

Gainful trade: **A new economics**

Chapter 1: *The meaning of gainful trade*

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1 The meaning of gainful trade

1.1 Prelude

The verbal statement ‘trade is gainful’ may sound innocent and even seem obviously true. Indeed, the standard textbook presentation of trade theory does assert that trade is gainful. This is done without much discussion, probably because its validity seems self-evident.

However, a verbal statement may not reveal its deeper significance, and its far-reaching and wide-ranging implications. When stated formally, the concept of gainful trade leads to a new economic theory, with many results not expected by prevailing economics. It is verily different from classical and neoclassical economic theory. The familiar pictures either disappear or take totally new appearances, and hitherto unseen pictures emerge.

It would be quite unpleasant and ultimately futile to first discuss and criticize the previous literature in order to justify the new. Instead, it is much easier and more pleasant to just go ahead with the new economics from scratch. It is an exciting journey of discovery. One sees how simple it is to fuse micro and macroeconomics in a unified body of theory. One sees formerly intractable issues becoming plain and simple. This may be one rewarding journey.

1.2 Trade is gainful

The idea that trade is gainful is the starting point for the standard presentation of the theory of trade. Thus suppose that Crusoe lives in an isolated island. He lives under subsistence (autarky), until somehow a relation is established with a neighboring island of Defoe. Crusoe produces and consumes Fruits (F) and Grains (G) before trade.

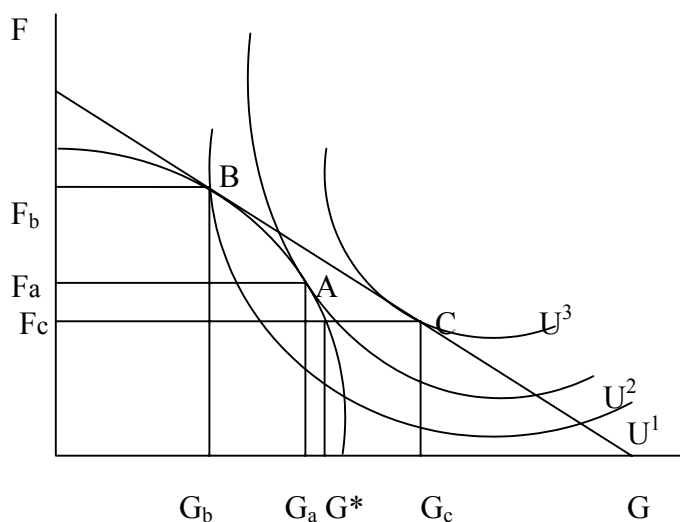


Figure 1: Gains from trade

In Figure 1, Crusoe’s autarkic production and consumption of (F, G) occur at point A at utility level U^1 . But once trade with Defoe is possible, he shifts production to point B but consumption

to point C. While production is still on his production possibility frontier (PPF), his consumption is outside the frontier. At world prices, his budget line allows him to reach the highest possible utility level U^3 , outside his PPF. His gains from trade may be expressed in terms of opportunity cost and price. He exports $F^x = (F_b - F_c)$ and imports $G^m = (G_c - G_b)$. Crusoe's opportunity cost of F^x is $(G^* - G_b)$ under autarky, but he makes a net profit $(G_c - G^*)$ by trade. The gain from trade is positive $[(G_c - G^*) > 0]$

1.2.1 Measurement of gains

It is worth repeating that the gain made by Crusoe from the trade with Defoe has been shown in two ways. First, it is a shift from a lower utility level U^1 to a higher utility level U^3 . Second, it is a net profit measured in units of output $(G_c - G^*)$. To measure the gains from trade, these two ways will help, as shown in Chapter 3 on Value

1.2.2 Formal statement of gains

The idea that trade is gainful may be called Menger Dictum in honor of Carl Menger (1871) who recognized this explicitly. He need not be the first to have done so. The honor is for his subjectivism: the gain must be ultimately understood in terms of human intentions. It is not necessary for the gain to come from increased material output, but it is necessary that the utility be increased.

In the trade between Crusoe and Defoe, let Crusoe sell Fruits (F) and buy Grains (G). Let U denote the level of utility, and a superscript denote the agent. Then Crusoe's gain from trade is shown by:

$$U^{\text{Crusoe}}(F^x) < U^{\text{Crusoe}}(G^m) \quad \{\text{Menger Dictum}\}$$

The dictum just says that Crusoe prefers what he buys (imports) to what he sells (exports). The increase in utility $[U^{\text{Crusoe}}(G^m) - U^{\text{Crusoe}}(F^x)] > 0$ is necessary for trade, but not sufficient. The other necessary part is the 'anomaly' in the original endowment of the goods, such that the individual prefer what he does not possess to what he does. Consider three possible scenarios:

1.2.2.1 Scenario 1: Peaceful coexistence without trade

Crusoe prefers fruits to grains, and already has the fruits. Defoe prefers grains to fruits and already has the grains. Trade is not possible, because there is no available gains. Both remain in subsistence, and co-exist peacefully.

