Cost effectiveness of R&D and the robustness of Strategic Trade Policy *

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Abstract

This paper analyzes the incentives for governments to impose export subsidies when firms invest in a cost saving technology before market competition. Governments first impose an export subsidy or a tax. After observing export policy, firms invest in cost reducing R&D and subsequently compete in the market. Governments subsidize exports under Cournot competition. Under Bertrand competition, export subsidies are positive whenever R&D is sufficiently cost-effective at reducing marginal costs, and negative otherwise. The trade policy reversal found in models without endogenous sunk costs disappears if R&D is sufficiently cost-effective. Output subsidies are more robust than implied by the recent literature.

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1 Introduction

Since Eaton and Grossman (1986), one of the major criticisms of the strategic trade literature has been its non-robustness to the mode of market competition. If trade policy is sensitive to the choice of strategic variable by firms and governments are uncertain about the mode of competition then strategic trade policy can be more harmful than beneficial. In this paper, we analyze export subsidies when firms invest in cost-reducing R&D before the market competition stage. Governments choose export subsidies first. After observing governments' choice, firms invest in R&D and then compete in a third market (in prices or quantities). We find that for sufficiently cost effective R&D[1] governments subsidize exports independently of the mode of competition. This suggests that export subsidies are more robust to the type of the market competition than implied by the recent literature.

Several authors have studied the robustness of strategic trade policy using two kinds of models. In the first kind, in a two-stage game, governments first commit to output subsidies and then firms compete in the market. Using this approach Brander and Spencer (1985) show that the optimal trade policy is an export subsidy under Cournot competition. Eaton and Grossman (1986), however, show that the optimal strategic trade policy reverses to an export tax if firms compete in prices.[2] This policy reversal highlights the lack of robustness of strategic trade policy when governments are uncertain about the mode of competition.

In the second kind of models, actions are chosen in a three-stage game: governments first commit to a policy, firms then invest in R&D and later compete in the market. In such models, investing in a strategic variable before the market competition stage captures entry barriers, a feature that is fundamental to oligopolistic market structures (see Sutton, 1991). A further appeal of these models is that they capture firm commitment to a strategic variable before the competition stage (Grossman, 1988). If firms can make sunk investments before the market competition stage then governments have two instruments at their disposal: output and R&D subsidies. If governments use only R&D policy Bagwell and Staiger (1994) show that governments subsidize R&D under both Cournot and Bertrand Competition.[3] Based on this, Brander (1995) suggests that R&D subsidies seem more robust than output subsidies. Neary and Leahy (2000), however, dispute Brander's claim.[4] They show that when governments use two instruments (an output and a R&D subsidy at the same time) then both instruments are not robust to the nature of market competition.[5]

This paper adds another argument against Brander's claim that R&D subsidies are more robust than output subsidies. If governments only subsidize exports and firms invest in R&D (before competing in the

[1]We refer to the cost-effectiveness of R&D as the effect of R&D on marginal costs relative to the cost of investing in R&D.

[2]The reversal in the optimal export policy is explained by the fact that outputs are strategic substitutes and prices are strategic complements. See Brander (1995) for a discussion on this.

[3]Spencer and Brander (1983) had shown the optimality of R&D subsidies under Cournot competition. Bagwell and Staiger (1994) develop a model where the effect of R&D investment is stochastic. In the case where R&D reduces the mean but does not affect the variance of costs (the closest case to deterministic R&D), they find that R&D should be subsidized under both Cournot and Bertrand competition. Maggi (1996) finds a similar result in a model where firms invest in capacities (instead of R&D) before the competition stage. The optimal policy in his model is to subsidize capacities.

[4]See Neary and Leahy (2000), page 505.

[5]Neary and Leahy (2000) show that under Cournot competition governments subsidize exports and tax R&D, a result found in Spencer and Brander (1983). However, under Bertrand competition, governments will tax exports and subsidize R&D. The intuition is that governments use export policy to shift profits from foreign firms (as in models without R&D) and use R&D policy to correct the distortion on R&D generated by the strategic behavior of firms. Therefore, if governments use two instruments, R&D is no longer robust to changes in the mode of competition.

market), we show that the optimal trade policy is an export subsidy under both Cournot and Bertrand competition, provided R&D is sufficiently cost-effective.[6] This means that output policy is more robust than previously considered by the literature. This is true especially in industries where the marginal cost of R&D is not too high relative to its effect on process innovation.

The paper closest to ours are Spencer and Brander (1983) and Neary and Leahy (2000). Spencer and Brander (1983) show that governments impose an output subsidy under Cournot competition when firms can invest in R&D before competing in the market. They analyze two cases that are different to ours. First, they show the optimality of output subsidies if they are set by governments *after* firms decide their R&D investment. Second, they show that output subsidies are optimal if they are *set jointly* with R&D subsidies before R&D is chosen by firms. In the first part of our paper, we extend their results to the case when output subsidies are chosen before firms invest in R&D.

In a numerical simulation, Neary and Leahy (2000) show that if governments only use output subsidies then the Eaton and Grossman trade policy reversal from Cournot to Bertrand competition is still observed when firms invest in R&D before the market competition stage. In this paper, we show that their result holds only when R&D is ineffective at reducing marginal costs. Our result becomes clear once one realizes that the effect of R&D on profits depends on the level of output. Due to output expansion, an export subsidy increases the ability of domestic R&D to shift profits from the foreign firm. Output expansion, due to the output subsidy, occurs under both Cournot and Bertrand competition. Therefore, only looking at R&D, governments have the incentive to subsidize exports both under price and quantity competition.

The sign of the optimal policy depends upon the net effect of the export subsidy on the R&D and the market competition stage. In a model without R&D, the sign of the strategic trade policy depends on the strategic complementarity or substitutability of the variables chosen by firms in the market competition stage. Under R&D and Cournot competition, a unilateral export subsidy increases welfare both in the R&D and in the market competition stage. This means that governments want to subsidize exports (Spencer and Brander, 1983). Under Bertrand competition, however, the two effects have the opposite sign. If R&D is sufficiently cost effective then R&D will be relatively elastic with respect to an export subsidy. This high elasticity of R&D will make the effect of the output subsidy on the R&D stage stronger than the effect on the price competition stage. In this case, governments subsidize output under Bertrand competition. Conversely, if R&D is not sufficiently cost-effective then the effect of an output subsidy on the price competition stage dominates the effect on the R&D stage and the optimal policy under Bertrand competition is an output tax.

We use the standard third country model of strategic trade as in Spencer and Brander (1983). Two firms, one located in each country, produce a differentiated good which is exported to a third country. There is no domestic consumption and welfare is measured as producer surplus (profits) net of subsidy costs.[7] In a three stage game, the domestic government first sets an output subsidy s^1. This is followed by both firms simultaneously deciding their investment in cost-reducing R&D (Δ^i and Δ^j). In the third stage, firms

[6]Spencer and Brander (1983) showed that governments impose an output subsidy under Cournot competition when firms can invest in R&D before competing in the market. However, they focus on two cases that are different to ours. They show the optimality of output subsidies if they are set by governments *after* firms decide their R&D investment. They also show that output subsidies are optimal if they are *set jointly* with R&D subsidies and before R&D is chosen by firms.

[7]Public funds may have an opportunity cost bigger than one (as in Neary [1994]). We abstract from this issue in this analysis.

compete in the product market simultaneously choosing quantities, or prices. At each stage, all players have complete information about the moves of the preceding stage. We also assume that governments commit to an export subsidy while firms commit to their investment in R&D.

We proceed as follows: in section 2 we analyze the benchmark free trade case under Cournot competition and then introduce the choice of an output subsidy by a domestic government. In section 3 we perform the same analysis under Bertrand competition. Section 4 presents a numerical simulation that highlights the effect of the convexity of the cost of R&D over the optimal trade policy. Section 5 concludes.

2 Cournot Competition

2.1 Final Stage: Quantity Competition

Firms first choose R&D investment. Output is chosen in the second stage of the game. R&D investment generates a process innovation of size Δ^i (by firm i), imposing a monetary cost of $\phi(\Delta^i)$ upon the firm. The monetary cost is increasing and convex in the extent of process innovation and reduces total and marginal costs of production. Denoting firms by superscripts and derivatives by subscripts these assumptions translate into:

$$C^i_\Delta = \frac{\partial C^i(x^i,\Delta^i)}{\partial \Delta^i} \leq 0, \qquad C^i_{\Delta\Delta} = \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial\left(\Delta^i\right)^2} \geq 0, \qquad C^i_{x\Delta} = \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial \Delta^i \partial x^i} \leq 0 \tag{1}$$

$$\phi^i_i(\Delta^i) > 0, \qquad \phi^i_{ii}(\Delta^i) > 0 \tag{2}$$

The choice of R&D investment is irreversible and simultaneous for both firms. We assume that goods are imperfect substitutes and that the own-price effect dominates the cross-price effect:[8]

$$\frac{\partial p^i(x^i,x^j)}{\partial x^j} < \frac{\partial p^i(x^i,x^j)}{\partial x^i} < 0 \tag{3}$$

The following assumptions concern the behavior of revenues $R^i(x^i,x^j) = x^i p^i(x^i,x^j)$:

$$R^i_j(x^i,x^j) = x^i \frac{\partial p^i(x^i,x^j)}{\partial x^j} < 0 \tag{4}$$

$$R^i_{ii}(x^i,x^j) = x^i \frac{\partial^2 p^i(x^i,x^j)}{\partial\left(x^i\right)^2} + 2\frac{\partial p^i(x^i,x^j)}{\partial x^i} < 0 \tag{5}$$

$$R^i_{jj}(x^i,x^j) = x^i \frac{\partial^2 p^i(x^i,x^j)}{\partial\left(x^j\right)^2} \geq 0 \tag{6}$$

$$R^i_{ij}(x^i,x^j) = x^i \frac{\partial p^i(x^i,x^j)}{\partial x^i \partial x^j} + \frac{\partial p^i(x^i,x^j)}{\partial x^j} < 0 \tag{7}$$

Assumption (4) states that goods i and j are imperfect substitutes, (5) states that the revenue is concave in own quantity, and is satisfied by demand functions that are not too convex. Assumption (6) states that revenue decreases (at a decreasing rate) with an increase in the other firm's output. This is true in particular for linear demands. Lastly, (7) states that an increase in sales of one good decreases marginal revenue of the other (again satisfied in the case of a linear demand).

[8] Strictly speaking, the condition for the own price effect to dominate the cross price effect is $\left(\frac{\partial p^i(x^i,x^j)}{\partial x^j}\right)^2 < \left(\frac{\partial p^i(x^i,x^j)}{\partial x^i}\right)^2$. In this paper we restrict our attention to the case of imperfect substitutes, that is $\frac{\partial p^i(x^i,x^j)}{\partial x^j} < 0$.

Firm i chooses output, x_i, to maximize profits, $\Pi^i(x^i, x^j, \Delta^i) = R^i(x^i, x^j) - C^i(x^i, \Delta^i) - \phi(\Delta^i) = x^i p^i(x^i, x^j) - C^i(x^i, \Delta^i) - \phi(\Delta^i)$, where $R^i(x^i, x^j)$ is firm i's revenue:

$$\max_{x^i} \Pi^i(x^i, x^j, \Delta^i) = R^i(x^i, x^j) - C^i(x^i, \Delta^i) - \phi(\Delta^i)$$

The first order conditions (FOC) to this maximization problem determine the reaction function $x^i(x^j)$ of firm i to changes in firm $j's$ output. These FOCs satisfy

$$\Pi^i_i = R^i_i(x^i, x^j) - C^i_x(x^i, \Delta^i) = 0 \tag{8}$$

with the second order condition

$$\begin{aligned} \Pi^i_{ii} &= R^i_{ii}(x^i, x^j) - C^i_{xx}(x^i, \Delta^i) < 0 \\ &= x_i \frac{\partial^2 p^i(x^i, x^j)}{\partial \left(x^i\right)^2} + 2\frac{\partial p^i(x^i, x^j)}{\partial x^i} - \frac{\partial^2 C^i(x^i, \Delta^i)}{\left(\partial x^i\right)^2} \leq 0 \end{aligned} \tag{9}$$

We assume that the second order condition is always satisfied. For later use we need to assume that the own effect of output on marginal profit is stronger (greater in absolute value) than the cross effect, that is, $\Pi^i_{ii} < \Pi^i_{ij}$. This then implies that,

$$\Pi^1_{11}\Pi^2_{22} - \Pi^1_{12}\Pi^2_{12} > 0 \tag{10}$$

The solution to the two equations in (8) gives us outputs as a function of R&D levels (chosen in the previous stage):

$$x^1 = q^1(\Delta^1, \Delta^2) \tag{11}$$

$$x^2 = q^2(\Delta^1, \Delta^2) \tag{12}$$

The slope of firm i's reaction function is computed by totally differentiating the first order condition (8) with respect to x^1 and x^2 (keeping Δ^1 and Δ^2 constant):

$$\left(R^i_{ii}(x^i, x^j) - C^i_{xx}(x^i, \Delta^i)\right) \mathrm{d}x^i + R^i_{ij}(x^i, x^j)\mathrm{d}x^j = 0 \tag{13}$$

This then gives us the slope of firm $i's$ reaction function, which is negative (i.e. *strategic substitutes*) from assumptions (7) and (9):

$$\frac{\mathrm{d}x^i}{\mathrm{d}x^j} = -\frac{R^i_{ij}(x^i, x^j)}{\left(R^i_{ii}(x^i, x^j) - C^i_{xx}(x^i, \Delta^i)\right)} = -\frac{R^i_{ij}(x^i, x^j)}{\Pi^i_{ii}(x^i, x^j, \Delta^i)} < 0 \tag{14}$$

2.1.1 Effect of R&D Investment on output

The effect of R&D investment on output can be seen by totally differentiating the system of two first order conditions (given by equation 8):

$$\left(R^1_{11}(x^1, x^2) - C^1_{xx}(x^1, \Delta^1)\right) \mathrm{d}x^1 + R^1_{12}(x^1, x^2)\mathrm{d}x^2 - C^1_{x\Delta}(x^1, \Delta^1)\mathrm{d}\Delta^1 = 0 \tag{15}$$

$$R^2_{12}(x^1, x^2)\mathrm{d}x^1 + \left(R^2_{22}(x^1, x^2) - C^2_{xx}(x^2, \Delta^2)\right) \mathrm{d}x^2 - C^2_{x\Delta}(x^2, \Delta^2)\mathrm{d}\Delta^2 = 0 \tag{16}$$

We thus have the system,

$$\begin{pmatrix} \Pi_{11}^1(x^1,x^2,\Delta^1) & R_{12}^1(x^1,x^2) \\ R_{12}^2(x^1,x^2) & \Pi_{22}^2(x^1,x^2,\Delta^2) \end{pmatrix} \begin{pmatrix} \mathrm{d}x^1 \\ \mathrm{d}x^2 \end{pmatrix} = \begin{pmatrix} C_{x\Delta}^1(x^1,\Delta^1)\mathrm{d}\Delta^1 \\ C_{x\Delta}^2(x^2,\Delta^2)\mathrm{d}\Delta^2 \end{pmatrix} \tag{17}$$

Using Cramer's rule we can write

$$\mathrm{d}x^i = \frac{\Pi_{jj}^j(x^i,x^j,\Delta^j)C_{x\Delta}^i(x^i,\Delta^i)\mathrm{d}\Delta^i - R_{ij}^i(x^i,x^j)C_{x\Delta}^j(x^j,\Delta^j)\mathrm{d}\Delta^j}{\Pi_{ii}^i(x^i,x^j,\Delta^i)\Pi_{jj}^j(x^i,x^j,\Delta^j) - R_{ij}^j(x^i,x^j)R_{ij}^i(x^i,x^j)} \tag{18}$$

From this we can obtain expressions that give us the effect on output of domestic ($\frac{\mathrm{d}x^i}{\mathrm{d}\Delta^i}$) and foreign R&D ($\frac{\mathrm{d}x^i}{\mathrm{d}\Delta^j}$):

$$q_i^i(\Delta^i,\Delta^j) = \frac{\mathrm{d}x^i}{\mathrm{d}\Delta^i} = \frac{\Pi_{jj}^j C_{x\Delta}^i}{\Pi_{ii}^i\Pi_{jj}^j - R_{ij}^j R_{ij}^i} = \frac{\Pi_{jj}^j C_{x\Delta}^i}{\Pi_{ii}^i\Pi_{jj}^j - \Pi_{ij}^j\Pi_{ij}^i} > 0 \tag{19}$$

$$q_j^i(\Delta^i,\Delta^j) = \frac{\mathrm{d}x^i}{\mathrm{d}\Delta^j} = \frac{-R_{ij}^i C_{x\Delta}^j}{\Pi_{ii}^i\Pi_{jj}^j - R_{ij}^j R_{ij}^i} = \frac{-\Pi_{ij}^i C_{x\Delta}^j}{\Pi_{ii}^i\Pi_{jj}^j - \Pi_{ij}^j\Pi_{ij}^i} < 0 \tag{20}$$

where the inequalities come from (1), (7) and (9). The intuition is straightforward: an increase in R&D expenditure reduces the marginal cost of production and thus shifts out the reaction curve of firm i. Given that reaction functions are downward sloping (14), this implies that firm i produces more output while, firm j produces less.

For later use we need to compute q_{ij}^j and q_{ii}^j. Differentiating, (20) we obtain

$$q_{ij}^j = \frac{R_{ij}^j C_{x\Delta}^i C_{xx\Delta}^j \Pi_{ii}^i}{\left(\Pi_{ii}^i\Pi_{jj}^j - R_{ij}^j R_{ij}^i\right)^2} \tag{21}$$

$$q_{ii}^j = \frac{\left(-R_{ij}^j C_{x\Delta\Delta}^i\right)\left(\Pi_{ii}^i\Pi_{jj}^j - R_{ij}^j R_{ij}^i\right) + R_{ij}^j C_{x\Delta}^i C_{xx\Delta}^i \Pi_{jj}^j}{\left(\Pi_{ii}^i\Pi_{jj}^j - R_{ij}^j R_{ij}^i\right)^2} \tag{22}$$

Notice that q_{ij}^j is equal to zero for constant marginal costs with respect to output and q_{ii}^j is zero for marginal costs that are constant with respect to output and linear with respect to R&D.

