Heterogeneous Students, Impartial Teaching and Optimal Allocation of Teaching Methods.

Carmen Zita Lamagna[1]

Vice Chancellor

American International University-Bangladesh

Sheikh Tareq Selim

Research Student

University of Southampton

Abstract:

This paper addresses the issue of identifying optimal mix of teaching methods for an instructor when students are of heterogeneous types. The exact student type cannot be identified *ex ante* which forces the instructor to act impartially and allocate teaching methods according to some pre-designed plan. In a simple model of instructor-student interaction, we show that if the instructor acts benevolent and impartially towards preparing the initial teaching method plan, there exists a unique optimal mix of teaching methods. We calibrate the impartial teaching model with data on the teaching of Business and Economics related undergraduate and postgraduate units, and find that the characterized optimal teaching method mix differs significantly across different units.

Keywords: Active teaching, Passive teaching, Impartial teaching.

JEL Classification Codes: *A20, B41, C72.*

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1.0 Introduction:

Ever since the notion of flexible learning has become trendy in university teaching and learning policy, it has allowed a wider choice of teaching methods and thereby posed a demanding challenge to instructors. A policy shift to flexible learning entails a switch of emphasis from authoritarian to the more democratic style of teaching, which otherwise resembles to the stylized transition from *teacher-centred* to *student-centred* learning. While there may be sceptics offering instantaneous arguments and evidences for and against the flexible learning method, its implementation and development has been rapid in most undergraduate programs across the global academia. This stimulates the present study, which addresses the issue of identifying an optimal mix of alternative teaching methods for university teaching and learning.

The marker 'flexible' is generally applied to the current practice of university teaching and learning because it allows a range of options for the students to learn outside the classroom. A rather complete survey of flexible learning methods can be found in Biggs (1999). A few examples may be web based learning, interactive workshops & discussions, study group discussions, debates etc., and by now there is an extensive research initiative amongst proponents of such methods that attempts to innovate *newer* effective learning devices. While relatively new universities are adopting these techniques more enthusiastically, there are evidences of introducing more activity-based and interactive workshops/tutorials, web-based learning tools such as games, simulations, on-line tutorials, discussion forums etc. in the once traditional and veteran type universities.

One of the important consequences of this shift towards flexible learning is that it in essence poses a challenge for instructors because it effectively generates a greater choice of teaching methods for the instructors. While it is a standard practice in most universities that teaching plans or unit descriptions are prepared and announced at the beginning of the semester (*ex ante*), an instructor with a wider choice of teaching methods at her disposal must also decide the optimal allocation of different teaching methods as part of the unit description. This issue in a standard instructor-student non-cooperative interaction model was proposed earlier by Guest (2001), while quite similar studies by Becker (1982), Correa & Gruver (1987), Oosterbeek (1995), Epstein & Spiegel (1996) and Bacdayan (1997) address the issue of optimal allocation of student and faculty time in maximizing learning outcomes. The modelling approach of the current paper can otherwise be compared to that of Guest (2001), since to our knowledge that is the only relevant study established in literature which presents a robust framework of instructor-student non-cooperative interaction. Guest (2001) derives the optimal teaching method mix as the solution to a non-cooperative game between the instructor and students. While Guest's (2001) modelling approach is more general, we consider a rather specific case where student type heterogeneity allows for varying student responses to alternative teaching methods, and students' achievement function possesses significant interaction effects of teaching methods and time devoted to study. In a purely analytical framework, these deviations from Guest (2001) may be considered unimportant in deriving the underlying principles. However, in a simple calibration we find that these empirically justified modifications can be instrumental in deriving consistent and reliable results.

In choosing the optimal allocation or mix of teaching methods, an instructor faces, among others, three main types of constraints. The first is the typical time constraint, since activity-based or the so-called student-oriented teaching method essentially requires more time per unit for preparation than its counterpart (traditional content delivery in lectures). This is tantamount to saying that the instructor faces different time cost weights associated with different available teaching methods (see for details, Biggs (1999), and Martin (1999)). The second constraint of student type heterogeneity is quite remarkably realistic in university (or any tertiary) level teaching. The fact that university students differ in type can be held as a working assumption with the realization of their ultimate career plans or underlying reasons for admission to the program. In this sense, a standard practice in relevant studies is to assume the two types of students to be academic and non-academic. We find this formulation reasonable, detailed and broad enough to generalize all possible student types. We leave the details of this heterogeneity for section 2.0. However, in assuming student type heterogeneity, we also assume that student type is irreversible, i.e. an *ex ante* type *j* student remains type *j ex post*. This strong assumption abstracts from the possibility that students graduate and thereafter re-realizes their true type. For our purpose of modelling optimal teaching method mix, this assumption is fairly innocuous, since a representative student's *ex post* decision does not affect the optimal teaching method mix of an instructor.

