frac{TR_{t+i}}{P_{t+i}} - \frac{TC_{t+i}}{P_{t+i}}\right]$$

Or, maximize,

$$(98)\qquad E_t \sum_{i=0}^{\infty} \left(\omega^H\right)^i \left(\beta^i \frac{C_t}{C_{t+i}}\right) \left[\frac{\tilde{p}_t^H(z)}{P_{t+i}} Y_{t+i}^H(z) - AC_{t+i}^H Y_{t+i}^H(z)\right]$$

Using equation (25) to eliminate $Y_{t+i}^H(z)$ we can rewrite the above equation as;

$$(99)\qquad E_t \sum_{i=0}^{\infty} \left(\omega^H\right)^i \left(\beta^i \frac{C_t}{C_{t+i}}\right) \left[\frac{\tilde{p}_t^H(z)}{P_{t+i}} \left(\frac{\tilde{p}_t^H(z)}{P_{t+i}}\right)^{-\varphi} Y_{t+i}^H - AC_{t+i}^H \left(\frac{\tilde{p}_t^H(z)}{P_{t+i}}\right)^{-\varphi} Y_{t+i}^H\right]$$

$$\Rightarrow E_t \sum_{i=0}^{\infty} \left(\omega^H\right)^i \left(\beta^i \frac{C_t}{C_{t+i}}\right) \left[\left(\frac{\tilde{p}_t^H(z)}{P_{t+i}^H}\right)^{1-\varphi} \frac{P_{t+i}^H}{P_{t+i}} - AC_{t+i}^H \left(\frac{\tilde{p}_t^H(z)}{P_{t+i}^H}\right)^{-\varphi}\right] Y_{t+i}^H$$

First order condition implies;

$$(100)\quad E_t\sum_{i=0}^{\infty}\left(\omega^H\right)^i\left(\beta^i\frac{C_t}{C_{t+i}}\right)\left[(1-\varphi)\left(\frac{\tilde{P}_t^H(z)}{P_{t+i}}\right)+\varphi MC_{t+i}^H\right]Y_{t+i}^H\left(\frac{\tilde{P}_t^H(z)}{P_{t+i}^H}\right)^{-\varphi}\frac{1}{\tilde{P}_t^H(z)}=0$$

$$(101)\qquad \tilde{P}_t^H(z)=\frac{\varphi}{1-\varphi}\frac{E_t\sum_{i=0}^{\infty}\omega^i\left(\beta^i\frac{C_t}{C_{t+i}}\right)MC_{t+i}^H\left(\frac{1}{P_{t+i}}\right)^{-\varphi}Y_{t+i}^H}{E_t\sum_{i=0}^{\infty}\omega^i\left(\beta^i\frac{C_t}{C_{t+i}}\right)\left(\frac{1}{P_{t+i}}\right)^{(1-\varphi)}Y_{t+i}^H}$$

Let $\tilde{P}_t^H$ denote the price set by a traded sector firm that gets to adjust its price in period t. Equation (35) shows that the price set by firms that get to revise them in period t will depend on the present and expected future aggregate price level, defined by equation (11). The aggregate price index of the traded sector goods is the average of the price charged by the fraction $\left(1-\omega^H\right)$ of firms that set their prices in period t and the average of the prices charged by the remaining firms. Since the firms that are able to set their prices in period t are chosen randomly, the average price of the firms that do not adjust in period t is just the average price in period t-1. Thus the average price of the traded good follows;

$$(102)\qquad P_t^{H^{(1-\varphi)}}=(1-\omega^H)\tilde{P}_t^{H^{(1-\varphi)}}+\omega^H P_{t-1}^{H^{(1-\varphi)}}$$

We can derive corresponding equations for the non-traded sector firms. The price chosen by non-traded sector firms that get to set a new price in period t satisfies the following equation.

$$(103)\qquad \tilde{P}_t^N(z)=\frac{E_t\sum_{i=0}^{\infty}\omega^i\left(\beta^i\frac{C_t}{C_{t+i}}\right)MC_{t+i}^N\left(\frac{1}{P_{t+i}^N}\right)^{\varphi}Y_{t+i}^N}{E_t\sum_{i=0}^{\infty}\omega^i\left(\beta^i\frac{C_t}{C_{t+i}}\right)\left(\frac{1}{P_{t+i}^N}\right)^{1-\varphi}Y_{t+i}^N}$$

Similarly, the average price of the non-traded good follows;

(104) $$P_t^{N^{(1-\varphi)}} = (1-\omega^N)\tilde{P}_t^{N^{(1-\varphi)}} + \omega^N P_{t-1}^{N^{(1-\varphi)}}$$

Appendix C:

We can derive the isoelastic demand for the traded goods output of a single traded sector firm with the following minimization problem.

(105) $$\min_{Y_t^H(z)} \int_0^1 p_t^H(z) Y_t^H(z)\, dz + \psi_t^H \left[Y_t^H - \left(\int_0^1 Y_t^H(z)^{\frac{\varphi-1}{\varphi}}\, dz \right)^{\frac{\varphi}{\varphi-1}} \right]$$

We can derive the first order condition;

(106) $$p_t^H(z) - \psi_t^H \left(\int_0^1 Y_t^H(z)^{\frac{\varphi-1}{\varphi}} \right)^{\frac{1}{\varphi-1}} Y_t^H(z)^{-\frac{1}{\varphi}} = 0$$

Substituting equation (21) into (24),

(107) $$\begin{gathered} p_t^H(z) - \psi_t^H Y_t^{H\frac{1}{\varphi}} Y_t^H(z)^{-\frac{1}{\varphi}} = 0 \\ \Rightarrow Y_t^H(z) = \left(\frac{p_t^H(z)}{\psi_t^H} \right)^{-\varphi} Y_t^H \end{gathered}$$

From equation (21) this implies;

