Econometric application of linear programming: a model of Russian large-scale farm

(the case of the Moscow Region)

Nikolai M. Svetlov[1]

Linear programming model and general reciprocity theorem in mathematical programming are used to approach utility functions of six large-scale Russian (the Moscow Region) case farms representing different production patterns. Technological coefficients of linear programmes are defined by means of linear regression. The data over 311 farms operating in the Moscow Region in 1999 are used. The utility functions include depreciation, wages and social costs. These attributes are about as desirable for the farms as profit. Milk production is associated with hidden utility amounting to a quarter of total utility. Vegetables market imperfections result from high price elasticity with respect to supply (–0.46). The scarcity of operating capital severely hampers agricultural production.

Keywords: linear programming, general reciprocity theorem, Leontieff technologies, farm behaviour, Russian agriculture, case farm, utility function, operating capital.

1. Introduction

The aim of this paper is to approach the utility function properties of case large-scale Russian farms located in the Moscow Region. Each case farm is assumed:

- to have average on the whole data set technological capabilities;
- to be specialised in accordance to one of the production patterns that are most widespread in this region.

There are three research hypotheses. The first one, which is of a methodological nature, is that a linear programme is a sufficient tool of micro-economic modelling, allowing a researcher to study the decision making process, utility and resources efficiency at the level of a firm. The other two hypotheses relate to the farms:

- The utility of the large-scale farms in the Moscow Region includes depreciation, wages and social costs;
- The essential limiting factor of agricultural production on these farms is a lack of operating capital (defined as a monetary representation of turnover means).

The research tasks are as follows:

- To develop the method of microeconomic modelling that deals with heterogeneous data, allows for complementary resources and is insensitive to correlated exogenous variables;
- To suggest the approach based on the general reciprocity theorem in mathematical programming, allowing a researcher to study the properties of an unobservable utility function;
- To develop and test an empirical model of a large-scale farm located in the Moscow Region;
- To estimate the utility function parameters for the case large-scale farms of the Moscow Region;
- To identify the policy guidelines facilitating stable economic growth of agricultural production in this region based on the research findings.

The relevance of the study is both theoretical and practical. First, it suggests an econometric technique that is very promising for any research applications concerning firm behaviour models. Second, it provides relevant information for policy makers and their advisers. That is expected to have a positive impact on the regional agricultural economy, which remains in a poor and uncertain state.

The importance of the study of the utility function of agricultural companies in transitional economics is justified by the results of Serova (1998), Bezemer (1999), Uzun (1999), Macours & Swinnen (1999). These economists agree that, under the specific conditions of transition, farm utility is likely to differ from profit maximising. Consequently, the contradictions between the true farm utility and agrarian policies could hamper agricultural production. To address this issue in details, one should approach the utility function.

To introduce readers to the problem I would like to address the following questions:

1) Why study large-scale farms?
2) Why in the Moscow Region?
3) Why use linear programming (LP)?

The answer to the first question is that, although Russian large-scale farms, also known as firm farms, agricultural corporations and collective farms[2], no longer produce the greatest part of agricultural production, they remain the leading suppliers of marketable agricultural production[3]. Another important aspect is that, as it is shown in Svetlov (2001a), other agricultural producers, namely family farms and household (subsistence) plots, widely use the resources belonging (at least formally) to the large-scale farms. Hence, the production by family farms and from household plots is highly dependent on the situation of the neighbouring large-scale farms.

The Moscow Region is chosen for this study because of data availability and of the author's personal experience in studying this region. In addition, this region is very interesting from a theoretical viewpoint, because it is characterised by a relatively intensive and diversified production and highly heterogeneous set of farms.

Commonly the utility functions are approached by means of either questioning respondents about their preferences (Viscusi & Evans, 1998) or analysing observed behaviour (Hazell & Scandizzo, 1975; Amador, 1998; Matzkin, 2000). The latter approach is advantageous as the reported preferences can differ from actual preferences. The idea to try LP to approach farm's utility function has its root in the analysis of similar studies based on the traditional regression technique. This analysis has shown that the results of the regression analysis, although being sometimes acceptable, greatly suffer from properties of firm models. First, regression is restricted in dealing with complementary resources, while LP is not. Second, a researcher performing regression analysis faces restrictions in choice of factors having to exclude those of them that correlate to others. Finally, the results of regression analysis greatly depend on the chosen functional form (see e.g. Oude Lansink & Thijssen, 1998). For this reason, one can expect that the LP microeconomic model would be more precise and informative than the model based on profit functions estimation. The cost of these advantages is that a researcher should explicitly specify a case farm for which the estimated utility function is valid.

