

# Big Mac parity, income, and trade

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## Abstract

Nontraded inputs account for the lion's share of a Big Mac price (Ong 1997, Parsley and Wei 2003). Major departures from Big Mac PPP may then be explained by the Balassa-Samuelson income differences effect, as shown e.g. by Click (1996). But it has been argued that Click's result is not robust to changing estimation methods, sample of countries, and time period (Fujiki and Kitamura 2003). Here we address a key theoretical distinction between high and low income countries for the Balassa-Samuelson effect to be properly evaluated. Since this distinction is missing in Click's analysis, we revisit his finding and take a sample which is distinct (in terms of both set of countries and time period) to meet Fujiki-Kitamura's criticism. We find that distinguishing high from low income makes no harm to Click's result. But we also find that openness to trade (viewed as a proxy for trade barriers) helps to explain departures from Big Mac PPP.

**JEL Classification:** F31, F41

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## 1. Introduction

Purchasing power parity (PPP) theory is the notion that a dollar should buy the same amount in all countries. PPP stands as the simplest nominal exchange rate theory. According to PPP, in a somewhat elusive long run exchange rates vary to accompany the price level differences in two countries. Thus the exchange rate between the two countries should ultimately move toward the rate that equalizes the prices of an identical basket of goods and services in each country.

*The Economist* has suggested that the "basket" is a McDonald's Big Mac, which is produced in about 120 countries. The Big Mac index has been released regularly by the magazine since its inception in September 1986. So the Big Mac PPP is the nominal exchange rate that would mean hamburgers cost the same in the United States as abroad. Given the price levels, comparing actual exchange rates with PPPs indicates whether a currency is under- or overvalued. (We display an amusing illustration at <http://www.angelfire.com/id/SergioDaSilva/bigmacppp.html>).

Wherever the relative merits of PPP, the Big Mac index can satisfactorily represent the theory (Pakko and Pollard 1996, 2003). That relative PPP sounds more reasonable if compared to absolute PPP, for instance, is something that the Big Mac index can capture well.

*The Economist* often stresses that its index is a "lighthearted" measure to make PPP more digestible. PPP is not to be taken too seriously as Big Mac prices are not identical across countries once converted into a common currency. PPP fails thanks to a number of reasons mainly related to trade barriers, nontradables, and pricing to market (Pakko and Pollard 2003). Here we deal with the first two reasons. As for pricing to market, Fujiki and Kitamura (2003) have suggested that McDonald's quickly responds to fluctuations in nominal exchange rates on an almost one-to-one basis. Anyway an enduring stylized fact of nominal exchange rate returns is "no PPP", i.e. neither form of PPP holds in the short run, while there is some evidence favoring (relative) PPP in the long run (De Vries 1994).

As for nontradables, Click (1996) shows that Big Mac prices conform fairly well to PPP in a time-series dimension, and that country specific deviations are explained by the Balassa-Samuelson effect rather than by chance. Some estimates show that nontraded goods (such as wages and rent) account for 94 percent of the price of a Big Mac (Ong 1997). Parsley and Wei (2003) estimate that labor, rent, and electricity alone are responsible for 55.3 percent of a Big Mac's cost share. However Engel (1999) finds no convergence to PPP even if one looks at only traded goods, i.e. all movements in real exchange rates are due to deviations from the law of one price. But Obstfeld and Rogoff (2000) ponder that this is because even traded goods have a large nontraded component. Parsley and Wei (2003) argue, too, that using a Big Mac basket brings advantages if compared to a Consumer Price Index basket. They then revisit Engel's result and find that reduced exchange rate volatility, lower transport cost, higher tariffs, and exchange rate pegs may dampen the Engel effect.

Thus because nontradables are included in the Big Mac index, high income countries will have overvalued currencies relative to low income countries. Fujiki and Kitamura (2003) employ standard statistical tests to evaluate Click's result that PPP holds conditional on the Balassa-Samuelson income differences effect. By taking both cross section and panel data, they conclude that the result is not robust to changing estimation methods, sample of countries, and time period.

