Policy Synchronization and Staggering in a Dynamic Model of Strategic Trade*

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(Preliminary and incomplete)

Abstract

This paper studies steady-state, Markov-Perfect strategic trade policy when (infinitely lived) governments are committed to trade policy during two periods. We compare the case in which both governments choose their export subsidies in the same periods (synchronization) versus the case in which they set them in alternate periods (staggering). We find that, under Cournot competition, welfare is higher when both governments synchronize their choice of export-promoting policy, since export subsidies are lower (closer to the cooperative solution) than when governments set them sequentially. Retaliation is therefore stronger and more harmful when governments alternate as (temporary) Stackelberg leaders. Synchronization is the equilibrium of the pre-commitment game in which both governments choose when to set their export subsidies. These results are robust to having stochastic length of commitment period and to allowing predetermined but *flexible* choice of subsidies by both governments. We also analyze the results of policy-staggering under alternative modes of competition and the possibility of firms investing in R&D.

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1 Introduction

An important contribution in international trade theory is the inclusion of imperfect competition in international trade. A natural result of this integration has been the study of trade policies, carried out by national governments in the presence of international imperfect competition. One particular type of model of international trade with imperfect competition is the "third country model," where home firms sell all their production to a third market. This particular simplification has stressed the potential role of governments to shift profits from foreign to domestic firms, highlighting how apparently modest interventions by governments can have large effects on profits, national welfare and export market shares.

The use of strategic trade policy by governments usually take place between a small number of players with oligopolistic industries, and extend over long periods of time through a protracted retaliation process.[1] Policy measures are usually well prepared and planned for a long time and generally last for several years or even government administrations, and the issue of the synchronization or not of these policy measures might become relevant. For example, governments can always retaliate a trade policy set up by a rival government. This retaliation will take time to be put into practice, and once set, will be in place for some time, even after the foreign government has had time to change his policy again. This means that both governments need to be aware that when they set their policy, they will influence foreign governments and at the same time, they will be inactive during some time after a possible retaliation. In static models, retaliation by the foreign government does not take into account that in turn the home government may counter-retaliate in (a not so distant) future.

In spite of these features, most of the literature on strategic trade policy has focused on static, one-period games, extended only to allow for multiple stages in which firms and governments interact. Even multi-stage models are, however, one-shot games in that firms and governments only make decisions about a particular variable (subsidies, output, R&D, etc.) only once.[2] These models cannot capture the continuous nature of rivalries between firms and between governments. Some models which have tried to extend the strategic trade literature to truly dynamic settings have adopted the repeated game framework for their analysis.[3] These models usually have the problem of multiplicity of equilibria which reduces their predictive power.

In this paper we develop an infinite horizon third-country model with two governments and two firms. Governments live forever, and firms live for one period, being replaced every period.[4] Therefore, all the action takes place at the government level. We restrict governments to use Markov Strategies, avoiding multiple equilibria as in models using repeated versions of the static games.

[1]See, for example the survey by Krishna and Thursby (1990).

[2]See, for example the models of Spencer and Brander (1983), Brander and Spencer (1985), Eaton and Grossman (1986), Bagwell and Staiger (1994), Neary (1994), Maggi (1996) and Neary and Leahy (2000). In all these models, key to the strategic trade literature, firms and governments only have to choose a particular variable once in the game.

[3]Some examples are Davidson (1984) and Rotemberg and Saloner (1989) who consider how trade policy (set once at the beginning of the game) may affect the ability of firms to collude using trigger strategies in a repeated game. Collie (1993) considers the opposite case in which long lived governments play an infinitely repeated version of the Brander and Spencer (1985) export-subsidy game where firms compete à la Cournot.

[4]Allowing for infinitely lived firms significantly increases the complexity of the model, increasing the state space to four dimensions. An alternative assumption could be that firms are "myopic" and therefore take decisions only for the current period.

The benchmark case is the case of Cournot competition every period at the firm level, preceded by government policy-setting. To introduce the possibility of retaliation, we assume, in the benchmark case, that governments commit to their trade policies for two periods. This ensures that by the time the foreign government reacts, the home government has not changed his policy already.[5]

We examine the cases in which both governments set their policy in the same periods (synchronization) and on alternating periods (staggering). Note that when governments alternate their policy setting (staggered game) they in practice alternate as Stackelberg leaders: when the foreign government decides on their next subsidy the domestic government is committed to its present subsidy, and therefore influences the choice of the foreign government. Governments switch their role of Stackelberg leader and follower in the following period.

We find that, when firms compete a la Cournot in the third market, governments have an incentive to set higher subsidies in the staggered game than in the synchronized game (which coincides with subsidies in the static game). The reason is simple: each government knows that it will play as a Stackelberg leader during one period. As output subsidies are strategic substitutes in this setting, they have an incentive to impose a subsidy beyond the static Nash equilibrium level, in order to induce the rival government to lower theirs and increase domestic welfare. As roles are reversed in each period, both governments are locked into this mutually damaging policy outcome. Therefore, we find that the introduction of dynamic considerations reinforces the case for strategic trade policy.

We extend this benchmark case in a number of directions. First, we try to endogenize the choice of synchronization versus staggering in a game of timing. We find that synchronization arises as the Nash equilibrium of the timing game played before the policy game starts. Second, we extend the model to different modes of competition and investment capabilities by firms. We find that government policy (either output subsidies or R&D subsidies) are higher or lower than the static equivalent depending on whether subsidies are strategic substitutes or complements. When subsidies are strategic complements, the staggered game yields lower subsidies than the static equivalent, therefore getting both governments closer to their joint welfare–maximum.

The third extension allows the length of commitment to be stochastic, instead of two periods. This reflects instances where governments are uncertain about their political process in their rival country's. We also find that subsidies in the Cournot case are higher than the static output subsidies. More interestingly, subsidies are decreasing in the expected length of the commitment period, showing that the expected time a government will be tied to policy determines how far a it wants to set its "dynamic" subsidy away from the static one.

The fourth extension allows for governments to remain inactive, as an alternative to setting a subsidy when their time to act arrives. As in the case of endogenous timing, we show that synchronization (but not staggering) is robust in the sense of Maskin and Tirole (1988b).

Finally, we extend the model making the choice of governments slightly more flexible. In particular, governments commit to their policy for the next two periods, although we allow them to set their subsidies at different levels for the two periods. We call this predetermined (but flexible) trade policy. We show that governments actually prefer to set their subsidies differently in their two

[5]Governments may want to commit their policies for more than one period because they face implementation costs or because they may want to gain a reputation through commitment.

periods when they stagger their timing. This is easy to see since governments will like to set different subsidies when they are acting as Stackelberg leaders or Stackelberg followers. Therefore we have endogenous policy cycles in the staggered game. As before, when we endogenize the timing of policy setting, governments prefer to synchronize their timing, as they get closer to the cooperative (joint welfare maximizing) solution.

There are few dynamic models of strategic trade beyond the repeated game framework. Dockner and Haug (1990) study trade policy where firms interact in a differential game, with governments setting their policy at the beginning of the game. Cheng (1987) studies a dynamic version of Spencer and Brander (1983) and finds that R&D subsidies are also optimal in the dynamic setting.

Perhaps the closest to our work is Tanaka (1988). He studies an infinite horizon game where firms stagger their choice of quantity produced and they have to commit to that choice for two periods. Governments only act once and they commit simultaneously to their whole path of export subsidies at the beginning of the game (open–loop equilibrium). He only analyzes Cournot competition between firms, the case of staggered quantity setting, and a model where firms only decide on short-term variables (output) without previous investment in long-term variables (e.g. capacity or R&D). He finds that firms are more sensitive to export subsidies in this dynamic setting, which lead to smaller equilibrium subsidies set by governments at the beginning of the game. As in Maskin and Tirole (1987), he finds that firms behave more competitively and therefore output and welfare are both higher than under the static version of the game.[6] In our paper, when governments stagger their subsidy setting, subsidies and output are higher than in the static Cournot game, but this means a lower welfare for both exporting countries.

The remainder of the paper is organized as follows. Section 2 presents the benchmark case with synchronized and staggered subsidy-setting, and develops the first approximation to a game of timing. Section 3 presents in detail the extensions outlined before. Section 4 concludes.

2 Model

We use the standard third-country model allowing for investment in R&D by firms before competing in the third market but after governments set their policy (as in Spencer and Brander (1983)). Each of two exporting countries has a firm competing for exports to a third country. We assume that the third country's consumers, where exports are sold by both countries, have a linear demand, and firms have a constant marginal cost of production:

$$p_i(x_i, x_j) = a - b(x_i + \gamma x_j) \tag{1}$$

$$C(x_i, \Delta_i) = (c - \theta\Delta_i)\, x_i \tag{2}$$

Firm i's profits are, therefore:

$$\Pi_i^x(x_i, x_j, \Delta_i) = [(a - b(x_i + \gamma x_j) - (c - \theta\Delta_i - s_i)]\, x_i - \phi\frac{\Delta_i^2}{2} + z_i\Delta_i \tag{3}$$

[6] The equilibrium subsidy in Tanaka (1988) is lower than under the static version of the model, and therefore this increases welfare in both exporting countries.

where γ measures the degree of product differentiation, Δ_i is investment in R&D and $\eta = \frac{\theta^2}{\phi b}$ measures the effectiveness of R&D as in Leahy and Neary (1996).

Each period, governments and firms play a three stage game in which governments simultaneously choose their subsidies, firms then choose the level of R&D (Δ_i) and then compete in the market by choosing simultaneously their strategic variable (prices or quantities). This gives rise to four possible scenarios depending on whether firms compete in quantities (Cournot) or in prices (Bertrand) and on whether governments subsidize exports ($z_i = 0$) or subsidize R&D ($s_i = 0$). We also examine other two possible scenarios in which firms do not invest in R&D before competing in the market (in prices or quantities), which is equivalent to set $\Delta_i = 0$ for both firms.

Firms only live for one period and a new firm is reborn each period, while governments are infinitely lived.[7] Notice that, since governments act first in every period and firms live for only one period, quantities, prices and R&D will be chosen by firms as a function of subsidies set by governments in the first stage of that period. Therefore, from the perspective of governments, the profit function in (3) becomes a function of the subsidies chosen by both governments.[8]. If governments set subsidies to output, then $z_i = 0$ in (3) and we can write government i's utility function as the domestic firm's profits minus the cost of the subsidy:

$$f_i(s_i, s_{-i}) = \hat{\Pi}_i^x(s_i, s_{-i}) - s_i \cdot x_i(s_i, s_{-i})$$

where $\hat{\Pi}_i^x(s_i, s_{-i}) = \Pi_i^x(x_i^*(s_i, s_{-i}), x_j^*(s_i, s_{-i}), \Delta_i^*(s_i, s_{-i}))$ is the domestic firm's profit after replacing the equilibrium values of quantities and R&D investment. Similarly, if governments set subsidies to R&D, then $s_i = 0$ and

$$f_i(z_i, z_{-i}) = \hat{\Pi}_i^x(z_i, z_{-i}) - z_i \cdot \Delta_i(z_i, z_{-i})$$

As in Maskin and Tirole (1988 a,b) we assume that long lived agents (in this case, both governments) have to set their strategic variable (output or R&D subsidies) for the subsequent two periods.[9] Since firms live only for one period, the relevant payoff function will be $f_i(.,.)$ described before. Specifically, let each government i choose a subsidy s_t^i for the next two periods. Time is discrete, indexed by t, and the horizon is infinite. As argued before, at time t, government i's instantaneous payoff (domestic benefit) is a function of the subsidies of both governments:

$$f_t^i = f(s_t^i, s_t^{-i})$$

where $f(.)$ is quadratic.[10] The following table summarizes the properties of the $f(.,.)$ function for

[7] Alternatively, one may think that both governments and firms are infinitely lived, but firms behave myopically, while governments a far-sighted.

[8] Notice that, even though the subsidies chosen by the foreign government do not affect domestic profits directly, they affect them indirectly through their effect on the R&D, quantities or prices chosen by the foreign firm, which do affect domestic profits.

[9] One can think of big costs of adjusting policy in the short run, which makes the government unable to change his policy once it has been set. Jun and Vives (2001) analyze the case of (finite) adjustment costs between periods in a structure similar to Maskin and Tirole (1987, 1988b). Their qualitative results for the Markov perfect equilibria of that game are the same as the case in which actions have to be the same in both periods (i.e. infinite adjustment costs). Of course, the assumption of a fixed policy for two periods is a simplification, as adjustment costs would imply state-dependent policies instead of the time-dependent policies used in this paper.

[10] This reduced form $f(.)$ is obtained, for example, in a differentiated goods model with linear demands and constant marginal costs.

the different types of market structures and policy instrument used by governments[11]

	Cournot			Bertrand		
	without R&D investm't ($\Delta_i = 0$)	with R&D investm't		without R&D investm't ($\Delta_i = 0$)	with R&D investm't	
	Output subsidies ($z_i = 0$)	Output subsidies ($z_i = 0$)	R&D subsidies ($s_i = 0$)	Output subsidies ($z_i = 0$)	Output subsidies ($z_i = 0$)	R&D subsidies ($s_i = 0$)
$Sign[f_2(\bar{s}, \bar{s})]$	–	–	–*	–	–*	–*
$Sign[f_{11}]$	–	–	–*	–	–*	–*
$Sign[f_{12}]$	–	–*	–	+	–/+*	–
$Sign[f_{22}]$	+	+*	+*	+	+*	+*
$\|f_{11}\| > \|f_{12}\|$	yes	yes*	yes*	yes	yes*	yes*
Sign of optimal subsidy in static game ($\hat{s}$)	+[12]	+[13]	+[14]	–[15]	+/–[16]	+[17]
* for permissible (i.e.low) values of η and γ						

As can be observed from the table, some market structures and policy instruments require a sufficiently big degree of product differentiation (i.e. low γ) or R&D that is not too cost-effective (i.e. low η). This is necessary to ensure that we have an interior solution (i.e. both firm exporting to the third country) and that second-order conditions are satisfied.