1.2.2.2 Scenario 2: Violent war

Crusoe prefers fruits to grains and has the fruits. Defoe also prefers fruits to grains, but has the grains. Crusoe cannot gain from trade, and indeed must endure a loss. So he refuses to trade. Defoe stands to gain if Crusoe would give the fruits. But as Crusoe refuses, Defoe may undertake plunder. Resistance to plunder imposes a cost, and the rational decision compares the costs and benefits. Trade is preferred to war if the cost of paying is lower than the cost of plundering.

1.2.2.3 Scenario 3: Peaceful trade

Crusoe prefers grains to fruits, but has fruits. Defoe prefers fruits to grains, but has grains. Trade is possible, because it is gainful for both. There is an anomaly in endowments. Without this, the

opposite preferences alone would not lead to trade, as in Scenario 1. Also note that something must be present to ensure the seller's right to get payment.

1.3 Allocation model denies gains!

The main body of microeconomics apparently denies the existence of gainful trade. A core belief in microeconomics is that in equilibrium, the net profit is zero. To dispute this core theorem, one must dispute much in microeconomics. Instead, let us accept the Menger Dictum as an axiom and see where it takes us. One can then look back at previous economics and decide if it is right.

It is possible to resolve the apparent contradiction between the trade model and the core idea of zero equilibrium profit, if a clear distinction is made between allocation by an optimizer and an exchange by an entrepreneur. The issue may be clarified with Figure 2. It is really the same as Figure 1, except for deletion and addition of some curves.

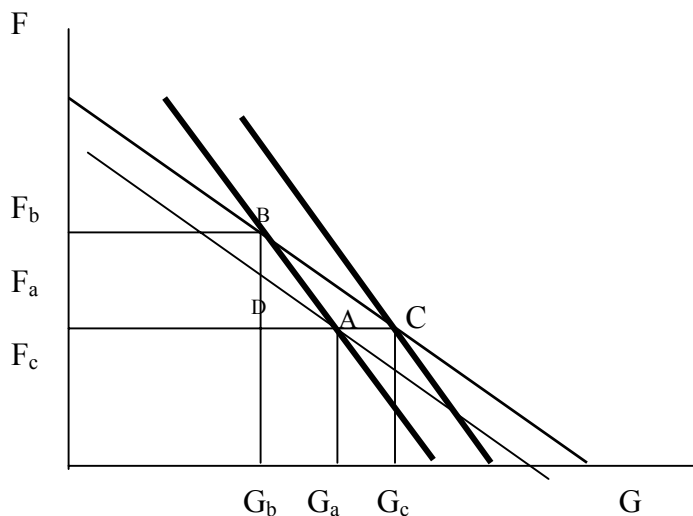


Figure 2: Income effect of trade

Here, points B and A belong to the same PPF. The straight line passing through both B and A can be seen as the domestic resource budget. Crusoe could reallocate resources to produce an extra amount $(G_a - G_b)$. His ability to reach point C at world prices may therefore be said to have a price effect, by making G cheaper abroad than at home. By standard practice, the price effect is split into an income effect and a substitution effect. The income effect shifts the budget line outward to reach point C, giving a net gain of $(G_c - G_a)$. The substitution effect $(G_c - G_b)$ is a shift from a lower utility level at point B to a higher utility level at point C, along the budget line given by the world prices.

While the Slutsky decomposition of a price effect into an income effect and a substitution effect is a standard part of microeconomics, the income effect seems to be promptly forgotten. It also loses sight of entrepreneurship.

This happens by way of imputation of the income effect in the budget constraint. At world prices, the marginal cost of exports F^x is $[(G_c - G_a) + (G_a - G_b)]$ rather than just $(G_a - G_b)$,

because it imputes the net income ($G_c - G_a$) to the factors. A movement along the budget line does not change the income.

The imputation of the pure entrepreneurial income from trade seems to take place without clear awareness. The standard supply function does not specify a fixed factor endowment, but merely fixed factor prices. The world price of course already includes the net income effect. The imputation presumes that the firm can procure the factors from the market at the given factor prices. This effectively removes the factor endowment constraint, because there is nothing to ensure that global factor endowments are not exceeded. It means that the imputation of net income and the presumption of the firm's ability to buy factors hides the gains from trade.

For theoretical progress, it is necessary to recognize the income effect and acknowledge the presence of net profit in equilibrium. It is also necessary to take note of the risk of loss taken by the supplier. Crusoe's optimal production is at point A under autarky, but he moves to point B, which is inferior in terms of utility, in the expectation of successful trade. If for some reason Crusoe is unable to sell the output meant for export, he will suffer a net loss. In a dynamic model, the possibility of net losses in equilibrium adjustment must be acknowledged. The possibility of losses must be an integral part of a proper model of business cycle.

The analytical issue raised by the presence of pure entrepreneurial gain is that this cannot be explained with an allocation model. The net gain has no cost or budget constraint. It is a free bonus from trade. Its analysis must be based on entrepreneurship. That cannot be done with the tool of optimization analysis. There is a need for a new analytical tool.