The third constraint, which can be considered as a result of the second, is that the instructor must remain impartial, or more technically, unbiased, in designing her teaching method mix *ex ante*. This in other words, implies that while choosing the mix of teaching methods, the instructor cannot bias her decision towards a particular type of students. This constraint is automatically satisfied if we assume that student type is not identified by the instructor *ex ante*, and therefore the instructor's objective is to maximize the un-weighted sum of student achievements. With these three constraints, and given the assumption that all students independent of type maximize utility derived from learning achievement and leisure, the optimal teaching method mix problem for an instructor becomes a simple programming problem. Solution to this programming problem produces the corresponding reaction functions, and simultaneous solution of these reaction functions gives the optimal allocation of teaching methods for the instructor. With the notion of non-cooperation between the instructor and students, and with the induced impartial teaching method design, the problem of optimal teaching method mix becomes algebraically more complicated and as a consequence analytical results lacks straightforward explanations. In order to characterize the robustness and usefulness of these analytical results, a calibration of the model becomes necessary.

This is exactly where this paper is intended to contribute and is likely to seize attention of its audience. Our main motivation is to present some strong empirical results derived from calibration of this seemingly useful analytical model, and hence list the optimal mix of teaching methods as benchmarks for a range of units typically taught in a university degree program in Business Studies. Our choice of Business Studies for this particular experiment is in no way arbitrary. The flexible learning method has been increasingly popular in units related to Business Studies, and therefore its practice and exploitation has been rather widespread in the Business faculty. The model is however quite flexible, and can conveniently be applied to more or less any university degree program. For the purpose of this paper, we first propose a simple instructor-student model with two alternative teaching methods at the instructor's disposal and two types of students which remain unidentified *ex ante*, and formulate the maximization problems of the student and the instructor. The solution to these problems leads to a non-cooperative game between the instructor and the students, and its corresponding equilibrium deduces the optimal teaching method mix for the instructor. The label *optimal* here refers to the particular teaching method mix that generates maximum welfare for both the instructor and the students, subject to the set of constraint each class faces. We use a relatively large survey data of School of Business of the American International University-Bangladesh (AIUB, hereafter) to deduce the necessary parameters for calibrating the model. Calibration of the model provides us the ultimate result of optimal mix of teaching methods for a range of units taught. The instructed survey is conducted by the Office of Research & Publication (ORP, hereafter) of AIUB.

The remainder of the paper is organized as follows. Section 2.0 illustrates the student type heterogeneity we consider. Section 3.0 presents details of the alternative teaching methods and their particular characteristics that we consider in our model. Section 4.0 presents the analytical model. Section 5.0 derives the analytical solutions from the programming problems. Section 6.0 presents the calibration and insightful results from the model, and section 7.0 concludes. All data are collected from a survey of School of Business of AIUB, and is available on request.

2.0 Student Type Heterogeneity:

We assume that students can be either 'academic' or 'non-academic' but not both. Hereafter, we use the subscript j for $j = 1, 2$ to denote academic type students and non-academic type students, respectively. In defining university student types, these terms and their meanings are due to Biggs (1999). The student type heterogeneity proposition developed by Biggs (1999) therefore acts as the benchmark classification of student types in this paper. The academic student is assumed to have (or more likely to have) sound background knowledge, and is highly motivated to search for meanings, explanations and clarifications. These are the students who join university in order to learn, practice and disseminate knowledge, and thereafter pursue higher education and obtain research based higher degrees in future. The non-academic student is motivated only by the desire to pass in order to obtain a qualification for a particular targeted job or career. These students possess (or more likely to possess) less relevant background knowledge, and their key motivation in joining university education is to obtain a diploma with a targeted result in order to qualify for a job. Their practice of learning in university is therefore solely result/grade oriented with less emphasis on explanations, meanings and further horizons of particular concepts taught.

In constructing the model (in section 4.0), we assume that students are aware of their types but the instructor is not. At the beginning of each semester, an instructor cannot identify the exact proportion of academic (or non-academic) students registered in the unit. Implicitly, therefore, we assume that student types are private information to students, and revealing student type is costly for a student. From a static point of view this is quite reasonable to assume so, since revealed student types may lead to biased teaching which is never desired by students. If an instructor cannot identify student types *ex ante*, she must maximize un-weighted sum of student achievements, which forces her to act impartially. We acknowledge the sketchy evidence that not all instructors possess utility functions where utility is derived from un-weighted sum of student achievements. Some instructors spends more time ensuring that almost every person in the class is made convinced with a particular point made during lectures, while some others become frustrated with slow learners and thereby allows the fast learners to set the pace.

Such possibilities are explored analytically using alternative utility functions for the instructor by Guest (2001). These utility functions places specific weights on achievements of particular type of students. We argue that such characterization of instructor's utility implicitly assumes that the instructor has knowledge of student types *ex ante*, and therefore is not forced to act impartially[2]. The assumption that student types are private information to students and revealing type is costly is therefore a strong working assumption of this paper, which restricts all teaching to be strictly impartial.