In all the applications we examine, an increase in the subsidy set by the foreign government reduces welfare of the domestic government. We therefore have that

$$f_2(s_t^i, s_t^{-i}) < 0$$

Both governments have the same discount factor δ between 0 and 1. Government i's intertemporal payoff is therefore defined as

$$\phi_t^i = \sum_{\tau=0}^{\infty} \delta^\tau f(s_{t+\tau}^i, s_{t+\tau}^{-i}) \tag{4}$$

Since governments commit to subsidies for two periods, we can have two types of patterns of subsidy-setting by the two governments. We will say that governments *synchronize* their choice of subsidies if they choose policy in the same periods (odd or even). Alternatively, we will say that governments *stagger* their choice of subsidies if they set them in different periods, acting sequentially over an infinite horizon.

[11]The mathematica program used to simulate all these scenarios can be obtained from the authors upon request.
[12]Brander and Spencer (1985)
[13]Spencer and Brander (1983), Neary and Leahy (2000), Kujal and Ruiz (2002).
[14]Spencer and Brander (1983), Bagwell and Staiger (1994), Maggi (1996).
[15]Eaton and Grossman (1986)
[16]Neary and Leahy (2000) develop a numerical simulation where the optimal output subsidy is negative when firms invest in R&D and compete à la Bertand. However, Kujal and Ruiz (2002) show analytically that the sign of the optimal output subsidy depends on the effectiveness of R&D, measured by η. The optimal subsidy is indeed negative if R&D is not too effective (i.e., low η), but *positive* for high (though "permissible") values of η.
[17]Bagwell and Staiger (1994), Maggi (1996).

We restrict ourselves to Markov Strategies, and will look for Markov Perfect Equilibria as developed in Maskin and Tirole (1988 a,b).[18] As a result, governments only react to the *current* policy of the other government. The use of Markov Strategies means that government i's strategy specifies a subsidy at time t that can only depend on the subsidy of the other government entering into its instantaneous benefit function for that period f_t^i. That is, government i's strategy only depends on the "payoff relevant" history.

We restrict attention to Markov strategies for two reasons. First, limiting the strategy space of players reduces the set of equilibria of dynamic games. No role is given to past history that does not affect current payoffs. This may seem like a sensible assumption in many instances. For example, it rules out trigger strategies, and therefore the multiplicity of equilibria in repeated games implied by the folk theorem. Second, the appeal of Markov strategies lies in its simplicity. They have been justified mainly on the basis of limited rationality and complexity costs: Markov strategies are the simplest strategies that can be treated as rational. Bhaskar and Vega-Redondo (2002) show that, if we require strategies to have finite (although arbitrarily long) memory and players bear a complexity cost related to memory length, then every Nash equilibrium of a staggered games (similar to ours) must be in Markov strategies.[19] Maskin and Tirole (2001) show that the concept of Markov perfect equilibrium is robust to a small perturbation in payoffs, and show how learning and complexity costs give rise to Markov strategies.

2.1 Synchronized Subsidy-setting

If governments set their subsidies in a synchronized way, the optimal Markov Strategy of government i is defined by:

$$\underset{s^i}{Max} f(s^i, s^{-i}) + \delta f(s^i, s^{-i}) \tag{5}$$

Notice that since both governments set their subsidies in the same periods and those subsidies are fixed for two periods, the program repeats itself every two periods. This is why we only need to analyze the optimal subsidy set for two periods (as in (5)).

The first and second order conditions for this problem are

$$f_1(s^i, s^{-i}) = 0 \tag{6}$$

$$f_{11}(s^i, s^{-i}) < 0 \tag{7}$$

which we assume are satisfied.

From (6) and using the implicit function theorem we derive the reaction function of government i, which we denote as $s^i = s(s^{-i})$. We restrict attention to globally stable Markov Perfect Equilibria

[18] Maskin and Tirole develop the first applications of Markov perfect equilibrium to industrial organization, studying Cournot competition (1987), the emergence of Edgeworth price cycles and the kinked demand curve under Bertrand competition (1988b), and quantity competition with large fixed costs (1988a).

[19] Bhaskar and Vega-Redondo (2002) also show a stronger result: if memory length is uniformly bounded, every rationalizable strategy must be a Markov strategy.

such that $|s'(.)| < 1$. Using the implicit function theorem we have that $s'(s^{-i}) = -\frac{f_{12}}{f_{11}}$. Therefore, to satisfy the stability condition we need the following restriction on government's benefits:

$$|f_{11}| > |f_{12}| \tag{8}$$

Which implies that the own effect of a subsidy over the government's benefits be greater that the effect of the other government's subsidy.

If equations (6), (7) and (8) are fulfilled,[20] then there exists a unique symmetric equilibrium subsidy $\hat{s}$ for the synchronized game satisfying:

$$\hat{s} = s^i = s(s^{-i}),\ i = 1, 2.$$

$$f_1(\hat{s}, \hat{s}) = 0 \tag{9}$$

Let $\hat{s}^i$ be the solution to (5). Notice that $\hat{s}^i$ is the Nash Equilibrium of the one-period static game where governments choose their subsidies simultaneously.

2.2 Staggered Subsidy-setting

Without loss of generality, assume that government 1 sets its subsidy in odd periods and government 2 sets it in even periods. Following Maskin and Tirole (1988 a,b), we define one reaction function for each player. We restrict ourselves to Markov strategies, which we call $R^1(s^2)$ and $R^2(s^1)$. Note that these dynamic reaction functions depend only on the current subsidy of the other government, which is the one affecting its current period payoff.

As in Maskin and Tirole (1988a) we solve the Markov Perfect Equilibrium of the staggered game by using dynamic programming. Given an equilibrium pair of Markov Strategies $R^1(s^2)$ and $R^2(s^1)$, let $V^1(s^2)$ be the present discounted value of government 1's benefits given that last period government 2 set s^2 and henceforth both governments play optimally, according to their Markov strategies $R^1(s^2)$ and $R^2(s^1)$.[21] Also, given the pair $R^1(s^2)$ and $R^2(s^1)$, let $W^1(s^1)$ be the present discounted value of government 1's benefits given that last period firm 1 played s^1 and that henceforth both firms play optimally, according to their strategies $R^1(s^2)$ and $R^2(s^1)$.[22] We define $V^2(s^1)$ and $W^2(s^2)$ symmetrically.

For the reaction and value functions $\left(R^1(s^2), R^2(s^1), V^1, V^2, W^1, W^2\right)$ to correspond to an equilibrium, the following conditions need to be satisfied:

$$\begin{aligned} V^1(s^2) &= \underset{s^1}{Max}\left\{f(s^1, s^2) + \delta W^1(s^1)\right\} && (10)\\ &= f\left(R^1(s^2), s^2\right) + \delta W^1\left(R^1(s^2)\right) && (11) \end{aligned}$$

[20] These equations are fulfilles for all the cases considered in the paper: Cournot or Bertrand competition, with and without R&D investment by firms in each period and when governments use output or R&D subsidies (the latter if firms invest in R&D).

[21] Recall that governments set their subsidies for two periods. Therefore, if government 2 set s^2 in the previous period, then government 1 faces the same subsidy s^2 in this period, when it has to make his choice of s^1.

[22] In this case, if government 1 set s^1 in the previous period, it means that this period it has no choice but to stick to s^1. In response to s^1, government 2 will choose this period its subsidy optimally according to $R^2(s^1)$.

$$W^1(s^1) = f\left(s^1, R^2(s^1)\right) + \delta V^1(R^2(s^1)) \tag{12}$$

with symmetric equations for V^2 and W^2.

Since the benefit function $f(.)$ is quadratic, we will restrict attention to linear dynamic reaction functions of the type $R^i(s^{-i}) = \alpha^i + \beta^i s^{-i}$. As in the synchronized game, we will also restrict attention to stable Markov Perfect Equilibria, such that $\left|\beta^i\right| < 1$.

We can rewrite equation (11) to state more clearly the definition of $R^1(s^2)$:

$$R^1(s^2) = \arg \underset{s^1}{Max} \left\{f(s^1, s^2) + \delta W^1(s^1)\right\}$$

and therefore the dynamic reaction function $R^1(s^2)$ has to satisfy the first order conditions of the problem:

$$f_1\left(R^1(s^2), s^2\right) + \delta W_1^1\left(R^1(s^2)\right) = 0 \tag{13}$$

We will also assume that the second order condition is satisfied:

$$f_{11} + \delta W_{11}^1\left(R^1(s^2)\right) < 0 \tag{14}$$

Following the results in Lau (1997) we know that in any symmetric Markov Perfect Equilibrium to the game defined in equations (10) to (12), the (common) slope of the dynamic reaction function $R^i(s^{-i})$ is the solution β to the equation

$$\beta^4\delta^2 f_{12} + \beta^3\delta(1+\delta)f_{22} + \beta^2 2\delta f_{12} + \beta f_{11}(1+\delta) = -f_{12} \tag{15}$$

Notice that in order to have a *stable* Markov Perfect Equilibrium, we also need the solution to equation (15) to be less than one in absolute value. In the remainder of the paper we will assume that this condition is satisfied, and that this polynomial has a solution in the interval $(-1,1)$. In fact, as the next table shows, this condition is satisfied for the six cases we analyze here, provided some standard restrictions on γ and η are satisfied:

	Cournot			Bertrand		
	without R&D investm't ($\Delta_i = 0$)	with R&D investm't		without R&D investm't ($\Delta_i = 0$)	with R&D investm't	
	Output subsidies ($z_i = 0$)	Output subsidies ($z_i = 0$)	R&D subsidies ($s_i = 0$)	Output subsidies ($z_i = 0$)	Output subsidies ($z_i = 0$)	R&D subsidies ($s_i = 0$)
$\exists\beta \in (-1,1)$ in eq (15)	yes	yes*	yes*	yes	yes*	yes*
* for permissible (i.e.low) values of η and γ						

Another thing to notice in (15) is that β, the slope of the dynamic reaction functions does not depend on the levels of the subsidies s^1 or s^2, since for $f(.)$ quadratic, all second derivatives are constant.

In the steady state of the staggered game, both governments will set the same subsidies $s^1 = s^2 = \bar{s}$. The following proposition characterizes the steady state stable Markov Perfect Equilibrium of the staggered game. The proof can be found in the appendix.

Proposition 1 *The steady state stable Markov Perfect Equilibrium subsidy in the staggered game is the value of $\bar{s}$ satisfying*

$$f_1(\bar{s},\bar{s}) + \delta\beta f_2(\bar{s},\bar{s}) = 0 \tag{16}$$

with $\beta \in (-1,1)$ given by the solution to equation (15).

In order to understand condition (16), it is helpful to consider the effect on the present value of government benefits $\phi_t^i = \sum_{\tau=0}^{\infty} \delta^\tau f(s_{t+\tau}^i, s_{t+\tau}^{-i})$ of a slight change of the level of the subsidy $\hat{s}$. The subsidy is set for two periods, so the direct effect is given by $(1+\delta)f_1(\bar{s},\bar{s})$. However, there is also a strategic effect to be considered: a change in today's subsidy will also lead the other government to change its subsidy in the next period according to β, the slope of the dynamic reaction function. Since the other government's action is also set for two periods, then it has to be multiplied by $(1+\delta)$ and since the effect on the other government starts next period, it has to be discounted using δ. The total strategic effect is therefore $\delta(1+\delta)\beta f_2(\bar{s},\bar{s})$. The effect of this slight change in the level of the subsidy only lasts for three periods since the government that effected the small change reoptimizes after two periods.

2.3 Comparison and welfare implications

Given the properties of the equilibrium subsidy in the synchronized and staggered game, we can proceed to compare $\hat{s}$ and $\bar{s}$. The following proposition shows that they will be ordered depending on whether subsidies are strategic substitutes or complements in the static game.

Proposition 2 *If subsidies are strategic substitutes (complements), the output subsidy $\bar{s}$ chosen in the staggered game is higher (lower) than the output subsidy $\hat{s}$ chosen in the synchronized game. That is:*

$$\begin{aligned} f_{12} &< 0 \rightarrow \bar{s} > \hat{s} \\ f_{12} &> 0 \rightarrow \bar{s} < \hat{s} \end{aligned}$$

The intuition for this result is simple. Suppose we are in and odd period in the staggered game and subsidies are strategic substitutes ($f_{12} < 0$). Government 1 has to decide on its subsidy, knowing that government 2 is tied up by its choice of the subsidy in the previous (even) period and can only respond in the next period (also an even period). Government 1 is therefore a (temporary) leader. This does not happen in the synchronized game. We already pointed out that the synchronized game is equivalent to the static game with simultaneous moves.