2.2 R&D investment

In the R&D stage the profit function of firm i can be written as $\pi^i(\Delta^i,\Delta^j) = \Pi^i(q^i(\Delta_i,\Delta_j), q^j(\Delta_i,\Delta_j),\Delta^i) = R^i(q^i,q^j) - C^i(q^i,\Delta^i) - \phi^i(\Delta^i)$. Thus, firm $i's$ FOC in the simultaneous choice of R&D is[9],

$$\pi_i^i(\Delta^i,\Delta^j) = \frac{\partial \pi^i}{\partial \Delta^i} = \frac{\partial \Pi^i}{\partial \Delta^i} + \frac{\partial \Pi^i}{\partial x^j}\frac{\partial q^j}{\partial \Delta^i} = 0 \tag{23}$$

where $\frac{\partial \Pi^i}{\partial \Delta^i} = -C_\Delta^i(x^i,\Delta^i) - \phi_i^i(\Delta^i)$, $\frac{\partial \Pi^i}{\partial x^j} = x^i\frac{\partial p^i(x_i,x_j)}{\partial x^j} < 0$, and

$$\frac{\partial q^j}{\partial \Delta^i} = \frac{-\Pi_{ij}^j C_{x\Delta}^i}{\Pi_{ii}^i\Pi_{jj}^j - \Pi_{ij}^j\Pi_{ij}^i} < 0. \tag{24}$$

[9] Using the envelope theorem $\frac{\partial \Pi^i}{\partial x^i} = 0$.

Note that this is just a reflection of the effect mentioned before: an increase in R&D by firm i reduces its marginal cost and increases the quantity produced. Given that quantities are strategic substitutes, this results in a reduction in the quantity produced by the other firm (j).

The strategic effect $\frac{\partial \Pi^i}{\partial x^j}\frac{\partial q^j}{\partial \Delta^i}$ is positive. As a result, firms have an incentive to *overinvest in R&D*. This decreases rival output and increases own profits. This can be easily seen from (23) where firms choose R&D to set $\frac{\partial \Pi^i}{\partial \Delta^i} < 0$.

Noting that $R^i_i(x^i, x^j) - C^i_x(x^i, \Delta^i) = \Pi^i_i = 0$, the first order condition for a Nash equilibrium in the choice of R&D levels can be rewritten as,

$$\pi^i_i(\Delta^i, \Delta^j) = R^i_j(x^i, x^j)q^j_i(\Delta^i, \Delta^j) - C^i_\Delta(x^i, \Delta^i) - \phi^i_i(\Delta^i) = 0 \tag{25}$$

with the second order condition,

$$\pi^i_{ii}(\Delta^i, \Delta^j) = R^i_j q^j_{ii} + q^j_i \frac{\mathrm{d}R^i_j(x^i, x^j)}{\mathrm{d}\Delta^i} - C^i_{x\Delta}q^i_i - C^i_{\Delta\Delta} - \phi^i_{ii} < 0. \tag{26}$$

Where, $\frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^i} = R^i_{ij}(x^i, x^j)q^i_i + R^i_{jj}(x^i, x^j)q^j_i < 0$ (by (6), (7), (19) and (20)) and $q^j_i\frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^i} - C^i_{x\Delta}q^i_i > 0$. Even if we assume that marginal costs are constant with respect to output and linear with respect to R&D, (i.e. $q^j_{ii} = 0$) we still need to ensure that $C^i_{\Delta\Delta} + \phi^i_{ii}$ is big enough for (26) to hold. This implies that as R&D increases its cost-effectiveness has to decline fast enough.

We now assume a condition similar to (10). It refers to the effect of R&D on profits. Again, assuming that own effect of R&D on marginal profits is stronger (bigger in absolute value) than the cross effect (i.e. $\pi^i_{ii} < \pi^i_{ij}$) we get

$$\pi^1_{11}\pi^2_{22} - \pi^1_{12}\pi^2_{12} > 0. \tag{27}$$

Where, $\pi^i_{ij} = R^i_j q^j_{ij} + q^j_i \frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^j} - C^i_{x\Delta}q^i_j$. In this case $\frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^j} = R^i_{ij}(x^i, x^j)q^i_j + R^i_{jj}(x^i, x^j)q^j_j > 0$ (from (6), (7), (19) and (20)). This, together with (4), (19), (20), (1), and the assumption that marginal costs are constant with respect to output (so that $q^j_{ij} = 0$), gives us

$$\pi^i_{ij} = q^j_i \left(R^i_{ij}(x^i, x^j)q^i_j + R^i_{jj}(x^i, x^j)q^j_j\right) - C^i_{x\Delta}q^i_j < 0. \tag{28}$$

In order to compute the slope of firm i's reaction function, we differentiate the first order condition (25):

$$\frac{\mathrm{d}\Delta^i}{\mathrm{d}\Delta^j} = -\frac{\pi^i_{ij}}{\pi^i_{ii}} < 0 \tag{29}$$

where the inequality comes from (26) and (28). Thus, R&D expenditures are *strategic substitutes*.

To understand (28), notice that firm i sets its R&D Δ^i to satisfy (25). An infinitesimal increase in Δ^i increases profits for firm i since the total cost of production is reduced. Further, the quantity produced by firm j in the last stage also declines which, in turn, increases the revenues of firm i.[10] This increase in revenues has to be compared with the cost of increasing R&D $\phi^i_i(\Delta^i)$.

[10] Because of the envelope theorem, the effect of an infinitesimal change on firm i's R&D on profits through its effect on the quantity produced by firm i can be ignored

Consider now an increase in R&D by firm j (Δ_j). An increase in R&D by firm j increases its own quantity and reduces the quantity of firm i. The most important effect is the reduction in x^i (for linear demands the effect on x^j vanishes), since a lower output implies that own R&D (Δ^i) is less effective at increasing profits. Since the marginal cost of R&D for firm 1 does not change, this implies that the optimal level of R&D for firm 1 has to be lower after an increase in Δ_j. Hence $\frac{\mathrm{d}\Delta^i}{\mathrm{d}\Delta^j} < 0$.

2.3 Output subsidies

Suppose that the government 1 subsidizes exports giving a per-unit output subsidy, s^1, to its domestic firm. The profit function of firm 1 and firm 2 can then be written as,

$$\bar{\Pi}^1(x^1, x^2, \Delta^1, s^1) = R^1(x^1, x^2) - C^1(x^1, \Delta^1) - \phi(\Delta^1) + s^1 x^1 = \Pi^1(x^1, x^2, \Delta^1) + s^1 x^1 \tag{30}$$

$$\bar{\Pi}^2(x^1, x^2, \Delta^2) = \Pi^2(x^1, x^2, \Delta^2) = R^2(x^1, x^2) - C^2(x^2, \Delta^2) - \phi(\Delta^2) \tag{31}$$

The net domestic benefit of country 1 is simply the profit of the domestic firm minus the cost of the subsidy,

$$\begin{aligned} \bar{B}^1(s^1) &= \bar{\Pi}^1(x^1, x^2, \Delta^1, s^1) - s^1 x^1 \qquad (32) \\ &= R^1(x^1, x^2) - C^1(x^1, \Delta^1) - \phi(\Delta^1) \\ &= \Pi^1(x^1, x^2, \Delta^1) \end{aligned}$$

In the first stage, the problem is similar to the one under free trade. Choosing output, x^i, firms maximize profits, $\bar{\Pi}^i(x^1, x^2, \Delta^i, s^1)$. The first order condition for the two firms gives us the following expression,

$$\bar{\Pi}^1_1 = R^1_1(x^1, x^2) - C^1_x(x^1, \Delta^1) + s^1 = 0 \tag{33}$$

$$\bar{\Pi}^2_2 = R^2_2(x^1, x^2) - C^2_x(x^2, \Delta^2) = 0 \tag{34}$$

with the same second order condition (9).[11] The solution of the two equations in (33) gives us equilibrium outputs (as a function of R&D levels chosen in the second stage and the output subsidy chosen by government 1 in the first stage):

$$x^i = \bar{q}^i(\Delta^i, \Delta^j, s^1) \tag{35}$$

Total differentiation of the first order conditions (33) and (34) (keeping Δ^i, Δ^j and s^1 constant) gives us the slope of firm i's reaction function, which is the same as (14). To see the effect of R&D investment and subsidies over output, we can totally differentiate the two first order conditions (33) and (34):

$$\begin{aligned} \left(R^1_{11}(x^1, x^2) - C^1_{xx}(x^1, \Delta^1)\right) \mathrm{d}x^1 + R^1_{12}(x^1, x^2)\mathrm{d}x^2 - C^1_{x\Delta}(x^1, \Delta^1)\mathrm{d}\Delta^1 + \mathrm{d}s^1 &= 0 \qquad (36a) \\ R^2_{12}(x^1, x^2)\mathrm{d}x^1 + \left(R^2_{22}(x^1, x^2) - C^2_{xx}(x^2, \Delta^2)\right) \mathrm{d}x^2 - C^2_{x\Delta}(x^2, \Delta^2)\mathrm{d}\Delta^2 &= 0. \qquad (36b) \end{aligned}$$

[11] Note that $\bar{\Pi}^i_{ii} = \Pi^i_{ii}$ and $\bar{\Pi}^i_{ij} = \Pi^i_{ij}$ are the same as under free trade.

The effect of R&D on output (keeping the output subsidy constant) is the same as under free trade[12] [equations (19) and (20)]. The effect of the subsidy (s^1) on output is also determined by the effect the output subsidy has on R&D of both firms. Keeping R&D levels Δ^1 and Δ^2 fixed, the *partial* effects are,

$$\bar{q}^1_{s^1}(\Delta^1, \Delta^2, s^1)\big|_{\Delta^1, \Delta^2 \text{ constant}} = \frac{-\Pi^2_{22}}{\Pi^1_{11}\Pi^2_{22} - R^2_{12}R^1_{12}} > 0 \tag{37}$$

$$\bar{q}^2_{s^1}(\Delta^1, \Delta^2, s^1)\big|_{\Delta^1, \Delta^2 \text{ constant}} = \frac{R^2_{12}}{\Pi^1_{11}\Pi^2_{22} - R^2_{12}R^1_{12}} = \frac{\Pi^2_{12}}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} < 0 \tag{38}$$

The partial effects state that own output is increasing in own (subsidy) and decreasing in the other subsidy. However, R&D levels are influenced by the choice of output subsidies. Therefore, the *total* effect of a change in s^1 should take this into account. (Expressions for $\bar{q}^1_{s^1}$ and $\bar{q}^2_{s^1}$ above would be relevant if output subsidies are chosen *after* R&D levels are set.) Using equation (14) we have:

$$\bar{q}^2_{s^1} = \bar{q}^1_{s^1}\frac{\mathrm{d}x^2}{\mathrm{d}x^1} \tag{39}$$

Moreover, from (19), (20) we have that,

$$\bar{q}^1_{s^1} = -\frac{\bar{q}^1_{\Delta^1}}{C^1_{x\Delta}} \tag{40}$$

$$\bar{q}^2_{s^1} = -\frac{\bar{q}^2_{\Delta^1}}{C^1_{x\Delta}} \tag{41}$$

In order to study the effect of imposing an output subsidy before R&D takes place, we now turn to the R&D stage.

2.3.1 R&D Investment with output subsidies

In the second stage, we can rewrite the profit of a firm as a function of R&D and output subsidies: $\bar{\pi}^i(\Delta^i, \Delta^j, s^1) = \bar{\Pi}^i(\bar{q}^i(\Delta_i, \Delta_j, s^1), \bar{q}^j(\Delta_i, \Delta_j, s^1), \Delta^i, s^1) = R^i(\bar{q}^i, \bar{q}^j) - C^i(\bar{q}^i, \Delta^i) - \phi^i(\Delta^i) + s^1\bar{q}^i$. As before, we can decompose the effect of a change in R&D (for given output subsidies) into a direct and a strategic effect. The strategic effect $\frac{\partial \Pi^i}{\partial x^j}\frac{\partial \bar{q}^j}{\partial \Delta^i}$ is still positive. Therefore, for a given output subsidy, firms overinvest in R&D.

The first order condition for a Nash equilibrium in the choice of R&D is given by the same first order condition as in the case of free trade:

$$\bar{\pi}^1_{\Delta^1}(\Delta^1, \Delta^2, s^1) = R^1_2(x^1, x^2)\bar{q}^2_{\Delta^1}(\Delta^1, \Delta^2, s^1) - C^1_{\Delta}(x^1, \Delta^1) - \phi^1_1(\Delta^1) = 0 \tag{42}$$

$$\bar{\pi}^2_{\Delta^2}(\Delta^2, \Delta^1, s^1) = R^2_1(x^2, x^1)\bar{q}^1_{\Delta^2}(\Delta^2, \Delta^1, s^1) - C^2_{\Delta}(x^2, \Delta^2) - \phi^2_1(\Delta^2) = 0 \tag{43}$$

With the second order condition, $\bar{\pi}^i_{\Delta^i\Delta^i}(\Delta^i, \Delta^j, s^1) < 0$. In order to see the effect of output subsidies over R&D investment, we totally differentiate the two first order conditions. The following proposition states the effect of an output subsidy on the equilibrium R&D of both firms.

[12] Therefore, $\bar{q}^i_{\Delta^i}(\Delta^i, \Delta^j, s^i, s^j) = q^i_i(\Delta^i, \Delta^j)$ and $\bar{q}^j_{\Delta^i}(\Delta^i, \Delta^j, s^i, s^j) = q^j_i(\Delta^i, \Delta^j)$

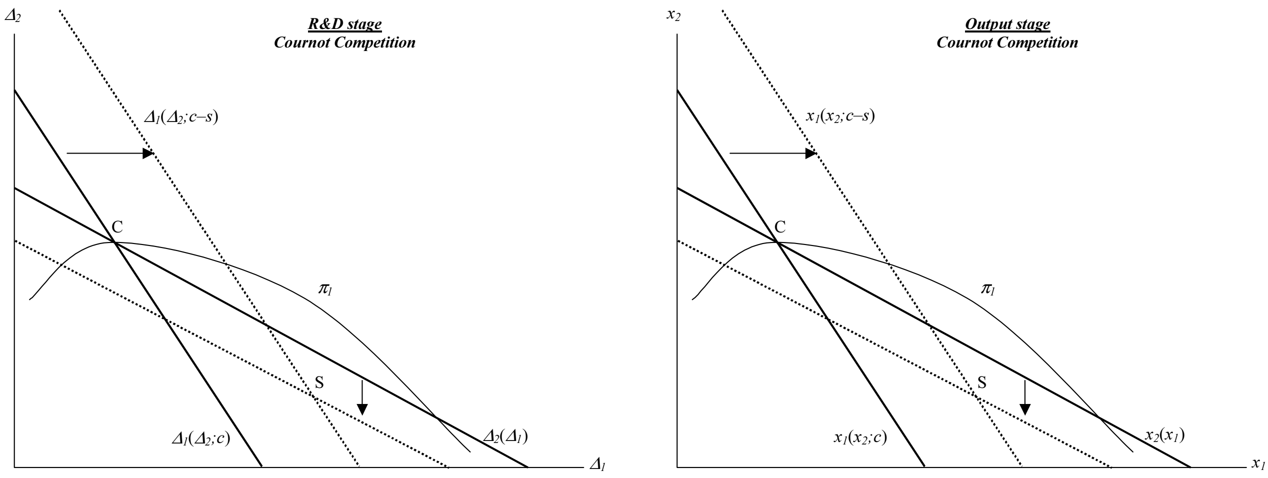

Figure 1: Cournot competition: Effect of an output subsidy s imposed by government 1.

Proposition 1 *An output subsidy by the domestic government increases the equilibrium level of R&D chosen by the domestic firm and reduces the R&D level chosen by the foreign firm. That is,*

$$\frac{d\Delta^1}{ds^1} > 0 \tag{44}$$

$$\frac{d\Delta^2}{ds^1} < 0. \tag{45}$$

Proof. See appendix. ■

Proposition 1 states that an increase in the subsidy s^1 shifts the reaction function of both firms in R&D space. The reaction function of firm 1 shifts outwards while the reaction function of firm 2 shifts inwards. This is illustrated in the left half of figure 1. An output subsidy s^1 moves the equilibrium in the R&D space from point C (free trade) to point S. This implies that for a small increment in its output subsidy, firm 1 will be inside its isoprofit contour (π_1) passing through the free trade equilibrium point C. This analysis, however, does not take into account the effect of output subsidies in output space (i.e. in the third stage). The effect in the output competition stage is illustrated on the right side of figure 1. Notice that an output subsidy s^1, imposed by government 1, increases domestic R&D and lowers foreign R&D (as seen in the left half of figure 1). This reduces domestic marginal costs beyond the direct effect of the subsidy and increases foreign marginal costs. In output space, this means that the domestic output reaction function shifts out and the foreign reaction function shifts in. The resulting equilibrium is at point S, which is inside the isoprofit contour (π_1) that passes through the free trade equilibrium at point C. Therefore, an output subsidy increases welfare for the domestic country both in the R&D and the output stage.