Here we revisit the findings of Click and take a sample that is also distinct in terms of both set of countries and time period. We departure from Click's analysis and make a distinction between high and middle income countries. Such a distinction is theoretically needed if the Balassa-Samuelson effect is to be taken consistently. Indeed the Balassa-Samuelson's theory is useless if countries have similar per capita incomes (see a naked-eye piece of evidence on this at Table 2 of Pakko and Pollard 2003). Thus one needs to consider different income groups of countries to properly evaluate the role of nontradables in departures from Big Mac PPP. Here we do that only to find that neglecting such a distinction makes no harm to Click's result. Unlike Click (and following Fujiki and Kitamura) we also employ Hausman test (to be described below) as a criterion for model selection.

Arguably the Balassa-Samuelson effect might not be the sole reason for departures from Big Mac PPP (as seen). Thus we further consider trade barriers as modeled by the degree of openness to trade. We find that trade openness does help to explain departures from Big Mac PPP as well.

The rest of this paper is organized as follows. Section 2 presents data and our econometric model, Section 3 analyzes the data, and Section 4 concludes.

## 2. Data and model

Data on Big Mac prices are collected from various issues of *The Economist*, and range from 1995 to 2003. (Pakko and Pollard 2003 provide them at <http://research.stlouisfed.org/publications/review/03/11/0311mpd.xls>). Data on incomes are taken from the World Bank Development Indicators database for the time period 1995–2001. High income countries include Australia, Britain, Canada, Denmark, Hong Kong, Japan, Singapore, South Korea, Sweden, Switzerland, and Taiwan. For the second group of middle income countries we have merged upper middle and lower middle income countries. Mid income countries in this study then include Argentina, Brazil, Chile, China, Hungary, Malaysia, Mexico, Poland, Russia, and Thailand. And data on trade openness are for the time period from 1995 to 2000, and are collected from the Penn World Table version 6.1 (Heston, Summers, and Aten 2002), which is available at [pwt.econ.upenn.edu](http://pwt.econ.upenn.edu).

We employ panel data to test the Big Mac PPP hypothesis, and models are estimated by searching for fixed and random effects. A clear advantage of such an approach is to highlight individual heterogeneity.

Our econometric model is

$$\ln\left(\frac{P}{P^*}\right)_{it} = \beta_0 + \beta_1 \ln E_{it} + \varepsilon_{it} \quad (1)$$

$$\varepsilon_{it} = \alpha_i + u_{it} \quad (2)$$

where  $(P/P^*)_{it}$  is the ratio between a country's Big Mac price and the one in the United States,  $E_{it}$  is the nominal exchange rate (foreign currency per dollar),  $\alpha_i$  tracks individual effects and is fixed over time, and  $u_{it}$  is a random error. If the  $\alpha_i$ 's are correlated with the explanatory variables, the estimator of fixed effects is both consistent and efficient, whereas the estimator of random effects is not consistent. If the  $\alpha_i$ 's show no correlation with  $\ln E_{it}$ , so the random effects model generates consistent and efficient estimates, whereas the fixed effects model ends up consistent but not efficient (Hsiao 1986, Baltagi 1995, Greene 2003).

To learn about which model is appropriate for a given data set, we employ Hausman (1978) test. If a model is well specified, Hausman test contrasts the null hypothesis of suitability of the random effects model with the alternative hypothesis of appropriateness of the fixed effects model. In particular, Hausman test checks whether the coefficients estimated in the two models are equal. If the two estimates are not significantly distinct, the random effects model will be selected. Conversely, if the estimates are significantly distinct, the fixed effects model will be chosen.

The test statistic is

$$w = (\beta_f - \beta_r)' (V_f - V_r)^{-1} (\beta_f - \beta_r) \quad (3)$$

where the rows and columns of vectors and matrices that correspond to the intercept have been dropped together with the rows and columns of parameters that cannot be estimated by the fixed

effects model. Under the null hypothesis,  $w$  is  $\chi^2(k)$  distributed, where  $k$  is the number of estimated coefficients in  $\beta$  apart from the intercept and time invariant regressors.

For Big Mac PPP to hold, the null hypothesis  $\beta_0 = 0$  and  $\beta_1 = 1$  cannot be jointly rejected. Otherwise PPP fails to hold, and prices are not dependent on the exchange rate. This might imply that either prices are set at distinct base levels ( $\beta_0 \neq 0$ ), prices do not change as the exchange rate changes ( $\beta_1 \neq 1$ ), or both (Click 1996).