The challenge for the new analytical tool is to reconcile the balance in the budget with the imbalance in utility between points B and C. The market value of exports F^x must be the same as that of the imports G^m , and yet the imports must yield higher utility than the exports. Thus, let V denote the market value. Hence let $V(F^x) = P_f F^x$ where P_f is price of F . Then it must be true that both of the following relations hold:

$$\begin{array}{ll} V(F^x) = V(G^m) & \{\text{Equivalence}\} \\ U(F^x) < U(G^m) & \{\text{Crusoe's gains}\} \end{array}$$

The literature recognizes the substitution effect of moving along the same budget line, which keeps the budget intact and yet allows the consumer to move to a higher level of utility. Despite the clear recognition that the same budget line passes through many levels of utility, the idea that two baskets of the same market value may have different utility levels seems incredible on first intuition. The allocation model's habit of thinking is that if there is any further gains to be made, then the optimizer can seize it by changing the demand or supply until all opportunities for gain are exhausted. Apparently, the thinking is that if the marginal utility of G is higher than that of F , the optimal response must be to increase the supply of F (= reduce consumption) and the demand for G (= increase consumption) until the gap in marginal utility disappears. This allocation is not possible, because the agent is already on the boundary of the PPF. This thinking ignores that the choice of producing at B and consuming at C is already optimal. It forgets that the original endowment was anomalous, and trade has created gains by virtue of a move to point C from point B.

1.3.1 The challenge of Pareto Optimality

There is apparently some loss of memory of the whole exercise on splitting the price effect. This happens because an exercise in the moral quality of plunder, despite not being a part of economic analysis of exchange, came to occupy a permanent place in the consciousness of microeconomics. This is the notion of Pareto Optimality, originally formulated before the Slutsky decomposition of the price effect.

The task before Pareto Optimality is to evaluate the welfare effect of a transfer of wealth from one individual to another. A properly formulated model of transfer would look into the institutional context. In politics, taxpayers are plundered to feed the recipients of subsidy. There must be some political rationale to judge the merit of one-way transfers. Pareto Optimality did not see the issue as one of politics. Instead, it looked for some kind of economic optimum in which transfers may give benefits to some, as long as nobody is burdened with a loss. The result of the search is that there is a social optimum in which the parties neither gain nor lose anything. This social equilibrium fulfills Pareto Optimality. In it, the ratio of marginal utilities is the same for all individuals, since everybody equalizes it with the given ratio of market prices. Let the prices be (p_f, p_g) and let u denote the marginal utility for the good in the subscript. Then Pareto Optimality gives

$$(u_f/u_g)^{Crusoe} = (u_f/u_g)^{Defoe} = (p_f/p_g) \quad [\text{Pareto Optimality}]$$

In a standard allocation exercise, Crusoe would maximize $U^{Crusoe}(F, G) + \lambda(I - p_f F - p_g G)$. The first order condition for utility maximization gives

$$(u_f/\lambda) = p_f; \quad \text{and} \quad (u_g/\lambda) = p_g \quad \text{or} \quad (u_f/u_g) = (p_f/p_g)$$

There is no harm in redefining the physical units so that the price ratio is 1, giving $(u_f/u_g) = 1$ or

$$u_f = u_g$$

Notice also that since the entire sale of F^x is against the entire bundle of G^m , it is proper to regard the associated utility as marginal utility, namely

$$U(F^x) = u_f; \quad U(G^m) = u_g;$$

Then the contrast is evident from putting the Menger Dictum and Pareto Optimality side by side

Menger Dictum for Crusoe:	$u_f < u_g$
Menger Dictum for Defoe:	$u_f > u_g$
Pareto Optimality for both:	$u_f = u_g$

It is not possible to reconcile the above without changing the theory of price so that it allows income effects for both buyers and sellers! Let us suspend the issue here, and come back to it later in Chapter 3.

1.3.2 Can all parties gain?

The Menger Dictum applies to all individuals. The participation in exchange is not costless- it imposes transaction costs. Hence it is unlikely that exchange can occur even if one party finds it gainful while the other party merely gets a zero benefit. It is generally not possible, because exchange must necessarily occur between strangers who are not friends, and who cannot play a futile game merely to let somebody else win. Why transfers between members of family and friends cannot be regarded as exchange will be discussed in Chapter 2.

If exchange is gainful for both parties, the following must be true for Crusoe **and** Defoe.

$$\begin{array}{l} U^{\text{Crusoe}}(F^x) < U^{\text{Crusoe}}(G^m) \quad \{\text{Crusoe's gain}\} \\ U^{\text{Defoe}}(F^x) > U^{\text{Defoe}}(G^m) \quad \{\text{Defoe's gain}\} \end{array}$$

The above relations together define what is known as double coincidence. Jevons (1868) used the term, but its verbal meaning was soon repudiated by the formal statement of Pareto Optimality. Verbally, there is a double coincidence of wants in barter, if the good sold by the first agent is bought by the second and vice-versa. But its formal meaning was not available.

Double coincidence, stated formally as above, immediately reveals an inconsistency in the preference orders of the two agents. The intuitive reaction is to reject double coincidence for this apparent inconsistency.

1.4 Consistency of choice

The most important challenge to the double coincidence relation (Menger Dictum) comes from the highly remarkable idea known as Arrow's Impossibility Theorem, which is a generalization of Pareto Optimality. The theme is that it is impossible to formulate a social welfare function due to the linear inconsistency of preferences of different individual members of society. In short, double coincidence is inconsistent!