While the student type heterogeneity proposed in this paper stands simple, it is quite logical and empirically marked. In a survey from the School of Business of AIUB, we find that out of a total 688 respondents, approximately 42% students were reported to have the academic type. This was determined by two survey questions (from a comprehensive subset of many for the complete survey) regarding their preferred career plan after graduation and their principle reason behind joining the undergraduate program. A note on the survey conducted deserves attention. We conducted a comprehensive student and instructor survey on selected questions and generated the dataset required for calibration of the model. Completed survey questionnaires from a total of 722 undergraduate Business Studies students of AIUB and a total of 12 faculty members of the School of Business, AIUB, were collected during Fall 2004 academic semester. The data collected were for the three semesters starting from Fall 2003, hence the final dataset was a panel of three academic semesters. We screened out incomplete and/or unreasonable survey responses, all of which came from students. The final database was constructed using 688 student respondents and 12 instructors.

3.0 Teaching Methods:

We assume flexible learning for students, and hence assume that the instructor can choose a mix of teaching methods from a set of two available modules. Once again we follow Biggs (1999) and define the two available teaching methods on a

[2] This is justifiable if one considers the fact that the instructor herself was a student once, and therefore is of any one type (academic or non-academic). We cannot find any strong reason to believe that all instructors are academic type. Hence if the instructor identifies student types at the beginning of the semester, she is more likely to be biased towards any one type. We leave the characterization of this bias mechanism unattended, since it is not our main focus. Nevertheless, the idea that the instructor acts impartially in designing teaching methods since she cannot identify student types, is maintained throughout, which is tantamount to saying that attaching any weights to student achievement would imply the instructor is biased towards a particular type.

continuous scale as 'active' and 'passive'. This generality is quite popular in literature concerning allocation of student time or teaching method, and apart from Biggs (1999) may be found (either implicitly or explicitly) in Correa & Gruver (1987), Prosser & Trigwell (1999), Martin (1999) and Guest (2001). The terms active and passive refer to the level of student activity and involvement encouraged by the teaching method. An active teaching method requires more student participation and activity and hence is more student-oriented in type. Problem solving, group discussions, forums, web-based learning, debates etc. innovative teaching modules fall under this category. Passive teaching methods require less student activity and engagement, and the trivial example of such teaching method is traditional lecture. Hereafter, we use subscript *i* with *i* = *a, p,* to denote active and passive teaching methods, respectively.

We further assume that the learning objectives in a particular unit are divided into a large number of standard learning topics, and while teaching the instructor attaches equal weights (or importance) to each topic. Over the course of the semester, these topics can be addressed with either active or passive teaching methods. Let Γ_i denote the number of standard learning topics in a particular unit addressed with teaching method *i* with *i* = *a, p*. We define the teaching method mix for a particular unit, (Γ_a, Γ_p), as the number of learning topics in the unit addressed through active and passive methods, respectively. Each Γ_i is associated with a time cost $c_i > 0$ with $c_a > c_p$. This assumption is consistent with Guest (2001) which states that active teaching methods are strictly more time consuming than passive ones. It is also understandable from a realistic point of view. Active teaching methods are typically dynamic in nature and require continuous process of innovation by an instructor. It is sensible to think that an instructor will spend more time in developing such modules involving simulations, case studies, debates etc. The point is reinforced by Biggs (1999) with the intuitively appealing link between teaching methods and student type heterogeneity. Biggs (1999) illustrates that both non-academic and academic students achieve higher quality learning outcomes under active teaching methods than under passive methods, but that non-academic students stand to gain more in improved quality of learning from more active teaching methods than do academic students. For an instructor, therefore, it is always optimal to follow only active teaching methods if $c_a = c_p$, since it will maximize the joint quality of learning outcomes for all students.

From the survey on AIUB Business School we find that the evidence on active and passive teaching methods and their relative proportions in use is empirically stimulating. The following table 1.1 reveals the actual subset (Γ_a, Γ_p) which was found in the survey conducted on School of Business of AIUB.

Table 1.1: Actual active and passive teaching topics and hours allocated by AIUB Business School Faculty during 2003-2004 academic year.