(108) $$Y_t^H = \left[\int_0^1 \left[\left(\frac{p_t^H(z)}{\psi_t^H} \right)^{-\varphi} Y_t^H \right]^{\frac{\varphi-1}{\varphi}} dz \right]^{\frac{\varphi}{\varphi-1}}$$

or, $$Y_t^H = \left(\frac{1}{\psi_t^H} \right)^{-\varphi} \left(\int_0^1 p_t^H(z)^{\varphi-1} dz \right)^{\frac{\varphi}{\varphi-1}} Y_t^H$$

Thus solving for the Lagrange multiplier;

(109) $$\psi_t^H = \left(\int_0^1 p_t^H(z)^{\varphi-1} dz\right)^{\frac{1}{1-\varphi}} \equiv P_t^H$$

From equation (25) then the demand for the output of the traded goods produced by a single firm in the sector is;

(110) $$Y_t^H(z) = \left(\frac{p_t^H(z)}{P_t^H}\right)^{-\varphi} Y_t^H$$

φ here is the price elasticity of demand for the output of the firm's output, and as $\varphi \to \infty$ individual firms have less market power as their goods become closer substitutes.

We can derive a similar isoelastic demand function for the non-traded output of a single firm with the following minimization problem.

(111) $$\min_{Y_t^H(z)} \int_0^1 p_t^N(z) Y_t^N(z) dz + \psi_t^N \left[Y_t^N - \left(\int_0^1 Y_t^N(z)^{\frac{\varphi-1}{\varphi}} dz\right)^{\frac{\varphi}{\varphi-1}} \right]$$

We can derive the first order condition;

(112) $$p_t^N(z) - \psi_t^N \left(\int_0^1 Y_t^N(z)^{\frac{\varphi-1}{\varphi}}\right)^{\frac{1}{\varphi-1}} Y_t^N(z)^{-\frac{1}{\varphi}} = 0$$

Substituting equation (21) into (30),

(113) $$\begin{gathered} p_t^N(z) - \psi_t^N Y_t^{N\frac{1}{\varphi}} Y_t^N(z)^{-\frac{1}{\varphi}} = 0 \\ \Rightarrow Y_t^N(z) = \left(\frac{p_t^N(z)}{\psi_t^N}\right)^{-\varphi} Y_t^N \end{gathered}$$

From equation (22) this implies;

(114) $$Y_t^N = \left[\int_0^1 \left[\left(\frac{p_t^N(z)}{\psi_t^N} \right)^{-\varphi} Y_t^N \right]^{\frac{\varphi-1}{\varphi}} dz \right]^{\frac{\varphi}{\varphi-1}}$$

or, $$Y_t^N = \left(\frac{1}{\psi_t^N} \right)^{-\varphi} \left(\int_0^1 p_t^N(z)^{\varphi-1} dz \right)^{\frac{\varphi}{\varphi-1}} Y_t^N$$

Thus solving for the Lagrange multiplier;

(115) $$\psi_t^N = \left(\int_0^1 p_t^N(z)^{\varphi-1} dz \right)^{\frac{1}{1-\varphi}} \equiv P_t^N$$

From equation (31) then the demand for the output of the non-traded goods produced by a single firm in the sector is;

(116) $$Y_t^N(z) = \left(\frac{p_t^N(z)}{P_t^N} \right)^{-\varphi} Y_t^N$$

Appendix D:

Derivation of the New Keynesian Phillips curves (i.e equations 69 and 70) – AS

We can derive expressions for the deviations of trade and the non traded sector inflation rates around their steady-states levels, using the equations (46) and (47), and equations (48) and (49) respectively. These equations are rewritten as follows, for sector J.

(117)
$$\tilde{P}_t^J(z) = \frac{E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_{t+i}}{C_t} \right) MC_{t+i}^J \left(\frac{1}{P_{t+i}^J} \right)^{-\varphi} Y_{t+i}^J}{E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_{t+i}}{C_t} \right) \left(\frac{1}{P_{t+i}^J} \right)^{(1-\varphi)} Y_{t+i}^J}$$

(118)
$$P_t^{J\,(1-\varphi)} = (1-\omega^J)\,\tilde{P}_t^{J\,(1-\varphi)} + \omega^J P_{t-1}^{J\,(1-\varphi)}$$

Dividing both sides of equation (103) by P_t^H, we get,

(119)
$$\frac{\tilde{P}_t^J(z)}{P_t^J} = \frac{E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_{t+i}}{C_t} \right) MC_{t+i}^J \left(\frac{P_t^J}{P_{t+i}^J} \right)^{-\varphi} Y_{t+i}^J}{E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_{t+i}}{C_t} \right) \left(\frac{P_t^J}{P_{t+i}^J} \right)^{(1-\varphi)} Y_{t+i}^J}$$

We assume that steady state involves a zero inflation rate. Let $Q_t^J = \frac{\tilde{P}_t^J(z)}{P_t^J}$ be the relative price chosen by all firms adjusting their price in period t. The steady-state value of Q_t^J will then be equal to 1, which is also the value of Q_t^J when all the firms are able to adjust their prices every period. Dividing (104) by P_t^J, we obtain

$$1 = (1-\omega^J) Q_t^{J\,(1-\varphi)} + \omega^J \left(\frac{P_{t-1}^J}{P_t^J} \right)^{(1-\varphi)}$$

In terms of percentage deviations around the steady state this becomes,

$$1 = (1-\omega^J)\dot{q}_t^J + \omega^J \pi_t^J \Rightarrow \dot{q}_t^J = \left(\frac{\omega^J}{1-\omega^J}\right)\pi_t^J \tag{120}$$

Next, we can rewrite equation (105) as

$$\begin{aligned}&\left[E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_{t+i}}{C_t}\right)\left(\frac{P_t^J}{P_{t+i}^J}\right)^{(1-\varphi)} Y_{t+i}^J\right] Q_t^J \\ &\qquad = \mu^J \left[E_t \sum_{i=0}^{\infty} \omega^i \left(\beta^i \frac{C_{t+i}}{C_t}\right) MC_{t+i}^J \left(\frac{P_t^J}{P_{t+i}^J}\right)^{-\varphi} Y_{t+i}^J\right]\end{aligned} \tag{121}$$

