Derivation of Theory by Means of Factor Analysis or
Tom Swift and His Electric Factor Analysis Machine

J. Scott Armstrong

Abstract

Problems in the use of factor analysis for deriving theory are illustrated by means of an example in which the underlying factors are known. The actual underlying model is simple and it provides a perfect explanation of the data. While the factor analysis "explains" a large proportion of the total variance, it fails to identify the known factors in the model, The illustration is used to emphasize that factor analysis, by itself, may be misleading as far as the development of theory is concerned. The use of a comprehensive, and explicit à priori analysis is proposed so that there will be independent criteria for the evaluation of the factor analytic results.

It has not been uncommon for social scientists to draw upon analogies from the physical sciences in their discussions of scientific methods. They look with envy at some of the mathematical advances ,in the physical sciences and one gets the impression that the social sciences are currently on the verge of some major mathematical advances. Perhaps they are – but there are many social scientists who would disagree. Their position is that we really don't know enough about what goes into our mathematical models in order to expect results that are meaningfully-related to anything in the "real world." In other words, the complaint is not that the models are no good or that they don't really give us optimum results; rather it is that the assumptions on which the model is based do not provide a realistic representation of the world as it exists. And it is in this area where the social sciences differ from the physical sciences.

But now, thanks to recent advances in computer technology, and to refinements in mathematics, social scientists can analyze masses of data and determine just what the world is like. Armchair theorizing has lost some of its respectability. The computer provides us with objective results.

Despite the above advances, there is still a great deal of controversy over the relevant roles of theorizing and of empirical analysis. We should note that the problem extends beyond one of scientific methodology; it is also an emotional problem with scientists. There is probably no one reading this paper who is not aware of the proper relationship between theorizing and empirical analysis. On the other hand, we all know of *others* who do not understand the problem. We are willing to label others as either theorists or empiricists; and we note that these people argue over the relative merits of each approach.

It may be useful at this point to describe these mythical people. The theorist is a person who spends a great deal of time in reading and contemplation. He then experiences certain revelations or conceptual breakthroughs from which his theory is published. When others fail to validate his theory (that is, to demonstrate its usefulness) the problems are nearly always said to be due to improper specification or measurement.

The empiricist is a person who spends a great deal of time collecting data and talking to computers. Eventually he uncovers relationships that are significant at the 5% level and he publishes his findings. If he is very careful and reports only "what the data say," he will not even have to defend himself when the other 99 people in his line of work read his study.

While it would appear that the relationships between the theorist and the empiricist should be complementary, this is not always evident from the literature which is published. Everyone knows that theorists have existed (and probably much more comfortably) without empiricists; and one now gets the impression that the empiricist feels little need for the theorist. The data speak for themselves. There is no need for a predetermined

theory because the theory will be drawn directly from the data. An examination of the literature reveals many studies which seem to fit this category. For example, Cattell (1949) has attempted to discover primary dimensions of culture by obtaining data on 72 variables for each of 69 national cultures. The 12 basic factors which were obtained seemed to me to be rather mysterious. They included factors such as cultural assertion, enlightened affluence, thoughtful industriousness, bourgeois philistinism, and cultural disintegration.

Problem

I would now like to draw upon an analogy in the physical sciences[1] in order to indicate how science might have advanced if only computers had been invented earlier. More specifically, we'll assume that computer techniques have advanced to the stage where sophisticated data analysis can be carried out rather inexpensively. Our hero will be an empiricist.

Tom Swift is an operations researcher who has recently been hired by the American Metals Company. Some new metals have been discovered. They have been shipped to the American Metals Company and now sit in the basement. AMC is unfamiliar with the characteristics of these metals and it was Tom's job to obtain a short but comprehensive classification scheme.

Tom hadn't read the literature in geometry, in metallurgy, or in economics, but he did know something about factor analysis. He also had a large staff.

In fact, all of the 63 objects were solid metallic right-angled parallelepipeds of varying sizes – which is to say, they looked like rectangular boxes.

Tom instructed his staff to obtain measurements on all relevant dimensions. After some careful observations. the staff decided that the following measures would provide a rather complete description of the objects:

(a) thickness	(g) total surface area
(b) width	(h) cross-sectional area
(c) length	(i) total edge length
(d) volume	(j) length of internal diagonal
(e) density	(k) cost per pound
(f) weight	

Each of the above measurements was obtained independently (e.g., volume was measured in terms of cubic feet of water displaced when the object was immersed in a tub.)[2]

Being assured that the measurements were accurate.[3] Tom then proceeded to analyze the data in order to determine the basic underlying dimensions. He reasoned that factor analysis was the proper way to approach the

---

[1] The idea of using data from physical objects is not new. Demonstration analyses have been performed on boxes, bottles, geometric figures, cup; of coffee and balls. Overall (1964) provides a bibliography on this literature. The primary concern in these papers has been to determine which measurement models provide the most adequate description.

