Endogenous Growth Models in Open Economies:

A Possibility of Permanent Current Account Deficits

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Abstract

The paper explores the impacts of heterogeneity in degree of relative risk aversion on the balance on current account in a two-country endogenous growth model. It concludes that, like the heterogeneity of demographic changes, the heterogeneity in degree of relative risk aversion generates persisting current account deficits. The deficit continues permanently, but its ratio to output stabilizes. With evidence that the degree of relative risk aversion in Japan is relatively higher than that in the U.S., there is a possibility that the persisting bilateral trade deficit of the U.S. with Japan is partially generated by this mechanism.

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I. INTRODUCTION

The large current account deficit in the U.S. and the large current account surplus in Japan have continued during the past three decades (see Figure 1), and the large bilateral trade deficit of the U.S. with Japan has also persisted (see Figure 2). The conventional intertemporal approach to the current account can not explain these persisting large current account imbalances as Obstfeld and Rogoff (1995) argue, and thus it needs the help of overlapping-generations variants of the intertemporal models. The overlapping-generations variants of the intertemporal models explain persistent current account imbalances by heterogeneous demographic changes. For example, they explain that more rapidly aging Japan experiences persisting current account surpluses while less rapidly aging US experiences persisting current account deficits. There are voluminous works that conduct simulations based on the overlapping-generations variants of the intertemporal models and project the impacts of heterogeneous demographic changes in the U.S., Japan and other countries.[1] Although there are various types of models, the basic idea behind the explanation of persisting current account imbalances is common and simple: national savings moves heterogeneously under the influence of heterogeneous demographic changes while national investments are affected less by the heterogeneous demographic changes because they are determined basically by the world real interest rate, and thus heterogeneous demographic changes generate heterogeneous movements of the balance on current account, i.e., heterogeneous movements of national savings minus national investments.

However, although theoretical projections based on demographic changes have been numerously carried out, few systematic empirical examinations into the relationship between the

[1] See e.g. Kotlikoff, Smetters and Walliser (2001), Brooks (2003), or Faruqee (2003). Most of these simulations project that the current account in Japan will turn to deficits in near future due to the rapid demographical change in Japan.

balance on current account and demographic changes have reported. Among these few studies, Poterba (2001) conclude that although theoretical models generally suggest that equilibrium returns on financial assets will vary in response to changes in population age structure, it is difficult to find robust evidence of such relationships in the time series data. As a result, the conjecture that current account imbalances are generated by heterogeneous demographic changes has not been fully supported by empirical evidence and is still merely a theoretical possibility. If explanations based on heterogeneous demographic changes indicate only a theoretical possibility, heterogeneity that can be examined as the source of persistent current account imbalances may not necessarily be limited only to the heterogeneity of demographic changes. Other heterogeneous nature of economies may also theoretically generate discrepancy between national savings and investments.

The paper explores heterogeneity in degree of risk aversion as another possible source of persistent current account imbalances. The reason why the paper directs its attention to the degree of risk aversion is firstly that in endogenous growth models the degree of relative risk aversion plays a crucial role for growth rates and thus its heterogeneity significantly complicates movements of international transactions. Assume that there are only two countries in the world, only difference between which is the degree of relative risk aversion. The conventional growth models with exogenous technologies, the heterogeneous degrees of risk aversion in two countries do not matter for steady state growth rates because they are determined by the common exogenous growth rate of technologies in both countries. However, in endogenous growth models, the degree of relative risk aversion is the crucial parameter that determines growth rates, and thus steady state growth paths under its heterogeneity are not so simple. The familiar Euler condition in case of a Harrod neutral production function such that $y_t = \frac{Y_t}{L_t} = A_t^{\alpha} k_t^{1-\alpha} = A_t^{\alpha} \left(\frac{K_t}{L_t} \right)^{1-\alpha}$ is $\frac{\dot{c}_t}{c_t} = \frac{(1-\alpha)\left(\frac{A_t}{k_t} \right)^{\alpha} - n_t - \theta}{\varepsilon}$ where y_t is outputs per capita, c_t

is consumption per capita, k_t is capital inputs per capita, Y_t is outputs, K_t is capital inputs, L_t is labor inputs, A_t is knowledge/technology/idea, $n_t = \frac{\dot{L}_t}{L_t}$ is the growth rate of population, θ is the rate of time preference, ε is the degree of relative risk aversion, and α is a constant. In most endogenous growth models, $\frac{A_t}{k_t}$ is modeled to be constant, and thus the growth rate of consumption becomes constant.[2] Hence, in endogenous growth models, the constant growth rate of consumption $\frac{\dot{c}_t}{c_t} = \frac{(1-\alpha)\left(\frac{A_t}{k_t}\right)^{\alpha} - n_t - \theta}{\varepsilon}$ crucially depends on the value of degree of relative risk aversion ε, and thus its heterogeneity significantly complicates steady state growth paths in the world of free trade.

The second reason why the paper directs its attention to the degree of relative risk aversion is because it has been reported that the degree of relative risk aversion in Japan is relatively higher than that in the U.S. It is another important heterogeneity than demographic changes between the U.S. and Japan, and it implies a possibility that the large current account deficits in the U.S. and the large current account surpluses in Japan can be explained by the relatively higher degree of relative risk aversion in Japan than that in the U.S. In the well-known Szpiro (1986), it is reported that of the nine industrialized countries studied the Japanese have the highest degree of relative risk aversion, e.g. the degree of relative risk aversion in Japan is 2.76 while that in the U.S. is 1.19. It is a well known fact that compared with households in the U.S., households in Japan invest their financial assets much less in risky investments, which clearly indicates that the degree of risk aversion in Japan is much higher than that in the U.S. Furthermore, heterogeneity in risk aversive behavior has recently been reported from the medical or genetical point of view. Ono et al. (1997) and Nakamura et al. (1997) show that the

[2] See e.g. Romer (1990), Aghion and Howitt (1998), or Jones (2003).

genetic composition of the receptor for brain chemicals such as serotonin or dopamine differs widely among human races, and that most Japanese have inherited a certain type of receptor composition that produces more cautious and therefore more risk aversive characteristics, while many Americans have inherited the other type that produces less risk aversive characteristics. In addition, Harashima (1998) shows that the so-called "Japanese economic system" or "Japanese capitalism" originates in the higher degree of relative risk aversion in the Japanese.

The model in the paper, which is based on the solution in Sorger (2002) and Ghiglino (2002) to the problem of heterogeneous households raised by Becker (1980), shows that there is a possibility that heterogeneity in degree of relative risk aversion generates persistent, more correctly permanent, current account imbalances. In some circumstances, a country with lower degree of relative risk aversion experiences a permanent current account deficit, and in reverse a country with higher degree of relative risk aversion experiences a permanent current account surplus. Nevertheless, current account deficits and surpluses do not explode but the ratio of deficit and surplus to output asymptotically approach unique finite values and stabilize in both countries. Hence, the model predicts that if the degree of relative risk aversion in Japan is truly relatively higher than that in the U.S., there is a possibility that the current account surplus persists in Japan permanently and the current account deficit persists in the U.S. permanently.

The paper is organized as follows. In section II, a two-country endogenous growth model that incorporates international transactions is constructed, and the basic nature of the model is examined. It is shown that there is a steady state growth path on which the limits of growth rates of consumption, capital, knowledge/technology/idea, and output are all equal and they are equal in both countries. In section III, the balance on current account in the model is examined. It is shown that there is a possibility that a country with lower degree of relative risk aversion experiences a permanent current account deficit and in reverse a country with higher degree of relative risk aversion experiences a permanent current account surplus. In section IV, first, evidence that the degree of relative risk aversion in Japan is relatively higher than that in the

U.S. is presented. Secondly simple simulations with calibrated parameter values are carried out, results of which indicate a possibility of permanent trade imbalance. Finally, some concluding remarks are offered in section V.

II. THE MODEL

1. The basic model

In most endogenous growth models, the growth rate of consumption is commonly expressed as $\frac{\dot{c}_t}{c_t} = \frac{(1-\alpha)\left(\frac{A_t}{k_t}\right)^{\alpha} - n_t - \theta}{\varepsilon}$ where $\frac{A_t}{k_t}$ is kept constant by some mechanisms that are different according to the type of models, and thus in most models, the degree of relative risk aversion plays a crucial role for steady state growth rates. In this sense, many types of endogenous growth models that incorporate international transactions may be used for the sake of the analysis in the paper and may lead to the same conclusions. From among various endogenous growth models, however, the paper chooses a specific model that is examined in Harashima (2004), because this model has the advantage of being free from scale effects and the influence of population growth simultaneously, which seems very advantageous when a factor other than demographic changes is examined since we can extract the effect of the factor that are independent from effects of population.[3]

The production function is assumed to be $Y_t = F(A_t, K_t, L_t)$, where Y_t (≥ 0) is outputs, K_t (≥ 0) is capital inputs, L_t (≥ 0) is labor inputs, and A_t (≥ 0) is knowledge/technology/idea inputs in period t. The model is based on the following assumptions.