To obtain the optimal output subsidy these two effects need to be included. The net benefit of government 1 is $\bar{B}^1(s^1) = \bar{\pi}^1(\Delta^1, \Delta^2, s^1) - s^1 x^1$. Differentiating $\bar{B}^1(s^1)$ with respect to s^1 we obtain,

$$\frac{\partial \bar{B}^1}{\partial s^1} = \bar{\pi}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\pi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\pi}^1_{s^1} - x^1 - s^1\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1}. \tag{46}$$

Recall that, $\bar{\pi}^1_{s^1} = R^1_2(x^1, x^2)\bar{q}^2_{s^1}(\Delta^1, \Delta^2, s^1) + x^1$ and $\bar{\pi}^1_{\Delta^1} = 0$ from the R&D stage. Further,

$$\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1} = \bar{q}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{q}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{q}^1_{s^1} > 0. \tag{47}$$

This last inequality simply states that the total effect of an output subsidy over equilibrium output is positive, i.e. an output subsidy makes a firm in that country more competitive in the output stage ($\bar{q}^1_{s^1} > 0$). Further, an output subsidy reduces foreign R&D while increasing domestic R&D in the second stage. This in turn benefits domestic production, i.e. $\bar{q}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{q}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} > 0$. Given this $\frac{\partial \bar{B}^1}{\partial s^1}$ simplifies to

$$\begin{aligned} \frac{\partial \bar{B}^1}{\partial s^1} &= \bar{\pi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + R^1_2\bar{q}^2_{s^1} - s^1\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1} \\ &= R^1_2\bar{q}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + R^1_2\bar{q}^2_{s^1} - s^1\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1} \\ &= R^1_2\left(\bar{q}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{q}^2_{s^1}\right) - s^1\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1}. \end{aligned} \tag{48}$$

The first term reflects the effect of the output subsidy over domestic benefit in the second (R&D) stage.[13] The output subsidy reduces foreign R&D (Δ^2) resulting in an increase in domestic profits. As a result, the effect of a subsidy s^1 on benefits in the second stage is positive. The second term captures what happens in the third (output) stage: an increase in the subsidy s^1 reduces the quantity produced by the foreign firm resulting in an increase in domestic revenues (and profits). The third term reflects the increased subsidy expenditure brought about by an increased production for the domestic firm $s^1\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1}$. The sign of the expression $\frac{\partial \bar{B}^1}{\partial s^1}$ is determined by the net of the three effects pointed out above. Notice that, starting from a subsidy s^1 equal to zero, an infinitesimal increase in the output subsidy increases domestic benefit for the subsidizing country as both the output effect $\left(\bar{q}^2_{s^1}\right)$ and the R&D effect $\left(\bar{q}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}\right)$ move in the same direction.

$$\left.\frac{\partial \bar{B}^1}{\partial s^1}\right|_{s^1=0} = \bar{\pi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + R^1_2\bar{q}^2_{s^1} > 0 \tag{49}$$

To obtain the precise expression for the optimal output subsidy we set $\frac{\partial \bar{B}^1}{\partial s^1} = 0$:

Proposition 2 *When firms compete à la Cournot, the optimal output subsidy s^{1*} is positive:*

$$s^{1*} = \frac{\bar{\pi}^1_{\Delta^2}\frac{d\Delta^2}{ds^1} + R^1_2\bar{q}^2_{s^1}}{\frac{d\bar{q}^1}{ds^1}} = \frac{R^1_2\left(\bar{q}^2_{\Delta^2}\frac{d\Delta^2}{ds^1} + \bar{q}^2_{s^1}\right)}{\bar{q}^1_{\Delta^1}\frac{d\Delta^1}{ds^1} + \bar{q}^1_{\Delta^2}\frac{d\Delta^2}{ds^1} + \bar{q}^1_{s^1}} > 0 \tag{50}$$

Proof. Immediate from (48) ■

This proposition extends the results in Spencer and Brander (1983). They analyze the case when an output subsidy is set *after* firms invest in R&D and before they choose output. They find that the optimal output subsidy is positive. They also analyze the case of subsidies to R&D *and* output before the R&D stage, finding that output subsidies are also positive. Here we have shown that output subsidies are also positive under Cournot competition if subsidies are set before R&D investment.

Note that the separation into two effects related to each of the two stages in which firms play will be useful to characterize the solution in the case of Bertrand competition.

[13] Since firm 1 is choosing R&D, Δ^1, to maximize profits then an infinitesimal output subsidy s^1 will not affect benefits.

3 Bertrand Competition

3.1 Final Stage: Price Competition

Firms simultaneously choose R&D in the first stage and prices in the second. We assume that goods are imperfect substitutes and that the own-price effect dominates the cross-price effect,

$$\frac{\partial x^i(p^i,p^j)}{\partial p^i} < 0 < \frac{\partial x^i(p^i,p^j)}{\partial p^j} \tag{51}$$

$$\left|\frac{\partial x^i(p^i,p^j)}{\partial p^i}\right| > \left|\frac{\partial x^i(p^i,p^j)}{\partial p^j}\right|. \tag{52}$$

Using previous notation, revenues and costs can be written as, $\hat{R}^i(p^i,p^j) = x^i(p^i,p^j)\cdot p^i = R^i(x^i(p^i,p^j), x^j(p^i,p^j))$ and $\hat{C}^i(p^i,p^j,\Delta^i) = C^i(x^i(p^i,p^j),\Delta^i)$, respectively. Revenues are assumed to satisfy the following properties:

$$\hat{R}^i_j(p^i,p^j) = p^i\frac{\partial x^i(p^i,p^j)}{\partial p^j} > 0 \tag{53}$$

$$\hat{R}^i_{ii}(p^i,p^j) = p^i\frac{\partial^2 x^i(p^i,p^j)}{\partial (p^i)^2} + 2\frac{\partial x^i(p^i,p^j)}{\partial p^i} < 0 \tag{54}$$

$$\hat{R}^i_{jj}(p^i,p^j) = p^i\frac{\partial^2 x^i(p^i,p^j)}{\partial (p^j)^2} \geq 0 \tag{55}$$

$$\hat{R}^i_{ij}(p^i,p^j) = p^i\frac{\partial x^i(p^i,p^j)}{\partial p^i \partial p^j} + \frac{\partial x^i(p^i,p^j)}{\partial p^j} > 0 \tag{56}$$

Assumption (53) states that goods i and j are (imperfect) substitutes. (54) states that revenue is concave in its own price, a property which is satisfied by demand functions that are not too convex. Assumption (55) is the standard case where revenue is increasing, at a non-decreasing rate, in the other firm's price. This property, in particular, is satisfied by linear demand functions. Lastly, (56) states that an increase in the price of one good increases marginal revenue for the other firm. This is again satisfied in the case of linear demand.

We make the following assumptions about costs (which are equivalent to (1) and (2) in the Cournot case):

$$\begin{aligned}
\hat{C}^i_{\Delta} &= \frac{\partial \hat{C}^i(p^i,p^j,\Delta^i)}{\partial \Delta^i} \leq 0, \qquad \hat{C}^i_{\Delta\Delta} = \frac{\partial^2 \hat{C}^i(p^i,p^j,\Delta^i)}{\partial (\Delta^i)^2} \geq 0, \\
\hat{C}^i_{p^i\Delta} &= \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial \Delta^i \partial x^i}\frac{\partial x^i(p^i,p^j)}{\partial p^i} > 0, \qquad \hat{C}^i_{p^j\Delta} = \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial \Delta^i \partial x^i}\frac{\partial x^i(p^i,p^j)}{\partial p^j} < 0 \\
\phi^i_i(\Delta^i) &> 0, \qquad \phi^i_{ii}(\Delta^i) > 0
\end{aligned} \tag{57}$$

$$\begin{aligned}
\hat{C}^i_{p^i} &= \frac{\partial \hat{C}^i(p^i,p^j,\Delta^i)}{\partial p^i} = \frac{\partial C^i(x^i,\Delta^i)}{\partial x^i}\frac{\partial x^i(p^i,p^j)}{\partial p^i} < 0 \\
\hat{C}^i_{p^ip^i} &= \frac{\partial^2 \hat{C}^i(p^i,p^j,\Delta^i)}{(\partial p^i)^2} = \frac{\partial C^i(x^i,\Delta^i)}{\partial x^i}\frac{\partial^2 x^i(p^i,p^j)}{\partial (p^i)^2} + \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial (x^i)^2}\left(\frac{\partial x^i(p^i,p^j)}{\partial p^i}\right)^2 \geq 0 \\
\hat{C}^i_{p^ip^j} &= \frac{\partial^2 \hat{C}^i(p^i,p^j,\Delta^i)}{\partial p^i \partial p^j} = \frac{\partial C^i(x^i,\Delta^i)}{\partial x^i}\frac{\partial^2 x^i(p^i,p^j)}{\partial p^i \partial p^j} + \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial (x^i)^2}\frac{\partial x^i(p^i,p^j)}{\partial p^i}\frac{\partial x^i(p^i,p^j)}{\partial p^j} \leq 0.
\end{aligned} \tag{58}$$

Note that, assumption (52) implies

$$\left|\hat{C}^i_{p^i}\right| > \left|\hat{C}^i_{p^j}\right| \tag{59}$$

$$\left|\hat{C}^i_{p^i\Delta}\right| > \left|\hat{C}^i_{p^j\Delta}\right| \tag{60}$$

Firm i chooses p_i to maximize profits: $\Pi^i(p^i, p^j, \Delta^i) = \hat{R}^i(p^i, p^j) - \hat{C}^i(p^i, p^j, \Delta^i) - \phi(\Delta^i) = x^i(p^i, p^j) \cdot p^i - \hat{C}^i(p^i, p^j, \Delta^i) - \phi(\Delta^i)$, where, $\hat{R}^i(p^i, p^j)$ and $\hat{C}^i(p^i, p^j, \Delta^i) \equiv C^i(x^i(p^i, p^j), \Delta^i)$ are firm i's revenue and total cost:

$$\max_{p^i} \Pi^i(p^i, p^j, \Delta^i) = \hat{R}^i(p^i, p^j) - \hat{C}^i(p^i, p^j, \Delta^i) - \phi(\Delta^i) \tag{61}$$

The first order condition to this maximization problem gives us the reaction function $p^i(p^j)$ of firm i. The FOCs satisfy,

$$\Pi^i_i = \hat{R}^i_i(p^i, p^j) - \hat{C}^i_{p^i}(p^i, p^j, \Delta^i) = 0. \tag{62}$$

The second order conditions are the following:

$$\begin{aligned} \Pi^i_{ii} &= \hat{R}^i_{ii}(p^i, p^j) - \hat{C}^i_{p^i p^i}(p^i, p^j, \Delta^i) < 0 \qquad (63) \\ &= p^i \frac{\partial^2 x^i(p^i, p^j)}{\partial \left(p^i\right)^2} + 2\frac{\partial x^i(p^i, p^j)}{\partial p^i} - \frac{\partial^2 \hat{C}^i(p^i, p^j, \Delta^i)}{\partial \left(p^i\right)^2} \leq 0 \end{aligned}$$

We assume that the second order condition is satisfied.[14]

For later use we need to assume that own effects of output on marginal profits is stronger (bigger in absolute value) than the cross effect, that is $\left|\Pi^i_{ii}\right| > \left|\Pi^i_{ij}\right|$. This implies that,

$$\Pi^1_{11}\Pi^2_{22} - \Pi^1_{12}\Pi^2_{12} > 0 \tag{64}$$

The solution to the two equations in (62) gives us prices as a function of R&D levels chosen in the previous stage: $p^1 = \psi^1(\Delta^1, \Delta^2)$ and $p^2 = \psi^2(\Delta^1, \Delta^2)$. The slope of firm i's reaction function is obtained by total differentiation of the first order condition (62) with respect to p^1 and p^2 (keeping Δ^1 and Δ^2 constant):

$$\frac{\mathrm{d}p^i}{\mathrm{d}p^j} = -\frac{\hat{R}^i_{ij}(p^i, p^j) - \hat{C}^i_{p^i p^j}(p^i, p^j, \Delta^i)}{\hat{R}^i_{ii}(p^i, p^j) - \hat{C}^i_{p^i p^i}(p^i, p^j, \Delta^i)} = -\frac{\Pi^i_{ij}(p^i, p^j, \Delta^i)}{\Pi^i_{ii}(p^i, p^j, \Delta^i)} > 0 \tag{65}$$

Reaction functions are positively sloped (i.e. prices are *strategic complements*) from assumptions (56), (58) and (63)). This is a standard result for Bertrand games with differentiated products.

3.2 The Effect of R&D Investment

The effect of R&D investment can be seen by totally differentiating (62):

$$\left(\hat{R}^1_{11}(p^1, p^2) - \hat{C}^1_{p^1 p^1}(p^1, p^2, \Delta^1)\right)\mathrm{d}p^1 + \left(\hat{R}^1_{12}(p^1, p^2) - \hat{C}^1_{p^1 p^2}(p^1, p^2, \Delta^1)\right)\mathrm{d}p^2 - \hat{C}^1_{p^1\Delta}(p^1, p^2, \Delta^1)\mathrm{d}\Delta^1 = 0 \tag{66}$$

$$\left(\hat{R}^2_{12}(p^1, p^2) - \hat{C}^2_{p^1 p^2}(p^1, p^2, \Delta^2)\right)\mathrm{d}p^1 + \left(\hat{R}^2_{22}(p^1, p^2) - \hat{C}^2_{p^2 p^2}(p^1, p^2, \Delta^2)\right)\mathrm{d}p^2 - \hat{C}^2_{p^2\Delta}(p^1, p^2, \Delta^2)\mathrm{d}\Delta^2 = 0 \tag{67}$$

Solving the system using Cramer's rule we get,

$$\mathrm{d}p^i = \frac{\Pi^j_{jj}(p^1, p^2, \Delta^1)\hat{C}^i_{p^i\Delta}(p^1, p^2, \Delta^i)\mathrm{d}\Delta^i - \Pi^i_{ij}(p^1, p^2, \Delta^1)\hat{C}^j_{p^j\Delta}(p^1, p^2, \Delta^j)\mathrm{d}\Delta^j}{\Pi^i_{ii}(p^1, p^2, \Delta^i)\Pi^j_{jj}(p^1, p^2, \Delta^j) - \Pi^j_{ij}(p^1, p^2, \Delta^j)\Pi^i_{ij}(p^1, p^2, \Delta^i)}. \tag{68}$$

[14] Notice that $\frac{\partial \hat{C}^i(p^i, \Delta^i)}{\partial p^i} = \frac{\partial C^i(x^i, \Delta^i)}{\partial x^i}\frac{\partial x^i(p^i, p^j)}{\partial p^i} < 0$. $\frac{\partial^2 \hat{C}^i(p^i, \Delta^i)}{\left(\partial p^i\right)^2} = \frac{\partial C^i(x^i, \Delta^i)}{\partial x^i}\frac{\partial^2 x^i(p^i, p^j)}{\partial \left(p^i\right)^2} + \frac{\partial^2 C^i(x^i, \Delta^i)}{\partial \left(x^i\right)^2}\left(\frac{\partial x^i(p^i, p^j)}{\partial p^i}\right)^2$ is positive for convex demands and non-decreasing marginal costs. The second order condition $\Pi^i_{ii} < 0$ is satisfied for linear demands.

From the expression above we obtain,

$$\psi_i^i(\Delta^i, \Delta^j) = \frac{\mathrm{d}p^i}{\mathrm{d}\Delta^i} = \frac{\Pi_{jj}^j \hat{C}_{p^i\Delta}^i}{\Pi_{ii}^i \Pi_{jj}^j - \Pi_{ij}^j \Pi_{ij}^i} < 0 \tag{69}$$

$$\psi_j^i(\Delta^i, \Delta^j) = \frac{\mathrm{d}p^i}{\mathrm{d}\Delta^j} = \frac{-\Pi_{ij}^i \hat{C}_{p^j\Delta}^j}{\Pi_{ii}^i \Pi_{jj}^j - \Pi_{ij}^j \Pi_{ij}^i} < 0 \tag{70}$$

where the inequalities come from (56), (57), (58) and (63). The expressions above state that prices are decreasing both in domestic and foreign R&D. An increase in R&D expenditure reduces the marginal cost of production shifting the reaction curve of firm i downwards. Given that prices are strategic complements in (65) this implies that both firm i and firm j charge a lower price.

For later use we need to compute ψ_{ij}^j and ψ_{ii}^j. Differentiating (70) we obtain

$$\psi_{ij}^i = \frac{\Pi_{ij}^i \hat{C}_{p^j\Delta}^j \hat{C}_{p^i p^i \Delta}^i \Pi_{jj}^j}{\left(\Pi_{ii}^i \Pi_{jj}^j - \Pi_{ij}^j \Pi_{ij}^i\right)^2} \tag{71}$$

$$\psi_{jj}^i = \frac{\left(-\hat{R}_{ij}^i \hat{C}_{p^j\Delta\Delta}^j\right)\left(\Pi_{ii}^i \Pi_{jj}^j - \Pi_{ij}^i \Pi_{ij}^j\right) + \Pi_{ij}^i \hat{C}_{p^j\Delta}^j \hat{C}_{p^j p^j \Delta}^j \Pi_{ii}^i}{\left(\Pi_{ii}^i \Pi_{jj}^j - \Pi_{ij}^i \Pi_{ij}^j\right)^2} \tag{72}$$

Note that ψ_{ij}^j is zero for constant marginal costs with respect to output and ψ_{ii}^j is zero for marginal costs that are constant with respect to output and linear with respect to R&D.