### 3. Analysis

A simple test for Big Mac PPP is to evaluate whether  $\ln(P_{it}/P_t^*E_{it}) = 0$  holds on average. Table 1 shows that this cannot be rejected for mid and high income countries, and for the countries taken together. This result departs from Click's, where a zero mean is not found. But if we carry out the test for data of a given particular year, the null hypothesis is rejected for 2002 and 2003 for the mid income countries. (For 2002, mean of  $-39.3$ , standard deviation of  $18.4$ , and  $t$  statistic of  $-2.1$ ; for 2003, mean of  $-40.5$ , standard deviation of  $13.6$ , and  $t$  statistic of  $-2.97$ .)

Figure 1 displays  $\ln(P/P^*)$  against  $\ln E$ . A possible linear relationship between the variables emerges (intercept and angular coefficient of nil and one respectively). This suggests that Big Mac PPP might hold. Thus we test such a hypothesis using panel data.

Hausman test in Table 2 shows that the random effects model is significant. One cannot reject the null hypothesis of coefficient equivalence at the significance level of 5 percent for each group of countries and for the two groups merged. For each group the random effects model estimated according to equation (1) shows intercept and angular coefficient that are significant at the one percent level. For the groups taken together the intercept is not significant and the angular coefficient approaches one. The latter result is more in line with the hypothesis of PPP. The  $F/\chi^2$  statistic, which tests whether the constant and coefficients on the regressors are jointly zero, shows that the adjustment of the three models to the data is significant.

At this point one cannot jump to the conclusion that Big Mac PPP holds. One needs to further test whether condition  $\beta_0 = 0$  and  $\beta_1 = 1$  jointly holds. Table 2 shows that this is rejected. We then turn to check whether income differences provide a reason for this Big Mac PPP failure.

Balassa (1964) and Samuelson (1964) have warned that nontradables can systematically account for departures from PPP. So the Big Mac price in a country may convey more information than that in the prices of its ingredients because nontraded goods should be used to sell Big Macs. If nontradables are part of the cost of the sandwich, deviations from PPP may reflect possible different costs across countries. In Balassa-Samuelson's theory, labor in poor countries is less productive than labor in the traded sector of developed countries. Thus including nontradables in the basket that reckons price indices produces high income countries with relatively overvalued currencies. Higher productivity in these countries is the reason why that happens. It is then implied that the theory is useless for two countries with similar per capita incomes. One needs to consider different income groups of countries, and this is precisely what we have been doing in this paper.

Click (1996) argues that Big Mac PPP failures are due uniquely to the individual effects of countries, which are time invariant. And since nontraded inputs in a Big Mac vary across countries, this fact (he reasons) may well explain departures from PPP.

To consider the influence of differences in income, we then carry out new estimates of the models by taking  $\ln(Y/Y^*)$  as an explanatory variable. As before, Big Mac PPP continues to be rejected after adding  $\ln(Y/Y^*)$  to the regression (Table 3). Pakko and Pollard (2003) show in Table 2 of their paper that the Balassa-Samuelson effect is not clearly seen for advanced economies but a pattern of undervaluation emerges for developing countries (apart from Latin America). As theoretically expected, our Table 3 shows that the income variable is significant for mid income countries and the countries taken together; this is so because in these samples income differences

relative to that in the US are not negligible. (Taiwan has been dropped as information is missing.) Indeed for the high income country sample, the  $\ln(Y/Y^*)$  coefficient in the fixed effects model selected by Hausman test is not significant.

As observed, another classic reason for departures from PPP is trade barriers. Cassel (1922) has early warned that if a country adopts export restrictions, its currency will end up undervalued on a PPP basis. Similarly it can be inferred that if a country restricts its imports (through tariffs and quotas) local prices will be greater than foreign prices, thereby making its currency overvalued if compared with PPP. Now we aim to track such an effect by considering trade openness as an extra explanatory variable. The greater the openness, arbitrage is heightened and deviations from PPP abate. We make no distinction between income groups of countries once that should be relevant for evaluating the Balassa-Samuelson effect only.

Both fixed and random effects models present significant coefficients at the 5 percent level (Table 4). (Thailand is left out as information is missing.) The Big Mac PPP hypothesis is rejected for the fixed effects model selected by Hausman test at the one percent significance level. As expected, the trade openness coefficient presents the right negative sign throughout, which means that a Big Mac is cheaper wherever openness is greater.