The rationality of an individual's choice may be defined by linear consistency or transitivity. For example, let three points x, y and z lie on a line. Now, if x is preferred to y, and y is preferred to z, then x must be preferred to z. Let an arrow show the direction of preference. The line below shows linear consistency:

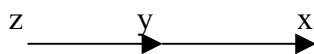


Figure 3: Linear consistency

In an indirect exchange with three or more agents, each with preferences inconsistent with others, it is indeed impossible to get linear consistency of social choice. However, as explained in detail later, indirect exchange does take place with rational choice of each agent. It does give the most preferred good to each agent, and hence it does maximize social welfare. And it does reveal social choice. All these happen without linear consistency. This may mean that the linear viewpoint of one agent may not properly judge a social decision. Another notion of consistency may be appropriate to study social choice. That notion may be called circular consistency, as represented by a circle.

In a circular consistency, the two ends of the line are joined to make a circle. The result is that if x is preferred to y , and y is preferred to z , then z is preferred to x . It sounds illogical and impossible at first, like the clock's dial, where 1 comes after 12 and also before 12. But the dial cannot work unless the day begins where the night ends and vice-versa. The day's 2 PM cannot be later than the day's 3 PM, but it can be later than the night's 3 AM. The independence of the day from the night, coupled with the continuation is the clue to circular consistency.

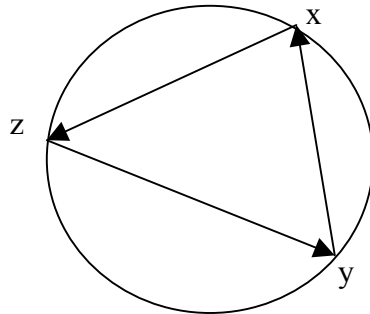


Figure 4: Circular consistency

Is circular consistency rational or logical? To search for an answer, a critical distinction must be made between the allocation decision of **one** agent and the exchange decision of **many** agents. The logical distinction is that the social decision may ignore the preferences of uninterested agents. There are three steps to see this. The first step recognizes that social decision is a sum of different parts made by different agents. The second step sees that each part ignores the agents who neither buy nor sell. The third step shows that each part has linear consistency, and yet when all parts are joined, the joint decision does not have linear consistency.

The logical issue is to consider the relevance of the preference of agents who neither buy nor sell. If the preferences of agents who neither buy nor sell can be ignored, then circular consistency is logical. To explore the issues, one may begin with linear consistency and see where it stops. One then proceeds further with a different notion of consistency.

Linear consistency defines rational choice from the viewpoint of a single agent. The simplest way to see its limitation is to consider an example of indirect exchange. Let there be 3 agents A, B and C who want to buy and sell three goods (x , y , z) among themselves. Let U denote the utility of the good to the agent as indicated by a superscript. Let the preference orders of the agents (A, B, C) over the three goods be:

$$\begin{array}{ll} \{U^a(y) < U^a(x) < U^a(z)\}; & y \longrightarrow x \longrightarrow z \\ \{U^b(z) < U^b(y) < U^b(x)\}; & z \longrightarrow y \longrightarrow x \\ \{U^c(x) < U^c(z) < U^c(y)\}, & x \longrightarrow z \longrightarrow y \end{array}$$

In the example, each agent has linear consistency. However, the group decision on social choice based on a unanimous majority of 3 cannot be formed at all. Next, a coalition of 2 cannot lead to linear consistency. A majority (A+B) prefers (x) to (y), a majority (B+C) prefers (y) to (z), and a majority (A+C) prefers (z) to (x). This means that there is no linear consistency. Indeed, each coalition fails to have linear consistency. Thus coalition (A+B) prefers x to y , but finds z both

superior and inferior to x or y or both. Coalition (B+C) finds x both superior and inferior to z and y. Coalition (A+C) finds y both superior and inferior to x and z.

What limits linear consistency? The two ends of the line are open, one missing a buyer and the other missing a seller. Thus, for the coalition (A+B), the line $(\overrightarrow{y \quad x \quad z})$ indicates that y has no buyer and z has no seller. If a third agent C joins them as both a buyer of y and a seller of z, then the points y and z must be connected such that y is preferred to z, giving circular consistency. In short, a linear consistency cannot possibly describe an exchange where all goods are both bought and sold. It cannot describe even direct exchange between two agents. Thus if A prefers z to x and C prefers x to z, the two agents can do barter, but linear consistency fails. The agents must ignore each other's preference if they want to get what they prefer.

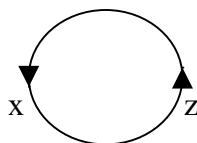


Figure 5: Circular consistency in barter

In indirect exchange, the preference orders reveal that each good is most preferred by one agent, least preferred by another agent and weakly preferred in between by the owner. In the example, the most preferred goods are z for A, x for B and y for C. But z, x and y are least preferred by B, C and A respectively. Linear consistency cannot possibly describe the same good being least preferred and most preferred in the same choice. And yet every agent is acting rationally. Their social decision is to be deemed rational if it gives the most preferred good to each agent. This is just what indirect exchange actually does: it gives each agent the good most preferred by that agent. This happens despite and **because** of the fact that each good is least preferred by one of the agents while it is most preferred by another.