This paper consists of seven sections, acknowledgements, references and appendices. The next section (the second) presents the theoretical framework used in this study. The empirical model is specified in Section 3. Data and estimation procedure are described in Section 4. In this paper special attention is paid to testing the empirical model. The testing issues are presented in Section 5. Section 6 presents the results the case farms analysis. In the last section the reader will find conclusions, discussion and outlook of this model applications.

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[1] Dr. Nikolai M. Svetlov is an associate professor at the department of Economic Cybernetics at Moscow Timiryazev Agricultural Academy. E-mail: *svetlov@mnts.msk.su*, Internet: *http://svetlov.value.da.ru*.

[2] The last term is the least precise, as only a small part of them consists of the collective farms indeed, meanwhile a majority is the joint-stock companies.

[3] Large-scale farms produce 40.3% of agricultural production while the household plots produce 57.2%. But the share of marketable production on the former farms varies from 41.1 to 100% depending on the kind of production, on the latter it is in the range 4.1-23.4%. The figures are based on data of Goskomstat, Russian federal statistical agency.

2. Theoretical framework

A theoretical background of this study consists of the following results:

- A theoretical model of a farm maximising its utility under constrained resources, some of which are available on the market and some are not available;
- A concept of von Neumann and Leontieff technologies;
- The general reciprocity theorem in mathematical programming;
- Lagrangean function and Lagrangean multipliers for vector programming problem (also known as multi-objective programming problem);
- An approach to estimating unobserved parameters of a utility function based on the theorem mentioned above.

This study originates in the theoretical model of a firm that makes a decision in an environment of an imperfect market. We assume that some goods can be obtained (traded) at the market unrestrictedly at their equilibrium price, which is assumed to be independent of the volume of a good bought (sold) by the individual firm. Other goods are assumed to be totally absent from the market. An appropriate model has the following form:

$$\begin{aligned}&\max f(\mathbf{x},\mathbf{y},\mathbf{v},\mathbf{w})\\ &\mathbf{q}(\mathbf{x}) \leqq \mathbf{q}^*\\ &\mathbf{r}(\mathbf{x}) \leqq \mathbf{y}\\ &\mathbf{x} \geqq \mathbf{0}, \mathbf{y} \geqq \mathbf{0}, \mathbf{v} \geqq \mathbf{0}, \mathbf{w} \geqq \mathbf{0}\end{aligned} \qquad (1)$$

where **x** is an outputs vector, **y** is a marketable (variable) inputs vectors, **q*** is a vector of fixed inputs, **v** is a vector of output prices and **w** is a vector of input prices; **q**(**x**) is a vector function of fixed inputs consumption on the outputs vector, **r**(**x**) is a vector function of marketable inputs consumption on the outputs vector; *f*(**x**,**y**,**v**,**w**) is a scalar utility function on outputs, variable inputs and prices vectors.

Von Neumann technology (introduced in von Neumann, 1937) is a technology having the following properties:

- It can be run at any non-negative intensity;
- Its consumption of inputs is proportional to its intensity.

Leontieff technology (first introduced in Dmitriev, 1904) is a special case of von Neumann technology having the following properties in addition to those mentioned above:

- It produces only one kind of output;
- No one other technology produces the same output.

If the firm represented with (1) runs only Leontieff technologies then (1) can be rewritten as

$$\begin{aligned}&\max f(\mathbf{x},\mathbf{y},\mathbf{v},\mathbf{w})\\ &\mathbf{Ax} \leqq \mathbf{q}^*\\ &\mathbf{Bx} \leqq \mathbf{y}\\ &\mathbf{x} \geqq \mathbf{0}, \mathbf{y} \geqq \mathbf{0}, \mathbf{v} \geqq \mathbf{0}, \mathbf{w} \geqq \mathbf{0}\end{aligned} \qquad (2)$$

where **A** and **B** are input matrices for fixed and variable inputs respectively.