In flexible price equilibrium with zero inflation rate, $Q_t^J = \mu^J MC_t^J = 1$. Approximating the above equation and setting $\mu^J MC_t^J = 1$, we get

$$\begin{aligned}&\left(\frac{1}{1-\omega^J\beta}\right)\dot{q}_t^J + \sum_{i=0}^{\infty} \omega^{J^i} \beta^i \left[E_t \hat{c}_{t+i} - (\varphi-1)\left(\hat{p}_t^J - E_t \hat{p}_{t+i}^J\right)\right] \\ &\qquad = \sum_{i=0}^{\infty} \omega^{J^i} \beta^i \left[m\hat{c}_t^J + E_t \hat{c}_{t+i} - \varphi\left(\hat{p}_t^J - E_t \hat{p}_{t+i}^J\right)\right]\end{aligned}$$

Solving the above equation further, we get

$$\begin{aligned}\left(\frac{1}{1-\omega^J\beta}\right)\hat{q}_t^J &= \sum_{i=0}^{\infty} \omega^{J^i} \beta^i \left(E_t m\hat{c}_{t+i}^J + E_t \hat{p}_{t+i}^J - \hat{p}_t^J\right) \\ &= \sum_{i=0}^{\infty} \omega^{J^i} \beta^i \left(E_t m\hat{c}_{t+i}^J + E_t \hat{p}_{t+i}^J\right) - \left(\frac{1}{1-\omega\beta}\right)\hat{p}_t^J\end{aligned} \tag{122}$$

Multiplying both sides by $1-\omega^J\beta$ and adding $\hat{p}_t^J$, we get

$$\begin{aligned}\hat{q}_t^J &= \left(1-\omega^J\beta\right) m\hat{c}_t^J + \omega^J\beta\left(E_t \hat{q}_{t+1}^J + E_t \hat{p}_{t+1}^J - \hat{p}_t^J\right) \\ &= \left(1-\omega^J\beta\right) m\hat{c}_t^J + \omega^J\beta\left(E_t \hat{q}_{t+1}^J + E_t \pi_{t+1}^J\right)\end{aligned}$$

Using equation (106) to eliminate $\hat{q}_t^J$,

$$\left(\frac{\omega^J}{1-\omega^J}\right)\hat{\pi}_t^J = \left(1-\omega^J\beta\right)m\hat{c}_t^J + \omega\beta\left(\left(\frac{\omega^J}{1-\omega}\right)E_t\hat{\pi}_{t+1}^J + E_t\hat{\pi}_{t+1}^J\right)$$

$$= \left(1-\omega^J\beta\right)m\hat{c}_t^J + \omega\beta\left(\frac{1}{1-\omega^J}\right)E_t\hat{\pi}_{t+1}^J$$

Now multiplying both sides by $\frac{1-\omega^J}{\omega^J}$, we can derive the forward-looking new Keynesian Phillips curves given by equations (69) and (70).

(123) $$\hat{\pi}_t^J = \varsigma^J m\hat{c}_t^J + \beta E_t\hat{\pi}_{t+1}^J + \hat{e}_t$$

where,

$$\varsigma^J = \frac{\left(1-\omega^J\right)\left(1-\omega^J\beta\right)}{\omega^J}$$

The equations (69) and (70) for marginal costs faced by firms in sector J are derived as follows.

Substituting equations (78) and (79) into equation (87) and solving for marginal cost gives the following.

The marginal cost for firms in the traded sector can be derived by substituting equations (79) and (78) in equation (87).

$$\begin{aligned} m\hat{c}_t^H &= (\kappa-1)\hat{L}_t + \hat{C}_t - \hat{Y}_t^H + \hat{L}_t^H \\ &= (\kappa-1)\left(\frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N\right) + \hat{C}_t - \hat{Y}_t^H + \hat{L}_t^H \end{aligned}$$

Substituting euqtions (71) and (72) into the above equations:

$$\begin{aligned} m\hat{c}_t^H &= (\kappa-1)\left(\frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N\right) + \gamma\left[\phi\hat{C}_t^H + (1-\phi)\left(\hat{C}_t^F\right)\right] \\ &\qquad + (1-\gamma)\hat{C}_t^N - \hat{Y}_t^H + \hat{L}_t^H \\ &= (\kappa-1)\left(\frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N\right) + \gamma\left[\phi\hat{C}_t^H + (1-\phi)\left(\hat{C}_t^H - \left(\hat{s}_t - \hat{p}_t^H\right)\right)\right] \\ &\qquad + (1-\gamma)\hat{C}_t^N - \hat{Y}_t^H + \hat{L}_t^H \end{aligned}$$

The production functions (81) and (82) can be used to eliminate $\hat{L}_t^H$ and $\hat{L}_t^N$ and the resource constraints (88) and (89) to eliminate $\hat{C}_t^H$ and $\hat{C}_t^N$ in the above equation. Taking terms together, marginal cost can be expressed as follows.

$$\begin{aligned} m\hat{c}_t^H &= \left(\frac{1}{1-\alpha}\left((\kappa-1)\frac{L^H}{L} + 1\right) + \gamma\frac{Y_H}{C_H}\right)\hat{Y}_t^H + \left(\frac{1}{1-a}\left((\kappa-1)\frac{L^N}{L}\right) + (1-\gamma)\frac{Y_N}{C_N}\right)\hat{Y}_t^N \\ &- \left(\frac{1}{1-\alpha}\left((\kappa-1)\frac{L^H}{L} + 1\right)\right)\left(\alpha\hat{K}_t^H + \hat{z}_t^H\right) - \left(\frac{1}{1-a}\left((\kappa-1)\frac{L^N}{L}\right)\right)\left(a\hat{K}_t^N + \hat{z}_t^N\right) \\ &- \gamma\frac{I^H}{C^H}\hat{I}_t^H - (1-\gamma)\frac{I^N}{C^N}\hat{I}_t^N - \left(\zeta\frac{C^H}{C^{H^*}} - \gamma(1-\phi)\right)\hat{s}_t \end{aligned}$$