Assumption:

[3] See e.g. Jones (1995), Aghion and Howitt (1998), and Peretto and Smulders (2002).

(A1) Accumulations of capital and knowledge/technology/idea are $\dot{K}_t = Y_t - C_t - v\dot{A}_t - \delta K_t$, where $v(>0)$ is a constant and a unit of K_t and $\frac{1}{v}$ of a unit of A_t are produced using the same amounts of inputs, and δ is the rate of depreciation.[4]

(A2) Every firm is identical and has the same size, and for any period, $m = \frac{M_t^{\rho}}{L_t} = \text{constant}$ where M_t is the number of firms and $\rho(>1)$ is a constant.

(A3) $\frac{\partial Y_t}{\partial K_t} = \frac{1}{M_t^{\rho}} \frac{\partial Y_t}{\partial (vA_t)}$ and thus $\frac{\partial y_t}{\partial k_t} = \frac{1}{mv} \frac{\partial y_t}{\partial A_t}$.

Assumption (A1) is standard one in the literature of endogenous growth. Assumption (A2) simply assumes that the number of population and the number of firms in an economy are positively related, which seems intuitively natural. In assumption (A3), the paper assumes that returns on investing in K_t and investing in A_t for a firm are kept equal. However it is also assumed in (A3) that a firm that invents a new technology can not obtain all the returns on investing in A_t. This means that investing in A_t increases Y_t but returns of an individual firm that invests in A_t is only a fraction of the increase of Y_t such that $\frac{1}{M_t^{\rho}} \frac{\partial Y_t}{\partial (vA_t)} = \frac{1}{mL_t} \frac{\partial Y_t}{\partial (vA_t)}$. The reason why only a fraction of the increase in Y_t the returns of an individual firm is, is uncompensated knowledge spillovers to other firms.

More specifically, the production function is assumed to have the following functional form: $Y_t = F(A_t, K_t, L_t) = A_t^{\alpha} f(K_t, L_t)$, where $\alpha(0 < \alpha < 1)$ is a constant. Let $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $c_t = \frac{C_t}{L_t}$ and $n_t = \frac{\dot{L}_t}{L_t}$ and assume that $f(K_t, L_t)$ is homogenous of degree one. Thereby

[4] Hence, like Jones' (1995) non-scale model, A_t, as well as K_t, is produced less as A_t and L_t increase if the usual production function of homogeneous of degree one is assumed.

$y_t = A_t^\alpha f(k_t)$, and $\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t - \delta k_t$. By assumptions (A2) and (A3), $A_t = \frac{\alpha f(k_t)}{m v f'(k_t)}$ because $\frac{\partial y_t}{m v \partial A_t} = \frac{\partial y_t}{\partial k_t} \Leftrightarrow \frac{\alpha}{mv} A_t^{\alpha-1} f(k_t) = A_t^\alpha f'(k_t)$. Since $A_t = \frac{\alpha f}{mvf'}$, then $y_t = A_t^\alpha f = \left(\frac{\alpha}{mv}\right)^\alpha \frac{f^{1+\alpha}}{f'^\alpha}$ and $\dot{A}_t = \frac{\alpha}{mv}\dot{k}_t\left(1 - \frac{f f''}{f'^2}\right)$.

2. The model in open economies

For simplicity, it is assumed that in the world there are only two countries, i.e., country 1 and country 2, in which parameters as well as population are identical except the degrees of relative risk aversion, and the growth rate of population is zero, i.e., $n_t = 0$. Let the degree of relative risk aversion in the country 1 be ε_1 and that in the country 2 be ε_2. Goods and services and capital are freely traded but labor is immobilized in each country.

The production function in the country 1 is $y_{1t} = A_t^\alpha f(k_{1t})$, and that in the country 2 is $y_{2t} = A_t^\alpha f(k_{2t})$ where y_{it} is outputs, k_{it} is capital inputs $(i = 1,2)$ in each country. In the paper, only the case of Harrod neutral technological progress such that $y_{it} = A_t^\alpha k_{it}^{1-\alpha}$ and thus $Y_{it} = K_{it}^{1-\alpha}(A_t L_t)^\alpha$ $(i = 1,2)$ is examined.[5]

Here, since both countries are free open economies, returns on investments in both countries are kept equal through international arbitration such that $\frac{\partial y_{1t}}{\partial k_{1t}} = \frac{1}{mv}\frac{\partial(y_{1t} + y_{2t})}{\partial A_t} = \frac{\partial y_{2t}}{\partial k_{2t}}$. An increase in A_t enhances outputs in both countries because of knowledge spillovers and thus returns on investing in A_t is described as $\frac{1}{mv}\frac{\partial(y_{1t} + y_{2t})}{\partial A_t}$. Thereby, $A_t = \frac{\alpha[f(k_{1t}) + f(k_{2t})]}{mvf'(k_{1t})}$

[5] As is well known, only Harrod neutral technological progress matches the stylized facts presented by Kaldor (1961).

$= \frac{\alpha[f(k_{1t}) + f(k_{2t})]}{mvf'(k_{2t})}$. Because this equation is always held through international arbitration, the following equations are also held: $k_{1t} = k_{2t}$, $y_{1t} = y_{2t}$ and $\dot{A}_{1t} = \dot{A}_{2t}$. Hence, $A_t = \frac{2\alpha f(k_{1t})}{mvf'(k_{1t})}$ $= \frac{2\alpha f(k_{2t})}{mvf'(k_{2t})}$.

Here, the balance of payments is introduced in the model. The balance on current account in the country 1 is τ_t and the balance on current account in the country 2 is $-\tau_t$. The sequence of τ_t is determined by the interaction of strategic behavior of both countries' households as Ghiglino (2002) and Sorger (2002) argue and thus each country can not control the sequence of τ_t independently. How the sequence of τ_t is determined is explained later.

The optimization problem in the country 1 is;

Max $E_0 \int_0^\infty u_1(c_{1t}) \exp(-\theta t) dt$,

subject to

(1) $\dot{k}_{1t} = y_{1t} + \left(\frac{\partial y_{2t}}{\partial k_{2t}} - \delta \right) \int_0^t \tau_s ds - \tau_t - c_{1t} - \frac{v \dot{A}_{1t}}{L_t} - \delta k_{1t}$,

and the optimization problem in the country 2 is;

Max $E_0 \int_0^\infty u_2(c_{2t}) \exp(-\theta t) dt$,

subject to

(2) $\dot{k}_{2t} = y_{2t} - \left(\frac{\partial y_{1t}}{\partial k_{1t}} - \delta \right) \int_0^t \tau_s ds + \tau_t - c_{2t} - \frac{v \dot{A}_{2t}}{L_t} - \delta k_{2t}$,

where $u_{it}\,(i = 1,2)$ is the utility function in each country, L_t is the population, and $\dot{A}_{it}\,(i = 1,2)$ is the increase of A_t by R&D in each country and $\dot{A}_t = \dot{A}_{1t} + \dot{A}_{2t}$. The accumulated current account balance $\int_0^t \tau_s ds$ mirrors international capital flows due to current account imbalances, i.e. a country with current account surpluses invest them in the other country. Since $\frac{\partial y_{1t}}{\partial k_{1t}}\left(=\frac{\partial y_{2t}}{\partial k_{2t}}\right)$ are returns on investments, $\left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t \tau_s ds$ and $\left(\frac{\partial y_{2t}}{\partial k_{2t}}-\delta\right)\int_0^t \tau_s ds$ represent international income receipts on assets or income payments on assets. Hence, $\tau_t-\left(\frac{\partial y_{2t}}{\partial k_{2t}}-\delta\right)\int_0^t \tau_s ds$ is the balance on goods and services in the country 1, and $\left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t \tau_s ds-\tau_t$ is the balance on goods and services in the country 2. Equations (1) and (2) implicitly assume that at $t = 0$ each country does not have any foreign asset.