Unit	Active Teaching Topics (Total topics)	Passive Teaching Topics (Total topics)	AT hrs/ PT hrs
Strategic Management	*4 (10)*	*6 (10)*	*0.83*
Accounting 1	*6 (8)*	*4 (8)*	*1.5*
Accounting 2	*8 (12)*	*5 (12)*	*0.80*
Money & Banking	*2 (8)*	*8 (8)*	*0.07*
Consumer Behaviour	*5 (10)*	*5 (10)*	*1.15*
Organizational Behaviour	*6 (15)*	*10 (15)*	*1*
Entrepreneurship	*6 (8)*	*2 (8)*	*5*
Principles of Management	*6 (8)*	*3 (8)*	*2.11*
Business Mathematics 1	*7 (7)*	*7 (7)*	*0.27*
Global Marketing	*7 (8)*	*1 (8)*	*5*
Marketing Communication	*6 (8)*	*2 (8)*	*2.25*
Microeconomics	*3 (10)*	*10 (10)*	*0.27*
Pricing	*3 (8)*	*8 (8)*	*0.27*
Retailing	*4 (10)*	*10 (10)*	*0.34*
Sales Management	*10 (10)*	*10 (10)*	*1*
Business Communication	*6 (14)*	*8 (14)*	*0.75*
Project Management	*3 (13)*	*10 (13)*	*0.55*
Operations Management	*4 (14)*	*10 (14)*	*0.90*

Source: ORP, AIUB survey November 2004.

Table 1.1 in its second and third columns presents data for number of teaching topics, or standard learning topics covered using active and passive teaching methods, during the three academic semesters under consideration, with total topics covered for that unit in parentheses. The last column presents the ratio of active and passive teaching hours for a particular unit during the sampling period. The instructor database was created using the survey report on 12 Business Studies faculty members who have established reputation in teaching (the best performing 12 faculty members, assessment based on teaching evaluation reports), and have been consistently teaching these units during the sample period. Total contact hours (including standard lecture hours) of these instructors are over 100 hours during a regular academic semester. Except for Principles of Management, rest of the classes had a passing rate of 85% or over on an average. The minimum class size of all units was recorded for the unit titled Money & Banking, which was well below the average of 35. The standard learning topics taught using active and passive methods may be overlapping (and hence is difficult to distinguish, since often the same topic is taught using a mix). The last column therefore adds the advantage of correct interpretation of the relative weight attached to different teaching methods by individual instructors.

4.0 An Instructor-Student Interaction Model:

In this section we propose a simple static model of instructor-student non-cooperative interaction. The environment we consider is one where at the beginning of each semester a representative instructor assigned in designing a particular unit is unaware of student type, and allocates (Γ_a, Γ_p) as the number of learning topics in the unit to be addressed through active and passive methods, respectively. The university policy is such that this announcement of teaching method mix is binding, i.e. the initially announced (Γ_a, Γ_p) is maintained throughout the semester. The instructor is impartial and benevolent, i.e. her objective is to maximize an un-weighted sum of students' achievements. The quality of learning outcomes for a representative student can be measured by an index of student achievement for a particular unit. Hence it is implicitly assumed that the assessment methods used precisely measure the quality of student achievement by rewarding superior quality learning with a proportionately higher achievement score. The two representative students of type *j, j = 1, 2,* have similar preferences over student achievement and leisure, but different abilities in generating learning outcomes. Total time endowment per individual is normalized to one. Students are allowed to freely allocate their total time endowment between learning and leisure. The representative student *j* maximizes utility defined as a function of her *j* specific academic achievement and leisure.

We assume that student *j*'s academic achievement in a particular unit is a strictly concave function of teaching method mix (Γ_a, Γ_p) and student *j*'s time allocation (t_j) for that particular unit. Considering the possible interaction effects amongst the two teaching methods and student *j*'s time allocation in generating learning outcomes, we assume that the student *j*'s achievement frontier is one of Translog form, as follows:

$$\ln \mathrm{H}_j = \phi_0 + \sum\nolimits_k \phi_k \ln \mathrm{G}_k + \sum\nolimits_k \phi_{kk} (\ln \mathrm{G}_k)^2 + \sum\nolimits_s \sum\nolimits_m \phi_{sm} \ln(\mathrm{G}_s) \ln(\mathrm{G}_m) \qquad (1)$$

Where H_j is student *j*'s learning achievement, and G is a vector of inputs *k*, with *k = 1, 2, 3,* which are considered to be teaching methods Γ_i with *i = a, p*, and student *j*'s allocation of learning time (t_j), respectively. All parameters are *j* specific, but we omit subscript *j* associated with each ϕ to avoid notational cluttering. The parameters ϕ_{sm} represent the interaction effects amongst the three arguments of the achievement frontier, where $s \neq m$ and

$$m = \begin{cases} 2,3 & \mathit{for} \quad s = 1 \\ 3 & \mathit{for} \quad s = 2 \end{cases}$$

It is straightforward to show that the traditional Cobb-Douglas representation of achievement frontier *(1)* is:

$$\mathbf{H}_j = \xi_j (\Gamma_a)^{\alpha_j} (\Gamma_p)^{\delta_j} (t_j)^{\beta_j} \qquad (2)$$

Where the technical parameters (or elasticity) of student achievement are determined by:

$$\alpha_j = \phi_{1j} + \phi_{11j} \ln(\Gamma_a) \qquad (2.1)$$

$$\delta_j = \phi_{2j} + \phi_{12j} \ln(\Gamma_a) + \phi_{22j} \ln(\Gamma_p) \qquad (2.2)$$

$$\beta_j = \phi_{3j} + \phi_{13j}\,\mathbf{ln}(\Gamma_a) + \phi_{23j}\,\mathbf{ln}(\Gamma_p) + \phi_{33j}\,\mathbf{ln}(t_j) \qquad (2.3)$$