Substituting equations (79) and (78) in equation (87) for the non-traded a sector gives the following expression for marginal cost:

$$
\begin{aligned}
m\hat{c}_t^N &= (\kappa - 1)\hat{L}_t + \hat{C}_t - \hat{Y}_t^N + \hat{L}_t^N \\
&= (\kappa - 1)\left(\frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N\right) + \hat{C}_t - \hat{Y}_t^N + \hat{L}_t^N
\end{aligned}
$$

Substituting equations (71) and (72) into the above equations:

$$
\begin{aligned}
m\hat{c}_t^N &= (\kappa - 1)\left(\frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N\right) + \gamma\left[\phi\hat{C}_t^H + (1-\phi)\left(\hat{C}_t^F\right)\right] \\
&\qquad + (1-\gamma)\hat{C}_t^N - \hat{Y}_t^N + \hat{L}_t^N \\
&= (\kappa - 1)\left(\frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N\right) + \gamma\left[\phi\hat{C}_t^H + (1-\phi)\left(\hat{C}_t^H - \left(\hat{s}_t - \hat{p}_t^H\right)\right)\right] \\
&\qquad + (1-\gamma)\hat{C}_t^N - \hat{Y}_t^N + \hat{L}_t^N
\end{aligned}
$$

The marginal cost for firms in the traded sector can be derived by substituting equations (79) and (78) in equation (87).

$$m\hat{c}_t^H = (\kappa - 1)\hat{L}_t + \hat{C}_t - \hat{Y}_t^H + \hat{L}_t^H$$
$$= (\kappa - 1)\left(\frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N\right) + \hat{C}_t - \hat{Y}_t^H + \hat{L}_t^H$$

Substituting euqtions (71) and (72) into the above equations:

$$m\hat{c}_t^H = (\kappa - 1)\left(\frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N\right) + \gamma\left[\phi\hat{C}_t^H + (1-\phi)\left(\hat{C}_t^F\right)\right]$$
$$+ (1-\gamma)\hat{C}_t^N - \hat{Y}_t^H + \hat{L}_t^H$$
$$= (\kappa - 1)\left(\frac{L^H}{L}\hat{L}_t^H + \frac{L^N}{L}\hat{L}_t^N\right) + \gamma\left[\phi\hat{C}_t^H + (1-\phi)\left(\hat{C}_t^H - \left(\hat{s}_t - \hat{p}_t^H\right)\right)\right]$$
$$+ (1-\gamma)\hat{C}_t^N - \hat{Y}_t^H + \hat{L}_t^H$$

The production functions (81) and (82) can be used to eliminate $\hat{L}_t^H$ and $\hat{L}_t^N$ and the resource constraints (88) and (89) to eliminate $\hat{C}_t^H$ and $\hat{C}_t^N$ in the above equation. Taking terms together, marginal cost can be expressed as follows.

$$m\hat{c}_t^H = \left(\frac{1}{1-\alpha}\left((\kappa-1)\frac{L^H}{L}+1\right)+\gamma\frac{Y_H}{C_H}\right)\hat{Y}_t^H + \left(\frac{1}{1-a}\left((\kappa-1)\frac{L^N}{L}\right)+(1-\gamma)\frac{Y_N}{C_N}\right)\hat{Y}_t^N$$

$$-\left(\frac{1}{1-\alpha}\left((\kappa-1)\frac{L^H}{L}+1\right)\right)\left(\alpha\hat{K}_t^H+\hat{z}_t^H\right)-\left(\frac{1}{1-a}\left((\kappa-1)\frac{L^N}{L}\right)\right)\left(a\hat{K}_t^N+\hat{z}_t^N\right)$$

$$-\gamma\frac{I^H}{C^H}\hat{I}_t^H-(1-\gamma)\frac{I^N}{C^N}\hat{I}_t^N-\left(\zeta\frac{C^H}{C^{H^*}}-\gamma(1-\phi)\right)\hat{s}_t$$

Substituting equations (79) and (78) in equation (87) for the non-traded sector gives the following expression for marginal cost:

$$\begin{aligned} m\hat{c}_t^N &= (\kappa-1)\hat{L}_t+\hat{C}_t-\hat{Y}_t^N+\hat{L}_t^N \\ &= (\kappa-1)\left(\frac{L^H}{L}\hat{L}_t^H+\frac{L^N}{L}\hat{L}_t^N\right)+\hat{C}_t-\hat{Y}_t^N+\hat{L}_t^N \end{aligned}$$

Substituting equations (71) and (72) into the above equations:

$$\begin{aligned} m\hat{c}_t^N &= (\kappa-1)\left(\frac{L^H}{L}\hat{L}_t^H+\frac{L^N}{L}\hat{L}_t^N\right)+\gamma\left[\phi\hat{C}_t^H+(1-\phi)\left(\hat{C}_t^F\right)\right] \\ &\qquad +(1-\gamma)\hat{C}_t^N-\hat{Y}_t^N+\hat{L}_t^N \\ &= (\kappa-1)\left(\frac{L^H}{L}\hat{L}_t^H+\frac{L^N}{L}\hat{L}_t^N\right)+\gamma\left[\phi\hat{C}_t^H+(1-\phi)\left(\hat{C}_t^H-\left(\hat{s}_t-\hat{p}_t^H\right)\right)\right] \\ &\qquad +(1-\gamma)\hat{C}_t^N-\hat{Y}_t^N+\hat{L}_t^N \end{aligned}$$

The production functions (81) and (82) can be used to eliminate $\hat{L}_t^H$ and $\hat{L}_t^N$, and the resource constraints (88) and (89) to eliminate $\hat{C}_t^H$ and $\hat{C}_t^N$ in the above equation. Taking terms together, marginal cost can be expressed as follows.