The general reciprocity theorem (Lourier, 1966; Aganbegian & Bagrinovsky, 1967) states that, given a general mathematical programming problem

$$\begin{aligned}&\max f(\mathbf{x})\\ &\mathbf{q}(\mathbf{x}) \leqq \mathbf{q}^*\\ &q_0(\mathbf{x}) \le q_0^*\\ &\mathbf{x} \geqq \mathbf{0}\end{aligned} \qquad (3)$$

where **q*** is a constant vector, q_0^* is a constant, all the components of the vector function **q**(**x**), as well as $\mathbf{q}_0(\mathbf{x})$, are differentiable functions and, given any optimal solution $\mathbf{x}^* \in X$, where *X* is a set of optimal solutions, the constraint $q_0(\mathbf{x}) \le q_0^*$ is bound, then the mathematical programming problem

$$\begin{aligned}&\min q_0(\mathbf{x})\\ &\mathbf{q}(\mathbf{x}) \leqq \mathbf{q}^*\\ &f(\mathbf{x}) \ge z\\ &\mathbf{x} \geqq \mathbf{0}\end{aligned} \qquad (4)$$

where $z = f(\mathbf{x}^*)$, $\mathbf{x}^* \in X$, has

- the same set of optimal solutions *X*;
- the set of optimal Lagrangean multipliers vectors that is defined as $P_0 = \{\mathbf{p}_0 \mid \mathbf{p}_0 = (p_i / p_0 \mid 1 / p_0)'\}$, where $(\mathbf{p} = (p_i \mid p_0)) \in P$, *P* is a set of optimal Lagrangean multipliers vectors for (3);
- $q_0(\mathbf{x}^*) = q_0^*$ for each $\mathbf{x}^* \in X$.

The problem (4) is called *reciprocal* with respect to (3).

This very elegant result of Lourier, which unfortunately remains virtually unknown to economists outside Russia, allows us to introduce Lagrangean function and Lagrangean multipliers for vector programming problems (Svetlov, 2001).

Any Pareto optimal solution of the vector programming problem

$$\begin{aligned}&\max \mathbf{f}(\mathbf{x})\\ &\mathbf{q}(\mathbf{x}) \leqq \mathbf{q}^*\\ &\mathbf{x} \geqq \mathbf{0}\end{aligned}$$

is located at the Kuhn-Tucker point of the following Lagrangean function:

$$\langle \boldsymbol{\pi}, \mathbf{f}(\mathbf{x}) - \mathbf{z} \rangle - \langle \mathbf{p}, \mathbf{q}(\mathbf{x}) - \mathbf{q}^* \rangle, \qquad (5)$$

where **π** and **p** are the vectors of Lagrangean multipliers of objective functions and constraints respectively, **z** is any vector. If, given some **z**, the Kuhn-Tucker point (**x***,**π***,**p***) exists and is Pareto optimal, then **z** is a Pareto optimal value of **f**(**x**). The economic meaning of a Pareto optimal vector of objective function Lagrangean multipliers is that they express the interchange ratio of any two objectives that does not affect other objectives and resource usage.

If (**x***,**π***,**p***) is a Kuhn-Tucker point then (**x***,*k***π***,*k***p***), where *k* is any positive value, is also a Kuhn-Tucker point for (5). Hence, it is reasonable to somehow normalise vectors **π*** and **p***, for instance, assuming that the first components of **π*** are equal to unity[4].

[4] Vector **π*** is such that $\boldsymbol{\pi}^* > \mathbf{0}$.

The general reciprocity theorem in mathematical programming suggests that, given the Pareto optimum **x***, the normalised vectors **π*** and **p*** can be found by means of the ordinary mathematical programming problem

$$\max f_1(\mathbf{x})$$
$$\mathbf{f}_1(\mathbf{x}) \geqq \mathbf{z}_1$$
$$\mathbf{q}(\mathbf{x}) \leqq \mathbf{q}^*$$
$$\mathbf{x} \geqq \mathbf{0}$$

where $f_1(\mathbf{x})$ is the first component of $\mathbf{f}(\mathbf{x})$, $\mathbf{f}_1(\mathbf{x})$ is obtained from $\mathbf{f}(\mathbf{x})$ by means of exclusion of the first component, $\mathbf{z}_1$ is obtained from **z** in the same way. Then **p*** is equal to the (ordinary) Lagrangean vector of $\mathbf{q}(\mathbf{x})$, the first component of **π*** equals to unity, other components are equal to the corresponding components (starting from the first one) of Lagrangean vector of $\mathbf{f}_1(\mathbf{x})$.