[2] Actually, the data for length. width and thickness were determined from the following arbitrary rules:

(a) Random integers from 1 to 4 were selected to represent width and thickness with the additional provision that the width ≥ thickness.
(b) A random integer from 1 to 6 was selected to represent length with the provision that length ≥width.
(c) A number of the additional variables are merely obvious combinations of length, width, and thickness.

The physical characteristics of the metals were derived from the *Handbook of Chemistry and Physics*. Nine different metals were used (aluminum, steel, lead, magnesium, gold. copper, silver, tin, and zinc.) Seven parallellepipeds of each type of metal were created.

problem since he was interested in reducing the number of descriptive measures from his original set of 11 and he also suspected that there was a great deal of multicollinearity in the original data. The California Biomedical 03M program was used to obtain a principal components solution. The procedure conformed with the following conventions:

(a) Only factors having eigenvalues greater than 1.0 were used. (This yielded three factors which summarized 90% of the information contained in the original 11 variables.)

(b) An orthogonal rotation was performed. This was done since Swift believed that basic underlying factors are statistically independent of one another.

(c) The factors were interpreted by trying to minimize the overlap of variable loadings on each factor. (The decision rule to use only those variables with a loading greater than 0.70 utilized all 11 variable. with no overlap in the 3 factor rotation.)

Principal components was used since this is the recommended factor analytic method when one is interested in generating hypotheses from a set of data.

Results

The factor loadings are shown in Table 1.

Table 1. Three Factor Results

Factor I			Factor II	
Variable	Loading		Variable	Loading
(a) Thickness	-.94		(e) Density	.96
(d) Volume	-.93		(k) Cost/lb.	.92
(g) Surface area	-.86			
(b) Width	-.74			
(f) Weight	-.72			
(i) Edge length	-.70			

Factor III	
Variable	Loading
(c) Length	-.95
(j) D. length	-.88
(h) C. S. area	-.74

Tom had a great deal of difficulty in interpreting the factors. Factor II was clearly a measure of the intensity of the metal. Factor III appeared to be a measure of shortness. But Factor I was only loosely identified as a measure of compactness.

To summarize then, the three basic underlying factors of intensity, shortness, and compactness summarize over 90% of the variance found in the original 11 variables. Tom felt that this finding would assist him in some of his coming projects – one of which was to determine just how the total cost of each of the metallic objects was derived. In other words, he could develop a regression model with Total Cost as the dependent variable and the three basic factors as his independent variables.

---

[3] Another variation would have been to trace the development of the science by having the data be collected first in ordinal form. Then another researcher skilled in the latest measurement techniques would come along, recognize the failure of the first study as a "measurement problem" – obtain interval data – then replicate the study.

Let us step back now and analyze what contribution Tom Swift has made to science. Those people who have read the literature in metallurgy. Geometry, and economics will recognize that, in the initial study, all of the information is contained in five of the original 11 variables – namely length, width, height, density, and cost per pound. The remaining six variables are merely built up from the five "underlying factors" by additions and multiplications. Since a rather simple model will give a perfect explanation, it is difficult to get excited about a factor analytic model which "explains" 90.7% of the total information.

The factor analysis was unable to uncover the basic dimensions. It determined that there were three rather than five basic factors. And the interpretation of these factors was not easy. In fact, one suspects that, had the field followed alone the lines advocated by Swift (by measuring intensity, shortness, and compactness), progress would have been much slower! The Swift study could easily mislead other researchers.

As one other example of how the researchers could be misled we can take the following. Both volume and surface area load heavily on Factor 1. We could go back to the original matrix and find that the correlation coefficient between surface area and volume is .969. We conclude that, allowing for some measurement error, these variables are really measuring the same thing and we are just as well off if we know either one of them as when we know both. This statement is, of course, a good approximation to this set of data. But if we tried to go beyond our data it is easy to see where the reasoning breaks down. That is. one can construct a *very* thin right-angled parallelepiped with surface area equal to that of a cube but with volume *much* smaller.

If Mr. Swift had not followed his original "rules" he might have done a little better. Let us say that he dropped the rule that the eigenvalues must be greater than 1.0. The fourth factor has an eigenvalue of .55; the fifth is .27 and the sixth is .09. He then rotates four factors, then five, etc. In this case the rotation of five factors showed that he had gone too far as none of the variables achieved a high loading (.70) on the fifth factor.

The four factor rotation is interesting. however. Thus is shown in Table 2.

Table 2. Four Factor Results

Factor I			Factor II			Factor III			Factor IV	
Variable	Loading		Variable	Loading		Variable	Loading		Variable	Loading
(a) Thickness	-.96		(e) Density	.96		(c) Length	-.99		(b) Width	-.90
(b) Volume	-.85		(k) Cost/lb	.93		(j) D. Length	-.84		(h) C.S. Area	-.72
(g) Surface area	-.73									
(f) Weight	-.71									

Swift's solution includes all variables except total edge length and there is no overlap (in the sense that one variable loads heavily on more than one factor). The fact that edge length is not included seems reasonable since it is merely the total of the length + width + thickness factors (and multiplied by the constant 4, of course).