#### **4. Conclusion**

This paper shows that Big Mac PPP fails to hold and considers both income and trade effects for explaining such a failure. We find that less productive countries tend to have currencies that are undervalued on a PPP basis. This finding is consistent with previous work. We also find that countries that are more opened to trade show less deviations from PPP. Although this is theoretically well known for long, our result is novel for PPP in terms of a Big Mac basket.

Balassa-Samuelson's theory is useless if two countries have similar per capita incomes. Thus one needs to consider different income groups of countries to properly evaluate the role of nontradables in departures from Big Mac PPP. This has not been done by previous works, such as that of Click (1996). So here we make this necessary distinction only to show that it makes no harm to Click's result.

Sample of Countries, 1995–2003	Number of Observations	Mean (Standard Deviation)	<i>t</i> Statistic
Mid Income	90	–0.28852 (0.234888)	–1.22834
High Income	99	0.03 (0.388604)	0.073736
Both Groups	189	–0.12238 (0.360646)	–0.33934

Table 1. Simple test for Big Mac PPP.

Notes

The hypothesis that  $\ln(P_{it}/P_t^* E_{it}) = 0$  holds on average cannot be rejected, and this presents casual evidence for PPP.

Model		Number of Observations	$\beta_0$	$\beta_1$	$R^2$	F/Chi2	Hausman Test	Null Hypothesis $\beta_0 = 0, \beta_1 = 1$
Mid Income	Fixed	90	-0.4059351 (0.0845053)*	1.00523 (.0286998)*	Within=0.9395 Between=0.9890 Overall=0.9816	F(1, 79)=1226.80 Prob>F=0.0000	Chi2(1)=0.01 Prob>Chi2=0.9129	F(2, 79)=123.94 Prob>F=0.0000
	Random	90	-0.4005017 (0.1014949)*	1.0033 (0.0226242)*	Within=0.9395 Between=0.9890 Overall=0.9816	Chi2(1)=1966.60 Prob>Chi2=0.0000		Chi2(2)=24.52 Prob>Chi2=0.0000
High Income	Fixed	99	2.053191 (0.0925464)*	-0.012332 (0.0446649) <sup>NS</sup>	Within=0.0009 Between=0.9795 Overall=0.9724	F(1, 87)=0.08 Prob>F=0.7831	Chi2(1)=0.00 Prob>Chi2=1.0000	F(2, 87)=269.54 Prob>F=0.0000
	Random	99	1.088173 (0.2296808)*	0.4549253 (0.0611109)*	Within=0.0009 Between=0.9795 Overall=0.9724	Chi2(1)=55.42 Prob>Chi2=0.0000		Chi2(2)=79.59 Prob>Chi2=0.0000
Both	Fixed	189	-0.1218337 (0.0636308) <sup>NS</sup>	0.9652622 (0.025478)*	Within=0.8958 Between=0.9775 Overall=0.9712	F(1, 167)=1435.36 Prob>F=0.0000	Chi2(1)=0.33 Prob>Chi2=0.5663	F(2, 167)=88.87 Prob>F=0.0000
	Random	189	-0.1429422 (0.089278) <sup>NS</sup>	0.9739777 (0.020449)*	Within=0.8958 Between=0.9775 Overall=0.9712	Chi2(1)=2268.57 Prob>Chi2=0.0000		Chi2(2)=9.31 Prob>Chi2=0.0095

Table 2. Regression  $\ln\left(\frac{P}{P^*}\right) = \beta_0 + \beta_1 \ln E + u$

Notes

\* significant at 1%, \*\* significant at 5%, NS non-significant, ( ) standard error