Proposition 2 shows how this difference leads to different levels of the output subsidy: if subsidies are strategic substitutes ($f_{12} < 0$), government 1 is aware that, by increasing his subsidy s^1 relative to the synchronized subsidy $\bar{s}$, government 2 will reduce s^2 next period. Therefore dynamic reaction functions are downward sloping ($\beta < 0$).[23] This in turn implies a positive effect on government

[23] Recall that the subsidy s^1 set today (odd period) by government 1 will still be in place next period, and would therefore condition government 2 to respond as a Stackelberg follower lowering his choice of subsidy s^2 in that (even) period

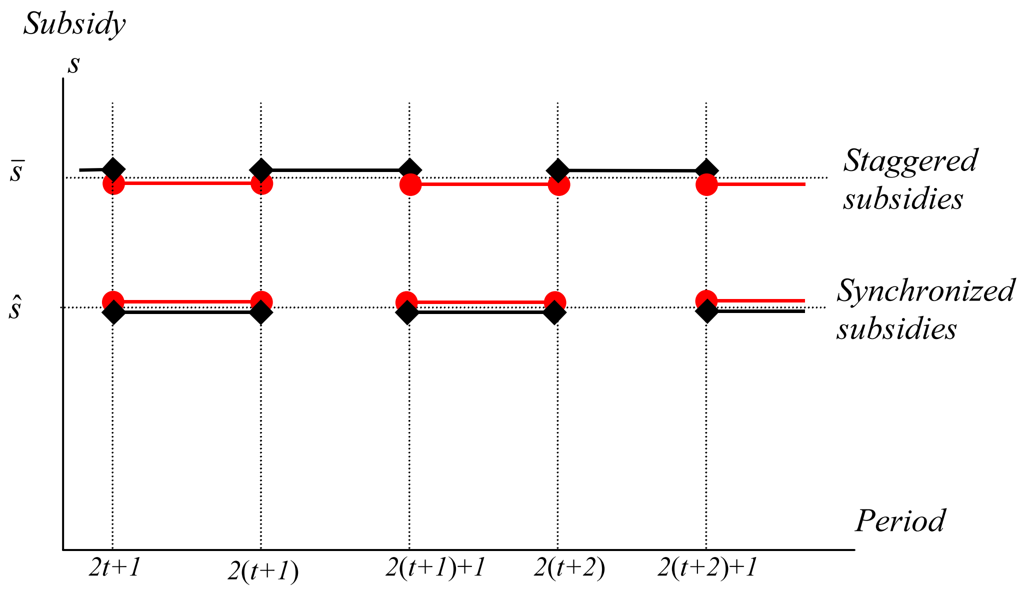

Figure 1: Time pattern of equilibrium subsidies in the synchronized ($\hat{s}$) and the staggered ($\bar{s}$) game (the relative level of steady state subsidies $\hat{s}$ and $\bar{s}$ is represented here for the case of strategic substitutes)

1's benefit because of the negative spillover of government 2's subsidy ($f_2(s^i, s^{-i}) < 0$). Therefore government 1 decides to increase its subsidy beyond the static equilibrium $\hat{s}$. The same reasoning is true when it is government 2's time to decide on even periods. The end result is that both governments set a subsidy above the equilibrium subsidy in the static game. This strategic incentive is captured by the term $\beta f_2(.)$ in (16), where β is the dynamic analog of the slope of the static reaction function $\left(-\frac{f_{12}}{f_{11}}\right)$.[24] Since this strategic effect only appears in the period after the choice of subsidy is done, it has to be discounted by δ. Notice that we can use a similar line of reasoning to conclude that, if subsidies are strategic complements, the steady-state subsidy in the staggered game is lower than the subsidy in the synchronized game.

Figure 1 represents the time pattern of equilibrium subsidies in the synchronized and the staggered game, where subsidies set in the same period are interconnected by a straight line and governments are represented by gray–round or black–square patterns. For instance, in this example, both governments set their subsidies in odd periods in the synchronized game. In the staggered game, the government represented by grey–round patterns sets its subsidies in odd periods, whereas the other government sets its subsidy in even periods. In the steady state, staggered subsidies are higher than synchronized subsidies if subsidies are strategic substitutes, and lower if subsidies are strategic complements.

Given that subsidies are higher in the staggered game under strategic substitutes, we would expect that welfare will also be lower than in the synchronized equilibrium. The opposite result is obtained in the case of strategic complements. This is summarized in the following proposition and

[24] See the proofs of lemma **??** and proposition 2 for the exact relationship between β and f_{12}.

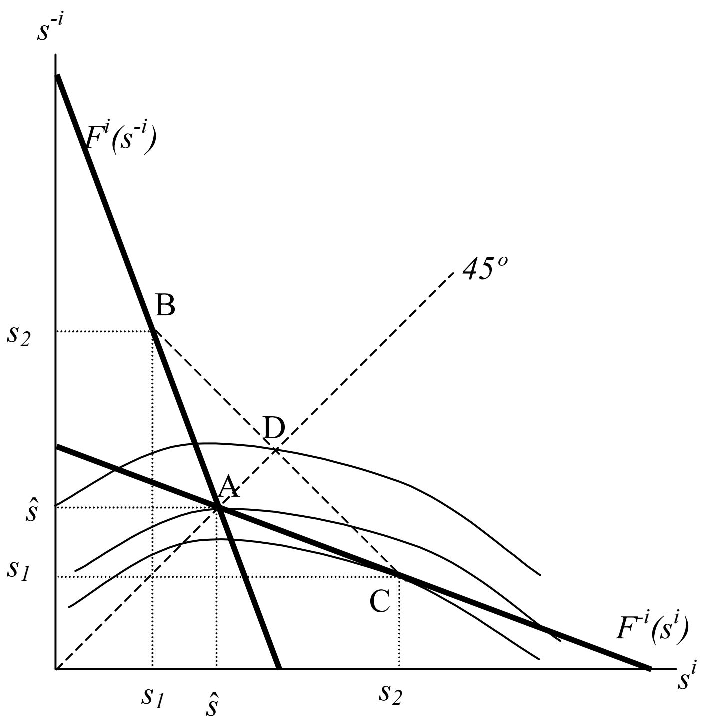

Figure 2: Subsidies as strategic substitutes.

proved in the appendix.

Proposition 3 *If subsidies are strategic substitutes, government benefits ϕ_t^i $(i = 1, 2)$ are higher when subsidies are set simultaneously than when they are set sequentially. If subsidies are strategic complements, government benefits are higher when subsidies are set sequentially.*

The intuition for this result becomes clear in figures 2 and 3. in both cases, since $f_2 < 0$, then iso-benefit lines for government i are concave lines, where, for a given domestic subsidy s^i, domestic benefits decrease on the foreign subsidy. The reaction functions of the corresponding static game are denoted $F^i(s^{-i})$ and $F^{-i}(s^i)$ for the domestic and foreign governments. Note that, given the first order condition (6), these are also the reaction functions for the synchronized game. Figure 2 shows the case when subsidies are strategic substitutes (downward sloping static reaction functions). Under synchronization, the equilibrium subsidy is $\hat{s}$ and both governments are at point A in every period. Under staggering, both governments set the same subsidy $\bar{s}$ above $\hat{s}$. This would imply that one is on the 45 degree line above and to the right of A. Given that the iso-benefit line has a slope of zero at A, staggering implies a lower benefit than synchronization in each period for both governments. Figure 3 shows the case of strategic complements. Again, under synchronization, the equilibrium subsidiy $\hat{s}$ and both governments are in point A every period (at the intersection of both static reaction functions). Staggering implies a lower subsidy $\bar{s}$ than under synchronization, which

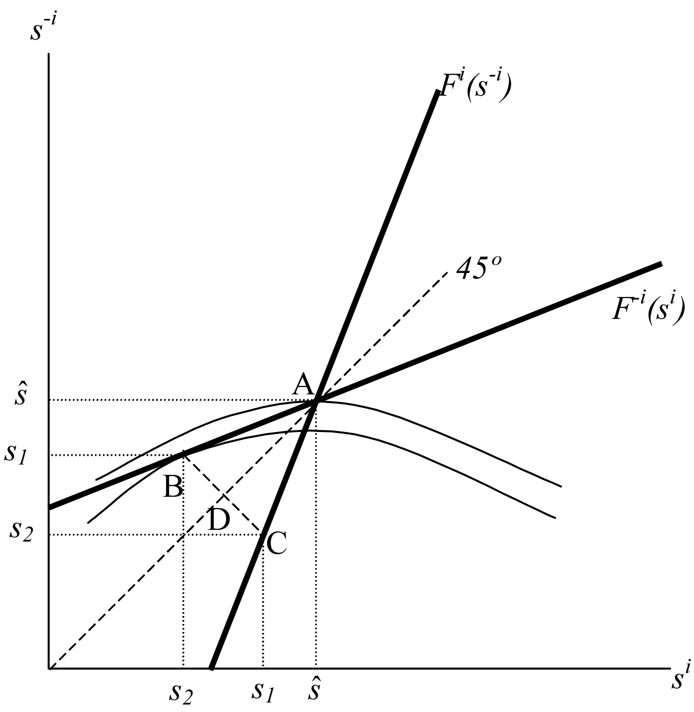

Figure 3: Subsidies as strategic complements.

means being on the 45 degree line below and to the left of A. Given that the iso-benefit line has a slope of zero at A, staggering implies a higher benefit than synchronization each period, as it leaves both governments inside their iso-benefit lines passing through A.

2.4 Endogenous timing of subsidy-setting

Lastly, we endogenize the timing of both governments. There are various ways of doing this. In section 2.6 we allow governments to remain inactive if they are uncommitted to a subsidy from the previous period. This creates the possibility of an endogenous switch between synchronization and staggering. Here, we present a simpler attempt to endogenizing the timing of subsidy-setting. Suppose that before subsidies are chosen, both governments have to decide simultaneously whether they will set their subsidies in odd or in even periods. After that decision is taken, time starts and they collect their benefits according to (4). In order to avoid problems defining the start of the game, and to maintain symmetry between the choices of even and odd periods, we will assume that nature decides whether the first period is odd or even after choices are made.

Proposition 4 *In equilibrium, governments set subsidies in the same periods (synchronization) if subsidies are strategic substitutes. Governments alternate (staggering) if subsidies are strategic complements.*

Proof. See appendix. ■

2.5 Stochastic length of commitment to a trade policy

Here we may suppose that time is continuous and payoffs are discounted at rate r. Instantaneous benefits $f(s^i, s^{-i})$ now represents a flow per unit of time. When a government sets a subsidy, its period of commitment to that action is stochastic (for example, its time in office). For simplicity we can assume that commitment length follows a Poisson process. In the interval $\triangle t$, the probability that commitment will end is $\lambda \triangle t$. Although we expect that the results would not change considerably with respecto to section 1, this allows us to see the effect of government instability over trade policy.

Due to the Poisson assumption, the length of time the other government has been committed to his action is not relevant, since the probability of ending his commitment does not depend on the length of time elapsed so far. Therefore, a Markov strategy only depends on the subsidy set by the other government. The dynamic programming equations are now:

$$V^1(s^2) = \underset{s^1}{Max} \left\{ f(s^1, s^2) \triangle t + e^{-r\triangle t} \left(\lambda \triangle t W^1(s^1) + (1 - \lambda \triangle t) V^1(s^2) \right) \right\} \tag{17}$$

$$W^1(s^1) = f\left(s^1, R^2(s^1)\right) \triangle t + e^{-r\triangle t} \left\{ \lambda \triangle t V^1(R^2(s^1)) + (1 - \lambda \triangle t) W^1(s^1) \right\} \tag{18}$$

which can rewritten as[25]

$$\begin{aligned} V^1(s^2) &= \underset{s^1}{Max} \left\{ \frac{f(s^1, s^2)}{r + \lambda} + \frac{\lambda}{r + \lambda} W^1(s^1) \right\} && (19) \\ &= \frac{f(R^1\left(s^2\right), s^2)}{r + \lambda} + \frac{\lambda}{r + \lambda} W^1(R^1\left(s^2\right)) && (20) \end{aligned}$$

$$W^1(s^1) = \frac{f\left(s^1, R^2(s^1)\right)}{r + \lambda} + \frac{\lambda}{r + \lambda} V^1(R^2(s^1)) \tag{21}$$

where $V^1\left(s^2\right)$ is the net present value of benefits to government 1 given that it is his moment to decide on a new subsidy and government 2 was committed to s^2 in the previous instant and $W^1(s^1)$ is the net present value of benefits to government 1 assuming that it is currently committed to s^1 and it is government 2's chance to act and set a subsidy s^2 optimally ($s^2 = R^2(s^1)$).

Dynamic reaction functions $R^i(s^{-i})$ are defined as before. They have to satisfy the first and second order conditions of (19):

$$f_1(R^1\left(s^2\right), s^2) + \lambda W_1^1(R^1\left(s^2\right)) = 0 \tag{22}$$

$$f_{11} + \lambda W_{11}^1(R^1\left(s^2\right)) < 0 \tag{23}$$

[25]For example, take the definition of $V^1(s^2)$ in (17), multiply both sides by $e^{r\triangle t}$ and reorder to get:

$$V^1(s^2) e^{r\triangle t} = \underset{s^1}{Max} \left\{ e^{r\triangle t} f(s^1, s^2) \triangle t + \lambda \triangle t W^1(s^1) \right\} + (1 - \lambda \triangle t) V^1(s^2)$$

$$V^1(s^2) \left(e^{r\triangle t} - 1 + \lambda \triangle t \right) = \underset{s^1}{Max} \left\{ e^{r\triangle t} f(s^1, s^2) \triangle t + \lambda \triangle t W^1(s^1) \right\}$$

divide by $\triangle t$ and take the limit as $\triangle t$ goes to zero:

$$V^1(s^2)\,(r + \lambda) = \underset{s^1}{Max} \left\{ f(s^1, s^2) + \lambda W^1(s^1) \right\}$$

which results in (19). Similar steps lead to from (18) to (21).