The preference of the agent who finds the good least preferred is irrelevant, because that agent neither buys nor sells the good. It is impossible to assure linear consistency if the preference of the irrelevant agent is included. But if the preference of the uninterested agent is ignored, there is no reason to seek linear consistency. The concerned agents can take a direction of preference opposite to that of the unconcerned agent. This leads to circular consistency. Logically, if every good is both more preferred to one good and less preferred to yet another good, then they all belong to a circle. In a circle, each point is both higher and lower to any other point in any direction. A circle has no endpoint while a line has endpoints.

To explore circular consistency, the following concepts are needed: ruling coalition, changing composition of ruling coalition leading to changing direction of preference, dominance of ruling coalition, strong and weak preference, and exit of the uninterested agent.

Ruling coalition: The ruling coalition consists of the buyers and sellers. It makes the decision on what and how much is exchanged between whom. It excludes or ignores other agents who are neither buyers nor sellers.

Changing composition of the ruling coalition: After a decision to exchange one good is made, the buyers leave the coalition. They are not interested in the decisions on other goods

which they neither buy nor sell. The seller of the earlier coalition remains, and becomes the buyer of the next good. The sellers of the next good join the new coalition. Consider good x. Agent B wants to buy it while agent A wants to sell it. The ruling coalition is formed with A and B. It does not include C or ignores C, as C is neither the buyer nor the seller of x. The decision is made so that A sells x to B. Then B leaves the coalition and C enters. Now, the job is to decide on the good z. A wants to buy it and C wants to sell it. The decision is made so that C sells z to A. Then A leaves the market, having both bought z and sold x, and having no interest in y. But C wants to buy y and B wants to sell y. They form a coalition. The decision is made so that B sells y to C. All trades are completed by the three successive coalitions.

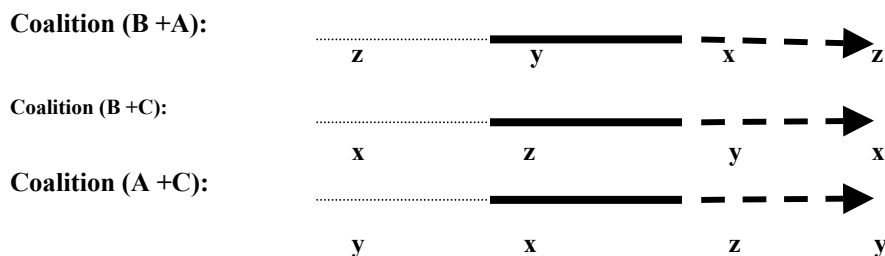
The result of the social decision is to give each agent the good most preferred by that agent. The important point is that the social decision on all three goods has been made by successive partial decisions with different ruling coalitions. Each individual and coalition behaves rationally. The process is rational as it gives each agent the most preferred good.

Dominance of coalition: Buyers dominate the coalition because they have the strongest preference for the good being considered for exchange. They get support from the sellers. The dominant agent chooses the direction of preference with the support of the seller. Buyers must overlook the preferences of irrelevant agents who dislike the good.

Strong and weak preference: The agent with the strong preference for the good is the buyer. Strong preference means that the buyer has no other good preferred to the one being bought. The seller's preference is weak, but consistent with the buyer's direction of preference. The term weak preference implies that the seller prefers some other good to the one being sold. For example, both A and B prefer x to y, but B prefers it strongly, while A prefers it weakly. A wants z strongly. The coalition has a segment over which the preferences overlap, so that they move in the same direction. The strong preference implies buying while weak preference implies selling. In short, the coalition is pair of a buyer and a seller of one good.

Exit of the uninterested agent: The agent who is neither a buyer nor a seller voluntarily leaves the coalition. It means that the preference of this uninterested agent can be ignored. The direction of preference is reversed due to the exit of the uninterested agent. If the agent refuses to exit, involuntary unemployment occurs due to a blockade from the irrelevant agent. (See Chapter 5)

The process of making a social decision may be shown by the three successive coalitions. Shown below are the preferences of two agents in a coalition. In each coalition the solid line shows where the preferences match. At the tail end, there is one good that is less preferred by one agent which is more preferred by the other. There is no linear consistency except in the middle segment.



It is possible to show how these coalitions can make partial decisions on one good at a time. They can successively make decisions on all goods. The combination of the partial decisions lead to a social decision with circular consistency. By successively adding the coalition lines as sides of a triangle, one gets a triangle of choice in a 3-good exchange. The circle is its generalization.

On each side of the triangle, there are two goods over which two agents have the same order of preference, but they have an opposite order of preference over a third good.

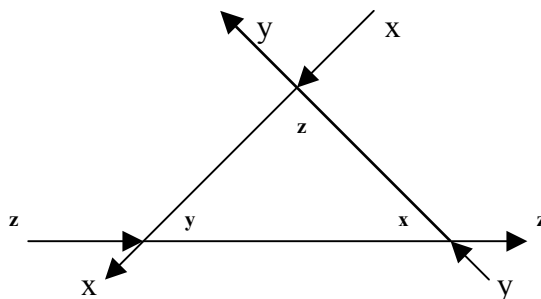


Figure 6: Social choice

Thus $(y \rightarrow x \rightarrow z)$ of A coincides with $(z \rightarrow y \rightarrow x)$ of B only on segment $(y \rightarrow x)$ on the line $z \rightarrow y \rightarrow x \rightarrow z$. The line $(y \rightarrow x \rightarrow z)$ of A matches with line $(x \rightarrow z \rightarrow y)$ of C in segment $(x \rightarrow z)$ on line $y \rightarrow x \rightarrow z \rightarrow y$. Lastly, agent C's $(x \rightarrow z \rightarrow y)$ matches segment $(z \rightarrow y)$ with agent B's $(z \rightarrow y \rightarrow x)$ on line $x \rightarrow z \rightarrow y \rightarrow x$. The matching segments $(y \rightarrow x)$, $(x \rightarrow z)$ and $(z \rightarrow x)$ together constitute circular consistency, but violate linear consistency.