This property of **π*** can be used for recovering local properties of a utility function of an economic agent from existing data describing its behaviour. Assume that the set of utility attributes are known, the *i*th utility attribute value is expressed by the function $f_i(\mathbf{x})$, $\mathbf{f}(\mathbf{x}) = (f_i(\mathbf{x}))$ and the true utility function that is maximised by the economic agent is $u(\mathbf{f}(\mathbf{x}))$. In a vicinity of any **x** this function can be approximated as $\langle \mathbf{a}(\mathbf{x}), \mathbf{f}(\mathbf{x}) \rangle$. If an observed state **x*** of the economic agent is assumed to be optimal with respect to $\langle \mathbf{a}(\mathbf{x}), \mathbf{f}(\mathbf{x}) \rangle$, then this state can be expressed as one of Pareto optima with respect to $\mathbf{f}(\mathbf{x})$, namely that one which yields the Lagrangean vector $\boldsymbol{\pi}^* = \mathbf{a}(\mathbf{x}^*)$. If it is possible to observe an actual value of all or all except one components of $\mathbf{f}(\mathbf{x})$, we can find $\mathbf{a}(\mathbf{x}^*)$ by means of maximising one component of $\mathbf{f}(\mathbf{x})$ (for an unobserved utility attribute, the component of $\mathbf{f}(\mathbf{x})$ should be maximised that corresponds to this attribute) subject to other components being not less than their observed values.

Applied to the problem (2), this approach yields the following formulation of a theoretical firm behaviour model:

$$\begin{aligned} &\max \mathbf{f}(\mathbf{x},\mathbf{y},\mathbf{v},\mathbf{w}) \\ &\mathbf{Ax} \leqq \mathbf{q}^* \\ &\mathbf{Bx} \leqq \mathbf{y} \\ &\mathbf{x} \geqq \mathbf{0}, \mathbf{y} \geqq \mathbf{0}, \mathbf{v} \geqq \mathbf{0}, \mathbf{w} \geqq \mathbf{0} \end{aligned} \qquad (6)$$

where $\mathbf{f}(\mathbf{x},\mathbf{y},\mathbf{v},\mathbf{w})$ is a vector of known utility attributes of the modelled firm.

3. Empirical model

To formulate an empirical model, I have to identify the components of the Moscow Region large-scale farm's utility function.

It is clear that profit reported by the farms in the state registry is not the only utility attribute. From (2) it follows that, if the farm's utility is nothing but profit, then the profit should be positive or, at least, zero. In the latter case no one technology should be being used. However, reported profits are commonly negative. Moreover, such poor profitability cannot be explained by an occasional failure due to either unfavourable circumstances or misinformation in decision making: many farms keep indicating negative profit in their business plans for several years, thus expressing their intent to prefer losses.

The present literature on this problem suggests that farm's utility includes the following attributes (Uzun, 1999):

- Wages;
- Social costs;

- Depreciation.

The relationship of depreciation to the needs to be clarified. Many Russian farms have inherited a surplus of fixed assets from the collective and state farms. Under the production decline they do not need to renovate those of them that are not actually used in the production, although the accounting regulations require them to keep reporting the depreciation. So, farms can use a large share of the depreciation like they would use their profit, which makes the utility of depreciation positive.

However, the previous versions of the empirical model have shown that the above mentioned attributes do not explain the observed intensity of milk and other production. If it were the case, the solutions obtained indicate that milk production should not take place on the large-scale farms of the studied region at all. Thus, I expand the farm's utility with two additional attributes:

- Amount of milk production;
- Amount of other production.

Empirical reasons to consider these attributes within the farm utility definition are also presented in Uzun (1999). In surveys conducted by RosAgroFond in 1997 and 1998 in the Nizhny Novgorod and Oryol Regions farm managers reported that one of the aims justifying their decisions was to save animal stock from being wholly slaughtered. However, it is very likely that this reasoning no longer holds. It is unlikely that a manager would love his cows so much that he would reject slaughtering even knowing for sure that they would bring only losses in the future. A more probable explanation, which is also supported by unbelievably low reported level of milk yields per dairy cow[5], is that the farms partially hide the milk yield and the corresponding benefits.

The set of fixed inputs includes:

- Hayfields and pastures as a source of feed;
- Arable land;
- Operating capital;
- Total production expenses;
- Buildings;
- Machinery;
- Amount of available labour (approximated as number of workers).

The market for agricultural land, including hayfields, pastures and arable land, is virtually missing in Russia. As it is shown in Svetlov (2001a), the underlying reason is the absence of agricultural land demand under the conditions of agricultural production decline. The price of land is so low that it even does not motivate trading unused land[6].