Notice that (22) and (23) are the analogs of the first and second order conditions for the deterministic case, except that δ is replaced by λ. An analog of condition (15) applies to this case:

Lemma 5 *In any symmetric Markov Perfect Equilibrium to the game defined in equations (19) to (21), the (common) slope of the dynamic reaction function $R^i(s^{-i})$ is the solution β to the equation*

$$\beta^4\lambda^2 f_{12} + \beta^3\lambda(r+2\lambda)f_{22} + \beta^2 2\lambda(r+\lambda)f_{12} + \beta(r+\lambda)(r+2\lambda)f_{11} + (r+\lambda)^2 f_{12} = 0 \tag{24}$$

Again, we will assume that the condition in the previous lemma is satisfied so there exists a solution β between zero and one in absolute value.[26]

In the steady state of this game, both governments will set the same subsidies $s^1 = s^2 = \widetilde{s}$. The following proposition characterizes the steady state stable Markov Perfect Equilibrium of the game with random length of commitment. The proof can be found in the appendix.

Proposition 6 *The steady state stable Markov Perfect Equilibrium subsidy in the game with random commitment is the value of $\widetilde{s}$ satisfying*

$$f_1(\widetilde{s},\widetilde{s}) + \left(\frac{\lambda}{r+\lambda}\right)\beta f_2(\widetilde{s},\widetilde{s}) = 0 \tag{25}$$

with $\beta \in (-1,1)$ given by the solution to equation (24) in lemma 5. Also, the subsidy $\widetilde{s}$ chosen in the game with random length of commitment is higher than the subsidy $\hat{s}$ chosen in the static game (i.e. $\widetilde{s} > \hat{s}$) under strategic substitutes and lower than the subsidy chosen in the static game under strategic complements. That is:

$$\begin{aligned} f_{12} &< 0 \rightarrow \widetilde{s} > \hat{s} \\ f_{12} &> 0 \rightarrow \widetilde{s} < \hat{s} \end{aligned}$$

Notice, from (25) that actually the subsidy $\widetilde{s}$ converges to the static subsidy $\hat{s}$ as λ goes to zero. This just reflects that, as λ goes to zero, the expected length of time during which the foreign government will remain committed to his policy goes to infinity. Since the domestic government will have no chance to act as a Stackelberg leader, then all dynamic strategic incentives dissapear, and we are back to the static equilibrium subsidy. Another way to look at this is to note that bringing λ to zero is equivalent to bringing the discount factor to zero in the deterministic case. If the future has no value for both governments, then they just play the static equilibrium. This leads to the conjecture that, as λ increases, strategic effects become more important, as the average time during which a government is committed to his action is shorter, therefore bringing forward the time when an uncommitted government will become a temporary stackelberg leader. We would therefore expect $\frac{\partial \widetilde{s}}{\partial \lambda} > 0$ if subsidies are strategic substitutes and $\frac{\partial \widetilde{s}}{\partial \lambda} < 0$ if subsidies are strategic complements: shorter commitment periods make governments use strategic trade policy more aggresively.[27]

[26] This condition is always fulfilled for the case of Cournot and Bertrand Competition without R&D investment. When firms can invest in R&D, the condition is fulfilled for permissible (i.e. low) values of η and γ in equations (1) to (3).

[27] Notice also that in this game with variable (random) length of commitment, governments behave quite similar to the staggering case in the deterministic model. In fact, this is reminiscent of a result showed in de Fraja (1993). In

2.6 Inactive Governments

In this extension, suppose that governments are not constrained to move in odd or even periods. If government i is currently not committed to a subsidy level, it may choose not to intervene, and therefore set $s^i = 0$. However, whenever government i decides to set a subsidy $s^i > 0$, it remains committed to it for two periods. As Maskin and Tirole (1988a) point out, the payoff relevant information in this case is, first, whether the other government is committed to a subsidy for the current period, and second, if it is committed, what is the level s^{-i} of that committed subsidy.

As in Maskin and Tirole (1988a), Markov strategies for government i can be described by the pair $\{R^i(.), \sigma^i\}$, where $R^i(.)$ is the reaction of government i to government $-i$'s current subsidy (when it was committed to in the previous period) and σ^i is government i's subsidy when the other government is not committed to a subsidy, either because it chose a subsidy two periods ago, or because it chose not to intervene ($s^{-i} = 0$) in the previous period. Notice that σ^i has no arguments because, if the other player is uncommitted, there is no payoff relevant variable. Notice also that when governments use $R(.)$ along the equilibrium path, they alternate in moves and a null action switches them into synchronization. When governments choose σ, they choose simultaneously, and therefore, the possibility of a null action can switch the game from synchronization to staggering. Figure 4 illustrates a possible time pattern of subsidy setting, where subsidies set together in the same period are displayed linked with a straight line, governments are differentiated according to round gray or square black patterns, and there exists a possibility of inaction (represented by a subsidy set equal to zero).

We check robustness of the synchronized or staggered equilibria in the sense of Maskin and Tirole (1988, section 9). The next proposition shows that, under strategic substitutes the synchronized equilibrium $(\hat{s}, \hat{s})$ is robust. Conversely, under strategic complements, the staggered equilibrium is robust.

Proposition 7 *Under strategic substitutes,*

1. *If the discount factor δ is sufficiently close to* 1, *then*
   (a) *the tuple $\{(R(.), \sigma), (R(.), \sigma)\}$, where $\{R(.), R(.)\}$ (played with probability $\omega = 0$) is the equilibrium of the staggered game and $\sigma = \hat{s}$ is the equilibrium subsidy in the synchronized game is a Markov perfect equilibrium of the game with endogenous timing*
   (b) *starting from the staggered mode (R, R), governments switch to the synchronized mode immediately and remain there indefinitely.*
2. *If the discount factor δ is sufficiently close to* 1, *then the Markov perfect equilibrium of the staggered game $\{R(.), R(.)\}$ is not robust to endogenous timing. In particular, there are no strategies $\{\sigma^*, \sigma^*\}$ for the simultaneous mode such that $\{(R(.), \sigma), (R(.), \sigma)\}$ is a Markov perfect equilibrium of the endogenous-timing game.*

an extension of his model of staggered wage settlements, he modifies the standard model to allow for commitment to $n > 2$ periods. In such a model, the number m of periods between moves of the two players can vary ($m < n$). He then asks what would be the effect of changing the number m of periods between moves of players on the equilibrium. He just shows that we just need to replace δ^n for δ in the equilibrium equations, so, surprisingly, m plays no role. All that matters is the length of commitment, but not how that time is divided between the two players.

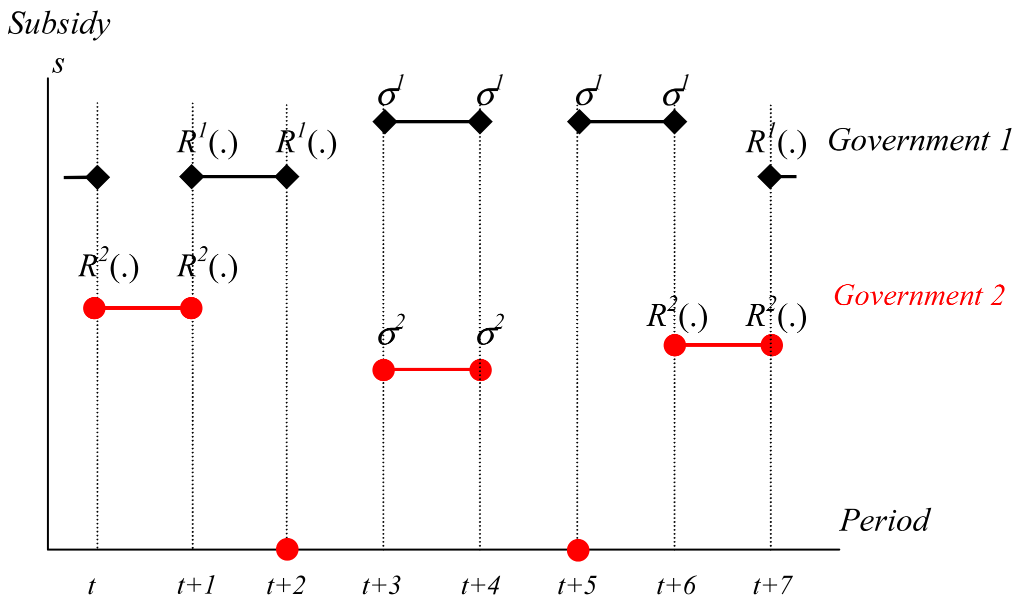

Figure 4: A possible time pattern of subsidy-setting when governments may choose to be inactive in some periods.

We also show that the staggered equilibrium $\{R(.), R(.)\}$ is robust under strategic complements:

Proposition 8 *Under strategic complements,*

1. *If the discount factor δ is sufficiently close to* 1, *then*
   (a) *the tuple $\{(R(.), \sigma), (R(.), \sigma)\}$, where $\{R(.), R(.)\}$ is the equilibrium of the staggered game and $\sigma = \hat{s}$ (played with probability $\omega = 0$) is the equilibrium subsidy in the synchronized game is a Markov perfect equilibrium of the game with endogenous timing*
   (b) *starting from the synchronized mode (σ, σ), governments switch to the staggered mode immediately and remain there indefinitely.*
2. *If the discount factor δ is sufficiently close to* 1, *then the Markov perfect equilibrium of the synchronized game $\{\sigma^*, \sigma^*\}$ (with $\sigma^* = \hat{s}$) is not robust to endogenous timing. In particular, there are no strategies $\{R(.), R(.)\}$ for the staggered mode such that $\{(R(.), \sigma), (R(.), \sigma)\}$ is a Markov perfect equilibrium of the endogenous-timing game.*

What these two propositions tell us is that synchronization seem more robust than staggering under strategic substitutes and vice-versa. Therefore, if we have strategic substitutes and we endogenize the timing of the choice of subsidies in this way, we could observe the type of switch observed in period $t + 2$ in figure 4, where governments move from staggering into synchronization. The second part of proposition 7 on the other hand states that the type of switch observed in period $t+5$ in figure 4 cannot happen in equilibrium under strategic substitutes: if we are in the synchronized mode, we remain there forever. The conclusions are reversed in the case of strategic complements in proposition 8.

2.7 Predetermined (but flexible) trade policy

In this extension, suppose that governments are committed to choose subsidy levels for two consecutive periods, but subsidies in both periods may differ. For example, in the staggered game, government 1 has to decide, in odd periods, its subsidy for that period s_t^1 and its subsidy for the next period s_{t+1}^1. In the original setup we were restricted to $s_t^1 = s_{t+1}^1$. Here we eliminate that restriction and allow for $s_t^1 \neq s_{t+1}^1$. Therefore subsidies are pre-committed but not fixed.

As before we will assume that government 1 sets its subsidy in odd periods and government 2 in even periods. If t is an odd period government 1 has to set its subsidies for the next two periods s_t^1 and s_{t+1}^1, and similarly for government 2 in even periods.

2.7.1 Flexible synchronized Subsidy-setting

Markov strategies for government 1 in the synchronized game will depend on the two subsidies chosen by the other government for the next two periods. Formally, government i solves

$$\underset{\{s_t^i, s_{t+1}^i\}}{Max} f(s_t^i, s_t^{-i}) + \delta f(s_{t+1}^i, s_{t+1}^{-i})$$

given that the two periods are alike, both subsidies s_t^i and s_{t+1}^i are identical. Let s^i denote the choice of government i, then the first order conditions imply

$$f_1(s^i, s^{-i}) = 0 \tag{26}$$

with the same second order and stability conditions as in the synchronized game in section 2.1. Therefore the synchronized game yields the same equilibrium subsidy $\hat{s}$.