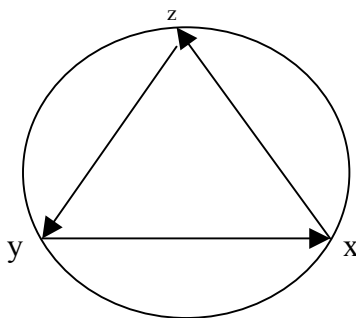


Figure 7: Triangles and circles of choice

The idea is that the society may ignore the preference of the agent who neither buys nor sells. In Figure 6, the tail of the coalition's preference line comes from the preference of the agent who does not buy or sell the product. That preference is irrelevant. The tail can be chopped off.

Circular consistency means that every good has both a buyer and a seller. But a buyer has a strong preference for it, while the seller has a weak preference. Therefore for each good, there is one good more preferred and another good less preferred to it. For this to happen to every good, it must be so that each good is both more preferred and less preferred. It is logical, because each good is less preferred to one good but more preferred to a different good. In isolation, each good meets linear consistency. Also, each agent's preference has linear consistency. But the combination lacks linear consistency. This happens because the uninterested agent's preference

can be ignored or reversed. If everybody has acted rationally, then the social decision must be rational. Indeed, it would not be rational to judge the merit of a decision according to the preferences of the agents who are not concerned with the decision.

To sum up the discussion, a single agent's linear viewpoint is not suitable to study social decisions. There is no need for independent agents to have the same preference order. As long each agent can act rationally to participate in social decision, the social choice is rational. Circular consistency is required to describe social choice such that each good has both a buyer and a seller. Without this circularity, there would be one good without a buyer and another without a seller at the two endpoints of the line. That would fail one seller and one buyer. Linear consistency would not reveal social choice or maximize welfare or establish equilibrium.

1.4.1 Arrow's Impossibility theorem

It is important to notice that Arrow sought to solve a problem that does not exist in economics. It is **not** a problem for society to choose only one good out of many when different people prefer different goods. There is nothing wrong with producing every kind of good. Diversity of goods is a market fact.

Even the political issue of choosing a single leader out of many candidates is misleading. If in a society there are different voting blocks supporting many different candidates with no clear choice for one leader, an efficient solution is a parliament where many leaders represent the many blocks. The problem of choosing a single leader for the whole nation has no parallel in economic analysis of allocation or exchange. Even in politics, the scenario with multiple candidates or parties with divided loyalties means lack of political equilibrium, and may involve fragile governments built on unstable coalitions. If the underlying preferences of people display a divided society, there is no a priori reason to propose a centralized society rather than a confederation. But that is not an economist's task.

Neither does culture present a problem of a whole society choosing a singular ideal. The history of culture is a history of diversity. In the end, the monomaniac search for a single good or a single leader or a single cultural norm for a whole society animated by a single mind may be misguided.

The real economic problem is the existence of an anomaly in endowment so that an individual may prefer something that does not belong to him or her. The problem for society is to find out how best to organize a transfer of goods to those who desire them most out of a given collection of goods, from those who rank the same goods on a lower scale but happen to own them. Thus if agent A prefers x most but owns y, while B prefers y most but owns z, and C prefers z most but owns x, then it is **not** a problem of deciding which of the three goods the whole society should consume. The problem is to allow A to have x in exchange for y, B to have y in exchange for z, and C to have z in exchange for x. Arrow is thinking of an allocation problem in place of an exchange problem. In an allocation problem, one single individual controls the choices. In an exchange problem, several independent agents have their own preferences such that they may or may not find options for gainful exchange.

It is important to recognize the institutional background of exchange and plunder. A political authority may impose taxes and provide subsidies according to some political rationale. May be politics considers some balancing of power through one-way transfers. Economics has nothing to say about political acts of one-way transfer of wealth.

More importantly, economic science has no reason to study welfare as such. Exchange happens to be gainful, whether one likes it or not. The focus on welfare imports value judgments and violates the sanctity of science. The job of science is to explain, and not judge.

Economics can deal only with voluntary exchange in which the buyer pays instead of plundering the seller. Arrow's theorem does not clarify who owns the resources and why many agents must make a single choice. Even if the resources were jointly owned, the solution would lie in producing all three goods according to some rule to decide who gets how much. To set up the allocation problem properly, the budget of each agent must be specified. Arrow's theorem does not specify the budget. Sen (1982) sees the issue correctly as a political one of entitlement.

1.4.2 Double coincidence is necessary

The notion of circular consistency says that it is logically valid to admit the existence of double coincidence in exchange. The linear inconsistency is not a problem. Indeed, it is necessary for trade to be gainful for both parties. As shown later, money arises as an artificial means of payment to fulfill the double coincidence requirement. This changes **all** of macroeconomics. It has special relevance for central issues of economic policy on (involuntary) unemployment, (unintended) indebtedness and (undue) instabilities unrelated to tastes, technologies, and endowments.