$$m\hat{c}_t^N = \left(\frac{1}{1-\alpha}\left((\kappa-1)\frac{L^H}{L}\right)+\gamma\frac{Y_H}{C_H}\right)\hat{Y}_t^H + \left(\frac{1}{1-a}\left((\kappa-1)\frac{L^N}{L}+1\right)+(1-\gamma)\frac{Y_N}{C_N}\right)\hat{Y}_t^N$$

$$-\left(\frac{1}{1-\alpha}\left((\kappa-1)\frac{L^H}{L}\right)\right)\left(\alpha\hat{K}_t^H+\hat{z}_t^H\right)-\left(\frac{1}{1-a}\left((\kappa-1)\frac{L^N}{L}+1\right)\right)\left(a\hat{K}_t^N+\hat{z}_t^N\right)$$

$$-\gamma\frac{I^H}{C^H}\hat{I}_t^H-(1-\gamma)\frac{I^N}{C^N}\hat{I}_t^N-\left(\zeta\frac{C^H}{C^{H^*}}-\gamma(1-\phi)\right)\hat{s}_t$$

Appendix E:

Derivation of the expectational IS curves (i.e equations 59 and 60) – AD relation

The aggregate demand relationship for the traded sector can be derived as follows.

$$\hat{C}_t = \gamma \hat{C}_t^T + (1-\gamma)\hat{C}_t^N$$

$$\hat{\mathrm{C}}_{\mathrm{t}} = \gamma \hat{C}_t^T + (1-\gamma)\left(\hat{C}_t^T - \hat{p}_t^N + \hat{p}_t^T\right)$$

$$\hat{C}_t = \hat{C}_t^T - (1-\gamma)\left(\hat{p}_t^N - \hat{p}_t^T\right)$$

$$\hat{C}_t = \phi \hat{C}_t^H + (1-\phi)\hat{C}_t^F - (1-\gamma)\left(\hat{p}_t^N - \hat{p}_t^T\right)$$

$$\hat{C}_t = \phi \hat{C}_t^H + (1-\phi)\left(\hat{p}_t^H - \hat{s}_t + \hat{C}_t^H\right) - (1-\gamma)\left(\hat{p}_t^N - \hat{p}_t^T\right)$$

$$\hat{C}_t = \hat{C}_t^H + (1-\phi)\left(\hat{p}_t^H - \hat{s}_t\right) - (1-\gamma)\left(\hat{p}_t^N - \hat{p}_t^T\right)$$

$$\hat{C}_t = \hat{C}_t^H + (1-\phi)\left(\hat{p}_t^H - \hat{s}_t\right) - \left(\hat{p}_t - \gamma \hat{p}_t^T\right) + (1-\gamma)\,\hat{p}_t^T$$

$$\hat{C}_t = \hat{C}_t^H + (1-\phi)\left(\hat{p}_t^H - \hat{s}_t\right) - \hat{p}_t + \hat{p}_t^T$$

$$\hat{C}_t = \hat{C}_t^H + (1-\phi)\left(\hat{p}_t^H - \hat{s}_t\right) - \hat{p}_t + \left(\phi \hat{p}_t^H + (1-\phi)\hat{s}_t\right)$$

$$\hat{C}_t = \hat{C}_t^H + (1-\phi)\left(\hat{p}_t^H - \hat{s}_t\right) - \hat{p}_t + \hat{p}_t^H - (1-\phi)\left(\hat{p}_t^H - \hat{s}_t\right)$$

$$\hat{C}_t = \hat{C}_t^H - \hat{p}_t + \hat{p}_t^H$$

$$\hat{C}_t + \hat{p}_t = \hat{C}_t^H + \hat{p}_t^H$$

From the euler equation

$$E_t \hat{C}_{t+1} = \hat{C}_t + E_t \left(\hat{i}_t - \left(\hat{p}_{t+1} - \hat{p}_t \right) \right)$$

$$E_t \hat{C}^H_{t+1} = \hat{C}^H_t + E_t \left(\hat{i}_t - \left(\hat{p}^H_{t+1} - \hat{p}^H_t \right) \right)$$

$$E_t \hat{C}^H_{t+1} = \hat{C}^H_t + \hat{i}_t - E_t \left(\pi^H_{t+1} \right)$$

$$E_t \left(\frac{Y^H}{C^H} \hat{Y}^H_{t+1} - \frac{I^H}{C^H} \hat{I}^H_{t+1} - \frac{C^{H^*}}{C^H} C^{H^*}_{t+1} \right) = \frac{Y^H}{C^H} \hat{Y}^H_t - \frac{I^H}{C^H} \hat{I}^H_t - \frac{C^{H^*}}{C^H} \hat{C}^{H^*}_t + \hat{i}_t - E_t \left(\pi^H_{t+1} \right)$$

$$\hat{Y}^H_t = E_t \hat{Y}^H_{t+1} - \frac{I^H}{Y^H} \left(E_t \hat{I}^H_{t+1} - \hat{I}^H_t \right) - \frac{C^{H^*}}{Y^H} \left(E_t \hat{C}^{H^*}_{t+1} - \hat{C}^{H^*}_t \right) - \frac{C^H}{Y^H} \left(\hat{i}_t - E_t \pi^H_{t+1} \right)$$

$$\hat{Y}^H_t = E_t \hat{Y}^H_{t+1} - \frac{I^H}{Y^H} \left(E_t \hat{I}^H_{t+1} - \hat{I}^H_t \right) - \frac{C^{H^*}}{Y^H} \left(E_t \hat{C}^{H^*}_{t+1} - \hat{C}^{H^*}_t \right) - \frac{C^H}{Y^H} \left(\hat{i}_t - E_t \pi^H_{t+1} \right)$$

$$\hat{Y}^H_t = E_t \hat{Y}^H_{t+1} - \frac{I^H}{Y^H} \left(E_t \hat{I}^H_{t+1} - \hat{I}^H_t \right)$$

$$- \frac{C^{H^*}}{Y^H} \zeta \left(E_t \left(\hat{s}_{t+1} - \hat{p}^H_{t+1} \right) - \left(\hat{s}_t - \hat{p}^H_t \right) \right) - \frac{C^H}{Y^H} \left(\hat{i}_t - E_t \pi^H_{t+1} \right)$$