There is no explicit restriction on the sum of parameter values, such that we keep the possibility of increasing returns to scale in student achievement function open. Nevertheless the restriction $[\alpha_j, \delta_j, \beta_j] \in (0,1)$ holds to ensure that *(2)* is jointly strictly concave. Recall that academic students possess sound background of knowledge and hence would necessarily possess higher total factor productivity in achievement, i.e. $\xi_1 > \xi_2$. Since both types of students attain higher quality learning outcomes from active teaching methods, but the non-academic students stand to gain more from more active teaching methods than do academic students, $0 < \alpha_1 < \alpha_2 < 1$ must hold. Moreover, the sensible assumption of diminishing marginal utility of time spent in either activity implies $\beta_j < 1$. Since we assume that factors interact to contribute to student achievement, and that such interaction effects are unobserved but significant, it is specification *(1)* which must be used for empirical estimation. We use *(2)* for analytical modelling since it allows convenience in tractability of analytical results.

We assume, for analytical simplicity, that the time elasticity of student achievement is equal to the time elasticity of leisure. The representative student enjoys leisure produced by the following simple function:

$$\mathrm{L}_j = \kappa_j (1 - t_j)^{\beta_j} \qquad (3)$$

Where $\kappa_j > 0$ is a *j*-specific parameter. The total learning time (or class time) for a particular unit is normalized to one, such that with time cost $c_i > 0$ with $c_a > c_p$, the time budget constraint for the instructor is:

$$c_a \Gamma_a + c_p \Gamma_p = 1 \qquad (4)$$

The optimal teaching mix therefore balances the greater time cost of more active teaching methods against the increased student achievements that results. For a representative student of type *j*, the time budget constraint incorporates her time allocation to learning as functions of her achievement and learning methods, and time allocation to leisure. In this sense, we express learning time $t_j = \vartheta_j(\mathrm{H}_j, \Gamma_a, \Gamma_p)$ and leisure time $(1-t_j) = \mu_j(\mathrm{L}_j)$, where ϑ_j and μ_j are inverse functions of *(2)* and *(3)*, respectively. This formulation of the instructor's and student's time budget constraints is standard and can be compared to that proposed by Guest (2001). The representative type *j* student's time budget constraint can be expressed as:

$$\vartheta_j(\mathrm{H}_j, \Gamma_a, \Gamma_p) + \mu_j(L_j) = 1 \qquad (5)$$

We assume that the student's utility function is additively separable in achievement and leisure, and a representative student of type *j* maximizes utility which is a weighted sum of learning achievement and leisure. We consider the following utility function for students:

$$\mathrm{U}_j = \lambda_j \mathrm{H}_j + (1-\lambda_j)\mathrm{L}_j \qquad (6)$$

Where $\lambda_j \in (0,1)$ is the student type specific weight attached to learning achievement. Note that for all $\lambda_j \neq 0.5$, utility function *(6)* implies that learning achievement and leisure are imperfect substitutes of each other. Empirically it is rather ambiguous which type of students would attach relatively higher weight to learning achievements. However, it is sensible to assume that under intense learning situation which is typical in semester based teaching, all students irrespective of type attach relatively higher weight to learning achievement than leisure, such that $\lambda_j > 0.5$ holds. For a representative student of type *j*, the problem therefore is to maximize *(6)* subject to time budget constraint *(5)*.

As mentioned earlier, the instructor is unaware of student types *ex ante*, and hence is forced to act impartially. In this sense the instructor derives utility from an un-weighted sum of student achievements. We define the following simple utility function for the instructor:

$$V = \sum_j H_j \qquad (7)$$

The instructor's problem is to choose the teaching method mix (Γ_a, Γ_p) that maximizes *(7)* subject to the time budget constraint *(4)*. We leave the solutions to these two problems for the next section.

5.0 Equilibrium and Optimal Teaching Mix:

We first consider the representative student's problem. The consolidated necessary condition for an optimum for the representative student of type *j* in general form can be stated as:

$$\frac{U_{jH}}{U_{jL}} = \frac{\vartheta_{jH}}{\mu_{jL}} \qquad \forall j \qquad (8.1)$$