The studies of Swinnen & Gow (1997), Bezlepkina & Svetlov (2000) indicate a virtual absence of a market for short-term agricultural capital. The access to bank credit is very constrained. The terms of loans are seldom acceptable to borrowers. Under these conditions farms, as a rule, must rely on their own resources to finance production costs[7]. This justifies representing operating capital and total production expenses as fixed inputs.

Buildings and machinery are marketable, but at high transaction costs. Moreover, the farms do not have enough money to buy them when they suffer from their shortage. Under these conditions farms do not adjust the capacity of these resources to match current production demands. Instead, they mostly deal with them as with fixed inputs.

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[5] In 1999 the average milk yield per dairy cow in Russian farms of all types was 2432 kg (Goskomstat).

[6] Many farms use the existing gaps in legislation to sell the land to non-agricultural users at quite attractive prices, but such opportunities are occasional. They do not result in formation of a truly functioning land market.

[7] Surprisingly, that does not hold for long-term credits. Although they are sometimes distributed on the base of non-market criteria, the access of Russian large-scale agricultural farms to them is easier than to short-term credits, despite the demand for long-term financing is not as urgent. This issue will be addressed below in more details.

Wages in the agricultural sector are very low. They do not motivate people to join agricultural production. On the other hand, those people who reside in the rural area and lack money to move are virtually tied to the farm that employs them. Another factor tying employees to the farms is household plots that have accumulated a great amount of past labour and serve as a subsistence source. That is why the labour is not a freely tradable resource in rural area. Hence, the model represents it as a fixed input.

The only variable input in this model is fodder. Although fuel and fertilisers are also variable inputs, they are omitted due to unavailability of the corresponding data. That restricts the analytical value of this study and the precision of the model.

In this study we consider seven Leontieff technologies producing:

- Milk;
- Meat;
- Cereals;
- Fodder;
- Potatoes;
- Vegetables;
- Other production.

In case of milk and meat, the assumption of Leoniteff nature of the technologies is harmful, because dairy milk production cannot strictly be split from meat production. However, as the method of estimating matrices **A** and **B** in (6) that is used in this study is only applicable to the Leontieff technologies, the technologies of milk and meat production are both assumed to be Leontieff technologies. This makes it necessary to check the milk/meat ratio in the empirical model results for consistency with farm practice.

The outputs of all technologies except fodder production are assumed to be marketable. The fodder production is used for the internal consumption. It substitutes for the variable input of purchased fodder.

In this model it is assumed that a farm that does not run some of the six technologies (except for fodder production) is not able to run them in a short-term period because of absence of either required means of production or workers' experience. In other words, if the farm did not use a technology in 1999 then the model imposes this technology to have zero intensity despite its efficiency.

Among the Pareto optima set the model chooses that one for which the profit is maximal subject to other utility attributes being no less than actually observed. Hence, all the analysis presented below holds for this Pareto optimum.

Considering the above formulated assumptions and having defined the components of the utility function, sets of fixed inputs, variable inputs, outputs and technologies, the following empirical model results from (6):

$$\begin{aligned}
&\max \mathbf{c}'_{\pi}\mathbf{x} + c_y y + c_d d - \mathbf{1}'\mathbf{b}_1 - \boldsymbol{\psi}'\mathbf{b}_2 \\
&\mathbf{C}\mathbf{x} + \mathbf{c}'_y y + \mathbf{c}'_d d + \mathbf{T}_1\mathbf{b}_1 + \mathbf{T}_2\mathbf{b}_2 \geqq \boldsymbol{\omega}^* \\
&\mathbf{A}\mathbf{x} + \mathbf{a}'_y y + \mathbf{a}'_d d \leqq \mathbf{q}^* \\
&\mathbf{x} \geqq \mathbf{0}, y \geq 0, d \geq 0, \mathbf{b}_1 \geqq \mathbf{0}, \mathbf{b}_2 \geqq \mathbf{0}
\end{aligned} \tag{7}$$