$$\hat{Y}^H_t = E_t \hat{Y}^H_{t+1} - \frac{I^H}{Y^H} \left(E_t \hat{I}^H_{t+1} - \hat{I}^H_t \right) - \frac{C^H}{Y^H} \zeta \left(E_t \pi^F_{t+1} - E_t \pi^H_{t+1} \right) - \frac{C^H}{Y^H} \left(\hat{i}_t - E_t \pi^H_{t+1} \right)$$

$$\hat{Y}^H_t = E_t \hat{Y}^H_{t+1} - \frac{I^H}{Y^H} \left(E_t \hat{I}^H_{t+1} - \hat{I}^H_t \right) + \frac{C^H}{Y^H} \left(\zeta + 1 \right) E_t \pi^H_{t+1} - \left(\frac{C^H}{Y^H} \zeta \right) E_t \pi^F_{t+1} - \frac{C^H}{Y^H} \left(\hat{i} \right)_t$$

Similarly, the aggregate demand relationship for the non-traded sector can be derived as follows.

$$\hat{C}_t = \gamma \hat{C}_t^T + (1-\gamma)\hat{C}_t^N$$

$$\hat{\mathrm{C}}_{\mathrm{t}} = \gamma\left(\hat{C}_t^N + \hat{p}_t^N - \hat{p}_t^T\right) + (1-\gamma)\hat{C}_t^N$$

$$\hat{C}_t = \hat{C}_t^N + \gamma\left(\hat{p}_t^N - \hat{p}_t^T\right)$$

$$\hat{C}_t = \hat{C}_t^N + \gamma \hat{p}_t^N - \left(\hat{p}_t - (1-\gamma)\hat{p}_t^N\right)$$

$$\hat{C}_t = \hat{C}_t^N + \hat{p}_t^N - \hat{p}_t$$

$$\hat{C}_t + \hat{p}_t = \hat{C}_t^N + \hat{p}_t^N$$

From the euler equation

$$E_t \hat{C}_{t+1} = \hat{C}_t + E_t\left(\hat{i}_t - \left(\hat{p}_{t+1} - \hat{p}_t\right)\right)$$

$$E_t \hat{C}_{t+1}^N = \hat{C}_t^N + E_t\left(\hat{i}_t - \left(\hat{p}_{t+1}^N - \hat{p}_t^N\right)\right)$$

$$E_t \hat{C}_{t+1}^N = \hat{C}_t^N + \hat{i}_t - E_t\left(\pi_{t+1}^N\right)$$

$$E_t\left(\frac{Y^N}{C^N}\hat{Y}_{t+1}^N - \frac{I^N}{C^N}\hat{I}_{t+1}^N\right) = \frac{Y^N}{C^N}\hat{Y}_t^N - \frac{I^N}{C^N}\hat{I}_t^N + \hat{i}_t - E_t\left(\pi_{t+1}^N\right)$$

$$\hat{Y}_t^N = E_t\hat{Y}_{t+1}^N - \frac{I^N}{Y^N}\left(E_t\hat{I}_{t+1}^N - \hat{I}_t^N\right) + \frac{C^N}{Y^N}\left(\hat{i}_t - E_t\pi_{t+1}^N\right)$$

Table 1

Calibration

(Parameter Values)

Parameters	Value
β	0.99
$\gamma = \varphi$	0.5
δ	0.025
$\eta = \eta^*$	0.1
$\alpha = a$	0.35
$\varepsilon = e$	1
$\kappa = \kappa^N = \kappa^H$	3
C^{H*}/C^H	0.10
I^H/ Y^H	0.49
I^N/ Y^N	0.49
(Dh*+Dh)/Nh	0.52/0.48
Dn/Nn	0.52/0.48
Dh*/(Dh*+Dh)=Dh/(Dh*+h)	0.5
ξ^H	1
ε_ψ	1

Table 2

Volatility of exchange rate and interest rate for a given value of lambda

$(0 < \theta_H < 1,\ \theta_N = 1 - \theta_H,\ \lambda = 2)$

θ_H weight on non-traded output gap	$\theta_N = 1 - \theta_H$ weight on non-traded output gap	σ_S^2 var (interest rate)	σ_i^2 var (exchange rate)	θ_H / θ_N Relative weights	σ_i^2 / σ_S^2 Relative Variance
0.98	0.02	2.80E-05	0.000242	0.020408	8.621009
0.75	0.25	2.89E-05	0.000252	0.333333	8.737955
0.5	0.5	3.10E-05	0.000258	1	8.350026
0.25	0.75	3.34E-05	0.000258	3	7.708801
0.02	0.98	3.63E-05	0.000252	49	6.93764

Table 3

Volatility of exchange rate and interest rate for a given value of lambda

$(0 < \theta_H < 1,\ \theta_N = 1 - \theta_H,\ \lambda = 3)$

θ_H weight on non-traded output gap	$\theta_N = 1 - \theta_H$ weight on non-traded output gap	σ_S^2 var (interest rate)	σ_i^2 var (exchange rate)	θ_H / θ_N Relative weights	σ_i^2 / σ_S^2 Relative Variance
0.98	0.02	2.73E-05	0.000235	0.020408	8.58183
0.75	0.25	2.74E-05	0.000249	0.333333	9.078454
0.5	0.5	3.01E-05	0.000258	1	8.56318
0.25	0.75	3.38E-05	0.000257	3	7.618075
0.02	0.98	3.85E-05	0.000249	49	6.459021

Table 4

Volatility of exchange rate and interest rate for a given value of lambda

$(0 < \theta_H < 1,\ \theta_N = 1 - \theta_H,\ \lambda = 4)$

θ_H weight on non-traded output gap	$\theta_N = 1 - \theta_H$ weight on non-traded output gap	σ_S^2 var (interest rate)	σ_i^2 var (exchange rate)	θ_H / θ_N Relative weights	σ_i^2 / σ_S^2 Relative Variance
0.98	0.02	2.67E-05	0.00023332	0.020408	8.75333
0.75	0.25	2.64E-05	0.00024636	0.333333	9.343143
0.5	0.5	2.94E-05	0.00025793	1	8.779101
0.25	0.75	3.42E-05	0.00025706	3	7.526938
0.02	0.98	4.10E-05	0.00024568	49	5.996583