Where X_Y denotes the partial derivative of function X (Y, Z) with respect to Y. The necessary condition *(8.1)* states that the representative student maximizes utility at the point where the ratio of the marginal utilities of achievement and leisure equates the ratio of the time costs per unit of achievement and leisure. With $\vartheta_j = H_j^{\frac{1}{\beta}} \xi_j^{-\frac{1}{\beta}} \Gamma_a^{-\frac{\alpha}{\beta}} \Gamma_p^{-\frac{\delta}{\beta}}$, and $\mu_j = L_j^{\frac{1}{\beta}} \kappa_j^{-\frac{1}{\beta}}$ where the parameters α, β, δ are *j*-specific[3], and considering *(6)*, condition *(8.1)* yields:

$$\frac{\lambda_j}{(1-\lambda_j)} = \left(\frac{\xi_j}{\kappa_j}\right)^{-1} \left(\frac{t_j}{1-t_j}\right)^{(1-\beta_j)} \Gamma_a^{-\alpha_j} \Gamma_p^{-\delta_j} \qquad \forall j \qquad (8.2)$$

Which in turn, give the reaction function for the representative student:

[3] Hereafter, this is maintained, i.e. these parameters as superscripts will appear without subscript *j*, although these are *j*-specific. However, in expressions where such parameters of both student types appear jointly, we use the subscripts 1 and 2 to avoid possible confusion.

$$t_j = \left[1 + \left(\frac{\kappa_j}{\xi_j} \right)^{\frac{1}{1-\beta}} \Gamma_a^{-\frac{\alpha}{1-\beta}} \Gamma_p^{-\frac{\delta}{1-\beta}} \left(\frac{1-\lambda_j}{\lambda_j} \right)^{\frac{1}{1-\beta}} \right]^{-1} \qquad \forall j \qquad (8.3)$$

The reaction function *(8.3)* gives the optimal time that each student of type j will spend engaged in learning in terms of j-specific parameter values, relative weight of leisure in utility and the teaching method mix (Γ_a, Γ_p). Hence for unique values of an optimal teaching mix, a unique optimum of the student's maximization problem exists.

Now consider the instructor's maximization problem. Recall that students differ in their achievement function according to type and the instructor faces $c_a > c_p$ by assumption, and hence the trivial necessary condition for an instructor's optimum with $c_a = c_p$ and homogeneous student achievement function is ruled out[4]. With the underlying assumptions, the consolidated necessary condition for an instructor's optimum, in general form is:

$$\frac{c_a}{c_p} = \frac{\sum_j \mathbf{H}_{j\Gamma_a}}{\sum_j \mathbf{H}_{j\Gamma_p}} \qquad \forall j \qquad (9.1)$$

Condition *(9.1)* is fairly intuitive. It states that the instructor's utility is maximum where the ratio of time cost of active and passive teaching methods equals the ratio of un-weighted aggregate marginal achievement of students from additional topics delivered using active and passive teaching methods. Considering the functional form *(2)* proposed earlier, the necessary condition can be restated as:

$$\frac{c_a}{c_p} \left[\frac{\delta_1 \xi_1 \Gamma_a^{\alpha_1} \Gamma_p^{\delta_1 - 1} t_1^{\beta_1} + \delta_2 \xi_2 \Gamma_a^{\alpha_2} \Gamma_p^{\delta_2 - 1} t_2^{\beta_2}}{\alpha_1 \xi_1 \Gamma_a^{\alpha_1 - 1} \Gamma_p^{\delta_1} t_1^{\beta_1} + \alpha_2 \xi_2 \Gamma_a^{\alpha_2 - 1} \Gamma_p^{\delta_2} t_2^{\beta_2}} \right] = 1 \qquad (9.2)$$

[4] The consolidated necessary condition for an instructor optimum with the assumption that $c_a = c_p$ and if students have identical achievement function is simply $\frac{\Gamma_a}{\Gamma_p} = \frac{\alpha}{\delta}$. This is ruled out since we assume students' achievement functions and time costs differ.

Which in turn can be simplified to derive the instructor's reaction function:

$$\frac{\Gamma_a}{\Gamma_p} = \frac{c_p}{c_a}\left[\frac{\delta_1 \xi_1 t_1^{\beta_1} - \delta_2 \xi_2 t_2^{\beta_2}}{\alpha_1 \xi_1 t_1^{\beta_1} - \alpha_2 \xi_2 t_2^{\beta_2}}\right] \qquad (9.3)$$

The instructor's reaction function *(9.3)* defines the instructor's optimal mix of teaching methods in terms of parameter values and the time each student allocates to learning. It balances the greater cost of more active methods against the increased quality of the resulting learning outcomes. Both the reactions functions *(9.3)* and *(8.3)* show that the optimal teaching method mix and the optimal time allocated to learning by each student are jointly determined by the parameters in the *j*-specific achievement function, *j*-specific weight attached to learning achievement in utility (assumed as given) and the relative cost of the teaching methods. Solution to the system comprising the two reaction functions, with the time budget constraint for the instructor to eliminate the cost parameters, provides unique equilibrium solution for optimal mix of teaching methods.