Because the production function is Harrod neutral such that $y_{it}=A_t^{\alpha}k_{it}^{1-\alpha}$ and thus $Y_{it}=K_{it}^{1-\alpha}(A_tL_t)^{\alpha}$, and because $A_t=\frac{2\alpha f(k_{1t})}{mvf'(k_{1t})}=\frac{2\alpha f(k_{2t})}{mvf'(k_{2t})}$ and $f=k_{it}^{1-\alpha}$, then $A_t=\frac{2\alpha}{mv(1-\alpha)}k_{it}$, $\frac{f\,f''}{f'^2}=-\frac{\alpha}{1-\alpha}$ and $\frac{\partial y_{it}}{\partial k_{it}}=\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}$. Since $\dot{A}_{1t}=\dot{A}_{2t}$ and $\frac{\partial y_{1t}}{\partial k_{1t}}=\frac{\partial y_{2t}}{\partial k_{2t}}$,

$$\dot{k}_{1t}=y_{1t}+\left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t \tau_s ds-\tau_t-c_{1t}-\frac{v\dot{A}_{1t}}{L_t}-\delta k_{1t}$$

$$=y_{1t}+\left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t \tau_s ds-\tau_t-c_{1t}-\frac{v\dot{A}_t}{2L_t}-\delta k_{1t}$$

$$=\left(\frac{2\alpha}{mv}\right)^{\alpha}\frac{f^{1+\alpha}}{f'^{\alpha}}+\left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t \tau_s ds-\tau_t-c_{1t}-\frac{\alpha}{mL_t}\dot{k}_{1t}\left(1-\frac{f\,f''}{f'^2}\right)-\delta k_{1t}\text{. Hence,}$$

$$\dot{k}_{1t}=\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}\frac{f^{1+\alpha}}{f'^{\alpha}}+\left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t \tau_s ds-\tau_t-c_{1t}-\delta k_{1t}}{1+\frac{\alpha}{mL_t}\left(1-\frac{f\,f''}{f'^2}\right)}$$

$$= \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{1t}+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t \tau_s ds-\tau_t-c_{1t}\right\}.$$

Therefore the optimization problem in the economy 1 can be rewritten as

Max $E_0\int_0^{\infty} u_1(c_{1t})\exp(-\theta t)dt$,

subject to

$$\dot{k}_{1t} = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{1t}+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t \tau_s ds-\tau_t-c_{1t}\right\}.$$

Let Hamiltonian H_1 be

$$H_1 = u_1(c_{1t})\exp(-\theta t)+\lambda_{1t}\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{1t}+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t \tau_s ds-\tau_t-c_{1t}\right\}$$

where λ_{1t} is a costate variable, thus the optimality conditions for the economy 1 are

(3) $\dfrac{\partial u_1(c_{1t})}{\partial c_{1t}}\exp(-\theta t) = \dfrac{[mL_t(1-\alpha)+\alpha]}{mL_t(1-\alpha)}\lambda_{1t}$,

(4) $\dot{\lambda}_{1t} = -\dfrac{\partial H_1}{\partial k_{1t}}$,

(5) $\dot{k}_{1t} = \dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{1t}+\left[\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t \tau_s ds-\tau_t-c_{1t}\right\}$,

(6) $\lim_{t\to\infty}\lambda_{1t}k_{1t} = 0$.

Similarly, let Hamiltonian H_2 be

$$H_2 = u_2(c_{2t})\exp(-\theta t)+\lambda_{2t}\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{2t}-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t \tau_s ds+\tau_t-c_{2t}\right\}$$

where λ_{2t} is a costate variable, thus the optimality conditions for the economy 2 are

(7) $\dfrac{\partial u_2(c_{2t})}{\partial c_{2t}}\exp(-\theta t) = \dfrac{[mL_t(1-\alpha)+\alpha]}{mL_t(1-\alpha)}\lambda_{2t}$,

(8) $\dot{\lambda}_{2t} = -\dfrac{\partial H_2}{\partial k_{2t}}$,

(9) $\dot{k}_{2t} = \dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{2t} - \left[\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds + \tau_t - c_{2t}\right\}$,

(10) $\lim_{t\to\infty} \lambda_{2t} k_{2t} = 0$.

Since the problem of scale effects in endogenous growth model is not a focal point in the paper, it is assumed for simplicity that L_t is sufficiently large and thus $\dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha} = 1$ in the following sections. Hence, by the optimality conditions (3), (4) and (5), and by the optimality conditions (7), (8) and (9), the growth rates of consumption are

(11) $\dfrac{\dot{c}_{1t}}{c_{1t}} = \dfrac{\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta + \left[\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\dfrac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}} - \dfrac{\partial \tau_t}{\partial k_{1t}} - \theta}{\varepsilon_1}$ and

(12) $\dfrac{\dot{c}_{2t}}{c_{2t}} = \dfrac{\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \left[\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\dfrac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2t}} + \dfrac{\partial \tau_t}{\partial k_{2t}} - \theta}{\varepsilon_2}$.

3. The basic nature of the model

Before examining the balance of current account in the model, the basic nature of the model is investigated. The most important point that must be made clear beforehand is whether the model is an endogenous growth model that can achieve steady state growth paths and if it is such a model, in what condition steady state growth paths are achieved. To begin with, the transversality conditions are examined.

Lemma 1: The transversality conditions (6) $\lim_{t\to\infty} \lambda_{1t} k_{1t} = 0$ and (10) $\lim_{t\to\infty} \lambda_{2t} k_{2t} = 0$ are not satisfied if and only if $\lim_{t\to\infty}\left\{\left(\frac{\partial\tau_t}{\partial k_{1t}}-\frac{\tau_t}{k_{1t}}\right)-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\frac{\int_0^t\tau_s ds}{k_{1t}}\right]-\frac{c_{1t}}{k_{1t}}\right\}\geq 0$ or

$$\lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2t}}-\frac{\partial\tau_t}{\partial k_{2t}}\right)-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\int_0^t\tau_s ds}{k_{2t}}-\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2t}}\right]-\frac{c_{2t}}{k_{2t}}\right\}\geq 0\cdot$$

Proof: See Appendix 1.

Using lemma 1, an important nature of the model that the only growth path that satisfies all the optimality conditions is the path such that $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=$ constant is proved in the following lemma.

Lemma 2: If and only if $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=$ constant, all the optimality conditions are satisfied.

Proof: See Appendix 2.

Taking lemma 2 into consideration, it is highly likely that rational households behave so as to achieve a steady state growth path such that $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=$ constant. However, we must consider, beforehand, how the sequence of τ_t is determined. In the well-known paper of Becker (1980), it is proved that if households are purely price takers, the most patient household owns all wealth in the conventional Ramsey models if households have heterogeneous rates of time preference. Ghiglino (2002) predicts that it is likely that under appropriate assumptions the results in Becker (1980) still hold in endogenous growth models. Farmer and Lahiri (2004) show that in general, balanced growth equilibria do not exist in a multi-agent economy except for the special case where all agents have the same constant rate of time preference. The above results in the case of heterogeneous rates of time preference rate may hold in the case of heterogeneous degree of relative risk aversion.

Proposition 1: If each country sets τ_t without regarding the other countries optimality, then if and only if $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, all the optimality conditions for the representative households in the country 2 can be satisfied simultaneously.

Proof: See Appendix 3.

Proposition 1 may provide a possibility that the country 2 can escape the constraint of Becker (1980) if condition $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$ is satisfied. However, it is extremely difficult that the condition $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\dot{\tau}_t}{\tau_t}$ is satisfied because $\frac{\dot{c}_{2t}}{c_{2t}}$ and $\frac{\dot{\tau}_t}{\tau_t}$ are exogenously and independently given as shown in the proof, i.e., $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \theta}{\varepsilon_2}$ and $\frac{\dot{\tau}_t}{\tau_t} = \left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta$. Only in extremely lucky cases with the combination of exogenous parameters that satisfies the knife-edge condition $\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}[1-\varepsilon_2(1-\alpha)] - (1-\varepsilon_2)\delta - \theta = 0$, the condition $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\dot{\tau}_t}{\tau_t}$ is satisfied, which will be exceedingly rare. As a result, proposition 1 contends that virtually all the optimality conditions for the representative households in the country 2 can not be satisfied simultaneously. This result corresponds to the well-known result in case of Ramsey models with exogenous technologies and heterogeneous households shown in Becker (1980).

However, Sorger (2002) shows that in the case where a government levies a progressive income tax, or in the case where there are few households of each type and thus they are not simple price takers but play a Nash equilibrium, the results shown in Becker (1980) do not hold anymore. Ghiglino (2002) argues that the latter case in Sorger (2002) can be interpreted as a model of international trade with a common market simply by associating each household's type to a country with a national central planner or a representative household.

Based on the arguments in Sorger (2002) and Ghiglino (2002), in the model of two non-small countries with heterogeneous households in the paper, it is possible to assume that each representative household in two countries play a Nash equilibrium with regard to the sequence of τ_t in the optimization problems described in the previous sub-section. Because, by lemma 2, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}} =$ constant all the optimality conditions in each country are satisfied, which is a condition for a Nash equilibrium, then it is at least possible to assume the following behavior of households.