Model		Number of Observations	$\beta_0$	$\beta_1$	$\beta_2$	$R^2$	F/Chi2	Hausman Test	Null Hypothesis $\beta_0 = 0, \beta_1 = 1, \beta_2 = 0$
Mid Income	Fixed	70	-0.4289417 (0.0746984)*	1.028901 (0.0254672)*		Within=0.9651 Between=0.9858 Overall=0.9827	F(1, 59)=1632.25 Prob>F=0.0000	Chi2(1)=0.80 Prob>Chi2=0.3704	F(2, 59)=128.27 Prob>F=0.0000
	Random	70	-0.39554 (0.1123384)*	1.016996 (0.0217238)*		Within=0.9651 Between=0.9858 Overall=0.9827	Chi2(1)=2191.62 Prob>Chi2=0.0000		Chi2(2)=14.20 Prob>Chi2=0.0008
High Income	Fixed	70	2.036226 (0.1247817)*	-0.0747649 (0.0658607) <sup>NS</sup>		Within=0.0214 Between=0.9773 Overall=0.9724	F(1, 59)=1.29 Prob>F=0.2609	Chi2(1)=0.00 Prob>Chi2=1.0000	F(2, 59)=133.31 Prob>F=0.0000
	Random	70	0.8182858 (0.2608979)*	0.5696438 (0.0764603)*		Within=0.0214 Between=0.9773 Overall=0.9724	Chi2(1)=55.51 Prob>Chi2=0.0000		Chi2(2)=31.68 Prob>Chi2=0.0000
Both Groups	Fixed	140	-0.1788091 (0.05932)*	1.003125 (0.0244491)*		Within=0.9340 Between=0.9760 Overall=0.9729	F(1, 119)=1683.39 Prob>F=0.0000	Chi2(1)=0.30 Prob>Chi2=0.5846	F(2, 119)=65.74 Prob>F=0.0000
	Random	140	-0.1611345 (0.0947354) <sup>NS</sup>	0.9955967 (0.0202029)*		Within=0.9340 Between=0.9760 Overall=0.9729	Chi2(1)=2428.52 Prob>Chi2=0.0000		Chi2(2)=4.42 Prob>Chi2=0.1097
Mid Income	Fixed	70	1.684788 (0.2588307)*	0.9749416 (0.0185044)*	0.8716845 (0.1046602)*	Within=0.9841 Between=0.9681 Overall=0.9703	F(2, 58)=1796.50 Prob>F=0.0000	Chi2(2)=16.65 Prob>Chi2=0.0002	F(3, 58)=207.73 Prob>F=0.0000
	Random	70	1.080811 (0.2185497)*	0.9953577 (0.0167657)*	0.6288385 (0.0840841)*	Within=0.9826 Between=0.9849 Overall=0.9846	Chi2(2)=3891.63 Prob>Chi2=0.0000		Chi2(3)=76.44 Prob>Chi2=0.0000
High Income	Fixed	70	2.109259 (0.1771534)*	-0.1226178 (0.1053561) <sup>NS</sup>	-0.0720866 (0.123428) <sup>NS</sup>	Within=0.0271 Between=0.9806 Overall=0.9774	F(2, 58)=0.81 Prob>F=0.4508	Chi2(2)=170.96 Prob>Chi2=0.0000	F(3, 68)=87.99 Prob>F=0.0000
	Random	70	0.5429886 (0.2237251)**	0.8067857 (0.0777638)*	0.7159215 (0.142421)*	Within=0.0233 Between=0.9747 Overall=0.9719	Chi2(2)=108.34 Prob>Chi2=0.0000		Chi2(3)=65.04 Prob>Chi2=0.0000
Both Groups	Fixed	140	1.042708 (0.1151087)*	0.9625073 (0.0173384)*	0.9035564 (0.0795154)*	Within=0.9685 Between=0.8542 Overall=0.8626	F(2, 118)=1812.49 Prob>F=0.0000	Chi2(2)=63.61 Prob>Chi2=0.0000	F(3, 118)=134.05 Prob>F=0.0000
	Random	140	0.3769906 (0.1084613)*	0.99928 (0.0170269)*	0.4386979 (0.05409)*	Within=0.9592 Between=0.9725 Overall=0.9714	Chi2(2)=3494.33 Prob>Chi2=0.0000		Chi2(3)=70.96 Prob>Chi2=0.0000

Table 3. Regression  $\ln\left(\frac{P}{P^*}\right) = \beta_0 + \beta_1 \ln E + \beta_2 \ln\left(\frac{Y}{Y^*}\right) + u$

Notes

\* significant at 1%, \*\* significant at 5%, NS non-significant, ( ) standard error