Figure 1

Relative exchange rate volatility and the importance of traded sector

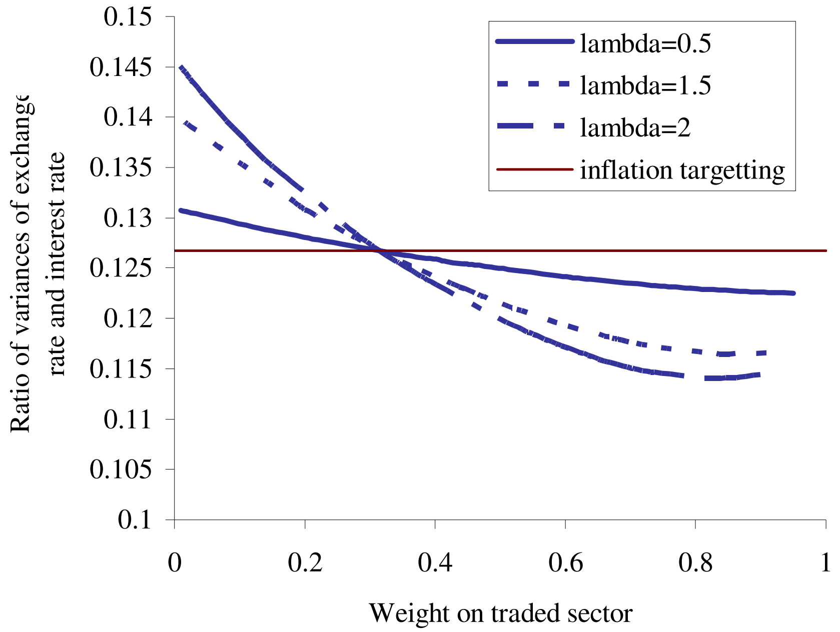


Figure 2

Sector Size and Relative Exchange Rate Volatility

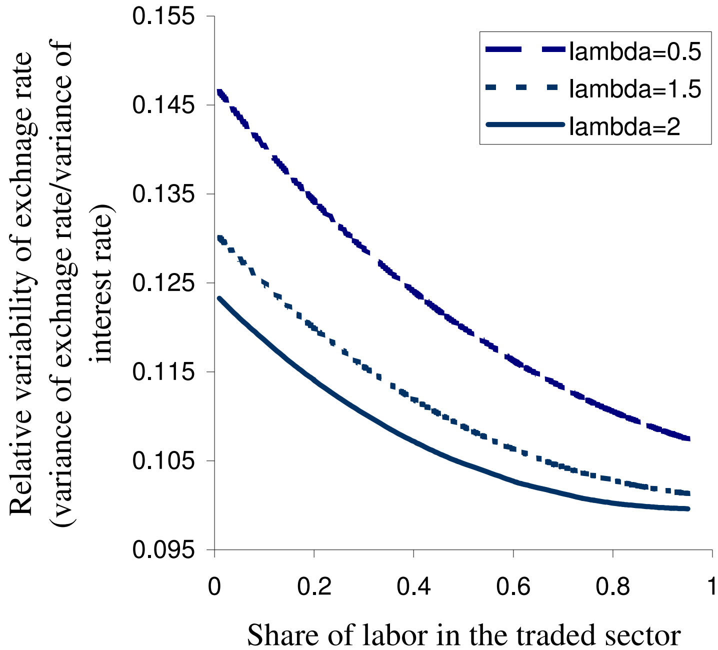


Figure 3

Response to foreign interest rate shock

(Weight on aggregate output (λ) = 2, Weight on traded sector output (θ_H) = 0.98,

Weight on non-traded sector output (θ_N) =0.02)

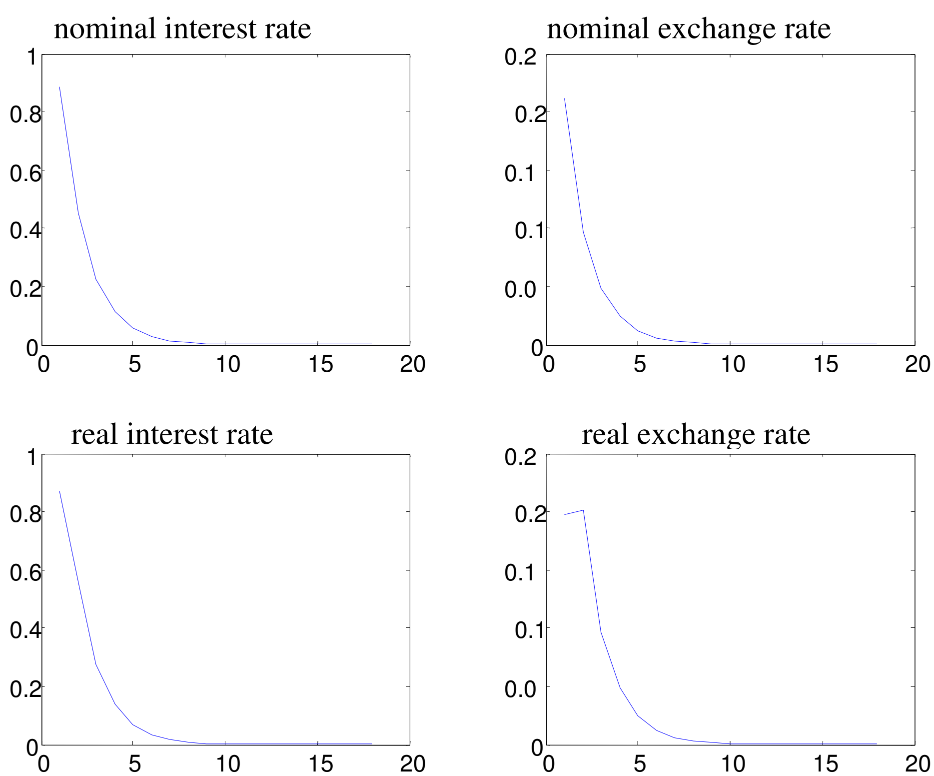


Figure 4

Response to foreign interest rate shock

(Weight on aggregate output (λ) = 2, Weight on traded sector output (θ_H) = 0.50,