Assumption: For the initial capital stocks $k_{10} = k_{20}$ and knowledge/technology/idea A_0, households in both countries select a sequence of τ_t and set the initial consumptions so as to achieve a growth path that satisfies all the optimality conditions, i.e. a growth path of $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}} =$ constant, while firms in both countries adjust k_t so as to achieve $\frac{\partial y_{1t}}{\partial k_{1t}} = \frac{1}{mv}\frac{\partial (y_{1t} + y_{2t})}{\partial A_t} = \frac{\partial y_{2t}}{\partial k_{2t}}$.[6]

[6] Because $A_t = \frac{2\alpha}{mv(1-\alpha)}k_{it}$, then $A_0 = \frac{2\alpha}{mv(1-\alpha)}k_{i0}$.

If households in both countries behave according to the above assumption, achieved steady state growth paths have the following important feature.

Proposition 2: $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}}=\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=$ constant.

Proof: See Appendix 4.

Because A_t will not decrease, in the paper only the case such that $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}}=\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}>0$ is examined. Therefore, it is assumed that $\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta>0$, which means that the rate of return on capital exceeds the rate of time preference plus the growth rate of population and is quite natural economically.

III. THE TRADE BALANCE

In this section, the balance on international transactions in the model is examined. Since trade imbalances will grow infinitely, what should be disclosed firstly is whether their ratios to output explodes or stabilizes. This question is answered by the following corollary.

Corollary 1: $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\int_0^t\tau_s ds}{dt}}{\int_0^t\tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}}=\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=$ a positive constant.

Proof: See Appendix 5.

Because current account imbalances grow at the same rate with output, consumption, or capital eventually, the ratio of the balance on current account to output do not explode but stabilizes, i.e., it approaches to a unique finite value.

Corollary 1 shows that one country experiences a permanent current deficit and the other country experiences a permanent current account surplus. The next natural question is which country experience current account deficits and which country experience current account surpluses; the country with lower degree of relative risk aversion or the other. This question is answered by the following proposition 3.

Proposition 3: If $1 \le \varepsilon_1 < \varepsilon_2$ and other parameters are equal, then if $\theta > \left[1-(1-\alpha)\frac{\varepsilon_1+\varepsilon_2}{2}\right]\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \left(1-\frac{\varepsilon_1+\varepsilon_2}{2}\right)\delta$, the country 1 experiences a permanent current account deficits such that $\lim_{t\to\infty}\frac{\tau_t}{y_{1t}} =$ a negative constant, and the country 2 experiences a permanent current account surpluses such that $-\lim_{t\to\infty}\frac{\tau_t}{y_{2t}} =$ a positive constant.

Proof: See Appendix 6.

Proposition 3 indicates that, in some circumstances, the country with lower degree of relative risk aversion will experience a permanent current account deficit, and in reverse the country with higher degree of relative risk aversion will experience a permanent current account surplus. Because in general $\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} > \delta$, and because the coefficient of $\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}$

is $1-(1-\alpha)\dfrac{\varepsilon_1+\varepsilon_2}{2}$ and the coefficient of δ is $1-\dfrac{\varepsilon_1+\varepsilon_2}{2}$ in the condition $\theta > \left[1-(1-\alpha)\dfrac{\varepsilon_1+\varepsilon_2}{2}\right]\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\left(1-\dfrac{\varepsilon_1+\varepsilon_2}{2}\right)\delta$, then in the reasonable range of ε_i, the larger ε_1 and ε_2 are, the more easily the condition will be satisfied.

It should be noted that proposition 3 needs the supplementary condition such that $\varepsilon_i \geq 1$ and ε_1 and ε_2 are not so different in order to stay in the situation such that $\lim_{t\to\infty}\dfrac{\left|\int_0^t \tau_s ds\right|}{k_{1t}}=\lim_{t\to\infty}\dfrac{\left|\int_0^t \tau_s ds\right|}{k_{2t}}<1$. It is shown more correctly in the following corollary.

Corollary 2: If $1 \leq \varepsilon_1 < \varepsilon_2$ and other parameters are equal, then if

$$-\frac{\varepsilon_1-\varepsilon_2}{2}\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}<\left|\left[1-(1-\alpha)\frac{\varepsilon_1+\varepsilon_2}{2}\right]\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\left(1-\frac{\varepsilon_1+\varepsilon_2}{2}\right)\delta-\theta\right|,$$ then

$$\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{1t}}=\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{2t}}<1.$$

Proof: See Appendix 7.

Next, the balance on goods and services is examined.

Corollary 3: If $1 \leq \varepsilon_1 < \varepsilon_2$ and other parameters are equal, then the country 1 experiences a permanent surplus in goods and services trade such that $\lim_{t\to\infty}\dfrac{\tau_t-\left[\left(\dfrac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t \tau_s ds}{y_{1t}}$ = a positive constant, and the country 2 experiences a permanent deficit in goods and services trade such that $\lim_{t\to\infty} \frac{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t \tau_s ds-\tau_t}{y_{2t}}$ = a negative constant.

Proof: See Appendix 8.

The reason of this result is because there is difference between the return on $\int_0^t \tau_s ds$, i.e., $\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta$ and the growth rate of τ_s.[7]

VI. DISCUSSION

1. The degree of risk aversion

As was mentioned in introduction, during the last three decades, the U.S. has experienced persisting current account deficits and in contrast Japan has experienced persisting current account surpluses. Since the U.S. economy is the world's largest and the Japanese economy is the second largest, the model in the paper will be applied well to the international transactions between the U.S. and Japan. The model in the paper predicts that, in some circumstances, if the degree of relative risk aversion in Japan is higher than that in the U.S., the U.S. will experience a permanent current account deficit and Japan will experience a permanent current account surplus.

[7] Although eventually a country with relatively lower degree of relative risk aversion experiences trade surpluses as corollary 3 shows, it is easily understood by the proof of corollary 3 that, if the conditions in proposition 3 are satisfied, in reverse a country with relatively lower degree of relative risk aversion experiences trade deficits and a country with relatively higher degree of relative risk aversion experiences trade surpluses in early transition periods such that $|\tau_t| >> \frac{\dot{\tau}_t}{\tau_t}\left|\int_0^t \tau_s ds\right|$.

Are the Japanese really more risk aversive than people in the U.S.? In the well-known Szpiro (1986) that carried out a comprehensive estimation of degrees of relative risk aversion around the world, of the nine industrialized countries studied the Japanese have the highest degree of relative risk aversion. Szpiro (1986) reports that the degree of relative risk aversion in Japan is 2.76 while that in the U.S. is 1.19, i.e., the degree of relative risk aversion in Japan is double that in the U.S.[8] It is a well known fact that compared with households in the U.S. the Japanese invest their financial assets less in risky investments, which clearly indicates that the degree of risk aversion in Japan is much higher than that in the U.S.[9] With these data, Nakagawa and Shimizu (1999) concludes that the degree of relative risk aversion in Japan is two or three times higher than that in the U.S. Although there are various estimates of the degree of relative risk aversion in the U.S. and Japan and thus their absolute values are still inconclusive[10], the results in Szpiro (1986) or Nakagawa and Shimizu (1999) at least indicate that if the same estimation method is used in both countries, the estimate of the degree of relative risk aversion in Japan will be relatively higher than that in the U.S.

Furthermore, there is indirect evidence that the Japanese are highly risk aversive. Firstly heterogeneity in risk-averse behavior has recently been reported from the medical or genetical point of view. See e.g. Ono et al. (1997) and Nakamura et al. (1997). Those researches have

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[8] The estimates of degree of relative risk aversion reported in Szpiro (1986) are as follows:

	minimum	mean	maximum
The U.S.	1.02	1.19	1.41
Japan	1.99	2.76	4.01.

[9] According to the data on flows of funds published by FRB and the Bank of Japan, households in the U.S. allocate about half of their personal financial assets to “risky assets” like equities, but in contrast households in Japan allocate only about 10 % of their personal financial assets to risky assets and instead allocate about 60 % of them to “safe assets” like deposits and trusts.

[10] As Lucas (1987) argues, the degree of relative risk aversion in the U.S. may be much higher than 1 but less than 20.

shown that the genetic composition of the receptor for brain chemicals such as serotonin or dopamine differs widely among human races. They also show that most Japanese have inherited a certain type of receptor composition that produces more cautious and therefore more risk aversive characteristics, while many Americans have inherited the other type that produces less risk aversive characteristics.[11] These recent results from medical researches strengthen the appropriateness of the results from past researches on estimation of degree of risk aversion.