1.5 Axioms of consistent economics

The new body of economic theory may be called consistent economics in view of the critical role played by the notion of consistent choice. This new economics stands on two axioms of gainful trade.

1.5.1 First axiom: Menger dictum

The theme that trade is gainful for both the buyer and the seller of each good is taken as an axiom called Menger Dictum. This is expressed formally as the condition of double coincidence.

To derive the implications of the Menger dictum, the need is to extend the analysis of choice over varying kinds of goods of given value. This leads to a new model of exchange in which choices have two dimensions: over varying quantities of goods of given kind, and over varying kinds of goods of given value. The formal analysis requires notions of offer, acceptance, and (single, double, multiple) coincidence to deal with the choice over kinds.

1.5.2 Second axiom: Circular consistency

The notion of gains for both parties cannot make sense without the notion of circular consistency of choice. This notion defines a rational outcome for social choice. This leads to a unified model in which individual choices are combined to arrive at social choice. This opens up a new vista and provides a new analytical tool . This tool may be called consistency analysis.

1.6 Allocation versus exchange

If economics must acknowledge that trade is gainful, it must make a clear distinction between allocation and exchange. The following four dimensions of the distinction may give a clue to the development of a tool to analyze exchange. These are (1) character of consistency in choice; (2) the presence of pure entrepreneurial gains and risks of loss; (3) the presence of artificial payments in indirect exchange and (4) the presence of intermediation.

1.6.1 Character of consistency in choice

Individual choice gives the individual **total control** on the choices over **substitutes** in **allocation**. However, social choice gives the individual **partial control** over **non-substitutes** in **exchange**. The three distinguishing elements are: preference relations between the goods (substitutes or not); degree of control (total or partial), and the integrity of the good (whether or not the buyers and sellers have agreed on the kind and quantity of the good).

1.6.1.1 Preference relations between goods

The first distinction between substitutes and non-substitutes shows why the equilibrium conditions under allocation must differ from those under exchange. Under allocation, an individual chooses among alternatives. The distinction in kind is not essential. This is because different kinds of goods are in essence the same source of utility or productivity, though they come in bundles of different sizes. If 2 apples are a substitute of 3 bananas, then a banana is just $2/3$ of an apple. In an exchange, the goods are not substitutes. They are preferred unequally so that one is bought while the other is sold. The reason exchange occurs is that each agent is able to pay for a more preferred good with a less preferred one. One cannot make the transition to an analysis of exchange without recognizing this distinction.

1.6.1.2 Degree of control over decision

The second distinction, between total and partial control, contains the seed of rationality of linear and circular consistency. When an individual has complete control over the decision, it is rational to abide by linear consistency, to reach the end of the line of preference. Thus if the arrowhead shows the direction of preference, an individual rationally decides to sell the good at the bottom end, and to buy the good at the top end. However, social choice can work only if the agents who have no interest in a good as either buyers or sellers exit from the coalition that makes the decision. This means that each agent has only partial control, in one segment of the circle. If an individual has total control on social decisions, Arrow's impossibility stops it from being rational. Freedom of choice cannot exist if each individual's preference is the same as that of others. In that case, the whole concept of rational choice becomes irrelevant. Instead, if all individuals have the same preferences, they might be regarded as nothing but inert objects with immutable physical properties. A mechanism would describe their mechanical behavior. The issue of gains would not arise at all in that case, as in physics.

1.6.1.3 Social integrity of choice

The third distinction between allocation and exchange concerns integrity of choice. An isolated individual may optimally choose the quantity to buy, but the good's quantity still lacks integrity. This is because there is nothing to say if this demand is consistent with its supply from other agents. Similarly, the quantity of supply chosen by an isolated individual lacks integrity, without

assurance of matching demands from others. Social choice must ensure the integrity of the good's kind and quantity. It means that the good's quantity demanded is matched exactly by its quantity supplied. This also means that the good's kind is offered by the seller and accepted by the buyer. The need to apply circular consistency becomes clear in regard to the determination of the kind. The seller must have weak preference for a good, but the buyer has a strong preference for the same good. Circular consistency in the double coincidence must exist to induce one to sell what the other wants to buy, to confer integrity of the good's kind.

The matching of the quantity hides the circularity in social choice. To the seller, the market value is higher than the cost. To the buyer, the market value is lower than the cost of production the buyer would incur if he or she had to produce. And yet they must be able to agree on a value of the good to give it integrity. Separately, the seller and the buyer both obey linear consistency. Together, their social choice has circular consistency. This circular consistency is the source of the integrity of the good's kind and quantity.

1.6.2 Entrepreneurial gains and losses

Exchange is an entrepreneurial action undertaken for the purpose of pure profit and does not involve optimization. Allocation involves optimization, but does not involve entrepreneurial gains or risks. Because the same individual may function in two different capacities, it is necessary to separate them analytically. In Figure 1, Crusoe's optimal choice before exchange is to produce and consume at autarky equilibrium point A. It involves efficiency gains from any other point on the PPF, but one does not presume which suboptimal allocation Crusoe might have used before choosing the optimal. A rational Crusoe would never choose any other point. Hence the efficiency gain would not be part of the analysis.