2.7.2 Flexible staggered subsidy-setting

In the staggered game, we need to modify the value functions and dynamic reaction functions used in section 2.2. In particular, since a government needs to specify two subsidies each time, we need to specify two reaction functions. Let $F^i(s_2^{-i})$ be the reaction function of government i for the first period for which it has to set a subsidy, depending on the subsidy set by government $-i$ for its second period of commitment s_2^{-i}. Also, let $S^i(s_2^{-i})$ be the reaction function of government i for the second period for which it has to set a subsidy, as a function of s_2^{-i}. Under these definitions, note that F^1 and S^2 are reaction functions for odd periods, and F^2 and S^1 are reaction functions for even periods. These four reaction functions are Markov perfect strategies if and only if there exists value functions V^1 and V^2 such that

$$\begin{aligned} V^i(s_2^{-i}) &= \underset{\{s_1^i, s_2^i\}}{Max} \left\{ f(s_1^i, s_2^{-i}) + \delta f\left(s_2^i, F^{-i}(s_2^i)\right) + \delta^2 V^i\left(S^{-i}(s_2^i)\right) \right\} &(27)\\ &= f(F^i\left(s_2^{-i}\right), s_2^{-i}) + \delta f\left(S^i\left(s_2^{-i}\right), F^{-i}(S^i\left(s_2^{-i}\right))\right) + \delta^2 V^i\left(S^{-i}(S^i\left(s_2^{-i}\right))\right) &(28) \end{aligned}$$

We can rewrite equation (27) to state more clearly the definition of $F^i(s_2^{-i})$ and $S^i(s_2^{-i})$

$$\begin{aligned} F^i(s_2^{-i}) &= \arg \underset{s_1^i}{Max} \left\{ f(s_1^i, s_2^{-i}) + \delta f\left(s_2^i, F^{-i}(s_2^i)\right) + \delta^2 V^i\left(S^{-i}(s_2^i)\right)\right\} \\ &= \arg \underset{s_1^i}{Max} f(s_1^i, s_2^{-i}) \end{aligned} \tag{29}$$

$$\begin{aligned} S^i\left(s_2^{-i}\right) &= \arg \underset{s_2^i}{Max} \left\{ f(s_1^i, s_2^{-i}) + \delta f\left(s_2^i, F^{-i}(s_2^i)\right) + \delta^2 V^i\left(S^{-i}(s_2^i)\right)\right\} \\ &= \arg \underset{s_2^i}{Max} \left\{ f\left(s_2^i, F^{-i}(s_2^i)\right) + \delta V^i\left(S^{-i}(s_2^i)\right)\right\} \end{aligned} \tag{30}$$

and therefore the dynamic reaction functions $F^i(s_2^{-i})$ and $S^i\left(s_2^{-i}\right)$ have to satisfy the first order conditions of the problem:

$$f_1(F^i(s_2^{-i}), s_2^{-i}) = 0 \tag{31}$$

$$\begin{aligned} & f_1\left(S^i\left(s_2^{-i}\right), F^{-i}(S^i\left(s_2^{-i}\right))\right) + f_2\left(S^i\left(s_2^{-i}\right), F^{-i}(S^i\left(s_2^{-i}\right))\right) \frac{\partial F^{-i}(S^i\left(s_2^{-i}\right))}{\partial S^i\left(s_2^{-i}\right)} + \\ & +\delta V_1^i\left(S^{-i}(S^i\left(s_2^{-i}\right))\right) \cdot \frac{\partial S^{-i}(S^i\left(s_2^{-i}\right))}{\partial S^i\left(s_2^{-i}\right)} \\ = {} & 0 \end{aligned} \tag{32}$$

We will also assume that the second order conditions are satisfied:

$$f_{11} < 0$$

$$\begin{aligned} & f_{11}\left(s_2^i, F^{-i}(s_2^i)\right) + f_{12}\left(s_2^i, F^{-i}(s_2^i)\right) \frac{\partial F^{-i}(s_2^i)}{\partial s_2^i} + \\ & + f_{21}\left(s_2^i, F^{-i}(s_2^i)\right) \frac{\partial F^{-i}(s_2^i)}{\partial s_2^i} + f_{22}\left(s_2^i, F^{-i}(s_2^i)\right) \left(\frac{\partial F^{-i}(s_2^i)}{\partial s_2^i}\right)^2 + \\ & + \delta V_{11}^i\left(S^{-i}(s_2^i)\right) \cdot \left(\frac{\partial S^{-i}(s_2^i)}{\partial s_2^i}\right)^2 \\ < {} & 0 \end{aligned}$$

where we have assumed that both $F^i(.)$ and $S^i(.)$ are linear functions.

Notice that equation (31) implicitly defines $F^i(s_2^{-i})$ and so

$$\frac{\partial F^i(s_2^{-i})}{\partial s_2^{-i}} = -\frac{f_{12}(F^i(s_2^{-i}), s_2^{-i})}{f_{11}(F^i(s_2^{-i}), s_2^{-i})} < 0 \tag{33}$$

Replacing this back into (32) we obtain

$$\begin{aligned} & f_1\left(S^i\left(s_2^{-i}\right), F^{-i}(S^i\left(s_2^{-i}\right))\right) - f_2\left(S^i\left(s_2^{-i}\right), F^{-i}(S^i\left(s_2^{-i}\right))\right) \frac{f_{12}(F^{-i}(s_2^i), s_2^i)}{f_{11}(F^{-i}(s_2^i), s_2^i)} + \\ & +\delta V_1^i\left(S^{-i}(S^i\left(s_2^{-i}\right))\right) \cdot \frac{\partial S^{-i}(S^i\left(s_2^{-i}\right))}{\partial S^i\left(s_2^{-i}\right)} \\ = {} & 0 \end{aligned} \tag{34}$$

which implicitly defines a reaction function for the subsidy for second period $s_2^i = S^i\left(s_2^{-i}\right)$. Note, however, that in (34) s_2^{-i} does not appear independently, and therefore $S^i\left(s_2^{-i}\right)$ does not depend on s_2^{-i}. This means that $\frac{\partial S^{-i}(s_2^i)}{\partial s_2^i} = 0$ and so s_2^i is independent of the choice of subsidy by the other government in the previous period (s_2^{-i}). Therefore (31) can be rewritten as

$$f_1\left(s_2^i, F^{-i}(s_2^i)\right) - f_2\left(s_2^i, F^{-i}(s_2^i)\right)\frac{f_{12}(F^{-i}(s_2^i), s_2^i)}{f_{11}(F^{-i}(s_2^i), s_2^i)} = 0 \tag{35}$$

The following proposition summarizes the results so far

Proposition 9 *The tuple* $\left\{s_1^1, s_1^2, s_2^1, s_2^2\right\}$ *with* $s_1^1 = F^1(s_2^2)$; $s_1^2 = F^2(s_2^1)$ *is a Markov Perfect Equilibrium of the subsidy game if there exists functions* F^1 *and* F^2 *such that*

$$f_1(F^i(s_2^{-i}), s_2^{-i}) = 0 \qquad\qquad i = 1,2 \tag{36}$$

and the following conditions are satisfied for s_2^1 *and* s_2^2*:*

$$f_1\left(s_2^i, F^{-i}(s_2^i)\right) - f_2\left(s_2^i, F^{-i}(s_2^i)\right)\frac{f_{12}(F^{-i}(s_2^i), s_2^i)}{f_{11}(F^{-i}(s_2^i), s_2^i)} = 0 \qquad\qquad i = 1,2 \tag{37}$$

$$f_{11}\left(s_2^i, F^{-i}(s_2^i)\right) - \left(f_{12}\left(s_2^i, F^{-i}(s_2^i)\right) + f_{21}\left(s_2^i, F^{-i}(s_2^i)\right)\right)\frac{\partial F^{-i}(s_2^i)}{\partial s_2^i} + f_{22}\left(s_2^i, F^{-i}(s_2^i)\right)\left(\frac{\partial F^{-i}(s_2^i)}{\partial s_2^i}\right)^2 < 0 \qquad i = 1,2$$

After characterizing the Markov perfect equilibrium of this game, we can investigate the relationship between the subsidies set in the first and second period by each government s_1^i and s_2^i and the subsidy set in the synchronized game $\hat{s}$. The following proposition states the main result of this section.

Proposition 10 *In a symmetric Markov perfect equilibrium:*

1. *Under strategic substitutes, subsidies set by both governments in their first periods are smaller than the subsidy set in the synchronized game. Subsidies set in their second periods are bigger than that set in the synchronized game:*

$$s_1^i < \hat{s} < s_2^i$$

2. *Under strategic complements, subsidies set by both governments in both periods are lower than subsidies set in the synchronized game. Subsidies set in their second periods are lower than those set in their first periods of commitment.*

$$s_2^i < s_1^i < \hat{s}$$

The intuition for the result is simple. Recall that $\hat{s}$, the equilibrium subsidy in the synchronized game, coincides with the equilibrium subsidy in the simultaneous move game. On the other hand, the equations in proposition 9 defining the equilibrium subsidies in the staggered game are exactly the same equations that define equilibrium subsidies in a sequential (Stackelberg) game. In particular, (36) states that $s_1^i = F^i\left(s_2^{-i}\right)$ is just the subsidy set by the government that acts second (the follower government) if they play a Stackelberg game during one period. It just responds to the choice of s_2^{-i} (decided in the previous period to be effective today) using its (static) reaction function $F^i(.)$. Equation (37) sets the value of s_2^i as arg $\underset{s_2^i}{Max} f\left(s_2^i, F^{-i}(s_2^i)\right)$. That is precisely the choice of subsidy by the leading government in the one-period Stackelberg game, knowing that the other government will respond using its reaction function $F^{-i}(s_2^i)$. If subsidies are strategic substitutes ($f_{12} < 0$), it is not surprising that leading subsidies (s_2^i), set in each "second period", are higher than those set in the static game ($\hat{s}$) and that the best response (s_1^i), set in each "first period" yields lower subsidies than in the static game. On the other hand, if subsidies are strategic complements ($f_{12} > 0$), then the "leading" government will choose lower subsidies (i.e. higher taxes) than in the static game. But then the optimal response of the other government is also to lower its subsidy below the static subsidy, along its reaction function. Since reaction functions have a slope smaller than 1, then subsidies set when the government acts as a "Stackelberg leader" (i.e. during the second period of commitment), are lower than subsidies set when the government acts as a follower (i.e. during the first period of commitment).

The pattern of equilibrium subsidies in both types of games is summarized in figure 5 for the case of strategic substitutes. In that figure, subsidies set in the same period are shown interconnected by a straight line and the two governments are shown with grey round or black square patterns. Notice that subsidies in the first period of commitment are lower than on the second period ($s_1 < s_2$) and they are set on each side of the synchronized subsidy $\hat{s}$. Figure 6 shows the time pattern for the case of strategic complements.

2.7.3 Welfare analysis and the pre-commitment game

We turn now to the analysis of welfare in the two types of games. In the synchronized game, the present discounted value of benefits for government i is equal to

$$\bar{\phi}_t^i = \sum_{\tau=0}^{\infty} \delta^\tau f(\hat{s}, \hat{s}) = \frac{f(\hat{s}, \hat{s})}{1-\delta}$$

In the staggered game, we need to distinguish whether a government is currently in a period when it is its turn to set subsidies for the next two periods or it is in a period when it is not deciding on policy. If we look for symmetric equilibria (where $s_j^i = s_j$ for $i, j = 1, 2$) then

$$\phi_t^{fi} = \sum_{\tau=0}^{\infty} \delta^{2\tau} \left(f(s_1, s_2) + \delta f(s_2, s_1)\right) = \frac{f(s_1, s_2) + \delta f(s_2, s_1)}{1-\delta^2} \qquad i = 1, 2$$

if government i is setting its subsidy in period t, and

$$\phi_t^{si} = \sum_{\tau=0}^{\infty} \delta^{2\tau} \left(f(s_2, s_1) + \delta f(s_1, s_2)\right) = \frac{f(s_2, s_1) + \delta f(s_1, s_2)}{1-\delta^2} \qquad i = 1, 2$$

if government i set its subsidy in period $t-1$ and therefore it is not setting policy in period t.

The following lemma (proved in the appendix) compares the three measures of government benefits:

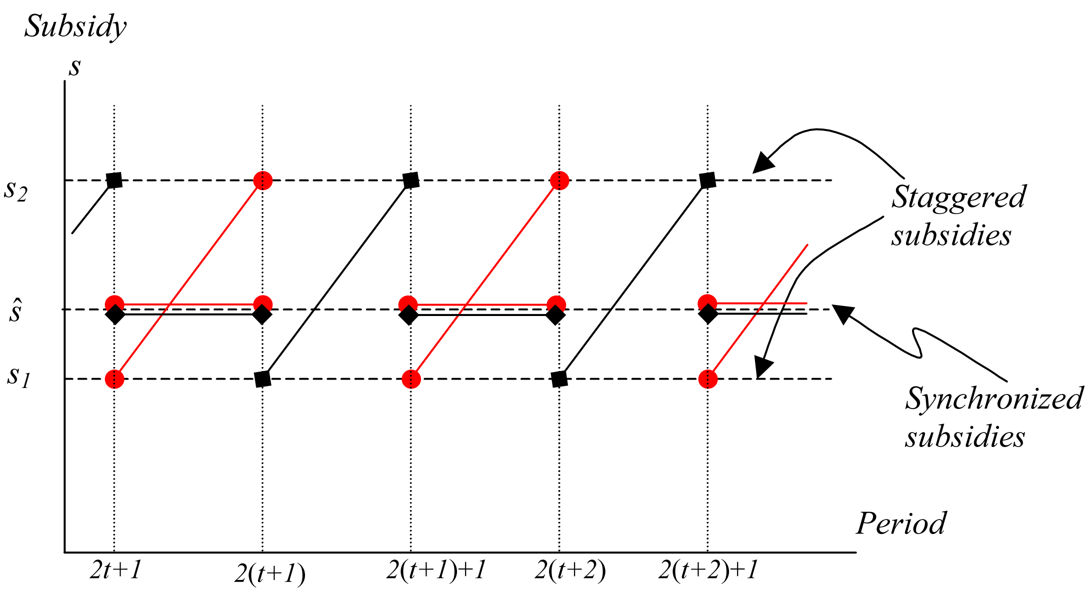

Figure 5: Flexible subsidy-setting under strategic substitutes: time pattern of equilibrium subsidies in the synchronized and the staggered game.