Secondly, Harashima (1998) shows that the so-called "Japanese economic system" or "Japanese capitalism" originates in the higher degree of risk aversion in the Japanese. The "Japanese economic system" can be regarded as a society with less open-minded convention in which relatively longer-term relationships within agents who are well-known each other, e.g. Keiretsu, are established and unknown agents are evaded. If the degree of risk aversion is higher, people will prefer maintaining the status quo to challenging new relationships, and thus existing relationships will continue relatively longer periods.

In sum, we can at least conclude that the Japanese are more cautious and have the relatively higher degree of risk aversion compared to the American, although the absolute values of degree of relative risk aversion in both countries are still inconclusive. The fact that two non-small countries have heterogeneous degrees of relative risk aversion is important, because the mechanism of trade imbalances shown in the paper may have worked between the U.S. and Japan.

2. Impacts on trade balances

The magnitude of impacts of heterogeneity in risk aversion on trade balances depends on

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[11] More precisely, the conclusions of Ono et al. (1997) and Nakamura et al. (1997) are as follows. There are two genetic types of serotonin transporter (5HTT): s-type and l-type. A person who inherits s-type tends to feel anxiety more and becomes more risk aversive. According to Ono et al. (1997), most Japanese have inherited only s-type and very few have l-type, but few Americans have inherited only s-type and over 30% have inherited only l-type.

parameter values, and which country experiences current account deficits is also determined by parameter values. In this sub-section, simple simulations with calibrated parameter values are carried out. By equations (17) and (18) in the proof of proposition 3 shown in Appendix 6,

$$(13)\quad \lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1-\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right]=-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}.$$

The limit of consumption growth rate $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}$ can be calculated by the relation such that $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\frac{\dot{c}_{1t}}{c_{1t}}+\frac{\dot{c}_{2t}}{c_{2t}}}{2}=\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta}{\frac{\varepsilon_1+\varepsilon_2}{2}}$. Thereby, with the appropriate calibrated parameters, i.e. the degrees of relative risk aversion in both countries ε_1 and ε_2, the rate of time preference θ, the rate of depreciation rate δ, the share of labor inputs α, and the output/capital ratio $\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}$, the limit of the balance on current account/capital ratio $\lim_{t\to\infty}\frac{\tau_t}{k_{1t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2t}}$ and thus the limit of the balance on current account/output ratio $\lim_{t\to\infty}\frac{\tau_t}{y_{1t}}=\lim_{t\to\infty}\frac{\tau_t}{y_{2t}}$ can be calculated by equation (13).

Here, the typical calibrated values of the deep parameters such that $\theta=0.05$, $\delta=0.05$, $\alpha=0.6$ and the capital/output ratio is 3 are selected.[12] The estimates of the limit of the balance on current account/capital ratio $\lim_{t\to\infty}\frac{\tau_t}{k_{1t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2t}}$ and thus the limit of the balance on current account/output ratio $\lim_{t\to\infty}\frac{\tau_t}{y_{1t}}=\lim_{t\to\infty}\frac{\tau_t}{y_{2t}}$ for three combinations of ε_1 and ε_2, i.e., (a) $\varepsilon_2=1.2\varepsilon_1$, (b) $\varepsilon_2=1.5\varepsilon_1$, and (c) $\varepsilon_2=2.0\varepsilon_1$, are shown in Table.

[12] The values are roughly same as those used for the calibration of the U.S. economy in Cooley and Prescott (1995).

The results make lucid some important features of the model. Firstly, if ε_1 is less than 3 in the cases of (a) and (b) and less than 2 in the case of (c), the country 1 experiences a permanent current account surplus, however, if ε_1 is 3 or more than 3 in the cases of (a) and (b) and 2 or more than 2 in the case of (c), the country 1 experiences a permanent current account deficits. Hence, whether the country 1 experiences a permanent current account surplus or deficit depends on the absolute value of the degree of relative risk aversion, which was predicted by proposition 3. In other words, there is a possibility that a country with relatively lower degree of relative risk aversion experiences a permanent current account deficit.

Secondly, several combinations of ε_1 and ε_2 result in unrealistic consequences: firstly, if the difference between ε_1 and ε_2 is large, e.g. the case of (c), accumulated balances on current account exceeds capital stocks, i.e., $\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{1t}} = \lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{2t}} > 1$, which was predicted by corollary 2, and secondly, if ε_1 is small, e.g. ε_1 is 3 or less than 3 in the cases of (a), the growth rate of consumption is unrealistically high. As a result, realistic results are observed in cases that ε_1 is more than 6 in the cases of (a) and (b). However, even in these cases, the magnitude of trade imbalance is several percent of GDP and thus larger than actually observed data, i.e., roughly 1 % of GDP (see figure 2). Hence, although the model shows a theoretical possibility of a permanent current account deficit, quantitatively the model may need some modifications to fit more closely with the actual data.

There are several possibilities that may solve this quantitative problem. Firstly, because there are many other factors that have influence on the trade balance and thus the observed actual data are the mix of effects of these many factors, there is a possibility that effects of heterogeneous degree of relative risk aversion are mostly canceled out by effects of other factors. Secondly, there is a possibility that the international financial market is not necessarily well integrated as the Feldstein-Horioka puzzle indicates.[13] In this situation, the volume of international financial transactions will be much smaller than that predicted in the model and thus trade imbalances will also be much smaller. Thirdly, there is a possibility that the difference of degree of relative risk aversion between the U.S. and Japan is not so large, e.g. that in Japan is 1.05 times that in the U.S. Hence trade imbalances may be much smaller because the model predicts that as the difference of degree of relative risk aversion becomes smaller, the trade imbalance becomes smaller. Fourthly, there is a possibility that there are other significant heterogeneities in deep parameters than the degree of relative risk aversion between the U.S. and Japan.

V. CONCLUDING REMARKS

The large current account deficit in the U.S. and the large current account surplus in Japan have continued during the past three decade, and the large bilateral trade deficit of the U.S. with Japan has also persisted. The conventional intertemporal approach to the current account can not explain these persisting large current account imbalances as Obstfeld and Rogoff (1995) note, and thus it needs the help of overlapping-generations variants of the intertemporal models with heterogeneous demographic changes. However, the conjecture that current account imbalances are generated by heterogeneous demographic changes has not been fully supported by empirical evidence and is still merely a theoretical possibility. The paper explores heterogeneity in degree of risk aversion as another possible source of persistent current account imbalances. The reason why the paper directs its attention to the degree of risk aversion is that in endogenous growth models the degree of relative risk aversion plays a crucial role for growth rates, and thus its heterogeneity significantly complicates movements of international

[13] See Feldstein and Horioka (1980).

transactions. Another reason is because it has been reported that the degree of relative risk aversion in Japan is relatively higher than that in the U.S., which implies a possibility that the large current account deficits in the U.S. and the large current account surpluses in Japan can be explained by the relatively higher degree of relative risk aversion in Japan than that in the U.S.

The model in the paper, which is based on the solution in Sorger (2002) and Ghiglino (2002) to the problem of heterogeneous households raised by Becker (1980), shows that the heterogeneity in degree of risk aversion generates persistent, more correctly permanent, current account imbalances. In some circumstances, a country with relatively lower degree of relative risk aversion experiences a permanent current account deficit, and in reverse a country with relatively higher degree of relative risk aversion experiences a permanent current account surplus. Nevertheless, current account deficits and surpluses do not explode but the ratios of deficits and surpluses to output asymptotically approach unique finite values and stabilize in both countries.

Hence, the model predicts that if the degree of relative risk aversion in Japan is truly relatively higher than that in the U.S., there is a possibility that the current account surplus persists in Japan permanently and the current account deficit persists in the U.S. permanently. The results of simple simulations with calibrated parameter values indicate that a country with relatively lower degree of relative risk aversion experience a permanent current account deficit for some combinations of degrees of relative risk aversion. However, the results also indicates that the magnitude of trade imbalance is larger than actually observed data, and thus although the model shows a possibility of a permanent current account deficit, quantitatively the model may need some modifications to fit more closely with the actual data. There are several possibilities that may solve this quantitative problem: 1) a possibility that effects of heterogeneous degree of relative risk aversion are mostly canceled out by effects of other factors, 2) a possibility that the international financial market is not necessarily well integrated as the Feldstein-Horioka puzzle indicates, 3) a possibility that the difference of degree of relative risk aversion between the U.S and Japan is not so large, and 4) a possibility that there are other significant heterogeneities in deep parameters than the degree of relative risk aversion between the U.S. and Japan.

Finally, the mechanism of trade imbalances presented in the paper of course does not deny the possibility of trade imbalances caused by heterogeneous demographic changes. Both mechanisms have probably worked simultaneously. Furthermore, there may be other heterogeneous parameters that play important roles for international transactions between other countries, e.g. heterogeneous technologies.