6.0 Calibration and Results:

As mentioned earlier, and as can be seen from the expressions of the reaction functions for the instructor and the students, the analytical results are less useful for interpretation and a calibration of the model is necessary. We conducted a comprehensive survey of students and instructors of School of Business of AIUB, and two panel datasets, one of 688 students for one academic year (comprising three regular semesters) and the other of 12 instructors for the same sample period, were constructed based on the survey responses. We collected information on active and passive teaching hours and topics, and these information were cross checked from both surveys and the *ex ante* submitted course outline of instructors. The course outlines, however, are not extremely strict *ex ante*, since AIUB academics allow its instructors to modify these outlines to certain extent given permission has been obtained from corresponding authority. We found that the students' response on instructors' teaching method allocation and the instructors' response on the same issue are more or less converging, and hence sensibly ignored the possibility of discretionary and unreported changes in teaching method plan in classrooms. We also rule out the possibility of major changes in teaching method mix during one academic year, since doing so would incur high adjustment costs for instructors.

The dataset which is used for estimation of the calibration parameters are primary data from a questionnaire survey, and hence may be subject to scepticism of reliability. Given the current task, this is the most reliable data source, since it is drawn from carefully chosen sample of standard business studies units taught by the rather most efficient instructors of AIUB. We have carefully excluded survey responses of irregularly enrolled students, students in the probation list and students who have not completed all course requirements of the units under consideration.

Data for students' time allocation to studies and leisure entirely relies on truthful responses of students, and it is no way possible to cross check these with established reports. We, however, investigated these responses across students, and across semesters by different students for the same unit. To our advantage these responses are reasonably consistent and hence reliable. We conducted a fixed effects panel estimation of specification *(1)* to derive parameter values, which resulted in $(\alpha_1, \delta_1, \beta_1) = (0.31, 0.36, 0.38)$ and $(\alpha_2, \delta_2, \beta_2) = (0.51, 0.33, 0.27)$, evaluated at means of the explanatory variables. The estimated coefficients from specification *(1)* and the estimation possessed overall statistical significance. We suppress the regression results considering parsimony of the current version of the paper. We acknowledge there may be variants of panel estimation, but in our case estimation with fixed effects was reasonable. We used simple growth accounting approach with the estimated parameters and thereby estimated the two total factor productivity parameters.

The following table 1.2 summarizes the calibration parameters, of which the value of parameter κ_j is due to bootstrapped estimation from specification *(3)*, and the student type specific weight attached to learning achievement is chosen from survey reports of students.

Table 1.2: List of calibration parameters for equilibrium optimal mix of teaching methods.

Parameter symbol	Description	Value
$(\alpha_1, \delta_1, \beta_1)$	Elasticity of academic students' achievement with respect to active teaching, passive teaching and time devoted to learning.	*(0.31,0.36,0.38)*
$(\alpha_2, \delta_2, \beta_2)$	Elasticity of non-academic students' achievement with respect to active teaching, passive teaching and time devoted to learning.	*(0.51,0.33,0.27)*
λ_1	Academic students' weight attached to learning outcome	*0.80*
λ_2	Non-academic students' weight attached to learning outcome	*0.60*
κ_1	Bootstrapped value of parameter in academic students' leisure function	*0.12*
κ_2	Bootstrapped value of parameter in non-academic students' leisure function	*0.34*
ξ_1	Total factor productivity parameter of academic students' learning achievement function	*0.83*
ξ_2	Total factor productivity parameter of non-academic students' learning achievement function	*0.67*

These parameter values were used to calibrate the optimal teaching method allocation for an instructor for individual units taught. The equilibrium optimal teaching method allocation rules are unique, which yields unique numerical values for the number of topics to be optimally taught for a particular unit. This identification is possible since the dataset constructed from the survey can be degenerated into particular units, and is large enough to conduct estimation of unit specific parameters. The table 1.3 presented in this section summarizes the calibrated optimal teaching method mix for the units considered in this particular study.

For the convenience of interpretation in terms of time allocated to different teaching methods, we converted the optimal allocation of teaching topics to two different methods for particular units into time devoted to teaching these topics using two alternative methods. Doing this task requires an approximation of time required to design one teaching topic into an active teaching module (and same for passive teaching module), which may vary by units and subjects. We accomplish this task from the observed time allocations in these units during our sample period. This is also verified by the total number of contact hours of each instructor. Instructors offering more active teaching hours in general, and in most cases in our sample, tend to have higher contact hours due to higher time in preparation. The table 1.3 therefore reports the optimal allocation of unit-specific teaching topics addressed by active and passive teaching methods, and the optimal ratio of active to passive teaching hours, with actual values in parentheses for comparison.

Table 1.3: Optimal teaching method allocations, and optimal ratio of active to passive teaching hours for AIUB Business School.