3.3 Choice of R&D investment

In the R&D stage the profit function of firm i can be written as, $\pi^i(\Delta^i, \Delta^j) = \Pi^i(\psi^i(\Delta_i, \Delta_j), \psi^j(\Delta_i, \Delta_j), \Delta^i) = R^i(p^i, p^j) - C^i(p^i, p^j, \Delta^i) - \phi^i(\Delta^i)$. Using the envelope theorem (i.e. $\frac{\partial \Pi^i}{\partial p^i} = 0$) we can write firm $i's$ FOC as,

$$\pi_i^i(\Delta^i, \Delta^j) = \frac{\partial \pi^i}{\partial \Delta^i} = \frac{\partial \Pi^i}{\partial \Delta^i} + \frac{\partial \Pi^i}{\partial p^j}\frac{\partial \psi^j}{\partial \Delta^i} = 0 \tag{73}$$

where $\frac{\partial \Pi^i}{\partial \Delta^i} = -\hat{C}_\Delta^i(p^i, p^j, \Delta^i) - \phi_i^i(\Delta^i)$ and $\frac{\partial \Pi^i}{\partial p^j} = p^i \frac{\partial x^i(p^i,p^j)}{\partial p^j} - C_{x^i}^i(x^i, \Delta^i)\frac{\partial x^i(p^i,p^j)}{\partial p^j} > 0$ (as long as prices are greater than marginal cost). Further, $\frac{\partial \psi^j}{\partial \Delta^i} = \psi_i^j = \frac{-\Pi_{ij}^j \hat{C}_{p^i\Delta}^i}{\Pi_{ii}^i \Pi_{jj}^j - \Pi_{ij}^j \Pi_{ij}^i} < 0$. This last inequality gives us the effect of an increase in R&D by firm i on firm j's price reaction curve. An increase in R&D by firm i reduces its marginal cost thereby reducing its price. Prices being strategic complements, this results in a decline in the price of firm (j). An increase in firm i's R&D shifts firm j's price reaction function inwards.

The strategic effect $\frac{\partial \Pi^i}{\partial p^j}\frac{\partial \psi^j}{\partial \Delta^i}$ is negative. Firms thus have an incentive to *underinvest in R&D* in order to increase the price charged by its rival and therefore increase their own profits. This can be easily seen from (73) where the optimal choice of R&D leaves $\frac{\partial \Pi^i}{\partial \Delta^i} > 0$.

Noting that $\hat{R}_i^i(p^i, p^j) - \hat{C}_{p^i}^i(p^i, p^j, \Delta^i) = \Pi_i^i = 0$, the first order condition for a Nash equilibrium in the choice of R&D levels can be rewritten as,

$$\pi_i^i(\Delta^i, \Delta^j) = \left[\hat{R}_j^i(p^i, p^j) - \hat{C}_{p^j}^i(p^i, p^j, \Delta^i)\right]\psi_i^j(\Delta^i, \Delta^j) - \hat{C}_\Delta^i(p^i, p^j, \Delta^i) - \phi_i^i(\Delta^i) = 0 \tag{74}$$

With the second order condition[15]

$$\pi^i_{ii}(\Delta^i,\Delta^j) = \left(\hat{R}^i_j - \hat{C}^i_{p^j}\right)\psi^j_{ii} + \psi^j_i\left(\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^i} - \frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^i}\right) - \hat{C}^i_{p^i\Delta}\psi^i_i - \hat{C}^i_{p^j\Delta}\psi^j_i - \hat{C}^i_{\Delta\Delta} - \phi^i_{ii} < 0. \tag{75}$$

We also assume a condition similar to (64). This refers to the effect of R&D on profits. We again assume that the own effect of R&D on marginal profits is stronger (bigger in absolute value) than the cross effect, that is, $\pi^i_{ii} < \pi^i_{ij}$. This implies that,

$$\pi^1_{11}\pi^2_{22} - \pi^1_{12}\pi^2_{12} > 0 \tag{76}$$

where,

$$\pi^i_{ij} = \left(\hat{R}^i_j - \hat{C}^i_{p^j}\right)\psi^j_{ij} + \psi^j_i\left(\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^j} - \frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^j}\right) - \hat{C}^i_{p^i\Delta}\psi^i_j - \hat{C}^i_{p^j\Delta}\psi^j_j \tag{77}$$

Here, $\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^j} = \hat{R}^i_{ij}(p^i,p^j)\psi^i_j + \hat{R}^i_{jj}(p^i,p^j)\psi^j_j < 0$ (from (55), (56), (69) and (70)) and $\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^j} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\psi^i_j + \hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\psi^j_j$. Both the second order condition (75) and the stability condition (76) impose bounds on ϕ^i_{ii} (this is discussed later in the determination of the optimal subsidy).

The cross partial derivative π^i_{ij} can be signed for the case of linear demand and constant marginal costs (i.e. $\psi^j_{ij} = \hat{C}^i_{p^ip^j} = \hat{C}^i_{p^jp^j} = \hat{R}^i_{jj} = 0$). The following proposition establishes that, under these circumstances, R&D expenditures are strategic substitutes even if firms compete in prices.

Proposition 3 *R&D expenditures are strategic substitutes for the case of linear demand and constant marginal costs:*

$$\pi^i_{ij} = \psi^j_i\psi^i_j\hat{R}^i_{ij}(p^i,p^j) - \hat{C}^i_{p^i\Delta}\psi^i_j - \hat{C}^i_{p^j\Delta}\psi^j_j < 0 \tag{78}$$

Proof. See Appendix. ■

Proposition 3 states that an increase in R&D by firm 2 reduces the marginal profitability of R&D by firm 1. To see how this occurs, notice that firm 1 sets its R&D, Δ^1, to satisfy (74). An infinitesimal increase in Δ^1 has two opposing effects on firm 1's profits. First, profits increase due to the reducion in total costs $\hat{C}^1$. On the other hand the decrease in p^2 (due to increased R&D, Δ^1) decreases firm revenues.[16] The first order condition (74) shows this trade off against the increase in the cost of R&D , $\phi^1_1(\Delta^1)$.

[15] Notice that $\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^i} = \hat{R}^i_{ij}(p^i,p^j)\psi^i_i + \hat{R}^i_{jj}(p^i,p^j)\psi^j_i < 0$ (by (55), (56), (69) and (70)) and $\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta_i)}{\mathrm{d}\Delta^i} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\psi^i_i + \hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\psi^j_i + \hat{C}^i_{p^j\Delta}(p^i,p^j,\Delta^i)$. In general, $\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta_i)}{\mathrm{d}\Delta^i}$ is hard to sign. However, in the case of linear demand it is equal to $\hat{C}^i_{p^j\Delta}(p^i,p^j,\Delta^i)$, which is negative. Assuming that marginal costs are constant with respect to output and linear with respect to R&D (i.e. $\psi^j_{ii} = 0$), we get $\pi^i_{ii}(\Delta^i,\Delta^j) = \psi^j_i\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^i} - \hat{C}^i_{p^i\Delta}\psi^i_i - 2\hat{C}^i_{p^j\Delta}\psi^j_i - \hat{C}^i_{\Delta\Delta} - \phi^i_{ii}$. This expression can only be negative (for (75) to hold) if $2\hat{C}^i_{p^j\Delta}\psi^j_i + \hat{C}^i_{\Delta\Delta} + \phi^i_{ii}$ is big enough. This is equivalent to saying that as R&D increases, its cost-effectiveness has to decline fast enough, a condition similar to the Cournot case.

[16] From the envelope theorem we can ignore the effect on firm 1's price on its profits.

Consider now an infinitesimal increase in R&D by firm 2. This reduces both p^1 and p^2. However, the fall in own price (p^2) is greater than the price decline for the rival.[17] A bigger price increase for firm 1 means that it now sells less. Lower output reduces the effectiveness of Δ^1 in reducing total costs for firm 1. This is captured by the last two terms of (78). The first term captures the effect of an increase in Δ^2 on the marginal effect of Δ^1 on firm 1's revenue. The fall in quantity (x^1), associated with an increase in Δ^2, makes the revenue loss of an increase in Δ^1 less important. This accounts for $\psi_i^j \psi_j^i \hat{R}_{ij}^i(p^i, p^j)$ being positive.

Note that, the (direct) effect over costs dominates the (indirect) effect over revenue (as shown in the proof of proposition 3). The positive effect of investing in R&D for firm 1 weakens due to an increase in Δ^2. Since the marginal cost of R&D $\phi_1^1(\Delta^1)$ is unaffected by a change in Δ^2, an increase in foreign R&D (Δ^2) makes own R&D less attractive. Therefore, firm 1 optimally invests less in R&D in response to an increase in Δ^1, implying that $\pi_{ij}^i < 0$.

A corollary of the previous proposition is that the slope of firm i's R&D reaction function is also negative:

$$\frac{\mathrm{d}\Delta^i}{\mathrm{d}\Delta^j} = -\frac{\pi_{ij}^i}{\pi_{ii}^i} < 0 \tag{79}$$

where the inequality comes from (75) and (78). Note that, R&D reaction functions are negatively sloped (i.e. strategic substiututes) both under Cournot and Bertrand competition because the main effect of R&D comes through total costs. In both cases an increase in R&D by firm 2 reduces firm 1's output thereby decreasing the capacity of Δ^1 to reduce firm 1's total costs. Under Cournot competition, the effect over marginal revenue adds to this effect over costs. With Bertrand competition the effect over marginal revenue dampens (but does not dominate) the effect over costs (as shown in proposition 3).

The next section describes the effect of output subsidies over R&D and price choices, under Bertrand competition.

3.4 Government Policy

Suppose that government 1 imposes an output subsidy s^1. The profit function of firm 1 and firm 2 can now be written as:

$$\begin{aligned} \bar{\Pi}^1(p^1, p^2, \Delta^1, s^1) &= \hat{R}^1(p^1, p^2) - \hat{C}^1(p^1, p^2, \Delta^1) - \phi(\Delta^1) + s^1 \cdot x^1(p^1, p^2) && (80) \\ &= \Pi^1(p^1, p^2, \Delta^1) + s^1 \cdot x^1(p^1, p^2) && (81) \end{aligned}$$

$$\bar{\Pi}^2(p^1, p^2, \Delta^2) = \Pi^2(p^1, p^2, \Delta^2) = \hat{R}^2(p^1, p^2) - \hat{C}^2(p^1, p^2, \Delta^2) - \phi(\Delta^2). \tag{82}$$

The net domestic benefit of country 1 is simply the profit of the domestic firm minus the cost of the subsidy,

$$\begin{aligned} \bar{B}^1(s^1) &= \bar{\Pi}^1(p^1, p^2, \Delta^1, s^1) - s^1 \cdot x^1(p^1, p^2) && (83) \\ &= \hat{R}^1(p^1, p^2) - \hat{C}^1(p^1, p^2, \Delta^1) - \phi(\Delta^1) \\ &= \Pi^1(p^1, p^2, \Delta^1). \end{aligned}$$

[17] This can be easily seen comparing ψ_i^i and ψ_j^i on (69) and (70), and recalling that own effects dominate cross effects in the price stage: $\left|\Pi_{jj}^j\right| > \left|\Pi_{ij}^j\right|$ by (64)

In the first stage, the problem is similar to the one under free trade: firms maximize $\bar{\Pi}^i(p^1, p^2, \Delta^i, s^1)$ choosing the price p^i. The first order condition to this problem is,

$$\bar{\Pi}_1^1 = \hat{R}_1^1(p^1, p^2) - \hat{C}_{p^1}^1(p^1, p^2, \Delta^1) + s^1 \frac{\partial x^1}{\partial p^1} = 0 \tag{84}$$

$$\bar{\Pi}_2^2 = \hat{R}_2^2(p^1, p^2) - \hat{C}_{p^2}^2(p^1, p^2, \Delta^2) = 0 \tag{85}$$

with the same second order condition as in (63).[18] The solution to the two equations in (84) gives us prices as a function of the R&D levels of both firms (chosen in the previous stage) and output subsidy s^1,

$$p^i = \bar{\psi}^i(\Delta^i, \Delta^j, s^1) \tag{86}$$

Differentiation of (84) and (85), (keeping Δ^i, Δ^j and s^1 constant), yields the slope of firm i's reaction function. This gives us the same expression as in (65). To see the effect of R&D investment and subsidies over prices, we differentiate the two first order conditions given in (84) and (85):

$$\left(\hat{R}_{11}^1(p^1, p^2) - \hat{C}_{p^1p^1}^1(p^1, p^2, \Delta^1) + s^1 \frac{\partial^2 x^1}{\partial \left(p^1\right)^2}\right) \mathrm{d}p^1 + \\ + \left(\hat{R}_{12}^1(p^1, p^2) - \hat{C}_{p^1p^2}^1(p^1, p^2, \Delta^1) + s^1 \frac{\partial^2 x^1}{\partial p^1 \partial p^2}\right) \mathrm{d}p^2 - \hat{C}_{p^1\Delta}^1(p^1, p^2, \Delta^1)\mathrm{d}\Delta^1 + \frac{\partial x^1}{\partial p^1}\mathrm{d}s^1 = 0$$

$$\left(\hat{R}_{12}^2(p^1, p^2) - \hat{C}_{p^1p^2}^2(p^1, p^2, \Delta^2)\right) \mathrm{d}p^1 + \\ + \left(\hat{R}_{22}^2(p^1, p^2) - \hat{C}_{p^2p^2}^2(p^1, p^2, \Delta^2)\right) \mathrm{d}p^2 - \hat{C}_{p^2\Delta}^2(p^1, p^2, \Delta^2)\mathrm{d}\Delta^2 = 0$$

The effect of R&D on prices (keeping output subsidy constant) is the same as in the case of free trade (except for the terms $s^1 \frac{\partial^2 x^1}{\partial (p^1)^2}$ and $s^1 \frac{\partial^2 x^1}{\partial p^1 \partial p^2}$). Notice, that these terms are equal to zero for linear demands.[19] Note that, the output subsidy is chosen before firms decide on their R&D. This implies that its effect on prices has to take into account how the subsidy affects the choice of R&D by both firms. The partial effects, keeping R&D levels (Δ^1 and Δ^2) constant, are:

$$\left.\bar{\psi}_{s^1}^1(\Delta^1, \Delta^2, s^1)\right|_{\Delta^1, \Delta^2 \text{ constant}} = \frac{-\Pi_{22}^2 \left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi_{11}^1 \Pi_{22}^2 - \Pi_{12}^2 \Pi_{12}^1} < 0 \tag{88}$$

$$\left.\bar{\psi}_{s^1}^2(\Delta^1, \Delta^2, s^1)\right|_{\Delta^1, \Delta^2 \text{ constant}} = \frac{\Pi_{12}^2 \left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi_{11}^1 \Pi_{22}^2 - \Pi_{12}^2 \Pi_{12}^1} < 0. \tag{89}$$

Notice that we assume that R&D levels are kept constant, while in fact they are influenced by the choice of output subsidies. The total effect of a change in s^1, therefore, has to also take this into account.[20] Using equation (65) we have:

$$\bar{\psi}_{s^1}^2 = \bar{\psi}_{s^1}^1 \frac{\mathrm{d}p^2}{\mathrm{d}p^1} \tag{90}$$

[18] Note that $\bar{\Pi}_{ii}^i = \Pi_{ii}^i$ and $\bar{\Pi}_{ij}^i = \Pi_{ij}^i$ are the same as in the case of free trade.

[19] Therefore, $\bar{\psi}_{\Delta^i}^i(\Delta^i, \Delta^j, s^i, s^j) = \psi_i^i(\Delta^i, \Delta^j)$ and $\bar{\psi}_{\Delta^i}^j(\Delta^i, \Delta^j, s^i, s^j) = \psi_i^j(\Delta^i, \Delta^j)$ for linear demands [equations (69) and (70)].

[20] Expressions for $\bar{\psi}_{s^1}^1$ and $\bar{\psi}_{s^1}^2$ (in (88) and (89)) would be relevant if output subsidies are chosen after R&D levels are set.

Moreover, from (69) and (70) we have

$$\bar{\psi}^1_{s^1} = -\frac{\bar{\psi}^1_{\Delta^1}}{\hat{C}^1_{p^1\Delta}}\left(\frac{\partial x^1}{\partial p^1}\right) < 0 \tag{91}$$

$$\bar{\psi}^2_{s^1} = -\frac{\bar{\psi}^2_{\Delta^1}}{\hat{C}^2_{p^2\Delta}}\left(\frac{\partial x^1}{\partial p^1}\right) < 0 \tag{92}$$

In order to obtain the effect of imposing an output subsidy (before R&D takes place), we turn now to the R&D stage.

3.4.1 R&D Investment with output subsidies

Rewriting the profit of the firm as a function of R&D and output subsidies:

$$\begin{aligned} \bar{\pi}^i(\Delta^i, \Delta^j, s^1) &= \bar{\Pi}^i(\bar{\psi}^i(\Delta_i, \Delta_j, s^1), \bar{\psi}^j(\Delta_i, \Delta_j, s^1), \Delta^i, s^1) \\ &= \hat{R}^i(\bar{\psi}^i, \bar{\psi}^j) - \hat{C}^i(\bar{\psi}^i, \bar{\psi}^j, \Delta^i) - \phi^i(\Delta^i) + s^1 \cdot x^i(\bar{\psi}^i, \bar{\psi}^j) \end{aligned} \tag{93}$$

We can decompose the effect of a change in R&D (given output subsidies) into its direct and strategic effect. The strategic effect has the same components as before, hence it is still negative. Therefore, firms underinvest in R&D for a given output subsidy, . The first order conditions for a Nash equilibrium in the choice of R&D are,

$$\bar{\pi}^1_{\Delta^1}(\Delta^1, \Delta^2, s^1) = \left[\hat{R}^1_2(p^1, p^2) - \hat{C}^1_{p^2}(p^1, p^2, \Delta^1) + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right]\psi^2_{\Delta^1}(\Delta^1, \Delta^2, s^1) - \hat{C}^1_{\Delta}(p^1, p^2, \Delta^1) - \phi^1_1(\Delta^1) = 0 \tag{94}$$

$$\bar{\pi}^2_{\Delta^2}(\Delta^1, \Delta^2, s^1) = \left[\hat{R}^2_1(p^1, p^2) - \hat{C}^2_{p^1}(p^1, p^2, \Delta^2)\right]\psi^1_{\Delta^2}(\Delta^2, \Delta^1, s^1) - \hat{C}^2_{\Delta}(p^1, p^2, \Delta^2) - \phi^2_1(\Delta^2) = 0. \tag{95}$$

With the second order condition $\bar{\pi}^i_{\Delta^i\Delta^i} < 0$.