APPENDIX

1. Proof of lemma 1

By the optimality condition (5),

$$\dot{k}_{1t}=\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{1t}+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds-\tau_t-c_{1t}\text{, and thus}$$

$$\frac{\dot{k}_{1t}}{k_{1t}}=\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\int_0^t\tau_s ds}{k_{1t}}-\frac{\tau_t+c_{1t}}{k_{1t}}.$$

On the other hand, by the optimality condition (4),

$$\dot{\lambda}_{1t}=-\lambda_{1t}\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\frac{\partial\tau_t}{\partial k_{1t}}\right\},$$

and thus

$$\frac{\dot{\lambda}_{1t}}{\lambda_{1t}}=-\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\frac{\partial\tau_t}{\partial k_{1t}}\right\}.$$

Here,

$$\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{1t}}{\lambda_{1t}}+\frac{\dot{k}_{1t}}{k_{1t}}\right)=-\lim_{t\to\infty}\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\frac{\partial\tau_t}{\partial k_{1t}}\right\}$$

$$+\lim_{t\to\infty}\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\int_0^t\tau_s ds}{k_{1t}}-\frac{\tau_t+c_{1t}}{k_{1t}}\right\}$$

$$=\lim_{t\to\infty}\left\{\left(\frac{\partial\tau_t}{\partial k_{1t}}-\frac{\tau_t}{k_{1t}}\right)-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\frac{\int_0^t\tau_s ds}{k_{1t}}\right]-\frac{c_{1t}}{k_{1t}}\right\}.$$

Thereby if $\lim_{t\to\infty}\left\{\left(\frac{\partial\tau_t}{\partial k_{1t}}-\frac{\tau_t}{k_{1t}}\right)-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\frac{\int_0^t\tau_s ds}{k_{1t}}\right]-\frac{c_{1t}}{k_{1t}}\right\}<0$, then $\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{1t}}{\lambda_{1t}}+\frac{\dot{k}_{1t}}{k_{1t}}\right)<0$.

Similarly if $\lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2t}}-\frac{\partial\tau_t}{\partial k_{2t}}\right)-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\int_0^t\tau_s ds}{k_{2t}}-\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2t}}\right]-\frac{c_{2t}}{k_{2t}}\right\}<0$, then $\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{2t}}{\lambda_{2t}}+\frac{\dot{k}_{2t}}{k_{2t}}\right)<0$.

Hence, the transversality conditions (6) $\lim_{t\to\infty}\lambda_{1t}k_{1t}=0$ and (10) $\lim_{t\to\infty}\lambda_{2t}k_{2t}=0$ are not satisfied if and only if $\lim_{t\to\infty}\left\{\left(\frac{\partial\tau_t}{\partial k_{1t}}-\frac{\tau_t}{k_{1t}}\right)-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\frac{\int_0^t\tau_s ds}{k_{1t}}\right]-\frac{c_{1t}}{k_{1t}}\right\}\geq 0$ or

$$\lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2t}}-\frac{\partial\tau_t}{\partial k_{2t}}\right)-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\int_0^t\tau_s ds}{k_{2t}}-\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2t}}\right]-\frac{c_{2t}}{k_{2t}}\right\}\geq 0.$$

Q.E.D.

2. Proof of lemma 2

(Step 1) By equations (11) and (12),

$$\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{1t}}-\theta}{\varepsilon_1}$$ and

$$\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2t}}+\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{2t}}-\theta}{\varepsilon_2}.$$

On the other hand,

$$\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta+\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1t}}-\lim_{t\to\infty}\frac{\tau_t+c_{1t}}{k_{1t}} \quad \text{and}$$

$$\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2t}}+\lim_{t\to\infty}\frac{\tau_t-c_{2t}}{k_{2t}}.$$

(Step 2) By equations (1) and (2), $c_{1t}-c_{2t}=2\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-n-\delta\right]\int_0^t \tau_s ds-\tau_t\right\}$. Hence, if $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=$ constant, then $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=$ $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$. Thus, by lemma 1 the transversality conditions (6) and (10) are satisfied, and also all the other optimality conditions are satisfied.

However, if $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}\neq\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}$, then $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}\neq\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$. Thus by lemma 1, for both countries to satisfy the transverality conditions, it is necessary that $\lim_{t\to\infty}\frac{c_{1t}}{k_{1t}}=\infty$ or $\lim_{t\to\infty}\frac{\dot{c}_{2t}}{k_{2t}}=\infty$, which violates the optimality conditions (5) or (9). If $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}\neq\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}$ or $\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}\neq\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, the transversality conditions (6) or (10), or the optimality conditions (5) or (9) is violated.

As a result, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=$ constant, all the optimality conditions are satisfied.

Q.E.D.

3. Proof of proposition 1

(Step 1) In this case, τ_t can be seen as a control variable for each country. Hence, the optimality condition

$$(14)\quad \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t}=1$$

is added to the optimality conditions for the country 1, and the optimality condition

$$(15)\quad \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t}=1$$

is added to the optimality conditions for the country 2. The optimality conditions (14) and (15) are identical, and by them, $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}}-\frac{\partial \tau_t}{\partial k_{1t}}=0$ and thus the optimal consumption growth rates are $\frac{\dot{c}_{1t}}{c_{1t}}=\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta}{\varepsilon_1}$ and $\frac{\dot{c}_{2t}}{c_{2t}}=\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta}{\varepsilon_2}$.

Thereby $\frac{\dot{c}_{1t}}{c_{1t}}>\frac{\dot{c}_{2t}}{c_{2t}}$.

On the other hand, by the optimality conditions (14) and (15),

$$\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t}=\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial t}}{\frac{\partial \tau_t}{\partial t}}=1 \quad \text{and thus}$$

$$\frac{\dot{\tau}_t}{\tau_t}=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta .$$

(Step 2) For the country 1, it is necessary to hold $\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}$ for the optimality conditions (3), (4), (5) and (6) to be satisfied, and for the country 2, it is necessary to hold $\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$ for the optimality conditions (7), (8), (9) and (10) to be satisfied.

Here, because the equations such that $k_{1t} = k_{2t}$, $y_{1t} = y_{2t}$ and $\dot{A}_{1t} = \dot{A}_{2t}$ are kept by firms at any time, the households in the country 2 must set the higher initial consumption level than that in the country 1 by importing goods and services from the country 1 to hold $\frac{\dot{c}_{1t}}{c_{1t}} > \frac{\dot{c}_{2t}}{c_{2t}}$.

Therefore it must be that $\tau_t > 0$ and $\frac{\dot{c}_{1t}}{c_{1t}} \geq \frac{\dot{k}_{1t}}{k_{1t}} = \frac{\dot{k}_{2t}}{k_{2t}} \geq \frac{\dot{c}_{2t}}{c_{2t}}$.

(Step 3) There are three cases: i) $\frac{\dot{\tau}_t}{\tau_t} > \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, ii) $\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, and iii) $\frac{\dot{\tau}_t}{\tau_t} < \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$.

In the cases of i), eventually $\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{2t}} > 1$ and thus the optimality condition (7) or (15) is violated. In the case of iii), because $\frac{\dot{c}_{1t}}{c_{1t}} \geq \frac{\dot{k}_{1t}}{k_{1t}} = \frac{\dot{k}_{2t}}{k_{2t}} \geq \frac{\dot{c}_{2t}}{c_{2t}}$, then $\frac{\dot{c}_{1t}}{c_{1t}} > \frac{\dot{\tau}_t}{\tau_t}$, and thus the ratio of c_{1t} to τ_t diminishes to zero as time passes and the trade balance becomes negligible for the country 1. In this situation the country 1 selects a path such that $\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}$, thereby it is not possible for the country 2 to hold $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} > \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}$. In the case of ii), because the country 2 must hold $\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$ for the optimality conditions (7), (8), (9) and (10) to be satisfied, only and only if $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, it is possible for the country 2 to satisfy all the optimality conditions by setting appropriate initial values of c_{10} and τ_0.

Q.E.D.

4. Proof of proposition 2

(Step 1) As for y_{1t}, because $y_{1t} = A_t^{\alpha} k_{1t}^{1-\alpha}$,

(16) $\dot{y}_{1t} = \left(\dfrac{A_t}{k_{1t}}\right)^{\alpha}\left[(1-\alpha)\dot{k}_{1t} + \alpha \dfrac{k_{1t}}{A_t}\dot{A}_t\right]$.