Unit	Optimal Active Teaching Topics (Actual)	Optimal Passive Teaching Topics (Actual)	Optimal AT hrs/ PT hrs (Actual)
Strategic Management	*5(4)*	*5 (6)*	*1.28(0.83)*
Accounting 1	*6(6)*	*4(4)*	*2.1(1.5)*
Accounting 2	*8(8)*	*5(5)*	*1.56(0.80)*
Money & Banking	*3(2)*	*8(8)*	*0.60(0.07)*
Consumer Behaviour	*5 (5)*	*5 (5)*	*1.08(1.15)*
Organizational Behaviour	*7 (6)*	*9 (10)*	*1.80(1)*
Entrepreneurship	*6 (6)*	*2 (2)*	*3(5)*
Principles of Management	*6 (6)*	*4 (3)*	*2.29(2.11)*
Business Mathematics 1	*7 (7)*	*7 (7)*	*1.50(0.27)*
Global Marketing	*7 (7)*	*2 (1)*	*5.05(5)*
Marketing Communication	*8 (6)*	*4 (2)*	*2.88(2.25)*
Microeconomics	*5 (3)*	*10 (10)*	*1.85(0.27)*
Pricing	*5 (3)*	*8 (8)*	*0.77(0.27)*
Retailing	*6 (4)*	*10 (10)*	*0.85(0.34)*
Sales Management	*10 (10)*	*10 (10)*	*1.81(1)*
Business Communication	*12 (6)*	*8 (8)*	*4.52(0.75)*
Project Management	*8 (3)*	*10 (10)*	*3.88(0.55)*
Operations Management	*8 (4)*	*10 (10)*	*3.56(0.90)*

We leave the analysis of these results for the next concluding section.

7.0 Concluding Remarks:

In order to characterize an optimal mix of teaching methods for units taught with a mixture of two alternatives, namely active and passive teaching methods, we constructed a simple model of student-instructor interaction where students are of heterogeneous types and the instructor acts impartially in designing the *ex ante* teaching plan. We solved the instructors' and type specific students' programming problems and derived optimal response functions for both students are instructors. We computed the equilibrium mix of teaching methods that maximizes students' achievement and benevolent instructors' utility. Due to the indistinctness of the derived analytical solutions, we calibrated the results using a panel data from AIUB's school of Business. The calibration characterized the optimal mix of teaching methods by computing the optimal learning topics to be addressed using active and passive teaching methods. Using the gathered information these optimal learning topics were converted into teaching hours to be spent using active and passive methods.

Results, as can be seen from table 1.3, strongly suggest that there is significant difference between the optimal and actual ratios of active to passive teaching hours, and except for the units titled Consumer Behaviour and Entrepreneurship, there is clear evidence that allocation of teaching time to active teaching methods for all units is sub-optimal. The actual time devotion to active teaching methods for all other units is significantly lower than the optimal that maximizes students' learning achievements. The actual number of topics covered using active teaching methods and actual number of teaching hours devoted to active teaching methods is far below the optimal levels (possibly the worst cases) for Business Communication, Project Management and Operations Management. The two Economics units, Money & Banking and Microeconomics, have allocation of active teaching topics reasonably close to the optimal, although the optimal time that should be devoted to active teaching methods for both these units are eight fold higher than the actual. This could be clearly a signal for academic authorities for comprehensive review of these units' teaching plans. The results, however, are better viewed as a guide to policy formulation. Interesting results are also found for the case of units titled Accounting 1 & 2, Business Mathematics and Sales Management, since for these, although the actual and optimal topics to be covered by alternative teaching methods converge, the optimal time that should be allocated to active teaching method is far above the actual allocated time (converse hold for units titled Consumer Behaviour and Entrepreneurship, where optimal time allocation is lower than the actual, although optimal and actual topics converge).

A possible variation of the modelling technique, as may be suggested by many, is to abstract from the impartial teaching assumption and thereby introduce the idea of instructors' bias in designing *ex ante* teaching plans. The probable *rationale* behind this potential extension may be that instructors design their *ex ante* teaching plans according to some bias which aids their convenience in accomplishing the task, which still maintains the realistic assumption that the instructors are unaware of student type *ex ante*. This extension, or modification, can be accomplished by introducing alternative utility functions; a practice which is attempted in Guest (2001). We abstract from these potential extensions, and leave it for future research. The primary source of the key calibration parameters is fixed effects panel estimation of a Translog achievement frontier, which, like most econometric models and estimation, is subject to possible suggestions of variations in specifications. We humbly acknowledge these allegedly important suggestions, but also hold the view that given the main purpose of the paper the specification used for estimation is robust and effective. For technical clarification, however, we mention that this specification was chosen from a set of probable and commonly used specifications on the basis of standard likelihood ratio test.

To the advantage of our study, we find significant difference of optimal teaching method mix across units, and significant difference between the observed and optimal teaching method mix. We also find that even in the case where actual and optimal numbers of teaching topics for active (and passive) teaching method converge, the optimal time that should be devoted to a type of teaching method may be different from the levels observed. The model and its empirical application, therefore, can conveniently be used in revising or constructing teaching plans for future which would enable instructors to design optimal mix of teaching methods that maximizes the learning outcomes of both types of students.

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