In order to see the effect of output subsidies over R&D investment, we totally differentiate the two first order conditions given by (94) and (95). The next proposition describes the effect of an output subsidy on the equilibrium R&D chosen by firms.

Proposition 4 *Under Bertrand competition, an output subsidy by the domestic government increases R&D of the domestic firm, and reduces R&D of the foreign firm:*

$$\frac{d\Delta^1}{ds^1} > 0 \tag{96}$$

$$\frac{d\Delta^2}{ds^1} < 0. \tag{97}$$

Proof. See appendix ■

The intuition for this proposition is straightforward once we consider how R&D influences profits. Recall (from the discussion of proposition 3) that the incentives to invest in R&D decrease if output declines: the beneficial effects of cost reduction are smaller if output is lower. Consider now an increase in the output subsidy s^1. The output subsidy results in a reduction in the price of both goods. However, p^1 declines by a

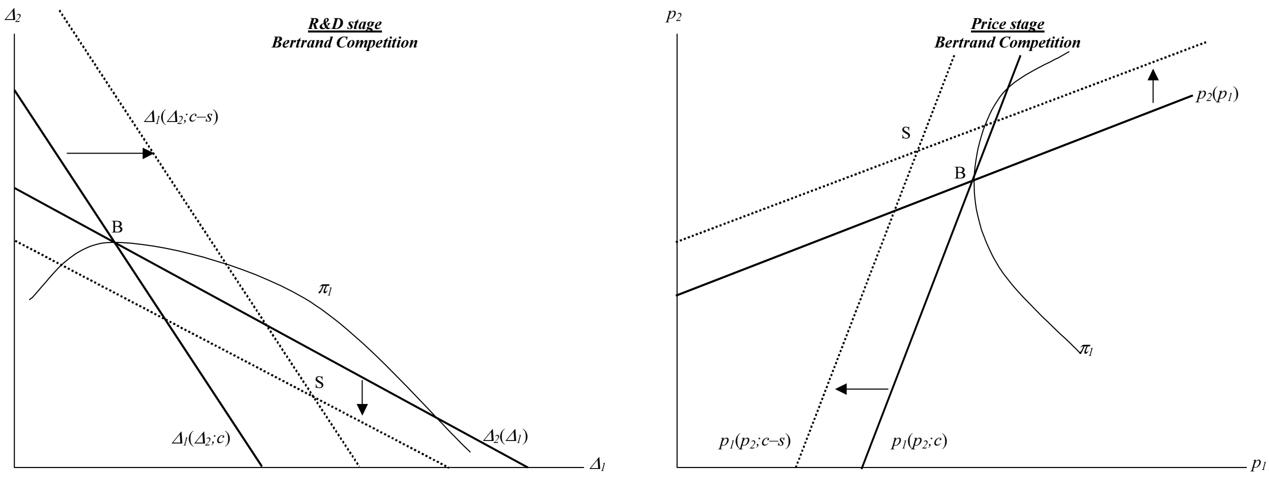

Figure 2: Bertrand Competition: Effect of an output subsidy s imposed by government 1.

greater amount than p^2. As a result, output of firm 1 increases while output of firm 2 decreases. The output expansion creates an even greater incentive for firm 1 to invest in R&D (shifts its R&D reaction function out). The effect on firm 2 is just the contratry: the incentives for firm 2 to invest in R&D decline (firm 2's R&D reaction function shifts in) due to the output subsidy, s^1.

This is the same type of effect as was observed under Cournot competition. An increase in the output subsidy increases quantity produced thereby positively affecting the incentives to invest in R&D for the home firm. In both cases the foreign firm reduces its R&D due to decreased foreign production. As one would expect, an output subsidy imposed by the domestic government affects domestic R&D more than foreign R&D. This result, formalized in the next corollary, is used later to determine the sign of the optimal output subsidy.

Corollary 5 *Under Bertrand competition, the effect of an output subsidy over own R&D expenditures is stronger than over foreign R&D expenditures:*

$$\left|\frac{d\Delta^1}{ds^1}\right| > \left|\frac{d\Delta^2}{ds^1}\right| \tag{98}$$

Proof. See appendix ■

We can conduct a graphical analysis similar to the Cournot case. As with quantity competition, an increase in output subsidy (s^1) shifts the R&D reaction function of firm 1 out and that of firm 2 in (left half of figure 2) This means that the equilibrium in R&D space moves from B (free trade) to S. For a small output subsidy, this leaves firm 1 inside its isoprofit contour (π_1) that passes through the free trade point B : just looking at the R&D stage an output subsidy increases welfare for the domestic country. However, as in the Cournot case, we have to also take into account the effect of the subsidy in the price competition stage. This is illustrated in the right half of figure 2. As in the case of Cournot competition, an output subsidy increases domestic and reduces foreign R&D, reducing domestic marginal costs beyond the direct effect of the subsidy and increasing foreign marginal costs. This means that the domestic price reaction function shifts in and the foreign price reaction function shifts out, moving the equilibrium from B to S. From corollary 5 we know that even if we only take into account the effect of R&D on the price stage, the reaction function of firm 1 will shift more that the reaction function of firm 2. This leaves point S outside the isoprofit contour π_1 passing through point B in the price space. Therefore an output subsidy reduces welfare for the home government in the price stage. The *net effect* on the two stages determines whether an output subsidy increases or reduces welfare.

Formally, define the net domestic benefit of government 1 as $\bar{B}^1(s^1) = \bar{\pi}^1(\Delta^1, \Delta^2, s^1) - s^1 x^1(\bar{\psi}^1, \bar{\psi}^2)$. Taking the derivative of $\bar{B}^1(s^1)$ with respect to s^1:

$$\frac{\partial \bar{B}^1}{\partial s^1} = \bar{\pi}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\pi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\pi}^1_{s^1} - x^1 - s^1\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} - s^1\frac{\partial x^1}{\partial p^2}\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1} \tag{99}$$

Recall that from the first order condition in the R&D stage, $\bar{\pi}^1_{\Delta^1} = 0$, and,

$$\begin{aligned} \bar{\pi}^1_{s^1} &= \left[\hat{R}^1_2(p^1,p^2) - \hat{C}^1_{p^2}(p^1,p^2,\Delta) + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right]\bar{\psi}^2_{s^1}(\Delta^1,\Delta^2,s^1) + x^1 \\ &= \left[p^1 - \frac{\partial C^1}{\partial x^1} + s^1\right]\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{s^1} + x^1 \end{aligned} \tag{100}$$

$$\begin{aligned} \bar{\pi}^1_{\Delta^2} &= \left[\hat{R}^1_2(p^1,p^2) - \hat{C}^1_{p^2}(p^1,p^2,\Delta) + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right]\bar{\psi}^2_{\Delta^2}(\Delta^1,\Delta^2,s^1) \\ &= \left[p^1 - \frac{\partial C^1}{\partial x^1} + s^1\right]\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{\Delta^2} \end{aligned} \tag{101}$$

$$\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} = \bar{\psi}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\psi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^1_{s^1} \tag{102}$$

and

$$\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1} = \bar{\psi}^2_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}. \tag{103}$$

The last two expressions capture the *total* effect of the output subsidy over prices. They take into account that the subsidy also affects the choice of R&D by both firms in the second stage (and these, in turn, affect prices). This effect (through R&D) is reflected in the first two terms of the expression.

With these expressions we can rewrite $\frac{\partial \bar{B}^1}{\partial s^1}$:

$$\begin{aligned} \frac{\partial \bar{B}^1}{\partial s^1} &= \left[p^1 - \frac{\partial C^1}{\partial x^1} + s^1\right]\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \left[p^1 - \frac{\partial C^1}{\partial x^1} + s^1\right]\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{s^1} - s^1\left[\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} + \frac{\partial x^1}{\partial p^2}\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1}\right] \\ &= \left[p^1 - \frac{\partial C^1}{\partial x^1}\right]\left(\frac{\partial x^1}{\partial p^2}\right)\left(\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}\right) - s^1\left[\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} + \frac{\partial x^1}{\partial p^2}\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1} - \left(\frac{\partial x^1}{\partial p^2}\right)\left[\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}\right]\right] \\ &= \left[p^1 - \frac{\partial C^1}{\partial x^1}\right]\left(\frac{\partial x^1}{\partial p^2}\right)\left(\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}\right) - s^1\left[\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} + \frac{\partial x^1}{\partial p^2}\left(\bar{\psi}^2_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}\right)\right] \end{aligned} \tag{104}$$

The first term on the right hand side of (104) shows the effect of the output subsidy over domestic benefit in the *second stage* (R&D investment). A domestic output subsidy reduces foreign R&D investment $\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} < 0\right)$, which in turn increases the foreign price p^2. The increase in p^2 increases domestic output x^1 and hence firm 1's profits. Notice that due to the envelope theorem, the effect of an infinitesimal increase in the subsidy s^1 on domestic benefit $\bar{B}^1$ (through domestic R&D) can be ignored.

The second term in (104) captures the effect of an output subsidy over domestic benefit in the *third stage* (price competition stage). A domestic output subsidy reduces the foreign price in the price competition stage ($\bar{\psi}^2_{s^1} < 0$). The reduction in the foreign price p^2 reduces domestic output and profits. Again the envelope theorem allows us to ignore the effect of the output subsidy on domestic benefits through the domestic price p^1.

Notice that, starting from a subsidy s^1 equal to zero, an infinitesimal increase in the subsidy increases, or decreases, domestic benefits if the *R&D stage effect* $\left(\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}\right)$ is stronger, or weaker, than the *price stage effect,* $\left(\bar{\psi}^2_{s^1}\right)$.

$$\left.\frac{\partial \bar{B}^1}{\partial s^1}\right|_{s^1=0} = \left[p^1 - \frac{\partial C^1}{\partial x^1}\right]\left(\frac{\partial x^1}{\partial p^2}\right)\left(\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}\right) \tag{105}$$

The third term in (104) captures the increase, or decrease, in the subsidy bill brought about by an increase, or decrease, in domestic output. It includes the direct effect of the subsidy in the price competition stage as well as the R&D stage effect and price stage effect. To obtain the expression for the optimal output subsidy we need to solve

$$\frac{\partial \bar{B}^1}{\partial s^1} = 0 \tag{106}$$

with the second order condition

$$\frac{\partial^2 \bar{B}^1}{(\partial s^1)^2} < 0. \tag{107}$$

Solving (106), the precise expression for the optimal output subsidy is obtained:

$$s^{1*} = \left[p^1 - \frac{\partial C^1}{\partial x^1}\right]\frac{\left(\frac{\partial x^1}{\partial p^2}\right)\left(\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}\right)}{\left[\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} + \frac{\partial x^1}{\partial p^2}\left(\bar{\psi}^2_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}\right)\right]} \tag{108}$$

Given that the denominator in (108) is positive,[21] the sign of the optimal subsidy depends on whether the effect on the R&D stage or on the price stage dominates in the numerator of (108). As we will see, the sign is

[21] Note that for linear demands and constant marginal costs:

$$\begin{aligned}
\left[\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} + \frac{\partial x^1}{\partial p^2}\left(\bar{\psi}^2_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}\right)\right] &= \frac{\partial x^1}{\partial p^1}\left[\bar{\psi}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\psi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^1_{s^1} - \gamma\left(\bar{\psi}^2_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}\right)\right] \\
&= \frac{\partial x^1}{\partial p^1}\left[\left(\bar{\psi}^1_{\Delta^1} - \gamma\bar{\psi}^2_{\Delta^1}\right)\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\psi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^1_{s^1}\right] \\
&= \frac{\partial x^i}{\partial p^i}\bar{\psi}^i_{\Delta^i}\left[\left(1 - \gamma\frac{\bar{\psi}^i_{\Delta^j}}{\bar{\psi}^i_{\Delta^i}}\right)\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \frac{\bar{\psi}^i_{\Delta^j}}{\bar{\psi}^i_{\Delta^i}}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \frac{\bar{\psi}^i_{s^1}}{\bar{\psi}^i_{\Delta^i}}\right] \\
&= \frac{\partial x^i}{\partial p^i}\bar{\psi}^i_{\Delta^i}\left[\left(1 + \gamma\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} - \frac{\Pi^i_{ij}}{\Pi^i_{ii}}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} - \frac{\frac{\partial x^i}{\partial p^i}}{\hat{C}^i_{p^i\Delta}}\right] \\
&= \frac{\partial x^i}{\partial p^i}\bar{\psi}^i_{\Delta^i}\left[\left(1 + \gamma\frac{\frac{\partial x^i}{\partial p^j}}{2\frac{\partial x^i}{\partial p^i}}\right)\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} - \frac{\frac{\partial x^i}{\partial p^j}}{2\frac{\partial x^i}{\partial p^i}}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \frac{1}{\theta}\right] \\
&= \frac{\partial x^i}{\partial p^i}\bar{\psi}^i_{\Delta^i}\left[\left(1 - \frac{\gamma^2}{2}\right)\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \frac{\gamma}{2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \frac{1}{\theta}\right] > 0
\end{aligned}$$

ambiguous and depends on the cost of R&D (ϕ_{11}^1) relative to the effectiveness of R&D at reducing marginal costs of production. Define $\theta = -\frac{\hat{C}^i_{p^i\Delta}}{\frac{\partial x^i}{\partial p^i}} = -\frac{\partial^2 C^i(x^i,\Delta^i)}{\partial \Delta^i \partial x^i}$ as the effectiveness of R&D. Notice from (69) that $\bar{\psi}^i_{\Delta^i}$ is independent of ϕ_{11}^1. Therefore, $\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}$ is the only term in the numerator of (108) that depends on ϕ_{11}^1. The following lemma helps to understand the role of the cost of R&D on the elasticity of R&D to output subsidies.

Lemma 6 *The influence of output subsidies over R&D decreases as the marginal cost of R&D increases. Specifically,*

$$\frac{\partial \left|\frac{d\Delta^1}{ds^1}\right|}{\partial \phi_{11}^1} < 0 \tag{109}$$

$$\frac{\partial \left|\frac{d\Delta^2}{ds^1}\right|}{\partial \phi_{11}^1} < 0. \tag{110}$$

Proof. See Appendix ■

An increase in ϕ_{11}^1 makes R&D investment more convex. As a result, R&D is less elastic to an output subsidy, and therefore the *R&D stage* effect of an output subsidy in (108) is weaker. Whenever the R&D stage effect is weak, the optimal output subsidy is influenced more by the *price stage* effect and should be optimally set below zero (an output tax).

The domestic government only takes into account the effect of an output subsidy over price competition when the effect of an output subsidy over foreign R&D is smaller (ϕ_{11}^1 becomes higher). Contrarily, the government only takes into account the effect of the output subsidy on the R&D stage when ϕ_{11}^1 is small enough. The following proposition formalizes this result, showing that we could have an output subsidy or a tax depending on the convexity of the cost of investment in R&D, i.e. ϕ_{11}^i.[22]

Proposition 7 *Under Bertrand competition, the optimal output subsidy s^{1*} can be positive or negative, depending on the convexity of the cost of R&D (ϕ_{11}^i). The optimal output subsidy is positive (an output subsidy) when the cost of additional investment in R&D is sufficiently low (low ϕ_{11}^i), and negative (an output tax) when ϕ_{11}^i is sufficiently high. Specifically,*

$$\exists \bar{\phi} < \infty \text{ such that if } \phi_{11}^i > \bar{\phi} \text{ then } s^{1*} < 0$$

$$\exists \underline{\phi} > \theta \bar{\pi}^1_{\Delta^1 s^1} - \pi^i_{\Delta_i \Delta_j} \text{ such that if } \phi_{11}^i < \underline{\phi} \text{ then } s^{1*} > 0.$$

Proof. See Appendix ■

As ϕ increases, the cost of investng in R&D becomes more convex. A steeper R&D cost function makes R&D less elastic with respect to an output subsidy. This reduces the effect of the subsidy over the foreign

where $\theta = -\frac{\hat{C}^i_{p^i\Delta}}{\frac{\partial x^i}{\partial p^i}} = -\frac{\partial^2 C^i(x^i,\Delta^i)}{\partial \Delta^i \partial x^i}$ measures the effectiveness of R&D at reducing marginal costs of production and $\gamma = -\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}$ is a measure of the degree of product differentiation. The inequality comes from $\left|\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}\right| > \left|\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}\right|$ (Corollary 5) and $\left(1 - \frac{\gamma^2}{2}\right) > \frac{\gamma}{2}$ for γ between zero and one.

[22] Notice, however, that ϕ_{11}^i is bounded below by the stability condition (76) and therefore cannot take values below $\theta\bar{\pi}^1_{\Delta^1 s^1} - \pi^i_{\Delta_i \Delta_j}$. See the proof of lemma 6.

firm's R&D reaction function, leaving the effect over the foreign firm price reaction function unaffected. This implies that the domestic government has an incentive to reduce the output subsidy, or even tax output, as in the standard Bertrand game without R&D investment.

The following section performs a numerical exercise to highlight the results of price and quantity competition.

4 A Numerical Example

In this example,[23] we consider linear demands and constant marginal costs with respect to output. In particular, assume that the inverse demand for good i is given by:

$$p^i = a - b(x^i + \gamma x^j). \tag{111}$$

With $0 < \gamma < 1$. Cost functions are linear in output,

$$C(x^i, \Delta^i) = \left(c - \theta\Delta^i\right) x^i \tag{112}$$

and the monetary cost of Δ^i units of R&D is quadratic:

$$\phi(\Delta^i) = \phi\frac{\left(\Delta^i\right)^2}{2}. \tag{113}$$

The optimal output subsidy is always positive under Cournot competition, as both the R&D stage effect ($\bar{q}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}$) and the price stage effect ($\bar{q}^2_{s^1}$) have the same sign (see proposition 2). R&D becomes more elastic with respect to the output subsidy as the cost of R&D becomes flatter (i.e. ϕ^i_{11} falls). In this case the government has greater incentives to subsidize output thereby reducing foreign R&D. Figure 3 shows the optimal subsidy as a function of the cost-effectiveness of R&D (defined as $\eta = \frac{\theta^2}{\phi b}$). The optimal subsidy is increasing in η.