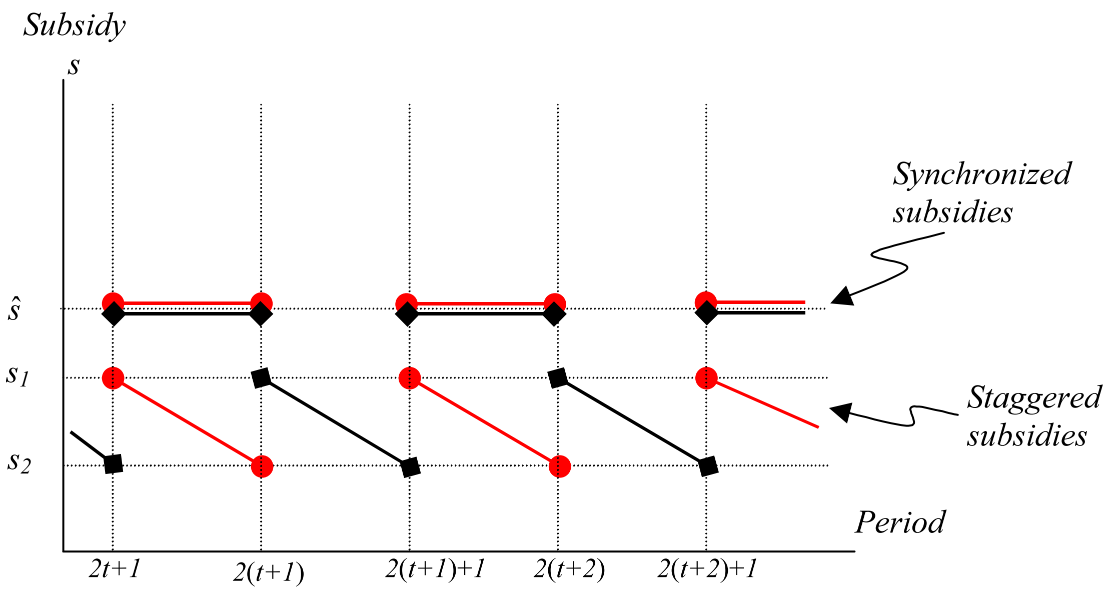

Figure 6: Flexible subsidy-setting under strategic complements: time pattern of equilibrium subsidies in the synchronized and the staggered game.

Proposition 11 *In a symmetric Markov perfect equilibrium, in period t*

1. *If subsidies are strategic substitutes*
   - (a) *the present discounted value of benefits of a government in the synchronized game is bigger than the present discounted value of benefits of the government which sets its subsidy at time t in the staggered game. That is,* $\bar{\phi}_t^i > \phi_t^{fi}$
   - (b) *for* δ *sufficiently close to* 1*, the present discounted value of benefits of a government in the synchronized game is bigger than the present discounted value of benefits of the government which will set its subsidy at time* $t+1$ *in the staggered game. That is,* $\exists \bar{\delta} < 1$ *s.t.* $\delta > \bar{\delta} \rightarrow \bar{\phi}_t^i > \phi_t^{si}$

2. *If subsidies are strategic complements*
   - (a) *the present discounted value of benefits of a government in the staggered game is bigger than the present discounted value of benefits of a government in the synchronized game. That is,* $\bar{\phi}_t^i < \phi_t^{fi}$ *and* $\bar{\phi}_t^i < \phi_t^{si}$

The intuition for this proposition is simple and is better explained looking at figures 2 for the case of strategic substitutes and 3 for the case of strategic complements.

Under strategic substitutes and in the synchronized game (figure 2), both governments set a subsidy $\hat{s}$ which is equivalent to the equilibrium in the static game with simultaneous choice. This is represented in the graph by point A at the intersection of both reaction functions $F^i(s^{-i})$ and $F^{-i}(s^i)$. In the staggered game, governments alternate being the Stackelberg leader and follower with the same reaction functions $F^i(s^{-i})$ and $F^{-i}(s^i)$. That means that when government i in period t has to set its policy, it will be acting as a Stackelberg follower (since it responds to the subsidy s_2 set by the other government in the previous period), and therefore it will be at point B. Next period, it will act as a Stackelberg leader and therefore set a subsidy equal to s_2, which brings the equilibrium outcome for that period at point C. Note that point C is just the mirror image of B along the 45 degree line. This dynamics repeats itself every two periods.

Suppose for a moment that δ is equal to 1. Then ϕ_t^{fi} and ϕ_t^{si} are just a multiple of the average benefit over this two period cycle. The average of subsidies set during this two period cycle is given by point D. Because of concavity of the benefit function $f(.)$, the benefit obtained at D is bigger than the average of benefits at B and C. Since government i's iso-benefit lines (shown as thin, continuous curves) have a slope equal to zero at their intersection with $F^i(s^{-i})$, this implies that in turn a government gets a higher benefit at A than at D. Therefore benefit at A is higher than the average of benefits at B and C.

If we introduce discounting between periods, we just have to realize that when a government starts in period t at point B in the staggered game (that is, if it has to set subsidies in period t), then it starts with a lower benefit at time t than under synchronization (A). Therefore, the result in this case is even stronger. If, on the contrary, a government starts the staggered game at point B (that is, if it set subsidies in period $t-1$) then the average subsidy between the two periods ($\frac{s_2+\delta s_1}{1+\delta}$)

will be somewhere between C and D and depending on how small δ is, we may get so close to C that the present value of benefits is bigger than at A.[28] However, as δ becomes closer to 1, that average will get closer to D, and therefore, for δ sufficiently big, we can still apply the same reasoning of the case $\delta = 1$.

The case of strategic complements (figure 3) is even simpler because when the government acts as a leader or as a follower, it gets benefits that are higher than under the static equilibrium. That can be seen in figure 3 since points B and C yiel a higher benefit than point A (the static equilibrium). Therefore, the convex combination of B and C is better for each government than A.

Suppose now that governments play a pre-commitment game before starting setting subsidies. Governments have to choose simultaneously whether they want to set their subsidies in odd or even periods. Of course, we have to modify slightly the previous setting to take care of the first period, in which it may be the case that one of the governments is not choosing a policy and it is not committed by a previous decision (for example in the staggered game). Therefore, we will assume that after choosing in which period to set their subsidies, uncommitted governments in the first period choose their subsidy for that first period and then follow the two-period setting as before. To avoid asymmetries in the choice of odd or even periods, we will also assume that after choices are made, nature tosses a fair coin to decide whether the first period is odd or even.

With this setting, if, for example, government 1 chooses to set their policy in odd periods and government 2 chooses to set it in even periods, we will have a staggered game. If nature decides that the first period is odd, then, in the first period, government 1 sets its subsidies for period 1 and 2 and government 2 will only set its subsidy for period 1.

The equilibrium in the first period of this augmented game can be easily derived. If both governments choose to set their subsidies in the same periods (synchronization), then, after nature decides, there are two possibilities. In the first case, they may have to set it in the first period, and therefore we have a synchronized game from the start, with equilibrium $(\hat{s}, \hat{s})$. The other possibility is that they have to set subsidies in the second period. In this last case, they will have to simultaneously choose their subsidies for the first period, and, since that choice does not affect future choices, we will have the static equilibrium $(\hat{s}, \hat{s})$.

Turn attention now to the case in which they are choosing their subsidies in different periods (staggering). Without loss of generality, assume that government 1 chooses odd periods, government 2 chooses even periods and nature chose that the first period would be odd. Therefore, government 2 will have to choose freely its subsidy for the first period. Government 1 will choose s_2 for the second period since that choice is independent of government 2's choice for the first period (see eq. (37)). In the first period, government 1's choice of subsidy does not depend on previous subsidies set by government 2 (this is the first period) and will not affect future choices of government 2 either. Therefore, the game in the first period becomes a simultaneous move game, with equilibrium $(\hat{s}, \hat{s})$. This result is summarized in the following

Lemma 12 *The equilibrium in the first period of the game is given by the subsidy pair* $(\hat{s}, \hat{s})$. *That*

[28]Notice that in general point C, as the Stackelberg point, always yields a higher benefit to government i than point A. If, for example, δ equals zero, then government i would get a higher benefit from staggering than from synchronization in this case.

is, regardless of the choice of timing by both governments, they will choose the same subsidy for the first period as in the synchronized game.

Given that both types of timing give the same payoff for the first period, we can compute the expected payoffs from each strategy pair starting from the second period. If we write the game in normal form we have the following payoff matrix

		Government 2	
		even periods	odd periods
Government 1	even periods	$\frac{f(\hat{s},\hat{s})}{1-\delta}, \frac{f(\hat{s},\hat{s})}{1-\delta}$	$\frac{f(s_1,s_2)+f(s_2,s_1)}{2(1-\delta)}, \frac{f(s_1,s_2)+f(s_2,s_1)}{2(1-\delta)}$
	odd periods	$\frac{f(s_1,s_2)+f(s_2,s_1)}{2(1-\delta)}, \frac{f(s_1,s_2)+f(s_2,s_1)}{2(1-\delta)}$	$\frac{f(\hat{s},\hat{s})}{1-\delta}, \frac{f(\hat{s},\hat{s})}{1-\delta}$

By a similar argument as in the proof of proposition 11, we know that[29] $\frac{f(\hat{s},\hat{s})}{1-\delta} > \frac{f(s_1,s_2)+f(s_2,s_1)}{2(1-\delta)}$ under strategic substitutes and the opposite under strategic complements. Therefore, under strategic substitutes we have two equilibria: (odd,odd) or (even,even). Both equilibria lead to synchronization. Under strategic complements we have two equilibria as well: (odd,even) or (even,odd). Both equilibria lead to staggering in this case. Therefore, we have proved the next proposition.

Proposition 13 *Under strategic substitutes, synchronization is the equilibrium outcome of the "pre-commitment" game between the two governments, which implies constant subsidies. Under strategic complements, staggering is the equilibrium of the "pre-commitment" game, which leads to cyclical subsidies.*

Therefore, introducing flexibility in the choice of governments only makes a difference in the case of strategic complements. In the case of strategic substitutes, the endogenous timing leads to synchronization, and thus to the return to the static equilibrium.

3 Conclusions

We have developed a dynamic model where governments can alternate their trade policy or they can synchronize the timing of their policy-setting. We have found that, when governments alternate (stagger) their choice of trade policy and firms compete á la Cournot in a third market, governments choose subsidies which are higher than under the corresponding static (one-shot) version of the model. Therefore, retaliation and the possibility of being a temporary Stackelberg leader, make governments pursue a more aggressive strategic trade policy to shift profits from foreign firms into domestic ones. We have extended the analysis in a number of ways, including endogenizing the timing of policy-setting. We have found that governments are then able to reach an equilibrium in which they set their policy simultaneously, therefore reducing the harm imposed on their attempt to divert profits from foreign firms.[30]

[29] Simply set $\delta = 1$ in equations (57) (58) to obtain the result.

[30] Note that the reduction in welfare (respect to free trade) caused by the simultaneous subsidy still remains.

A particular assumption we have used in this paper is that governments play the leading role in the dynamic interaction between the four players. Firms are assumed myopic or short lived. This raises the issue of whether trade policy would be closer to the cooperative solution if firms are allowed to play this dynamic game more actively, influencing not only the choice of the other firms, but also the choice of the other two governments. A partial step in this direction is Castro and Brandão (2000) which proves the existence of a Markov perfect equilibrium of a dynamic game between two firms and *one* government in a third country model like ours. However, they do not derive the properties of the equilibrium subsidy in that model. We plan to pursue this line of research in a separate project.

Appendix

A Proof of Proposition 1

Since $f(.)$ is quadratic, $f_{ij}(.)$ is constant so, abusing notation, we can forget about the arguments of f_{ij} altogether. Differentiating (13) we obtain:

$$\frac{\partial R^i(s^{-i})}{\partial s^{-i}} = \frac{-f_{12}}{f_{11} + \delta W_{11}^i\left(R^i(s^{-i})\right)} = \beta^i \tag{38}$$

where the last equality comes from our restriction to linear dynamic reaction functions $R^i(s^{-i}) = \alpha^i + \beta^i s^{-i}$. Therefore, $W_{11}^i(.)$ is a constant under this assumption. From now on we will also omit the arguments of this function.

The function $W^i(s^i)$ can be defined substituting (11) in (12):

$$W^i(s^i) = f\left(s^i, R^{-i}(s^i)\right) + \delta f\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right) + \delta^2 W^i\left(R^i\left(R^{-i}(s^i)\right)\right)$$

Using our assumption of linear dynamic reaction functions $R^i(s^{-i}) = \alpha^i + \beta^i s^{-i}$ we can differentiate W^i to obtain

$$\begin{aligned} W_1^i(s^i) \quad = \quad & f_1\left(s^i, R^{-i}(s^i)\right) + f_2\left(s^i, R^{-i}(s^i)\right)\cdot\beta^{-i} + \\ & +\delta f_1\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)\cdot\beta^{-i}\beta^i + \\ & +\delta f_2\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)\cdot\beta^{-i} + \\ & +\delta^2 W_1^i\left(R^i\left(R^{-i}(s^i)\right)\right)\cdot\beta^{-i}\beta^i \end{aligned} \tag{39}$$

$$\begin{aligned} W_{11}^i(s^i) \quad = \quad & f_{11}\left(s^i, R^{-i}(s^i)\right) + f_{12}\left(s^i, R^{-i}(s^i)\right)\cdot\beta^{-i} + f_{21}\left(s^i, R^{-i}(s^i)\right)\cdot\beta^{-i} + f_{22}\left(s^i, R^{-i}(s^i)\right)\cdot\left(\beta^{-i}\right)^2 + \\ & +\delta f_{11}\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)\cdot\left(\beta^{-i}\beta^i\right)^2 + \delta f_{12}\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)\cdot\left(\beta^{-i}\right)^2\beta^i + \\ & +\delta f_{21}\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)\cdot\left(\beta^{-i}\right)^2\beta^i + \delta f_{22}\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)\cdot\left(\beta^{-i}\right)^2 + \\ & +\delta^2 W_{11}^i\left(R^i\left(R^{-i}(s^i)\right)\right)\cdot\left(\beta^{-i}\beta^i\right)^2 \end{aligned}$$