However, when Crusoe becomes an entrepreneur, he seeks to break out of the constraint of the PPF. His first move is to undertake a risk by moving to point B, which is suboptimal under autarky. This move is entrepreneurial. An allocation model can make no sense of such a move. The next move is to consummate the actual trade itself. By selling F^x and buying G^m , Crusoe achieves a net gain. An allocation model looks at the budget line at world prices, and does not see the gain, nor the risk of loss in case the trade failed.

The analytical problem is that the allocation model has no clue how the world price is determined. Standard microeconomics makes a major mistake in this regard. On the one hand, it presumes that the prices are already given, and the agents optimally choose demands and supplies. But it then says that demands and supplies determine the price. This is not a valid proposition. Demands and supplies determine only the quantity, but not the price. Something else determines the price. That involves an entrepreneurial process that seeks pure profit and bears risks of losses. An equilibrium market price is determined by entrepreneurial arbitrage, which may leave pure profits or losses in equilibrium.

The conventional theory of price cannot be retained if gainful trade is to be admitted. In its place, a new theory of price is needed. That theory will allow positive gains from trade. Chapter 3 presents such a theory. Its gist is that setting a market price is a dynamic process led by arbitrageurs. In the first step, producers and consumers assume a given market price to choose

demands and supplies. In the second step, the arbitrageur clears the market by setting a price such that demand becomes equal to supply in value, not in quantity alone.

This market clearing is formally accomplished by taking equivalence as a necessary condition of exchange equilibrium. Equivalence is stated as the equality of value of the goods that pay for each other in exchange. Thus since Crusoe pays for G^m with F^x , it must be the case that

$$V(F^x) = V(G^m) \quad \{\text{Equivalence}\}$$

This condition imposes a constraint that the allocation model forgot. Chapter 3 discusses the significance of this equilibrium condition and shows how to use consistency analysis to impose this requirement. It avoids a major oversight in the standard multimarket model of allocation, often mistaken to be a model of exchange. The allocation model requires that for each good, demand must be equal to supply. This is a necessary condition, but not sufficient. It is further necessary to ensure that each agent's total income from sales must be equal to total expenses on purchases. Furthermore, this restriction must be true item by item. Thus, suppose that Crusoe sells fruits to many buyers. It is necessary that the value of fruits sold to any particular buyer be equal to the value of payment received from that buyer. It is possible that Crusoe sells fruits to Defoe, but does not buy anything back from Defoe, but instead buys something from Eto to whom Crusoe does not sell anything. This indirect trade messes up the budget balance and disables demands from being effective. Without the equivalence condition, there is nothing to impose the budget constraint.

Setting the market clearing price is the job of the arbitrageur. Crusoe may serve as an arbitrageur on his own behalf or for others. He can buy cheap and sell dear. He can buy the fruits from himself (at price equal to marginal cost, for example) and sell them to Defoe at a higher price. The pure profit is without a constraint. It is not an optimization exercise. (See Chapter 3).

1.6.3 1F3 Artificial payments in indirect exchange

The allocation model apparently does not recognize the existence of a payment. The Walrasian model has no money in it. It does not consider the issue of indirect exchange mentioned above where Crusoe sells to Defoe but buys nothing from Defoe, while Crusoe buys from Eto but sells nothing to Eto. Indeed, Arrow's impossibility theorem denies the possibility of indirect exchange implicitly, because of linear inconsistency in the preference orders. An exchange model must allow indirect exchange and must find out how payments are made. The formal problem is to acknowledge all of the following:

$$\begin{array}{ll} V(F^x) = V(G^m) & \{\text{Equivalence}\} \\ U(F^x) < U(G^m) & \{\text{Crusoe's gains}\} \\ U(F^x) > U(G^m) & \{\text{Defoe's gains}\} \end{array}$$

An analytical tool to acknowledge both equivalence and double coincidence may offer a breakthrough. Things of equal value may have unequal utility only if they are of different kinds. The analysis must find ways to study how the kinds of goods are determined, not just quantities. The solution lies in formally articulating the preferences over different kinds of goods of the same market value. (See Chapter 4)

1.6.4 Intermediation

Since an allocation model is essentially individualistic, it has no room to talk about intermediation between different agents. The attempts to study the buying and selling of information products, or the various probabilistic modes of search can not handle the essential character of entrepreneurship, or of transaction cost or of institutions as parts of intermediation. This is because there is nothing to optimize.

A theory of exchange cannot avoid talking about intermediation in indirect exchange. It includes issues of transaction cost outside the control of the producer and consumer. It covers entrepreneurial discovery of gainful arbitrage and seigniorage opportunities that arise in exchange. And it must address the issues of economic institutions. This is a virgin land with exciting features. (See Chapter 6).

1.7 The journey ahead

The journey ahead is exciting. It discovers a simple tool to study exchange. The tool is applied to reveal realities often unsuspected by previous economics.

The new discoveries bring a cheerful message. The proper use of money as a means of payment can solve the problems of unemployment, indebtedness and instability. It basically accomplishes the same entrepreneurial feat: create new wealth out of nothing, but by virtue of new knowledge or gainful ideas.