The case of Bertrand competition is a bit more complicated. We have to satisfy (107), the second order condition of the government maximization problem. It turns out that the optimal subsidy also depends on the cost–effectiveness of R&D (η). Figure 4 shows the optimal output subsidy, which is increasing in η (decreasing in ϕ^i_{11}). Note that as the R&D effect becomes stronger (η increases) the government reverses its policy from an output tax to an output subsidy.[24] Note also that, interestingly, there is a set of parameter values for which free trade ($s^{1*} = 0$) is an equilibrium in the Bertrand case, even in the presence of imperfect competition.

Tables 1 and 2 present numerical results for $\gamma = 0.3$ and two different values of η (0.3 and 0.7). Notice that all relevant quantities are positive and that the second order condition for the government's maximization problem is satisfied. Table 2 shows that, depending on the cost-effectiveness of R&D (η), there could be a policy reversal under Bertrand competition.[25]

[23]The mathematica code used to generate the numerical results is available from the authors upon request.

[24]For the Bertrand example in this section, (figure 4), the stability condition (76) translates into

$$\eta < \frac{\left(1-\gamma^2\right)\left(4-\gamma^2\right)^2}{2\left(2-\gamma^2\right)\left(2+\gamma-\gamma^2\right)}$$

For the value in the numerical example ($\gamma = 0.5$), we require $\eta < 1.33929$ to satisfy that condition.

[25]The numerical simulations presented in section 3 of Neary and Leahy (2000) assume, for the Cournot case, a set of

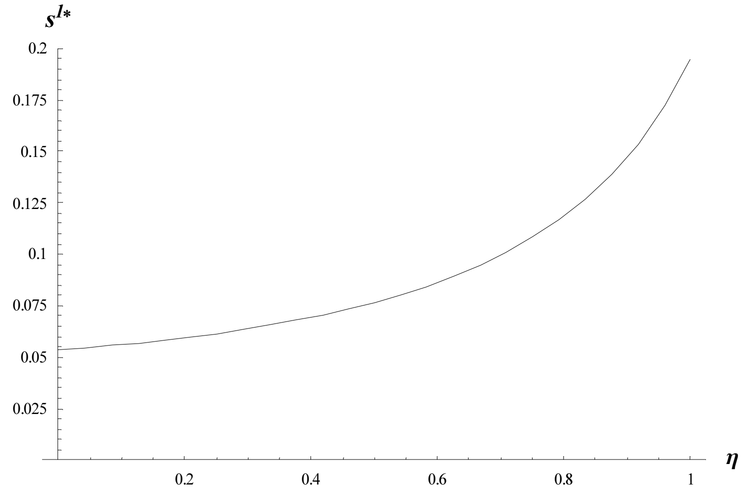

Figure 3: Cournot: Optimal output subsidy $\left(s^{1*}\right)$ as a function of the cost-effectiveness of R&D $\left(\eta = \frac{\theta^2}{b\phi}\right)$.
(for $a - c = 1$, $\gamma = 0.5$)

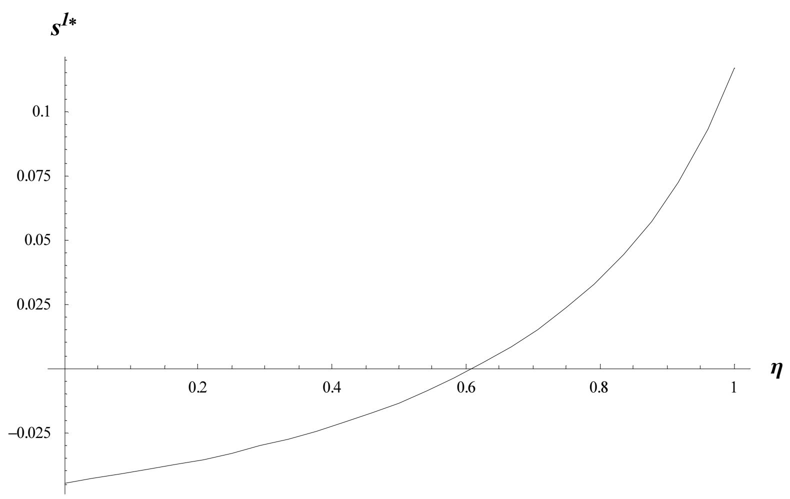

Figure 4: Bertrand: Optimal output subsidy $\left(s^{1*}\right)$ as a function of the cost-effectiveness of R&D $\left(\eta = \frac{\theta^2}{b\phi}\right)$.
(for $a - c = 1$, $\gamma = 0.5$)

Cournot Competition: numerical simulation			
Product differentiation	γ	0.5	0.5
Cost-effectiveness of R&D	$\eta = \frac{\theta^2}{\phi b}$	0.3	0.7
Price firm 1	p^1	$0.2765a + 0.7235c$	$0.0689a + 0.9311c$
Price firm 2	p^2	$0.3035a + 0.6965c$	$0.1349a + 0.8651c$
Output firm 1	x^1	$0.5004\left(\frac{a-c}{b}\right)$	$0.6648\left(\frac{a-c}{b}\right)$
Output firm 2	x^2	$0.4463\left(\frac{a-c}{b}\right)$	$0.5328\left(\frac{a-c}{b}\right)$
R&D firm 1	Δ^1	$0.1601\left(\frac{a-c}{\theta}\right)$	$0.4964\left(\frac{a-c}{\theta}\right)$
R&D firm 2	Δ^2	$0.1428\left(\frac{a-c}{\theta}\right)$	$0.3977\left(\frac{a-c}{\theta}\right)$
Unit profit firm 1	$m^1 = p^1 - c + s^1$	$0.3403\,(a-c)$	$0.1684\,(a-c)$
Unit profit firm 2	$m^2 = p^2 - c$	$0.3035\,(a-c)$	$0.1349\,(a-c)$
Total profits firm 1	π^1	$0.2076\frac{(a-c)^2}{b}$	$0.2659\frac{(a-c)^2}{b}$
Total profits firm 2	π^2	$0.1652\frac{(a-c)^2}{b}$	$0.1707\frac{(a-c)^2}{b}$
Benefits country 1	B^1	$0.1757\frac{(a-c)^2}{b}$	$0.1998\frac{(a-c)^2}{b}$
Benefits country 2	B^2	$0.1652\frac{(a-c)^2}{b}$	$0.1707\frac{(a-c)^2}{b}$
Optimal output subsidy	s^{1*}	$\mathbf{0.0638}\,(a-c)$	$\mathbf{0.0995}\,(a-c)$
Government's SOC	$\frac{\partial^2 B^1}{(\partial s^{1*})^2}$	$-\frac{0.5987}{b}$	$-\frac{0.8141}{b}$

Table 1: Numerical simulation under Cournot Competition in the third stage

Bertrand Competition: numerical simulations			
Product differentiation	γ	0.5	0.5
Cost-effectiveness of R&D	$\eta = \frac{\theta^2}{\phi b}$	0.3	0.7
Price firm 1	p^1	$0.2575a + 0.7425c$	$0.0483a + 0.9517c$
Price firm 2	p^2	$0.2422a + 0.7578c$	$0.0598a + 0.9401c$
Output firm 1	x^1	$0.4848\left(\frac{a-c}{b}\right)$	$0.6422\left(\frac{a-c}{b}\right)$
Output firm 2	x^2	$0.5154\left(\frac{a-c}{b}\right)$	$0.6191\left(\frac{a-c}{b}\right)$
R&D firm 1	Δ^1	$0.1357\left(\frac{a-c}{\theta}\right)$	$0.4196\left(\frac{a-c}{\theta}\right)$
R&D firm 2	Δ^2	$0.1443\left(\frac{a-c}{\theta}\right)$	$0.4044\left(\frac{a-c}{\theta}\right)$
Unit profit firm 1	$m^1 = p^1 - c + s^1$	$0.2278\,(a-c)$	$0.0621\,(a-c)$
Unit profit firm 2	$m^2 = p^2 - c$	$0.2422\,(a-c)$	$0.0598\,(a-c)$
Total profits firm 1	π^1	$0.1455\frac{(a-c)^2}{b}$	$0.1836\frac{(a-c)^2}{b}$
Total profits firm 2	π^2	$0.1645\frac{(a-c)^2}{b}$	$0.1706\frac{(a-c)^2}{b}$
Benefits country 1	B^1	$0.1599\frac{(a-c)^2}{b}$	$0.1747\frac{(a-c)^2}{b}$
Benefits country 2	B^2	$0.1645\frac{(a-c)^2}{b}$	$0.1706\frac{(a-c)^2}{b}$
Optimal output subsidy	s^{1*}	$\mathbf{-0.0297}\,(a-c)$	$\mathbf{0.0138}\,(a-c)$
Government's SOC	$\frac{\partial^2 B^1}{(\partial s^{1*})^2}$	$-\frac{0.8055}{b}$	$-\frac{1.1226}{b}$

Table 2: Numerical simulation under Bertrand Competition in the third stage

5 Conclusions

This paper shows that for sufficiently cost effective R&D the trade policy reversal in Eaton and Grossman (1986) is not observed. Our result suggests that output subsidies are more robust than otherwise implied by the literature on strategic trade. If exporting industries make long run investments before competing in the market then governments have a case for using output subsidies even if they are uncertain about the mode of competition in the market.

We show that a necessary condition for output subsidies to be robust is that R&D be sufficiently cost effective. If the cost of R&D is too convex then R&D expenditures will be relatively inelastic to the export subsidy. In this case, the effect of an export subsidy on R&D will be negligible and will thus be arbitrarily close to the case when there is no R&D investment (Brander and Spencer (1985), Eaton and Grossman (1986)). If R&D costs are not too convex then R&D is responsive to an output subsidy. In this case, the effect of the output subsidy on the R&D stage reinforces the effect of the output subsidy on the market competition stage under Cournot competition, and dominates it under Bertrand competition. Thus, regardless of the mode of competition, the optimal policy is an output subsidy if R&D is sufficiently cost-effective.

Our condition on the curvature of the cost of R&D is reminiscent of Maggi (1996). In his model, firms invest in capacity and then compete in prices in the product market. Maggi shows that going from Cournot to Bertrand competition the optimal policy changes from an output subsidy to a tax. The key parameter is his model is the convexity of the cost function. A more convex cost function (i.e. steeper marginal cost) results in firm behavior closer to price competition. The optimal trade policy in this case is an output tax. Contrarily, a flatter marginal cost implies that the optimal policy is an output subsidy. In contrast to Maggi (1996), in our model marginal costs are constant. Under Bertrand competition, whether the optimal policy is an output subsidy or a tax, depends on the convexity of the cost of R&D . Under Cournot competition, the optimal trade policy is always an output subsidy.

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parameters, which with our notation, imply $b = \gamma = \theta = a - c = 1$ and $\eta = \frac{1}{\phi} = 0.2$. For that set of parameters we obtain an optimal subsidy $s^{1*} = 0.3089$, which roughly corresponds to what they refer to as the second–best optimal output subsidy. This is represented by the intersection of the flatter line with the vertical axis in their figure 3.

For the Bertrand simulation, they use a set of parameters $b = \theta = a - c = 1$ and $\eta = \frac{1}{\phi} = 0.4$, with inverse demands

$$x^i = a - b(p^i - p^j)$$

which means that cross price effects are as strong as own price effects. Therefore we cannot compare directly with their results. They find that the optimal output subsidy is negative (point C in their figure 4). If we set $\gamma = 0.5$ with their other parameters, in our simulation we obtain a negative output subsidy (i.e. a tax) equal to $s^{1*} = -0.0224$. We only need to have a cost–effectiveness of R&D beyond 0.6 to obtain a positive output subsidy, as shown in figure 4.

Appendix

A Proof of Proposition 1

Differentiate totally the two first order conditions given by (42). These equations can be rewritten in a more compact way as:

$$\bar{\pi}^1_{\Delta^1\Delta^1}\mathrm{d}\Delta^1 + \bar{\pi}^1_{\Delta^1\Delta^2}\mathrm{d}\Delta^2 + \bar{\pi}^1_{\Delta^1 s^1}\mathrm{d}s^1 = 0 \tag{114}$$

$$\bar{\pi}^2_{\Delta^1\Delta^2}\mathrm{d}\Delta^1 + \bar{\pi}^2_{\Delta^2\Delta^2}\mathrm{d}\Delta^2 + \bar{\pi}^2_{\Delta^2 s^1}\mathrm{d}s^1 = 0 \tag{115}$$

and using Cramer's rule:

$$\frac{\mathrm{d}\Delta^i}{\mathrm{d}s^1} = \frac{-\bar{\pi}^j_{\Delta^j\Delta^j}\bar{\pi}^i_{\Delta^i s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^j_{\Delta^j s^1}}{\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^j_{\Delta^j\Delta^j} - \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^j_{\Delta^i\Delta^j}} \tag{116}$$

In order to obtain the value of the expressions in (116) we first need to sign the *total* effect of subsidies over marginal revenues (including the effect over the last stage (quantity competition). We therefore have

$$\frac{\mathrm{d}R^1_2(x^1,x^2)}{\mathrm{d}s^1} = R^1_{12}(x^1,x^2)\bar{q}^1_{s^1} + R^1_{22}(x^1,x^2)\bar{q}^2_{s^1} < 0 \tag{117}$$

$$\frac{\mathrm{d}R^2_1(x^1,x^2)}{\mathrm{d}s^1} = R^2_{11}(x^1,x^2)\bar{q}^1_{s^1} + R^2_{12}(x^1,x^2)\bar{q}^2_{s^1} > 0 \tag{118}$$

by (6), (7), (37) and (38). Using these signs we can now turn to the elements in (116)

$$\bar{\pi}^i_{\Delta^i\Delta^i} = R^i_j\bar{q}^j_{\Delta^i\Delta^i} + \bar{q}^j_{\Delta^i}\frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^i} - C^i_{x\Delta}\bar{q}^i_{\Delta^i} - C^i_{\Delta\Delta} - \phi^i_{ii} < 0 \tag{119}$$

$$\bar{\pi}^i_{\Delta^i\Delta^j} = R^i_j\bar{q}^j_{\Delta^i\Delta^j} + \bar{q}^j_{\Delta^i}\frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^j} - C^i_{x\Delta}\bar{q}^i_{\Delta^j} < 0 \tag{120}$$

$$\bar{\pi}^1_{\Delta^1 s^1} = R^1_2\bar{q}^2_{\Delta^1 s^1} + \bar{q}^2_{\Delta^1}\frac{\mathrm{d}R^1_2(x^1,x^2)}{\mathrm{d}s^1} - C^1_{x\Delta}\bar{q}^1_{s^1} > 0 \tag{121}$$

$$\bar{\pi}^2_{\Delta^2 s^1} = R^2_1\bar{q}^1_{\Delta^2 s^1} + \bar{q}^1_{\Delta^2}\frac{\mathrm{d}R^2_1(x^1,x^2)}{\mathrm{d}s^1} - C^2_{x\Delta}\bar{q}^2_{s^1} < 0 \tag{122}$$

where the first inequality is the second order condition of the maximization in the R&D stage, the second inequality repeats (28), and the last two inequalities are derived from (117), (118), (37), (38), (20) and noting that for linear demand and constant marginal costs, $\bar{q}^i_{\Delta^j}$ is independent of s^1. Therefore

$$\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} = \frac{-\bar{\pi}^2_{\Delta^2\Delta^2}\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^1_{\Delta^1\Delta^2}\bar{\pi}^2_{\Delta^2 s^1}}{\bar{\pi}^1_{\Delta^1\Delta^1}\bar{\pi}^2_{\Delta^2\Delta^2} - \bar{\pi}^1_{\Delta^1\Delta^2}\bar{\pi}^2_{\Delta^1\Delta^2}} > 0 \tag{123}$$

$$\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} = \frac{-\bar{\pi}^1_{\Delta^1\Delta^1}\bar{\pi}^2_{\Delta^2 s^1} + \bar{\pi}^2_{\Delta^2\Delta^1}\bar{\pi}^1_{\Delta^1 s^1}}{\bar{\pi}^2_{\Delta^2\Delta^2}\bar{\pi}^1_{\Delta^1\Delta^1} - \bar{\pi}^2_{\Delta^2\Delta^1}\bar{\pi}^1_{\Delta^2\Delta^1}} < 0 \tag{124}$$

■

B Proof of Proposition 3.