Using again the fact that f_{ij} $(i, j = 1, 2)$ and W_{11}^i are constant, we can collect terms to obtain

$$W_{11}^i = f_{11}\left(1 + \delta\left(\beta^{-i}\beta^i\right)^2\right) + 2f_{12}\left(\beta^{-i} + \delta\left(\beta^{-i}\right)^2\beta^i\right) + f_{22}\left((1+\delta)\left(\beta^{-i}\right)^2\right) + \delta^2 W_{11}^i\cdot\left(\beta^{-i}\beta^i\right)^2$$

and so

$$W_{11}^i = \left(\frac{f_{11}\left(1 + \delta\left(\beta^{-i}\beta^i\right)^2\right) + 2f_{12}\left(\beta^{-i} + \delta\left(\beta^{-i}\right)^2\beta^i\right) + f_{22}\left((1+\delta)\left(\beta^{-i}\right)^2\right)}{1 - \delta^2\left(\beta^{-i}\beta^i\right)^2}\right) \tag{40}$$

In the staggered game, the steady state equilibrium subsidy will be the same for both governments: $s^i = s^{-i} = \bar{s}$. Therefore, from (39), the expression for W_1^i at the steady state equilibrium becomes:

$$W_1^i(\bar{s}) = f_1(\bar{s},\bar{s}) + f_2(\bar{s},\bar{s})\cdot\beta^{-i} + \delta f_1(\bar{s},\bar{s})\cdot\beta^{-i}\beta^i + \delta f_2(\bar{s},\bar{s})\cdot\beta^{-i} + \delta^2 W^i(\bar{s})\cdot\beta^{-i}\beta^i$$

If we analyze the symmetric solution where $\beta^i = \beta^{-i} = \beta$, then

$$W_1^i(\bar{s}) = \frac{f_1(\bar{s},\bar{s})\left(1+\delta\beta^2\right) + f_2(\bar{s},\bar{s})\,\beta\,(1+\delta)}{1-\delta^2\beta^2}$$

Replacing this into the first order condition (13) yields

$$f_1(\bar{s},\bar{s}) + \delta\left(\frac{f_1(\bar{s},\bar{s})\left(1+\delta\beta^2\right) + f_2(\bar{s},\bar{s})\,\beta\,(1+\delta)}{1-\delta^2\beta^2}\right) = 0$$

that is:

$$f_1(\bar{s},\bar{s}) + \delta\beta f_2(\bar{s},\bar{s}) = 0 \tag{41}$$

with β given by (38). ■

B Proof of Proposition 2

Notice first that the stability condition (8) for the synchronized game and the concavity of $f(.)$ implies that $f_1(s,s)$ is a linear and decreasing function with constant slope:

$$\frac{\partial f_1(s,s)}{\partial s} = f_{11} + f_{12} < 0 \tag{42}$$

Suppose the subsidies are strategic substitutes, and so $f_{12} < 0$: an increase in the subsidy set by one government reduces the marginal benefit of a subsidy set by the other government. Also, an increase in the subsidy set by one government reduces the benefit of the other government, so that $f_2(s^i, s^{-i}) < 0$.

Notice also that $\beta = \frac{-f_{12}}{f_{11}+\delta W_{11}^i(R^i(s^{-i}))}$. Using the second order condition (14) of the maximization problem in (10) we can easily see that β and f_{12} have the same sign. So $f_{12} < 0$ implies that $\beta \in (-1,0)$, that is, dynamic reaction functions are downward sloping. Therefore $\delta\beta f_2(\bar{s},\bar{s}) > 0$ and thus $f_1(\bar{s},\bar{s}) < 0$ from (16).

Since we have established that $f_1(s,s)$ is a decreasing function, then $f_1(\bar{s},\bar{s}) < 0 = f_1(\hat{s},\hat{s})$ implies that $\bar{s} > \hat{s}$ as stated in the proposition.

Notice that the same line of reasoning can be used to show that $\bar{s} < \hat{s}$ if subsidies are strategic complements. ■

C Proof of Proposition 3

If s^* is the symmetric cooperative equilibrium value of the subsidy s, then it will satisfy

$$s^* = \arg \underset{s}{Max} f(s,s)$$

therefore s^* satisfies the first order condition

$$f_1(s^*, s^*) + f_2(s^*, s^*) = 0 \tag{43}$$

with second-order condition

$$f_{11} + 2f_{12} + f_{22} < 0 \tag{44}$$

Notice that the conditions that that define the synchronized equilibrium subsidy $\hat{s}$, the equilibrium staggered subsidy $\bar{s}$ and the cooperative solution s^* can be summarized as

$$\begin{aligned} f_1(s^*, s^*) + K^* f_2(s^*, s^*) &= 0 \\ f_1(\bar{s}, \bar{s}) + \bar{K} f_2(\bar{s}, \bar{s}) &= 0 \\ f_1(\hat{s}, \hat{s}) + \hat{K} f_2(\hat{s}, \hat{s}) &= 0 \end{aligned}$$

with $K^* = 1$, $\bar{K} = \delta\beta$ and $\hat{K} = 0$. Total differentiation of any of the conditions above lead to:

$$\begin{aligned} \frac{\partial s}{\partial K} &= \frac{-f_2(s,s)}{f_{11} + f_{12} + K(f_{12} + f_{22})} \\ &= \frac{-f_2(s,s)}{f_{11} + 2f_{12} + f_{22} - (1-K)(f_{12} + f_{22})} \end{aligned}$$

where the numerator is positive. Recall that the stability condition (8) implies $f_{11} + f_{12} < 0$. If subsidies are strategic substitutes, then $f_{12} < 0$, which implies $K = \delta\beta \in (-1, 0)$. We have two cases:

If $f_{12} + f_{22} > 0$ then $f_{11} + f_{12} + K(f_{12} + f_{22}) < 0$. If $f_{12} + f_{22} < 0$ then (44) implies $|f_{11} + f_{12}| > |f_{12} + f_{22}| > |K(f_{12} + f_{22})|$ and so $f_{11} + f_{12} + K(f_{12} + f_{22}) < 0$. Therefore, under strategic substitutes, $\frac{\partial s}{\partial K} < 0$.

If subsidies are atrategic complements, then $f_{12} > 0$, which implies $K = \delta\beta \in (0, 1)$. We also examine two cases:

If $f_{12} + f_{22} > 0$ then $f_{11} + f_{12} + K(f_{12} + f_{22}) = f_{11} + 2f_{12} + f_{22} - (1-K)(f_{12} + f_{22}) < 0$ because of (44) and $K \in (0, 1)$. If $f_{12} + f_{22} < 0$ then $f_{11} + f_{12} + K(f_{12} + f_{22}) < 0$. Therefore, under strategic complements we have that $\frac{\partial s}{\partial K} < 0$.

Under strategic substitutes, $\bar{K} = \delta\beta \in (-1, 0)$, and therefore $\bar{K} < \hat{K} < K^*$ which implies $s^* < \hat{s} < \bar{s}$. Under strategic complements, $\bar{K} = \delta\beta \in (0, 1)$, and therefore $\hat{K} < \bar{K} < K^*$ which implies $s^* < \bar{s} < \hat{s}$. Since the payoff function $f(.,.)$ is quadratic, synchronization yields higher payoffs under strategic substitutes, and staggering yields higher payoffs under strategic complements, as they move equilibrium subsidies closer to the joint maximum. ■

D Proof of Proposition 4

If we write the game in normal form we have the following payoff matrix:

		Government 2	
		even periods	odd periods
Government 1	even periods	$\sum_{\tau=0}^{\infty} \delta^\tau f(\hat{s}, \hat{s}), \sum_{\tau=0}^{\infty} \delta^\tau f(\hat{s}, \hat{s})$	$\sum_{\tau=0}^{\infty} \delta^\tau f(\bar{s}, \bar{s}), \sum_{\tau=0}^{\infty} \delta^\tau f(\bar{s}, \bar{s})$
	odd periods	$\sum_{\tau=0}^{\infty} \delta^\tau f(\bar{s}, \bar{s}), \sum_{\tau=0}^{\infty} \delta^\tau f(\bar{s}, \bar{s})$	$\sum_{\tau=0}^{\infty} \delta^\tau f(\hat{s}, \hat{s}), \sum_{\tau=0}^{\infty} \delta^\tau f(\hat{s}, \hat{s})$

Under strategic substitutes, $s^* < \hat{s} < \bar{s}$. Therefore $f(\hat{s},\hat{s}) > f(\bar{s},\bar{s})$, and the only equilibrium strategies in the game are . In any case, this implies that both governments prefer to set their policy in the same periods, leading to synchronization. Under strategic complements, $s^* < \bar{s} < \hat{s}$. Therefore $f(\hat{s},\hat{s}) < f(\bar{s},\bar{s})$ and the only equilibria are $(even, odd)$ or $(odd, even)$, that is, staggering.

E Proof of Lemma 5

Differentiating (22) we obtain:

$$\frac{\partial R^i(s^{-i})}{\partial s^{-i}} = \frac{-f_{12}}{f_{11} + \lambda W_{11}^i\left(R^i(s^{-i})\right)} = \beta^i \tag{45}$$

where the last equality comes from our restriction to linear dynamic reaction functions $R^i(s^{-i}) = \alpha^i + \beta^i s^{-i}$. Therefore, $W_{11}^i(.)$ is a constant under this assumption (see also Lau, 1997). From now on we will also omit the arguments of this function.

The function $W^i(s^i)$ can be defined substituting (20) in (21):

$$W^i(s^i) = \frac{f\left(s^i, R^{-i}(s^i)\right)}{r+\lambda} + \lambda\frac{f\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)}{(r+\lambda)^2} + \left(\frac{\lambda}{r+\lambda}\right)^2 W^i\left(R^i\left(R^{-i}(s^i)\right)\right)$$

Using our assumption of linear dynamic reaction functions $R^i(s^{-i}) = \alpha^i + \beta^i s^{-i}$ we can differentiate W^i to obtain

$$\begin{aligned} W_1^i(s^i) \quad = \quad & \frac{f_1\left(s^i, R^{-i}(s^i)\right)}{r+\lambda} + \frac{f_2\left(s^i, R^{-i}(s^i)\right)}{r+\lambda}\cdot\beta^{-i} + \\ & +\lambda\frac{f_1\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)}{(r+\lambda)^2}\cdot\beta^{-i}\beta^i + \\ & +\lambda\frac{f_2\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)}{(r+\lambda)^2}\cdot\beta^{-i} + \\ & +\left(\frac{\lambda}{r+\lambda}\right)^2 W_1^i\left(R^i\left(R^{-i}(s^i)\right)\right)\cdot\beta^{-i}\beta^i \end{aligned} \tag{46}$$

$$\begin{aligned} W_{11}^i(s^i) \quad = \quad & \frac{f_{11}\left(s^i, R^{-i}(s^i)\right)}{r+\lambda} + \frac{f_{12}\left(s^i, R^{-i}(s^i)\right)}{r+\lambda}\cdot\beta^{-i} + \frac{f_{21}\left(s^i, R^{-i}(s^i)\right)}{r+\lambda}\cdot\beta^{-i} + \frac{f_{22}\left(s^i, R^{-i}(s^i)\right)}{r+\lambda}\cdot\left(\beta^{-i}\right)^2 + \\ & +\lambda\frac{f_{11}\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)}{(r+\lambda)^2}\cdot\left(\beta^{-i}\beta^i\right)^2 + \lambda\frac{f_{12}\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)}{(r+\lambda)^2}\cdot\left(\beta^{-i}\right)^2\beta^i + \\ & +\lambda\frac{f_{21}\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)}{(r+\lambda)^2}\cdot\left(\beta^{-i}\right)^2\beta^i + \lambda\frac{f_{22}\left(R^i\left(R^{-i}(s^i)\right), R^{-i}(s^i)\right)}{(r+\lambda)^2}\cdot\left(\beta^{-i}\right)^2 + \\ & +\left(\frac{\lambda}{r+\lambda}\right)^2 W_{11}^i\left(R^i\left(R^{-i}(s^i)\right)\right)\cdot\left(\beta^{-i}\beta^i\right)^2 \end{aligned}$$

Using again the fact that f_{ij} $(i,j=1,2)$ and W_{11}^i are constant, we can collect terms to obtain

$$\begin{aligned} W_{11}^i \quad = \quad & \frac{1}{r+\lambda}\left\{f_{11}\left(1+\frac{\lambda}{r+\lambda}\left(\beta^{-i}\beta^i\right)^2\right) + 2f_{12}\left(\beta^{-i}+\frac{\lambda}{r+\lambda}\left(\beta^{-i}\right)^2\beta^i\right) + \right. \\ & \left. +f_{22}\left((1+\frac{\lambda}{r+\lambda})\left(\beta^{-i}\right)^2\right) + \left(\frac{\lambda^2}{r+\lambda}\right)W_{11}^i\cdot\left(\beta^{-i}\beta^i\right)^2\right\} \end{aligned}$$

and so

$$W_{11}^i = \left(\frac{f_{11}\left(1+\frac{\lambda}{r+\lambda}\left(\beta^{-i}\beta^i\right)^2\right) + 2f_{12}\left(\beta^{-i}+\frac{\lambda}{r+\lambda}\left(\beta^{-i}\right)^2\beta^i\right) + f_{22}\left((1+\frac{\lambda}{r+\lambda})\left(\beta^{-i}\right)^2\right)}{1-\left(\frac{\lambda^2}{r+\lambda}\right)\left(\beta^{-i}\beta^i\right)^2} \right) \tag{47}$$