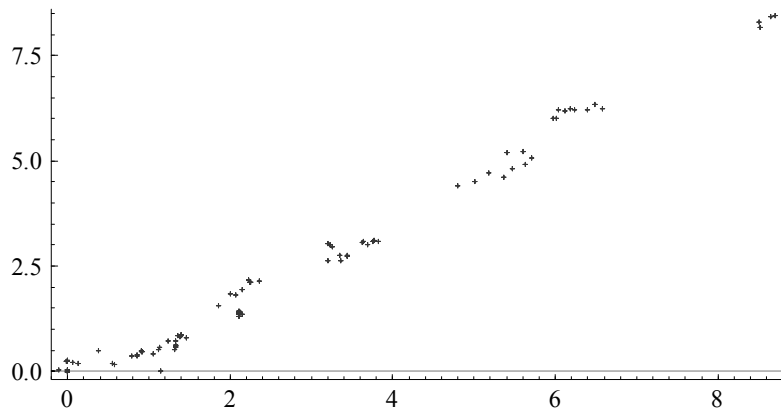


Model		Number of Observations	$\beta_0$	$\beta_1$	$\beta_2$	$R^2$	F/Chi2	Hausman Test	Null Hypothesis $\beta_0 = 0, \beta_1 = 1, \beta_2 = 0$
Both Groups	Fixed	114	2.789992 (0.6259376)*	0.9959564 (0.0241212)*	-0.6962616 (0.1482361)*	Within=0.9495 Between=0.9174 Overall=0.9194	F(2, 93)=875.18 Prob>F=0.0000	Chi2(2)=19.47 Prob>Chi2=0.0001	F(3, 93)=36.37 Prob>F=0.0000
	Random	114	0.8193366 (0.4186128)**	1.004758 (0.0218553)*	-0.2303715 (0.0987273)**	Within=0.9442 Between=0.9679 Overall=0.9664	Chi2(2)=2143.34 Prob>Chi2=0.0000		Chi2(3)=7.52 Prob>Chi2=0.0570

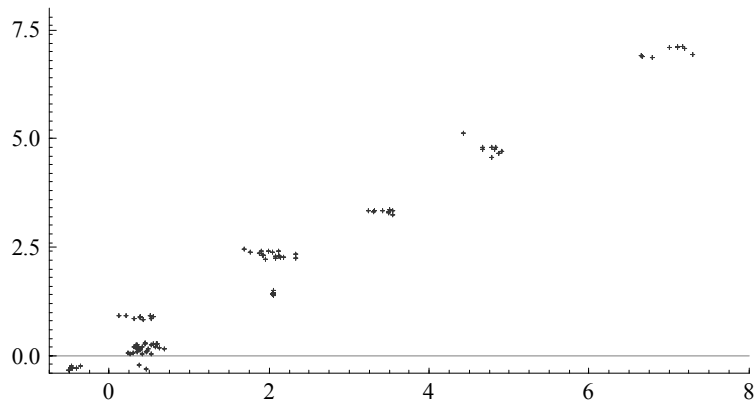
Table 4. Regression  $\ln\left(\frac{P}{P^*}\right) = \beta_0 + \beta_1 \ln E + \beta_2 \ln Open + u$

Notes

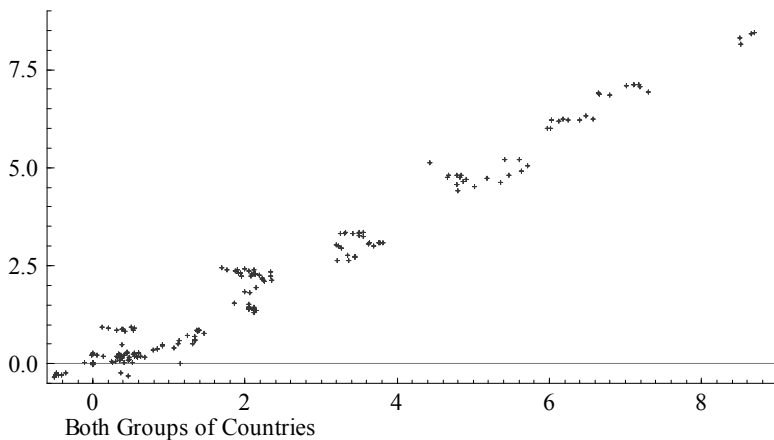
\* significant at 1%, \*\* significant at 5%, NS non-significant, ( ) standard error



Mid Income Countries



High Income Countries



Both Groups of Countries

Figure 1.  $\ln P/P^*$  versus  $\ln E$ .

Note

A linear relationship between the two variables is suggested by a zero intercept and an angular coefficient of one. This presents evidence for Big Mac PPP that is only casual.

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