Note that,

$$\begin{aligned}\pi^i_{ij} &= \psi^j_i \psi^i_j \hat{R}^i_{ij}(p^i,p^j) - \hat{C}^i_{p^i\Delta}\psi^i_j - \hat{C}^i_{p^j\Delta}\psi^j_j \\ &= \frac{-\Pi^j_{ij}\hat{C}^i_{p^i\Delta}}{\Pi^i_{ii}\Pi^j_{jj} - \Pi^j_{ij}\Pi^i_{ij}} \frac{-\Pi^i_{ij}\hat{C}^j_{p^j\Delta}}{\Pi^i_{ii}\Pi^j_{jj} - \Pi^j_{ij}\Pi^i_{ij}} \hat{R}^i_{ij} - \hat{C}^i_{p^i\Delta} \frac{-\Pi^i_{ij}\hat{C}^j_{p^j\Delta}}{\Pi^i_{ii}\Pi^j_{jj} - \Pi^j_{ij}\Pi^i_{ij}} - \hat{C}^i_{p^j\Delta} \frac{\Pi^i_{ii}\hat{C}^j_{p^j\Delta}}{\Pi^i_{ii}\Pi^j_{jj} - \Pi^j_{ij}\Pi^i_{ij}} \\ &= \frac{\Pi^j_{ij}\hat{C}^i_{p^i\Delta}\Pi^i_{ij}\hat{C}^j_{p^j\Delta}\hat{R}^i_{ij} + \hat{C}^i_{p^i\Delta}\Pi^i_{ij}\hat{C}^j_{p^j\Delta}\left(\Pi^i_{ii}\Pi^j_{jj} - \Pi^j_{ij}\Pi^i_{ij}\right) - \hat{C}^i_{p^j\Delta}\Pi^i_{ii}\hat{C}^j_{p^j\Delta}\left(\Pi^i_{ii}\Pi^j_{jj} - \Pi^j_{ij}\Pi^i_{ij}\right)}{\left(\Pi^i_{ii}\Pi^j_{jj} - \Pi^j_{ij}\Pi^i_{ij}\right)^2} \end{aligned} \tag{125}$$

Recall that, for linear demands, $\Pi^i_{ij} = \hat{R}^i_{ij}$. Notice also that in the case of linear demands, Π^i is quadratic and all second derivatives of $\Pi^i(p^1,p^2)$ are thus constant. Therefore, $\Pi^i_{ii} = \Pi^j_{jj}$ and $\Pi^j_{ij} = \Pi^i_{ij}$. If we also have constant marginal costs, all second derivatives of $\hat{C}^i(p^1,p^2,\Delta^i)$ are constant. Therefore $\hat{C}^i_{p^i\Delta} = \hat{C}^j_{p^j\Delta}$. Remember also that $\left|\Pi^i_{ii}\right| > \left|\Pi^i_{ij}\right|$, $\left|\hat{C}^i_{p^i\Delta}\right| > \left|\hat{C}^i_{p^j\Delta}\right|$ and $\left|\frac{\partial x^i}{\partial p^i}\right| > \left|\frac{\partial x^i}{\partial p^j}\right|$. This implies that

$$\begin{aligned}\pi^i_{ij} &= \frac{\left(\Pi^i_{ij}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2 + \left(\hat{C}^i_{p^i\Delta}\right)^2\Pi^i_{ij}\left(\Pi^i_{ii}\right)^2 - \left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ij}\right)^3 - \hat{C}^i_{p^j\Delta}\left(\Pi^i_{ii}\right)^3\hat{C}^i_{p^i\Delta} + \hat{C}^i_{p^j\Delta}\Pi^i_{ii}\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ij}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2} \\ &= \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\Pi^i_{ij}\left(\Pi^i_{ii}\right)^2 - \hat{C}^i_{p^j\Delta}\left(\Pi^i_{ii}\right)^3\hat{C}^i_{p^i\Delta} + \hat{C}^i_{p^j\Delta}\Pi^i_{ii}\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ij}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2} \\ &= \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{\Pi^i_{ij}}{\Pi^i_{ii}} - \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}} + \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^2\right] \\ &= \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}} - \frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}} + \frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right)^2\right] \\ &= \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right] < 0 \end{aligned} \tag{126}$$

since $\gamma = -\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}$ is between zero and one and $\Pi^i_{ii} < 0$. Notice that γ measures the degree of product differentiation and is bounded between 0 (independent goods) and 1 (perfect substitutes). ■

C Proof of Proposition 4

For the first part of the proof, we will follow similar steps as the proof of proposition 1. We start by differentiating totally the two first order conditions given by (94) and (95).

$$\bar{\pi}^1_{\Delta^1\Delta^1}\mathrm{d}\Delta^1 + \bar{\pi}^1_{\Delta^1\Delta^2}\mathrm{d}\Delta^2 + \bar{\pi}^1_{\Delta^1 s^1}\mathrm{d}s^1 = 0 \tag{127}$$

$$\bar{\pi}^2_{\Delta^2\Delta^2}\mathrm{d}\Delta^2 + \bar{\pi}^2_{\Delta^1\Delta^2}\mathrm{d}\Delta^1 + \bar{\pi}^2_{\Delta^2 s^1}\mathrm{d}s^1 = 0 \tag{128}$$

and using Cramer's rule:

$$\frac{\mathrm{d}\Delta^i}{\mathrm{d}s^1} = \frac{-\bar{\pi}^j_{\Delta^j\Delta^j}\bar{\pi}^i_{\Delta^i s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^j_{\Delta^j s^1}}{\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^j_{\Delta^j\Delta^j} - \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^j_{\Delta^i\Delta^j}} \tag{129}$$

To obtain the value of expressions in (116) we need to obtain the *total* effect of subsidies over marginal revenues (including the effect on the last (price competition) stage). We have

$$\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^i} = \hat{R}^i_{ij}(p^i,p^j)\bar{\psi}^i_{\Delta^i} + \hat{R}^i_{jj}(p^i,p^j)\bar{\psi}^j_{\Delta^i} < 0 \tag{130}$$

$$\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^j} = \hat{R}^i_{ij}(p^i,p^j)\bar{\psi}^i_{\Delta^j} + \hat{R}^i_{jj}(p^i,p^j)\bar{\psi}^j_{\Delta^j} < 0 \tag{131}$$

$$\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}s^1} = \hat{R}^i_{ij}(p^i,p^j)\bar{\psi}^i_{s^1} + \hat{R}^i_{jj}(p^i,p^j)\bar{\psi}^j_{s^1} < 0 \tag{132}$$

where the inequalities are obtained from (55), (56), (69), (70), (88) and (89).

Turn next to the *total* effect of R&D over marginal costs:

$$\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^i} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\bar{\psi}^i_{\Delta^i} + \hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\bar{\psi}^j_{\Delta^i} + \hat{C}^i_{p^j\Delta}(p^i,p^j,\Delta^i) < 0 \tag{133}$$

$$\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^j} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\bar{\psi}^i_{\Delta^j} + \hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\bar{\psi}^j_{\Delta^j} = 0 \tag{134}$$

$$\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}s^1} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\bar{\psi}^i_{s^1} + \hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\bar{\psi}^j_{s^1} = 0 \tag{135}$$

where the inequalities are derived from (57), (58), (69), (70), (88) and (89). We also assume linear demand and constant marginal cost with respect to output (so that $\hat{C}^i_{p^jp^j} = \hat{C}^i_{p^ip^j} = 0$). Finally, notice that for linear demands, the slope of the demand function is not influenced by R&D. Formally:

$$\frac{\mathrm{d}\left(\frac{\partial x^i}{\partial p^i}\right)}{\mathrm{d}\Delta^j} = \frac{\partial^2 x^i(p^i,p^j)}{\partial\left(p^i\right)^2}\bar{\psi}^i_{\Delta^j} + \frac{\partial^2 x^i(p^i,p^j)}{\partial p^i\partial p^j}\bar{\psi}^j_{\Delta^j} = 0 \tag{136}$$

Using these inequalities we can now turn to the elements of (129). As in the case of free trade (see proof of proposition 3), we will use the fact that, for linear demands, $\Pi^i_{ij} = \hat{R}^i_{ij}$ and both are quadratic with constant second derivatives with respect to prices. Therefore, $\Pi^i_{ii} = \Pi^j_{jj}$ and $\Pi^j_{ij} = \Pi^i_{ij}$. If we also have constant marginal costs, all second derivatives of $\hat{C}^i(p^1,p^2,\Delta^i)$ are constant. Therefore $\hat{C}^i_{p^i\Delta} = \hat{C}^j_{p^j\Delta}$. For linear demand and constant marginal costs we have $\bar{\psi}^i_{\Delta^j\Delta^j} = \bar{\psi}^i_{\Delta^i\Delta^j} = \frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^j} = \frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}s^1} = \frac{\mathrm{d}\left(\frac{\partial x^i}{\partial p^i}\right)}{\mathrm{d}\Delta^j} = \hat{C}^i_{\Delta\Delta} = 0$. Remember also that $\left|\Pi^i_{ii}\right| > \left|\Pi^i_{ij}\right|$, $\left|\hat{C}^i_{p^i\Delta}\right| > \left|\hat{C}^i_{p^j\Delta}\right|$ and $\left|\frac{\partial x^i}{\partial p^i}\right| > \left|\frac{\partial x^i}{\partial p^j}\right|$. All these imply that

$$
\begin{aligned}
\bar{\pi}^1_{\Delta^1\Delta^1} &= \left(\hat{R}^1_2 - \hat{C}^1_{p^2} + s^1\left(\frac{\partial x^2}{\partial p^1}\right)\right)\bar{\psi}^2_{\Delta^1\Delta^1} + \bar{\psi}^2_{\Delta^1}\left(\frac{\mathrm{d}\hat{R}^1_2(p^2,p^1)}{\mathrm{d}\Delta^1} - \frac{\mathrm{d}\hat{C}^1_{p^2}(p^2,p^1,\Delta^1)}{\mathrm{d}\Delta^1} + s^1\frac{\mathrm{d}\left(\frac{\partial x^2}{\partial p^2}\right)}{\mathrm{d}\Delta^1}\right) \\
&\quad -\hat{C}^1_{p^1\Delta}\bar{\psi}^1_{\Delta^1} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{\Delta^1} - \hat{C}^1_{\Delta\Delta} - \phi^1_{11} \\
&= \bar{\psi}^2_{\Delta^1}\left(\hat{R}^1_{12}\bar{\psi}^1_{\Delta^1} - \hat{C}^1_{p^2\Delta}\right) - \hat{C}^1_{p^1\Delta}\bar{\psi}^1_{\Delta^1} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{\Delta^1} - \phi^1_{11} \\
&= \frac{-\Pi^2_{21}\hat{C}^1_{p^1\Delta}}{\Pi^2_{22}\Pi^1_{11} - \Pi^1_{21}\Pi^2_{21}}\left(\frac{\hat{R}^1_{12}\Pi^2_{22}\hat{C}^1_{p^1\Delta}}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} - \hat{C}^1_{p^2\Delta}\right) - \hat{C}^1_{p^1\Delta}\frac{\Pi^2_{22}\hat{C}^1_{p^1\Delta}}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} - \hat{C}^1_{p^2\Delta}\frac{-\Pi^2_{21}\hat{C}^1_{p^1\Delta}}{\Pi^2_{22}\Pi^1_{11} - \Pi^1_{21}\Pi^2_{21}} - \phi^1_{11} \\
&= \frac{-\Pi^i_{ij}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2}\left(\Pi^i_{ij}\frac{\Pi^i_{ii}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} - \hat{C}^i_{p^j\Delta}\right) - \hat{C}^i_{p^i\Delta}\frac{\Pi^i_{ii}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} - \hat{C}^i_{p^j\Delta}\frac{-\Pi^i_{ij}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} - \phi^1_{11} \\
&= \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}} - \left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^3\frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}} - 1 + \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\frac{\Pi^i_{ij}}{\Pi^i_{ii}} - \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^3\right] - \phi^1_{11} \\
&= \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right)\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}} - \left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right)^3\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}} - 1 + \frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right) - \frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right)^3\right] - \phi^1_{11} \\
&= \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{1}{2}\gamma^2 - \frac{1}{8}\gamma^4 - 1 + \frac{1}{2}\gamma^2 - \frac{1}{8}\gamma^4\right] - \phi^1_{11} \\
&= \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right] - \phi^1_{11} \qquad (137)
\end{aligned}
$$

where $\gamma = -\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}$ measures the degree of product differentiation as in the proof of proposition 3.

$$
\begin{aligned}
\bar{\pi}^2_{\Delta^2\Delta^2} &= \left(\hat{R}^2_1 - \hat{C}^2_{p^1}\right)\bar{\psi}^1_{\Delta^2\Delta^2} + \bar{\psi}^1_{\Delta^2}\left(\frac{\mathrm{d}\hat{R}^2_1(p^1,p^2)}{\mathrm{d}\Delta^2} - \frac{\mathrm{d}\hat{C}^2_{p^1}(p^1,p^2,\Delta^2)}{\mathrm{d}\Delta^2}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{\Delta^2} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{\Delta^2} - \hat{C}^2_{\Delta\Delta} - \phi^2_{11} \\
&= \bar{\psi}^1_{\Delta^2}\left(\hat{R}^2_{12}\bar{\psi}^2_{\Delta^2} - \hat{C}^2_{p^1\Delta}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{\Delta^2} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{\Delta^2} - \phi^2_{11} \\
&= \bar{\pi}^1_{\Delta^1\Delta^1} \qquad (138)
\end{aligned}
$$

$$
\begin{aligned}
\bar{\pi}^1_{\Delta^1\Delta^2} &= \left(\hat{R}^1_2 - \hat{C}^1_{p^2} + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right)\bar{\psi}^2_{\Delta^1\Delta^2} + \bar{\psi}^2_{\Delta^1}\left(\frac{\mathrm{d}\hat{R}^1_2(p^1,p^2)}{\mathrm{d}\Delta^2} - \frac{\mathrm{d}\hat{C}^1_{p^2}(p^1,p^2,\Delta^1)}{\mathrm{d}\Delta^2} + s^1\frac{\mathrm{d}\left(\frac{\partial x^1}{\partial p^1}\right)}{\mathrm{d}\Delta^2}\right) \\
&\quad -\hat{C}^1_{p^1\Delta}\bar{\psi}^1_{\Delta^2} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{\Delta^2} \\
&= \bar{\psi}^2_{\Delta^1}\left(\hat{R}^1_{12}\bar{\psi}^1_{\Delta^2}\right) - \hat{C}^1_{p^1\Delta}\bar{\psi}^1_{\Delta^2} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{\Delta^2} \\
&= \frac{-\Pi^2_{21}\hat{C}^1_{p^1\Delta}}{\Pi^2_{22}\Pi^1_{11} - \Pi^1_{21}\Pi^2_{21}}\left(\hat{R}^1_{12}\frac{-\Pi^1_{12}\hat{C}^2_{p^2\Delta}}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}}\right) - \hat{C}^1_{p^1\Delta}\frac{-\Pi^1_{12}\hat{C}^2_{p^2\Delta}}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} - \hat{C}^1_{p^2\Delta}\frac{\Pi^1_{11}\hat{C}^2_{p^2\Delta}}{\Pi^2_{22}\Pi^1_{11} - \Pi^1_{21}\Pi^2_{21}} \\
&= \frac{-\Pi^i_{ij}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2}\left(\Pi^i_{ij}\frac{-\Pi^i_{ij}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2}\right) - \hat{C}^i_{p^i\Delta}\frac{-\Pi^i_{ij}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} - \hat{C}^i_{p^j\Delta}\frac{\Pi^i_{ii}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} \\
&= \frac{\left(\Pi^i_{ij}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2 + \left(\hat{C}^i_{p^i\Delta}\right)^2\Pi^i_{ij}\left(\Pi^i_{ii}\right)^2 - \left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ij}\right)^3 - \hat{C}^i_{p^j\Delta}\left(\Pi^i_{ii}\right)^3\hat{C}^i_{p^i\Delta} + \hat{C}^i_{p^j\Delta}\Pi^i_{ii}\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ij}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2} \\
&= \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{\Pi^i_{ij}}{\Pi^i_{ii}} - \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}} + \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^2\right] \\
&= \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right] < 0
\end{aligned} \tag{139}
$$

$$
\begin{aligned}
\bar{\pi}^2_{\Delta^2\Delta^1} &= \left(\hat{R}^2_1 - \hat{C}^2_{p^1}\right)\bar{\psi}^1_{\Delta^2\Delta^1} + \bar{\psi}^1_{\Delta^2}\left(\frac{\mathrm{d}\hat{R}^2_1(p^1,p^2)}{\mathrm{d}\Delta^1} - \frac{\mathrm{d}\hat{C}^2_{p^1}(p^1,p^2,\Delta^2)}{\mathrm{d}\Delta^1}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{\Delta^1} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{\Delta^1} \\
&= \bar{\psi}^1_{\Delta^2}\left(\hat{R}^2_{21}\bar{\psi}^2_{\Delta^1}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{\Delta^1} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{\Delta^1} \\
&= \bar{\pi}^1_{\Delta^1\Delta^2}
\end{aligned} \tag{140}
$$

$$
\begin{aligned}
\bar{\pi}^1_{\Delta^1 s^1} &= \left(\hat{R}^1_2 - \hat{C}^1_{p^2} + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right)\bar{\psi}^2_{\Delta^1 s^1} + \bar{\psi}^2_{\Delta^1}\left(\frac{\mathrm{d}\hat{R}^1_2(p^1,p^2)}{\mathrm{d}s^1} - \frac{\mathrm{d}\hat{C}^1_{p^2}(p^1,p^2,\Delta^1)}{\mathrm{d}s^1} + \left(\frac{\partial x^1}{\partial p^2}\right)\right) \\
&\quad -\hat{C}^1_{p^1\Delta}\bar{\psi}^1_{s^1} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{s^1} \\
&= \bar{\psi}^2_{\Delta^1}\left(\hat{R}^1_{12}\bar{\psi}^1_{s^1} + \left(\frac{\partial x^1}{\partial p^2}\right)\right) - \hat{C}^1_{p^1\Delta}\bar{\psi}^1_{s^1} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{s^1} \\
&= \frac{-\Pi^2_{21}\hat{C}^1_{p^1\Delta}}{\Pi^2_{22}\Pi^1_{11} - \Pi^1_{21}\Pi^2_{21}}\left(-\frac{\hat{R}^1_{12}\Pi^2_{22}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} + \left(\frac{\partial x^1}{\partial p^2}\right)\right) + \hat{C}^1_{p^1\Delta}\frac{\Pi^2_{22}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} - \hat{C}^1_{p^2\Delta}\frac{\Pi^2_{12}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\frac{\frac{\partial x^1}{\partial p^2}}{\frac{\partial x^1}{\partial p^1}} + \frac{\frac{\partial x^1}{\partial p^2}}{\frac{\partial x^1}{\partial p^1}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^3 + 1 - \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\frac{\Pi^i_{ij}}{\Pi^i_{ii}} + \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^3\right] \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{1}{2}\gamma^2 + \frac{1}{8}\gamma^4 + 1 - \frac{1}{2}\gamma^2 + \frac{1}{8}\gamma^4\right] \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\gamma^2 + \frac{1}{4}\gamma^4 + 1\right] > 0
\end{aligned} \tag{141}
$$

$$
\begin{aligned}
\bar{\pi}^2_{\Delta^2 s^1} &= \left(\hat{R}^2_1 - \hat{C}^2_{p^1}\right)\bar{\psi}^1_{\Delta^2 s^1} + \bar{\psi}^1_{\Delta^2}\left(\frac{\mathrm{d}\hat{R}^2_1(p^1,p^2)}{\mathrm{d}s^1} - \frac{\mathrm{d}\hat{C}^2_{p^1}(p^1,p^2,\Delta^2)}{\mathrm{d}s^1}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{s^1} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{s^1} \\
&= \bar{\psi}^1_{\Delta^2}\hat{R}^2_{21}(p^1,p^2)\bar{\psi}^2_{s^1} - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{s^1} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{s^1} \\
&= \frac{-\Pi^1_{12}\hat{C}^2_{p^2\Delta}}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}}\left(\hat{R}^2_{21}\frac{\Pi^2_{12}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}}\right) - \hat{C}^2_{p^2\Delta}\frac{\Pi^2_{12}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} - \hat{C}^2_{p^1\Delta}\frac{-\Pi^2_{22}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} \\
&= \frac{-\Pi^i_{ij}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2}\left(\frac{\left(\Pi^i_{ij}\right)^2\left(\frac{\partial x^1}{\partial p^1}\right)}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2}\right) - \hat{C}^i_{p^i\Delta}\frac{\Pi^i_{ij}\left(\frac{\partial x^1}{\partial p^1}\right)}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} - \hat{C}^i_{p^j\Delta}\frac{-\Pi^i_{ii}\left(\frac{\partial x^1}{\partial p^1}\right)}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} \\
&= \left(\frac{\partial x^1}{\partial p^1}\right)\frac{-\left(\Pi^i_{ij}\right)^3\hat{C}^i_{p^i\Delta} - \hat{C}^i_{p^i\Delta}\Pi^i_{ij}\left(\Pi^i_{ii}\right)^2 + \hat{C}^i_{p^i\Delta}\left(\Pi^i_{ij}\right)^3 + \hat{C}^i_{p^j\Delta}\left(\Pi^i_{ii}\right)^3 - \hat{C}^i_{p^j\Delta}\Pi^i_{ii}\left(\Pi^i_{ij}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2} \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{\Pi^i_{ij}}{\Pi^i_{ii}} + \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}} - \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^2\right] \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{1}{2}\gamma + \frac{1}{4}\gamma^3\right] < 0
\end{aligned} \tag{142}
$$