Substituting the value of W_{11}^i in (47) into (45) we obtain:

$$f_{11}\beta^i + \lambda\beta^i \left(\frac{f_{11}\left(1+\frac{\lambda}{r+\lambda}\left(\beta^{-i}\beta^i\right)^2\right) + 2f_{12}\left(\beta^{-i}+\frac{\lambda}{r+\lambda}\left(\beta^{-i}\right)^2\beta^i\right) + f_{22}\left((1+\frac{\lambda}{r+\lambda})\left(\beta^{-i}\right)^2\right)}{1-\left(\frac{\lambda^2}{r+\lambda}\right)\left(\beta^{-i}\beta^i\right)^2} \right) + f_{12} = 0$$

that is, we have a system of two equations:

$$f_{11}(r+\lambda)(r+2\lambda)\beta^1 + f_{12}\left(r+\lambda+\lambda\beta^1\beta^2\right)^2 + f_{22}\lambda(r+2\lambda)\beta^1\left(\beta^2\right)^2 = 0 \tag{48}$$

$$f_{11}(r+\lambda)(r+2\lambda)\beta^2 + f_{12}\left(r+\lambda+\lambda\beta^2\beta^1\right)^2 + f_{22}\lambda(r+2\lambda)\beta^2\left(\beta^1\right)^2 = 0 \tag{49}$$

which may have symmetric and asymmetric solutions. Substracting (49) from (48):

$$\left(\beta^1-\beta^2\right)\left(f_{11}(r+\lambda) - f_{22}\lambda\beta^1\beta^2\right) = 0 \tag{50}$$

Therefore we have two possible cases. If $f_{22}=0$, then (50) implies that there is only a symmetric solution $\beta^1 = \beta^2 = \beta$. If, on the contrary, $f_{22} \neq 0$ and there is an asymmetric solution (with $\beta^1 \neq \beta^2$), then

$$\beta^1\beta^2 = \frac{r+\lambda}{\lambda}\frac{f_{11}}{f_{22}}$$

If $\left|\frac{r+\lambda}{\lambda}\frac{f_{11}}{f_{22}}\right| > 1$ then at least one of the β^j $(j=1,2)$ has an absolute value greater than one and therefore the asymmetric equilibrium is unstable and can be ruled out.

The symmetric solution ($\beta^1 = \beta^2 = \beta$) can be derived from (48). It will be the solution to:

$$\beta^4\lambda^2 f_{12} + \beta^3\lambda(r+2\lambda)f_{22} + \beta^2 2\lambda(r+\lambda)f_{12} + \beta(r+\lambda)(r+2\lambda)f_{11} + (r+\lambda)^2 f_{12} = 0$$

which is the polynomial (24) in the statement of the lemma. ■

F Proof of Proposition 6

In the game with random commitment, the steady state equilibrium subsidy will be the same for both governments: $s^i = s^{-i} = \widetilde{s}$. Therefore, from (46), the expression for W_1^i at the steady state equilibrium becomes:

$$\begin{aligned} W_1^i(\widetilde{s}) &= \frac{f_1(\widetilde{s},\widetilde{s})}{r+\lambda} + \frac{f_2(\widetilde{s},\widetilde{s})}{r+\lambda}\cdot\beta^{-i} + \lambda\frac{f_1(\widetilde{s},\widetilde{s})}{(r+\lambda)^2}\cdot\beta^{-i}\beta^i + \\ &\quad +\lambda\frac{f_2(\widetilde{s},\widetilde{s})}{(r+\lambda)^2}\cdot\beta^{-i} + \left(\frac{\lambda}{r+\lambda}\right)^2 W_1^i(\widetilde{s})\cdot\beta^{-i}\beta^i \end{aligned}$$

If we analyze the symmetric solution where $\beta^i = \beta^{-i} = \beta$, then

$$W_1^i(\widetilde{s}) = \left(\frac{1}{r+\lambda}\right) \frac{f_1\left(\widetilde{s},\widetilde{s}\right)\left(1+\left(\frac{\lambda}{r+\lambda}\right)\beta^2\right) + f_2\left(\widetilde{s},\widetilde{s}\right)\beta\left(1+\left(\frac{\lambda}{r+\lambda}\right)\right)}{1-\left(\frac{\lambda}{r+\lambda}\right)^2\beta^2}$$

Replacing this into the first order condition (22) yields

$$f_1\left(\widetilde{s},\widetilde{s}\right) + \left(\frac{\lambda}{r+\lambda}\right)\left(\frac{f_1\left(\widetilde{s},\widetilde{s}\right)\left(1+\left(\frac{\lambda}{r+\lambda}\right)\beta^2\right) + f_2\left(\widetilde{s},\widetilde{s}\right)\beta\left(1+\left(\frac{\lambda}{r+\lambda}\right)\right)}{1-\left(\frac{\lambda}{r+\lambda}\right)^2\beta^2}\right) = 0$$

that is:

$$f_1\left(\widetilde{s},\widetilde{s}\right) + \left(\frac{\lambda}{r+\lambda}\right)\beta f_2\left(\widetilde{s},\widetilde{s}\right) = 0 \tag{51}$$

with β given by (45).

Also note that, since $\beta = \frac{-f_{12}}{f_{11}+\lambda W_{11}^i}$,(see (45)) then using the second order condition (23) and $f_{12} < 0$ we can conclude that $\beta < 0$. Therefore, using the same procedure as in the proof of proposition 2 we can show that $\widetilde{s} > \hat{s} > s^*$ if subsidies are strategic substitutes and $\hat{s} > \widetilde{s} > s^*$ if subsidies are strategic complements. ■

G Proof of Proposition 7

Consider a symmetric Markov perfect equilibrium $(\hat{s},\hat{s})$ of the synchronized game, and let $\bar{R}^1$, $\bar{R}^2$ be the mixed strategy which plays the equilibrium reaction functions $R^1(.)$ and $R^2(.)$ of the staggered game in eq. (10)-(12) with probability ω and the "inactive" strategy with probability $1-\omega$. We need to prove that $\left\{(\bar{R}^1,\hat{s}),(\bar{R}^2,\hat{s})\right\}$ is a Markov perfect equilibrium of the game with endogenous timing.

Note first that, in the synchronized mode, when governments are setting $(\hat{s},\hat{s})$ every two periods, since they are equilibrium strategies in the synchronized game, they never entail the choice of the "inactive strategy, and therefore, once governments are in the synchronized mode, they remain there forever.

Now turn to the staggered mode. Government i, when it is its time to act, will have an incentive to remain inactive during one period and turn the game into a synchronized game since $f(0,\bar{s}) + \delta\frac{f(\hat{s},\hat{s})}{1-\delta} > f(\bar{s},\bar{s}) + \delta\frac{f(\bar{s},\bar{s})}{1-\delta}$ for δ sufficiently close to 1. This implies that they would optimally set $\omega = 0$ in their mixed strategy $\bar{R}^i$

Left to show is that a government, when in the synchronized mode, has no incentive to play the "inactive" strategy, given that the other government is playing according to $(R^{-i}(.),\hat{s})$. If it did, they would be switching to staggering and therefore earning lower benefits since $f(0,\hat{s}) + \delta\frac{f(\bar{s},\bar{s})}{1-\delta} < f(\hat{s},\hat{s}) + \delta\frac{f(\hat{s},\hat{s})}{1-\delta}$ as $\bar{s} > \hat{s} > 0$. This means that once governments enter into the synchronized mode, they will never switch to staggering.

Combining these two claims, we complete the proof of parts 1a and 1b of the proposition.

To prove part 2, note that if there exists a Markov perfect equilibrium involving the equilibrium reaction functions $R^1(.)$ and $R^2(.)$ of the staggered game, then, since those strategies do not call for inaction as part of the equilibrium, once we reach staggering, both governments will continue in that mode forever, and thus earn $f(\bar{s},\bar{s})+\delta\frac{f(\bar{s},\bar{s})}{1-\delta}$.

If governments are in the simultaneous mode, they would have to play the equilibrium subsidy $\hat{s}$ (recall that this is the unique equilibrium subsidy of the simultaneous game). But then playing $R^i(.)$ when the other government is committed is not an equilibrium strategy: if government i plays "inactive" instead, can switch the game to synchronous mode and get $f(0,\bar{s})+\delta\frac{f(\hat{s},\hat{s})}{1-\delta} > f(\bar{s},\bar{s})+\delta\frac{f(\bar{s},\bar{s})}{1-\delta}$ for δ sufficiently close to 1.

For the same reason, when governments are in synchronized mode, they will never choose inaction and switch to staggering since that would imply, for the government that chooses inaction, a payoff equal to $f(0,\hat{s})+\delta\frac{f(\bar{s},\bar{s})}{1-\delta} < f(\hat{s},\hat{s})+\delta\frac{f(\hat{s},\hat{s})}{1-\delta}$ ■

H Proof of Proposition 8

****** COMPLETE HERE USING SAME LINE OF REASONING THAN PROOF OF PROPOSITION 7 ******* ■

I Proof of Proposition 10

Note that the conditions for a Markov perfect equilibrium imply that the synchronized subsidy $\hat{s}$ satisfies (see eq.(26))

$$f_1(\hat{s},\hat{s})=0 \tag{52}$$

Now turn to the staggered game. According to proposition 9, if s_1^i and s_2^i $(i=1,2)$ are equilibrium subsidies in the staggered game then

$$s_1^i = F^i(s_2^{-i}) \qquad i=1,2 \tag{53}$$

$$f_1(F^i(s_2^{-i}),s_2^{-i})=0 \qquad i=1,2 \tag{54}$$

$$\theta(s_2^i) \equiv f_1\left(s_2^i, F^{-i}(s_2^i)\right) - f_2\left(s_2^i, F^{-i}(s_2^i)\right)\frac{f_{12}(F^{-i}(s_2^i),s_2^i)}{f_{11}(F^{-i}(s_2^i),s_2^i)} = 0 \qquad i=1,2 \tag{55}$$

$$\theta_1(s_2^i)<0 \tag{56}$$

First, note that from (52) and (54) we have $F^i(\hat{s})=\hat{s}$.

Recall that $f_2<0$ and $f_{11}<0$. Under strategic substitutes $f_{12}<0$ and therefore $\theta(\hat{s})>0=\theta(s_2^i)$. Since the second order condition (56) implies that $\theta(.)$ is decreasing, it follows that $s_2^i<\hat{s}$ for all i. Finally since subsidies are strategic substitutes, $F(.)$ is decreasing (see eq (33)) and so $s_1^i=F^i(s_2^{-i})>F^i(\hat{s})=\hat{s}>s_2^i$.

********** COMPLETE PROOF HERE ALONG SAME LINES FOR STRATEGIC COMPLEMENTS ********* ■

J Proof of Proposition 11

To prove the proposition we just need to prove that $\bar{\phi}_t^i > \phi_t^{fi}$ for all δ and that $\bar{\phi}_t^i > \phi_t^{si}$ for δ sufficiently close to 1. These two statements are equivalent to prove that

$$f(\hat{s},\hat{s}) > \frac{f(s_1,s_2)+\delta f(s_2,s_1)}{1+\delta} \qquad \text{for all } \delta \in [0,1]$$

$$\exists\bar{\delta} < 1 \text{ s.t.} \qquad f(\hat{s},\hat{s}) > \frac{f(s_2,s_1)+\delta f(s_1,s_2)}{1+\delta} \qquad \text{for } \delta \in (\bar{\delta},1]$$

Note that, because of the linearity of the reaction function, $F^i(s_2) = F^i(\hat{s}) + F_1^i(\hat{s})[s_2 - \hat{s}]$. Since $F^i(s_2) = s_1$ (eq. (53)) and from (52) and (54) we have $F^i(\hat{s}) = \hat{s}$. therefore

$$s_1 - \hat{s} = -\frac{f_{12}}{f_{11}}(s_2 - \hat{s})$$

Using $f_{12} < 0$, $f_{11} < 0$ and the stability condition (8) we know that $-1 < -\frac{f_{12}}{f_{11}} < 0$ and therefore $\hat{s} < \frac{s_1+s_2}{2}$. Since $s_1 < \hat{s} < s_2$ (see proposition 10) then

$$\hat{s} < \frac{s_2+\delta s_1}{1+\delta} \qquad \text{for all } \delta \in [0,1]$$

Also, by continuity, there exists some $\bar{\delta} < 1$ such that

$$\hat{s} < \frac{s_1+\delta s_2}{1+\delta} \qquad \text{for } \delta \in (\bar{\delta},1]$$

Now turn to $f(\hat{s},\hat{s})$. From eq (42), $f(x,x)$ is decreasing in x for $x > s^*$. Since $\frac{s_2+\delta s_1}{1+\delta} > \hat{s} > s^*$ (see eq. (??)) then

$$f(\hat{s},\hat{s}) > f\left(\frac{s_2+\delta s_1}{1+\delta}, \frac{s_2+\delta s_1}{1+\delta}\right) > \frac{f(s_2,s_1)+\delta f(s_1,s_2)}{1+\delta} \tag{57}$$

where the last inequality comes from the concavity of $f(.)$. Similarly, since $\hat{s} < \frac{s_1+\delta s_2}{1+\delta}$ for $\delta \in (\bar{\delta},1]$ then

$$f(\hat{s},\hat{s}) > f\left(\frac{s_1+\delta s_2}{1+\delta}, \frac{s_1+\delta s_2}{1+\delta}\right) > \frac{f(s_1,s_2)+\delta f(s_2,s_1)}{1+\delta} \tag{58}$$

which completes the proof.

************** COMPLETE PROOF FOR STRATEGIC COMPLEMENTS ***************

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