Since, $\dot{A}_t = \dfrac{2\alpha}{mv}\dot{k}_{1t}\left(1 - \dfrac{f f''}{f'^2}\right) = \dfrac{2\alpha}{mv(1-\alpha)}\dot{k}_{1t}$, then $\dot{y}_{1t} = \dot{k}_{1t}\left(\dfrac{A_t}{k_{1t}}\right)^{\alpha}\left[(1-\alpha) + \dfrac{2\alpha^2}{mv(1-\alpha)}\dfrac{k_{1t}}{A_t}\right]$, and

thus $\dfrac{\dot{y}_{1t}}{y_{1t}} = \dfrac{\dot{k}_{1t}}{k_{1t}}\left[(1-\alpha) + \dfrac{2\alpha^2}{mv(1-\alpha)}\dfrac{k_{1t}}{A_t}\right]$. Because $A_t = \dfrac{2\alpha}{mv(1-\alpha)}k_{1t}$, $\dfrac{\dot{y}_{1t}}{y_{1t}} = \dfrac{\dot{k}_{1t}}{k_{1t}}\left[(1-\alpha)+\alpha\right] = \dfrac{\dot{k}_{1t}}{k_{1t}}$.

Hence $\lim_{t\to\infty}\dfrac{\dot{y}_{1t}}{y_{1t}} = \lim_{t\to\infty}\dfrac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\dfrac{\dot{k}_{1t}}{k_{1t}} > 0$.

Because $y_{1t} = y_{2t}$, then $\lim_{t\to\infty}\dfrac{\dot{y}_{1t}}{y_{1t}} = \lim_{t\to\infty}\dfrac{\dot{y}_{2t}}{y_{2t}} = \lim_{t\to\infty}\dfrac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\dfrac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\dfrac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\dfrac{\dot{k}_{2t}}{k_{2t}}$.

(Step 2) As for A_t, by equation (16) and $\dot{A}_t = \dfrac{2\alpha}{mv(1-\alpha)}\dot{k}_{1t}$, $\dot{y}_{1t} = \dot{A}_t\left(\dfrac{A_t}{k_{1t}}\right)^{\alpha}\left[\dfrac{mv(1-\alpha)^2}{2\alpha} + \alpha\dfrac{k_{1t}}{A_t}\right]$

and thus $\dfrac{\dot{y}_{1t}}{y_{1t}} = \dfrac{\dot{A}_t}{k_{1t}}\dfrac{mv(1-\alpha)^2}{2\alpha} + \alpha\dfrac{\dot{A}_t}{A_t}$. Because $\dot{A}_t = \dfrac{2\alpha}{mv(1-\alpha)}\dot{k}_{1t}$, then $\dfrac{\dot{y}_{1t}}{y_{1t}} = (1-\alpha)\dfrac{\dot{k}_{1t}}{k_{1t}} + \alpha\dfrac{\dot{A}_t}{A_t}$.

Hence, $\dfrac{\dot{y}_{1t}}{y_{1t}} = \dfrac{\dot{k}_{1t}}{k_{1t}} = (1-\alpha)\dfrac{\dot{k}_{1t}}{k_{1t}} + \alpha\dfrac{\dot{A}_t}{A_t}$ and thus $\dfrac{\dot{k}_{1t}}{k_{1t}} = \dfrac{\dot{A}_t}{A_t}$.

Since $k_{1t} = k_{2t}$, thus $\lim_{t\to\infty}\dfrac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\dfrac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\dfrac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\dfrac{\dot{k}_{2t}}{k_{2t}} = \lim_{t\to\infty}\dfrac{\dot{y}_{1t}}{y_{1t}} = \lim_{t\to\infty}\dfrac{\dot{y}_{2t}}{y_{2t}} = \lim_{t\to\infty}\dfrac{\dot{A}_t}{A_t}$.

Q.E.D.

5. Proof of corollary 1

By (step 2) in the proof of lemma 2, $\lim_{t\to\infty}\dfrac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\dfrac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\dfrac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\dfrac{\dfrac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds}$. Hence, by proposition 2, $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}}=\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=$ a positive constant.

Q.E.D.

6. Proof of proposition 3

(Step 1) Because $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}$, then by equations (11) and (12),

$$\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\varepsilon_2\lim_{t\to\infty}\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}+\varepsilon_1\lim_{t\to\infty}\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2t}}\right]-\left(\varepsilon_2\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{1t}}+\varepsilon_1\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{2t}}\right)$$

$$=(\varepsilon_1+\varepsilon_2)\lim_{t\to\infty}\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\frac{\partial\tau_t}{\partial k_{1t}}\right\}$$

$$\left(=(\varepsilon_1+\varepsilon_2)\lim_{t\to\infty}\left\{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2t}}-\frac{\partial\tau_t}{\partial k_{2t}}\right\}\right)=(\varepsilon_1-\varepsilon_2)\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}.$$

Hence, because $\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta>0$ by assumption, then

(17) $\lim_{t\to\infty}\left\{\frac{\partial\tau_t}{\partial k_{1t}}-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t\tau_t ds\right)}{\partial k_{1t}}\right\}=-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}$ = a positive constant.

(Step 2) By corollary 1, $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial t}}{\int_0^t\tau_s ds}$ = a positive constant. Hence, for sufficiently large $t = t'$, $\tau_t = \tau_{t'} \exp\left(\frac{\dot{\tau}_t}{\tau_t} t\right)$ and thus $\lim_{t\to\infty} \frac{\int_0^t \tau_s ds}{\tau_t} = \frac{1}{\lim_{t\to\infty} \frac{\dot{\tau}_t}{\tau_t}} = \frac{1}{\lim_{t\to\infty} \frac{\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial t}}{\int_0^t \tau_s ds}}$ = a positive constant.

Here, because $\lim_{t\to\infty} \frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty} \frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty} \frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty} \frac{\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial t}}{\int_0^t \tau_s ds}$ by corollary 1,

(18) $$\lim_{t\to\infty}\left\{\frac{\partial \tau_t}{\partial k_{1t}} - \left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}}\right\}$$

$$= \lim_{t\to\infty}\left\{\frac{\frac{\dot{\tau}_t}{\tau_t}}{\frac{\dot{k}_{1t}}{k_t}}\frac{\tau_t}{k_{1t}} - \left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\frac{\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial t}}{\int_0^t \tau_s ds}}{\frac{\dot{k}_{1t}}{k_t}}\frac{\int_0^t \tau_s ds}{k_{1t}}\right\}$$

$$= \lim_{t\to\infty}\left\{\frac{\tau_t}{k_{1t}} - \left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\int_0^t \tau_s ds}{k_{1t}}\right\} = \lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1 - \frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta}{\frac{\dot{\tau}_{1t}}{\tau_{1t}}}\right]$$

$$= \lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1 - \frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right] = \text{a positive constant.}$$

Here, $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\frac{\dot{c}_{1t}}{c_{1t}} + \frac{\dot{c}_{2t}}{c_{2t}}}{2} = \frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \theta}{\frac{\varepsilon_1 + \varepsilon_2}{2}}$. Hence, if

$$\frac{\frac{\varepsilon_1+\varepsilon_2}{2}\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]}{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta}>1$$ and thus if

$\theta>\left[1-(1-\alpha)\frac{\varepsilon_1+\varepsilon_2}{2}\right]\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\left(1-\frac{\varepsilon_1+\varepsilon_2}{2}\right)\delta$, then $\lim_{t\to\infty}\left[1-\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right]<0$.

Therefore if $1\le\varepsilon_1<\varepsilon_2$ and if $\theta>\left[1-(1-\alpha)\frac{\varepsilon_1+\varepsilon_2}{2}\right]\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\left(1-\frac{\varepsilon_1+\varepsilon_2}{2}\right)\delta$, $\lim_{t\to\infty}\frac{\tau_t}{k_{1t}}$ = a negative constant, and thus $\lim_{t\to\infty}\frac{\tau_t}{y_{1t}}$ = a negative constant and $-\lim_{t\to\infty}\frac{\tau_t}{y_{2t}}=$ a positive constant.

Q.E.D.