The second order condition (75) means that $\bar{\pi}^i_{\Delta^i\Delta^i} < 0$, whereas the stability condition (76) implies that $\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 > \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2$. All these mean:

$$\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} = \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^2_{\Delta^2 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} > 0 \tag{143}$$

and

$$\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} = \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^2_{\Delta^2 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^1_{\Delta^1 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} < 0 \tag{144}$$

Which is the statement of the proposition.

From (137), (139), (141) and (142) we can also derive the following relationships, to be used later:

$$\bar{\pi}^1_{\Delta^1 s^1}\left(-\frac{\hat{C}^i_{p^i\Delta}}{\frac{\partial x^1}{\partial p^1}}\right) - \phi^1_{11} = \theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11} = \bar{\pi}^i_{\Delta^i\Delta^i} \tag{145}$$

$$\bar{\pi}^2_{\Delta^2 s^1}\left(-\frac{\hat{C}^i_{p^i\Delta}}{\frac{\partial x^1}{\partial p^1}}\right) = \theta\bar{\pi}^2_{\Delta^2 s^1} = \bar{\pi}^i_{\Delta^i\Delta^j} \tag{146}$$

$$\frac{\bar{\pi}^1_{\Delta^1 s^1}}{\bar{\pi}^i_{\Delta^i\Delta^j}} = \frac{1}{\theta}\frac{\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right]}{\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right]} \tag{147}$$

$$\bar{\pi}^2_{\Delta^2 s^1}\theta\frac{\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right]}{\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right]} - \phi^1_{11} = \bar{\pi}^i_{\Delta^i\Delta^i} \tag{148}$$

■

D Proof of Corollary 5

Note, first, that from (141) and (142), $\left|\bar{\pi}^1_{\Delta^1 s^1}\right| > \left|\bar{\pi}^2_{\Delta^2 s^1}\right|$ for γ between 0 and 1. Also, from (143) and (144):

$$\begin{aligned}
\left|\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}\right| - \left|\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}\right| &= \frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} \\
&= \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^2_{\Delta^2 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} + \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^2_{\Delta^2 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^1_{\Delta^1 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\
&= \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\left(\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^2_{\Delta^2 s^1}\right) + \bar{\pi}^i_{\Delta^i\Delta^j}\left(\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^2_{\Delta^2 s^1}\right)}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\
&= \frac{-\left(\bar{\pi}^i_{\Delta^i\Delta^i} - \bar{\pi}^i_{\Delta^i\Delta^j}\right)\left(\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^2_{\Delta^2 s^1}\right)}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\
&= -\frac{\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^2_{\Delta^2 s^1}}{\bar{\pi}^i_{\Delta^i\Delta^i} + \bar{\pi}^i_{\Delta^i\Delta^j}} > 0 \qquad (149)
\end{aligned}$$

where the inequality comes from the denominator being negative ((75) and proposition 3), $\bar{\pi}^1_{\Delta^1 s^1} > 0$ by (141) and $\left|\bar{\pi}^1_{\Delta^1 s^1}\right| > \left|\bar{\pi}^2_{\Delta^2 s^1}\right|$. ■

E Proof of Lemma 6

Before proving the statement of the lemma, we need to derive the restrictions on ϕ^i_{11} implied by the second order condition (75) and the stability condition (76).

From the definition of $\bar{\pi}^i_{\Delta^i\Delta^i}$ in (137), in order to satisfy the second order condition $\bar{\pi}^i_{\Delta^i\Delta^i} < 0$ we need to ensure

$$\phi^i_{11} > \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right] = \theta\bar{\pi}^1_{\Delta^1 s^1} > 0 \qquad (150)$$

where $\theta = -\frac{\hat{C}^i_{p^i\Delta}}{\frac{\partial x^i}{\partial p^i}} = -\frac{\partial^2 C^i(x^i,\Delta^i)}{\partial\Delta^i\partial x^i}$ measures how fast marginal costs are reduced per unit of R&D.

On the other hand, the stability condition in (76) translates into:

$$\begin{aligned}
\frac{\bar{\pi}^i_{\Delta_i\Delta_i}}{\pi^i_{\Delta_i\Delta_j}} &= \frac{\frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2-\left(\Pi^i_{ij}\right)^2\right)^2}\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right] - \phi^1_{11}}{\frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2-\left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{1}{2}\gamma + \gamma - \frac{1}{4}\gamma^3\right]} > 1 \\
&= \frac{\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right]}{\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right]} - \frac{\phi^i_{11}}{\pi^i_{\Delta_i\Delta_j}} > 1 \qquad (151)
\end{aligned}$$

using (147):

$$\begin{aligned}
\phi^i_{11} + \pi^i_{\Delta_i\Delta_j} &> \pi^i_{\Delta_i\Delta_j}\frac{\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right]}{\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right]} \\
\phi^i_{11} &> \theta\bar{\pi}^1_{\Delta^1 s^1} - \pi^i_{\Delta_i\Delta_j} > 0 \qquad (152)
\end{aligned}$$

Since $\pi^i_{\Delta_i\Delta_j} < 0$, then only (152) is binding..

From the definition of $\frac{\mathrm{d}\Delta 1}{\mathrm{d}s^1}$ and $\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}$ (143), (144) and the identities (148), (146), (145) and (147) we have

$$\begin{aligned}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} &= \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^2_{\Delta^2 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\ &= \frac{\left(-\theta\bar{\pi}^1_{\Delta^1 s^1} + \phi^1_{11}\right)\bar{\pi}^1_{\Delta^1 s^1} + \frac{\left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2}{\theta}}{\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\ &= \frac{1}{\theta}\frac{\left(-\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 + \phi^1_{11}\theta\bar{\pi}^1_{\Delta^1 s^1} + \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)}{\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} > 0 \qquad (153)\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} &= \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^2_{\Delta^2 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^1_{\Delta^1 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\ &= \frac{\left(-\theta\bar{\pi}^1_{\Delta^1 s^1} + \phi^1_{11}\right)\frac{\bar{\pi}^i_{\Delta^i\Delta^j}}{\theta} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^1_{\Delta^1 s^1}}{\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\ &= \frac{1}{\theta}\frac{\phi^1_{11}\bar{\pi}^i_{\Delta^i\Delta^j}}{\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} < 0 \qquad (154)\end{aligned}$$

Notice that all the terms in the expressions above do not depend on ϕ^1_{11} except, of course ϕ^1_{11}. Taking the derivative with respect to ϕ^1_{11}

$$\begin{aligned}\frac{\partial\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}}{\partial\phi^1_{11}} &= \frac{1}{\theta}\frac{\theta\bar{\pi}^1_{\Delta^1 s^1}\left[\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right] + 2\left(-\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 + \phi^1_{11}\theta\bar{\pi}^1_{\Delta^1 s^1} + \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= \frac{1}{\theta}\frac{-\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^3 + \theta\bar{\pi}^1_{\Delta^1 s^1}\left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2 + 2\phi^1_{11}\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 - \left(\phi^1_{11}\right)^2\theta\bar{\pi}^1_{\Delta^1 s^1} - 2\phi^1_{11}\left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= -\frac{1}{\theta}\frac{\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 + \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\left(2\phi^1_{11} - \theta\bar{\pi}^1_{\Delta^1 s^1}\right)}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} < 0 \qquad (155)\end{aligned}$$

$$\begin{aligned}\frac{\partial\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}}{\partial\phi^1_{11}} &= \frac{1}{\theta}\frac{\bar{\pi}^i_{\Delta^i\Delta^j}\left[\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right] + 2\phi^1_{11}\bar{\pi}^i_{\Delta^i\Delta^j}\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= \frac{1}{\theta}\frac{\bar{\pi}^i_{\Delta^i\Delta^j}\left[\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 - 2\theta\phi^1_{11}\bar{\pi}^1_{\Delta^1 s^1} + \left(\phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right] + 2\phi^1_{11}\bar{\pi}^i_{\Delta^i\Delta^j}\theta\bar{\pi}^1_{\Delta^1 s^1} - 2\phi^1_{11}\bar{\pi}^i_{\Delta^i\Delta^j}\phi^1_{11}}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= \frac{1}{\theta}\frac{\bar{\pi}^i_{\Delta^i\Delta^j}\left[\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 - \left(\phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right]}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= -\frac{1}{\theta}\frac{\bar{\pi}^i_{\Delta^i\Delta^j}\left[\left(\phi^1_{11}\right)^2 - \left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \bar{\pi}^i_{\Delta^i\Delta^j}\right)\left(\theta\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\right)\right]}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &> 0 \qquad (156)\end{aligned}$$

where the inequalities are derived using (152). Since $\frac{d\Delta^1}{ds^1} > 0$ and $\frac{d\Delta^2}{ds^1} < 0$, the statement of the proposition follows ■

F Proof of Proposition 7

Rewrite the optimal subsidy as

$$s^{1*} = m^1 \left(\frac{\partial x^1}{\partial p^2}\right) \frac{\bar{\psi}^2_{\Delta^2}\frac{d\Delta^2}{ds^1} + \bar{\psi}^2_{s^1}}{\frac{\partial x^1}{\partial p^1}\frac{d\bar{\psi}^1}{ds^1} + \frac{\partial x^1}{\partial p^2}\frac{d\bar{\psi}^2}{ds^1}} \tag{157}$$

where $m^1 = p^1 - \frac{\partial C^1}{\partial x^1} + s^1 > 0$ is the gross benefit per unit sold, including the output subsidy. Of course, m^1 has to be positive (otherwise firm 1 would have negative profits).

Turn now to the sign of the denominator in (157). It is positive since

$$\begin{aligned}
\frac{\partial x^1}{\partial p^1}\frac{d\bar{\psi}^1}{ds^1} + \frac{\partial x^1}{\partial p^2}\frac{d\bar{\psi}^2}{ds^1} &= \frac{\partial x^1}{\partial p^1}\left(\bar{\psi}^1_{\Delta^1}\frac{d\Delta^1}{ds^1} + \bar{\psi}^1_{\Delta^2}\frac{d\Delta^2}{ds^1} + \bar{\psi}^1_{s^1} + \left(\frac{\frac{\partial x^1}{\partial p^2}}{\frac{\partial x^1}{\partial p^1}}\right)\left(\bar{\psi}^2_{\Delta^1}\frac{d\Delta^1}{ds^1} + \bar{\psi}^2_{\Delta^2}\frac{d\Delta^2}{ds^1} + \bar{\psi}^2_{s^1}\right)\right) \\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^1}{ds^1} + \frac{\bar{\psi}^i_{\Delta^j}}{\bar{\psi}^i_{\Delta^i}}\frac{d\Delta^2}{ds^1} + \frac{\bar{\psi}^1_{s^1}}{\bar{\psi}^i_{\Delta^i}} - \gamma\left(\frac{\bar{\psi}^i_{\Delta^j}}{\bar{\psi}^i_{\Delta^i}}\frac{d\Delta^1}{ds^1} + \frac{d\Delta^2}{ds^1} + \frac{\bar{\psi}^2_{s^1}}{\bar{\psi}^i_{\Delta^i}}\right)\right) \\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^1}{ds^1} - \frac{\Pi^i_{ij}}{\Pi^i_{ii}}\frac{d\Delta^2}{ds^1} - \frac{\frac{\partial x^i}{\partial p^i}}{\hat{C}^i_{p^i\Delta}} - \gamma\left(-\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\frac{d\Delta^1}{ds^1} + \frac{d\Delta^2}{ds^1} - \frac{\bar{\psi}^1_{s^1}}{\bar{\psi}^i_{\Delta^i}}\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)\right) \\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^1}{ds^1} - \frac{\frac{\partial x^i}{\partial p^j}}{2\frac{\partial x^i}{\partial p^i}}\frac{d\Delta^2}{ds^1} + \frac{1}{\theta} - \gamma\left(-\frac{\frac{\partial x^i}{\partial p^j}}{2\frac{\partial x^i}{\partial p^i}}\frac{d\Delta^1}{ds^1} + \frac{d\Delta^2}{ds^1} + \frac{\gamma}{2\theta}\right)\right) \\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^1}{ds^1} + \frac{\gamma}{2}\frac{d\Delta^2}{ds^1} + \frac{1}{\theta} - \gamma\left(\frac{\gamma}{2}\frac{d\Delta^1}{ds^1} + \frac{d\Delta^2}{ds^1} + \frac{\gamma}{2\theta}\right)\right) \\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^1}{ds^1}\left(1 - \frac{\gamma^2}{2}\right) - \frac{\gamma}{2}\frac{d\Delta^2}{ds^1} + \frac{1}{\theta}\left(1 - \frac{\gamma^2}{2}\right)\right) > 0
\end{aligned} \tag{158}$$

where the inequality comes from $\frac{d\Delta^1}{ds^1} > 0 > \frac{d\Delta^2}{ds^1}$ (Proposition 4) and $\left(1 - \frac{\gamma^2}{2}\right) > \frac{\gamma}{2} > 0$ for γ between zero and one.

Therefore the sign of s^{1*} is the same as the sign of $\bar{\psi}^2_{\Delta^2}\frac{d\Delta^2}{ds^1} + \bar{\psi}^2_{s^1}$. Recall that

$$\begin{aligned}
\bar{\psi}^2_{\Delta^2}\frac{d\Delta^2}{ds^1} + \bar{\psi}^2_{s^1} &= \bar{\psi}^2_{\Delta^2}\left(\frac{d\Delta^2}{ds^1} + \frac{\bar{\psi}^2_{s^1}}{\bar{\psi}^2_{\Delta^2}}\right) \\
&= \bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^2}{ds^1} + \frac{\bar{\psi}^2_{s^1}}{\bar{\psi}^i_{\Delta^i}}\right) \\
&= \bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^2}{ds^1} - \frac{\bar{\psi}^1_{s^1}}{\bar{\psi}^i_{\Delta^i}}\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right) \\
&= \bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^2}{ds^1} + \frac{\gamma}{2\theta}\right)
\end{aligned} \tag{159}$$

Since $\bar{\psi}^i_{\Delta^i}$ is independent of ϕ^i_{11}, then a change in ϕ^i_{11} only affects $\frac{d\Delta^2}{ds^1}$ (i.e. the R&D stage effect). From lemma 6, $\frac{\partial \frac{d\Delta^2}{ds^1}}{\partial \phi^1_{11}} > 0$ and so $\bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^2}{ds^1} + \frac{\gamma}{2\theta}\right)$ in decreasing on ϕ^i_{11}. Left to show is that $\bar{\psi}^i_{\Delta^i}\left(\frac{d\Delta^2}{ds^1} + \frac{\gamma}{2\theta}\right)$ can be positive or negative for permissible values of ϕ^i_{11}.

From (154) we have

$$\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} = \frac{1}{\theta}\frac{\phi_{11}^1 \bar{\pi}^i_{\Delta^i\Delta^j}}{\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} < 0 \tag{160}$$

And so $\lim_{\phi^1_{11}\nearrow\infty} \bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \frac{\gamma}{2\theta}\right) = \bar{\psi}^i_{\Delta^i}\frac{\gamma}{2\theta} < 0$ and $\lim_{\phi^1_{11}\searrow\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \pi^i_{\Delta_i\Delta_j}\right)} \bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \frac{\gamma}{2\theta}\right) = +\infty$. By continuity, the claim of the proposition follows. ■

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