The rotation of four factors appears to be very reasonable to us – since we know the theory. It is not clear, however, that Swift would prefer this rotation since he had no prior theory. Factor II once again comes through as intensity. Factors I, III, and IV may conceivably be named as thickness, length, and width factors. The factors still do not distinguish between density and cost per pound, however.

An Extension

Not being content with his findings, Swift called upon his staff for a more thorough study. As a result, the original set of 11 variables was extended to include:

(l)   average tensile strength
(m) hardness (Mohs scale)
(n)   molting point
(o)   resistivity

(p)   reflectivity
(q)   boiling point
(r)   specific heat at 20ºC
(s)   Young's modulus

The results of this principal components study are shown in Table 3.

Table 3. Five Factor Results

Factor I		Factor II		Factor III		Factor IV		Factor V	
Variable	Loading	Variable	Loading	Variable	Loading	Variable	Loading	Variable	Loading
(d) Volume	-.98	(l) T. strength	-.97	(e) Density	.96	(o) Resist.	-.93	(c) Length	-.92
(g) Surf. area	-.95	(s) Y. modulus	-.93	(r) Sp. heat	-.88	(p) Reflect.	.91	(j) D. Length	-.76
(a) Thickness	-.92	(m) Hardness	-.93	(t) Mol. wt.	.87				
(l) E. Length	-.82	(n) Melt pt.	-.91	(k) Cost/lb	.71				
(b) Width	-,80	(q) Boil pt.	-.70						
(f) Weight	-.76								
(h) C.S. area	-.74								

Five factors explain almost 90% of the total variance. Swift, with much difficulty, identified the factors Impressiveness, Cohesiveness, Intensity, Transference, and Length (reading from I to V respectively). There seem to be strange bedfellows within some of the factors. It is difficult to imagine how work in the field would proceed from this point.

Discussion

There are, of course, many other variations that Swift could have tried. Mostly these variations would be derived by using different communality estimates, obtaining different numbers of factors, making transformations of the original data, and experimenting with both orthogonal and oblique rotations. The point is, however, that without a prespecified theory Swift has no way to evaluate his results.

The factor analysis might have been useful in evaluating theory. For example, if one of the theorists had developed a theory that length, width, thickness, density, and cost-per-pound are all basic independent factors, then the four factor rotation above would seem to be somewhat consistent with the theory. Assuming that one was not able to experiment but just had to take the data as they came, this approach does not seem unreasonable.

If one does use the factor analytic approach, it would seem necessary to draw on existing theory and previous research as much as possible. That is to say, the researcher should make *prior* evaluations of such things as:

(a) What type of relationships exist among the variables? This should lead to a prior specification as to what transformations are reasonable in order to satisfy the fundamental assumptions that the observed variables are linear functions of the factor scores and also that the observed variables are not causally related to one another. Note that, in the example given above, the variables did not come from a linear model. One can hardly expect all of the variables in the real world to relate to each other in a linear fashion.

(b) How many factors are expected to show up in the solution?

(c) What types of factors are expected? The analyst should outline his conceptual model in sufficient detail so that he can make à priori statements about what combinations are reasonable and what combinations are unreasonable. In operational terms. the analyst should be in a position, to formulate indices on the basis of his theory before he examines the data.

(d) What set of variables should be considered in the original rata? Is each variable logically consistent with the theory?

(e) What relationships are expected to exist between the resulting factors? (e.g. should we expect them to be orthogonal?)

(f) What are the most meaningful communality estimates for the problem? (The choice here will influence the number of factors which are obtained).

Tom Swift's work would have been much more valuable if he had specified a conceptual model. He would have been able to present a more convincing argument for his resulting theory had it agreed with his prior model. Such agreement is evidence of construct validity. In addition, the model might have led to further testing (e.g., through the use of other sets of data or by means of other analytic techniques).

I would not like to argue that all factor analytic studies fall into the same category as the Swift study. On the other hand, there is a large number of published studies which do seem to fit the category. In these studies, where the data stand alone and speak for themselves, my impression is that it would be better had the studies never been published. The conclusion that "this factor analytic study has provided a useful framework for further research" may not only be unsupported – it may also be misleading.

Summary

The cost of doing factor analytic studies has dropped substantially in recent years. In contrast with earlier time,. it is now much easier to perform the factor analysis than to decide what you want to factor analyze. It is not clear that the resulting proliferation of the literature will lead us to the development of better theories.

Factor analysis may provide a means of evaluating theory or of suggesting revisions in theory. This requires, however, that the theory be explicitly specified prior to the analysis of the data. Otherwise, there will be insufficient criteria for the evaluation of the results. If principal components is used for generating hypotheses without an explicit a priori analysis. the world will soon be overrun by hypotheses.

References

Cattell, R. B. (1949), :The dimensions of culture patterns by factorization of national characters," *Journal of Abnormal and Social Psychology*, 44, 443-469.

Overall, J, (1964), "Note on the scientific status of factors," *Psychological Bulletin,* 61 (4), 270-276.