7. Proof of corollary 2

By equations (17) and (18),

$$\lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1-\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right]=-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}.$$

Because $\lim_{t\to\infty}\frac{\frac{\dot{\tau}_t}{\tau_t}\int_0^t\tau_s ds}{\tau_t}=1$ and $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}$,

$$\lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1-\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right]=\lim_{t\to\infty}\frac{\frac{\dot{\tau}_t}{\tau_t}\int_0^t\tau_s ds}{k_{1t}}\left[1-\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta}{\frac{\dot{\tau}_t}{\tau_t}}\right]$$

$$=\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1t}}\left[\frac{\dot{\tau}_t}{\tau_t}-\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}+\delta\right]=-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}.$$

Since $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{\dot{c}_{2t}}{c_{2t}}+\frac{\dot{c}_{2t}}{c_{2t}}}{2}=\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta}{\frac{\varepsilon_1+\varepsilon_2}{2}}$,

$$\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{1t}}=\frac{-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}}{\left|\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}-\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}+\delta\right|}=\frac{-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}}{\left|\frac{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta}{\frac{\varepsilon_1+\varepsilon_2}{2}}-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\right|}$$

$$=\frac{-\frac{\varepsilon_1-\varepsilon_2}{2}\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}}{\left|\left[1-(1-\alpha)\frac{\varepsilon_1+\varepsilon_2}{2}\right]\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\left(1-\frac{\varepsilon_1+\varepsilon_2}{2}\right)\delta-\theta\right|}.$$

Hence, if $-\frac{\varepsilon_1-\varepsilon_2}{2}\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}<\left|\left[1-(1-\alpha)\frac{\varepsilon_1+\varepsilon_2}{2}\right]\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\left(1-\frac{\varepsilon_1+\varepsilon_2}{2}\right)\delta-\theta\right|$,

then $\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{1t}}<1$.

Q.E.D.

8. Proof of corollary 3

By equation (18),

$\lim_{t\to\infty}\left\{\frac{\tau_t}{k_{1t}}-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\int_0^t \tau_t ds}{k_{1t}}\right\}=$ a positive constant. Hence,

$$\lim_{t\to\infty}\frac{\tau_t-\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds}{y_{1t}} = \text{a positive constant, and}$$

$$\lim_{t\to\infty}\frac{\left[\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds-\tau_t}{y_{2t}} = \text{a negative constant.}$$

Q.E.D.

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Figure 1: Trade Imbalances

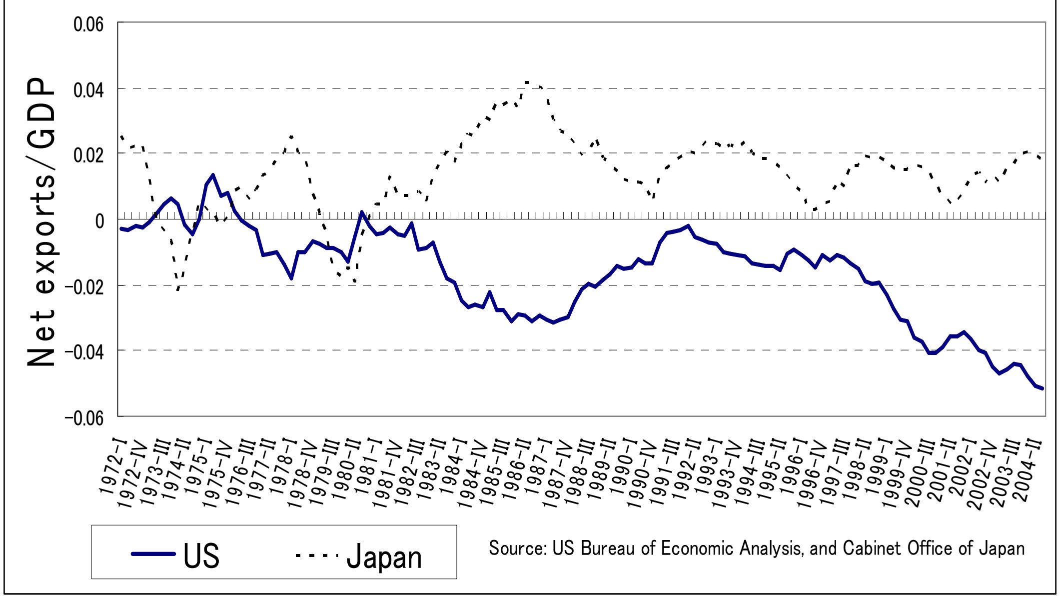
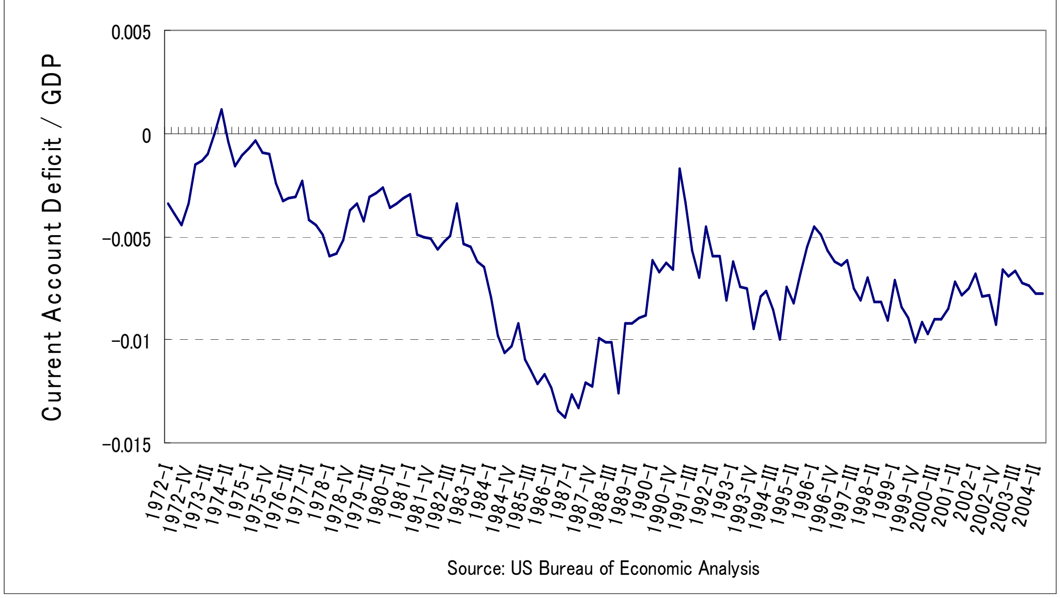


Figure 2: The US Current Account Deficit with Japan



Table

(a) $\varepsilon_2 = 1.2\,\varepsilon_1$

ε_1	ε_2,	$\lim_{t\to\infty} \frac{\dot{c}_{1t}}{c_{1t}}$	$\lim_{t\to\infty} \frac{\tau_t}{y_{1t}}$	$\lim_{t\to\infty} \frac{\left\lvert \int_0^t \tau_s ds \right\rvert}{k_{1t}}$
1.0	1.2	0.21	0.10	0.16
2.0	2.4	0.11	0.30	0.93
3.0	3.6	0.071	-0.36	-1.68
4.0	4.8	0.053	-0.11	-0.70
5.0	6.0	0.042	-0.066	-0.52
6.0	7.2	0.035	-0.047	-0.44
7.0	8.4	0.030	-0.036	-0.40
8.0	9.6	0.027	-0.030	-0.37
9.0	10.8	0.024	-0.025	-0.35
10.0	12.0	0.021	-0.022	-0.34

(b) $\varepsilon_2 = 1.5\,\varepsilon_1$

ε_1	ε_2,	$\lim_{t\to\infty} \frac{\dot{c}_{1t}}{c_{1t}}$	$\lim_{t\to\infty} \frac{\tau_t}{y_{1t}}$	$\lim_{t\to\infty} \frac{\left\lvert \int_0^t \tau_s ds \right\rvert}{k_{1t}}$
1.0	1.5	0.19	0.25	0.45
2.0	3.0	0.093	1.31	4.67
3.0	4.5	0.062	-0.41	-2.21
4.0	6.0	0.047	-0.18	-1.27
5.0	7.5	0.037	-0.11	-1.01
6.0	9.0	0.031	-0.083	-0.89
7.0	10.5	0.027	-0.066	-0.82
8.0	12.0	0.023	-0.054	-0.78
9.0	13.5	0.021	-0.046	-0.75
10.0	15.0	0.019	-0.040	-0.72

(c) $\boldsymbol{\varepsilon_2 = 2.0\,\varepsilon_1}$

ε_1	ε_2,	$\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}$	$\lim_{t\to\infty}\frac{\tau_t}{y_{1t}}$	$\lim_{t\to\infty}\frac{\left\lvert\int_0^t \tau_s ds\right\rvert}{k_{1t}}$
1.0	2.0	0.16	0.50	1.08
2.0	4.0	0.078	-3.27	-14.00
3.0	6.0	0.052	-0.38	-2.47
4.0	8.0	0.039	-0.20	-1.75
5.0	10.0	0.031	-0.14	-1.49
6.0	12.0	0.026	-0.11	-1.35
7.0	14.0	0.022	-0.085	-1.27
8.0	16.0	0.019	-0.071	-1.22
9.0	18.0	0.017	-0.061	-1.18
10.0	20.0	0.016	-0.054	-1.15