Weight on non-traded sector output (θ_N) = 0.50)

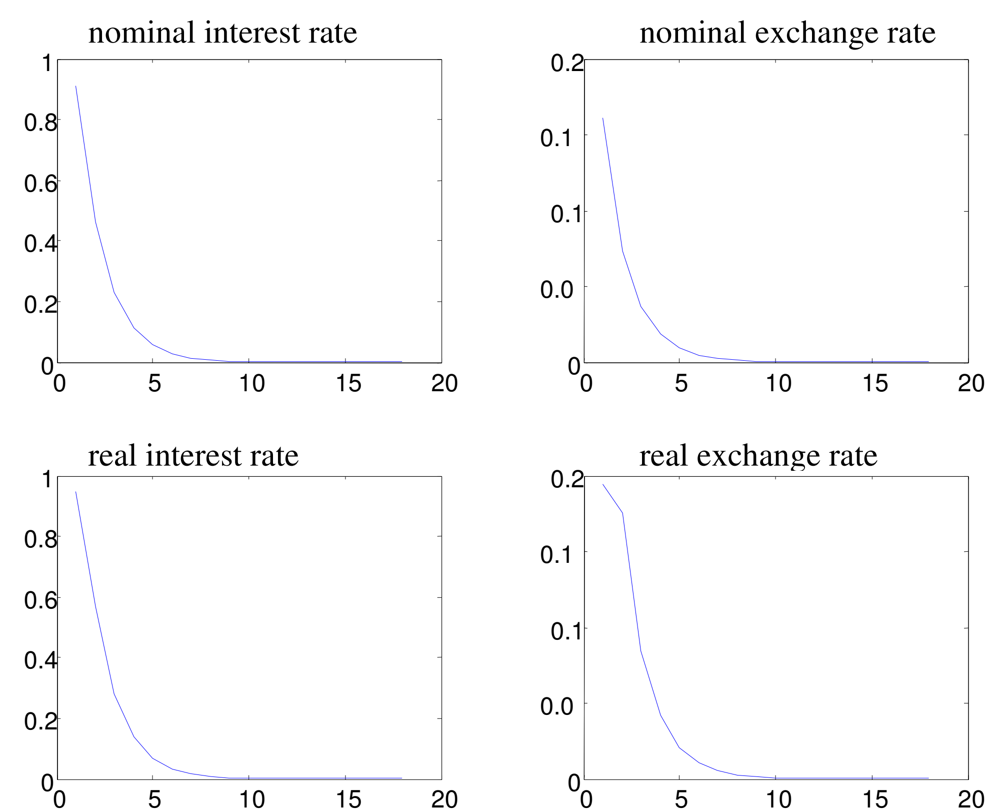
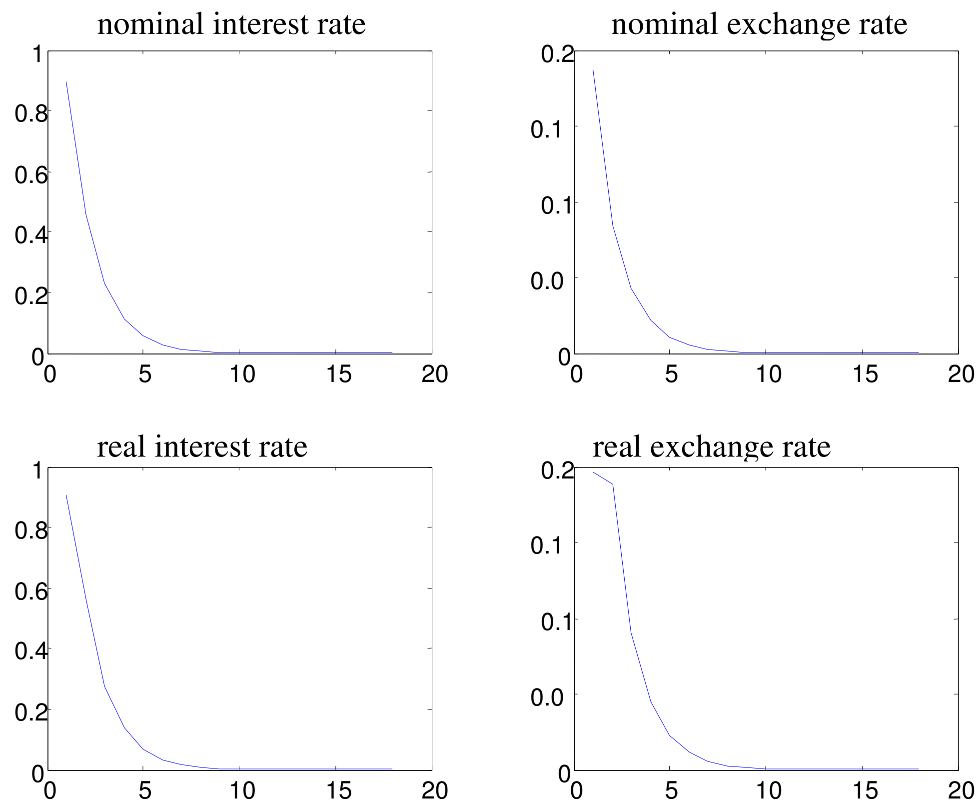


Figure 5

Response to foreign interest rate shock

(Weight on aggregate output (λ) = 2, Weight on traded sector output (θ_H) = 0.02,

Weight on non-traded sector output (θ_N) = 0.98)



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