The variables are denoted as follows: **x** is a vector of six outputs, y is a variable input (purchased fodder), d is an internal fodder production intensity, $\mathbf{b}_1$ is a vector of deductions from profit in favour of other attributes of utility, $\mathbf{b}_2$ is a vector of incremental outputs. The latter is used only in case of considering volume of some output as a utility attribute in order to prevent model infeasibility and to allow for lower resource usage on some farms. The right hand side constants **ω*** and **q*** are the vectors of actual values of the utility attributes and of fixed input volumes. Parameters of the model: $\mathbf{c}_\pi$ is a vector of profits per unit of output; c_y is a cost of purchased fodder; c_d is a cost of internal fodder production; **ψ** is a vector of penalties for usage of incremental outputs; **C** is a matrix of contributions of each technology to each utility attribute; $\mathbf{c}_y$ is a vector of contributions of purchased fodder to each utility attribute; $\mathbf{c}_d$ is a vector of contributions of internal fodder production to each utility attribute; $\mathbf{T}_1$ is a matrix of transfer rates of profit into other utility attributes; $\mathbf{T}_2$ is a distribution matrix assigning incremental outputs to the corresponding utility attributes; **A** is an input matrix of the technologies producing outputs; $\mathbf{a}_y$ and $\mathbf{a}_d$ are vectors of input-output rates for fodder purchase and production respectively. Symbol **1** denotes a vector of ones.

4. Data and estimation

I used the year 1999 data of 407 large-scale farms of the Moscow Region. The data set includes 18 variables:

- Outputs (milk, meat, cereals, potatoes, vegetables, value of other production);
- Inputs (grassland and pastures area, arable area, total feed cost, operating capital at the beginning of the year, book value of buildings at the beginning of the year, book value of machinery at the beginning of the year, average yearly number of workers);
- Efficiency indicators (total costs, profit after taxation);
- Supplementary variables (depreciation, wages, farm social expenses).

The sources of the data are the regional agriculture and food department of the Moscow Region (operating capital, book value of buildings and machinery, depreciation, wages and social costs) and Goskomstat, a federal statistical agency of Russian Federation (others). The data set is supplemented with average prices of the five outputs (except other production) on these 407 farms for 1999. Other production has a monetary measurement, so, its price equals unity.

Of those 407 records, 6 were struck out as they contained obvious unrecoverable errors. The resulting set was used for estimating the model parameters and testing the model. It was found that in the data set there is a large proportion of farms (25%) that produce more than 25% of "other" output, which probably is of a non-agricultural nature or may reflect barter operations. Clearly a model that concentrates on the agricultural aspects of production may not be relevant to these farms. So, although the results of testing the model on this set were acceptable and interpretable, it was decided that these farms should be excluded from the data set for the purposes of this paper. This decision left me with a data set of 311 farms.

This set is still very heterogeneous as it contains farms that differ considerably in specialisation, size and production intensity. However, one of the research hypotheses was that the applied methodology deals well with heterogeneous sets. To test this hypothesis, the set of 311 farms was used without splitting into smaller less heterogeneous groups.

The characteristics of the data set are presented in Appendix 1.

The farms in the data set can be classified according to the patterns of their production. In total there are 15 patterns. The definition of 6 patterns spanning together more than 90% of farms is presented in Appendix 2. The analysis in Section 6 will be concentrated on these 6 most common patterns. The average volumes of fixed inputs, outputs and utility attributes for each of these production patterns are presented in Appendix 3.

Estimating parameters for Leontieff technologies is very simple. In general, if the data of technology intensities **g** and of fixed inputs/utility attributes **h** are given, one can estimate the matrix **Ω** of technological coefficients from $\mathbf{h} = \boldsymbol{\Omega}\mathbf{g}$. Each line $\boldsymbol{\omega}'_i$ of **Ω** is estimated separately from an equation $h_i = \boldsymbol{\omega}'_i\mathbf{g}$. This is a simple linear regression with an absolute term imposed to be zero.

In our case **g** includes the six technologies producing tradable outputs and **h** includes all fixed inputs but hayfields and pastures and all utility attributes but profit. Fodder production is excluded from **g** because, given our data set, it is not possible to split fodder into purchased and produced internally. Hayfields and pastures are not accounted as a separate resource because they substitute fodder. To allow for that, we have to calculate fodder output per unit of these lands and add it to other sources of fodder.

In this study **U** is assumed to be independent on the production pattern. The results of its estimation are presented in Appendix 4.

To deal with internal fodder production, we assume that this technology is similar to that of cereals production. As the internal fodder production should be represented in the model as a separate technology, it is necessary to correspondingly adjust the milk and meat production technologies, which should not use arable land in this case. The non-zero values of arable land used by unit of meat and milk production technologies indicate the amount of internally produced fodder, measured in hectares, used in these technologies. Hence, the adjusted technologies can be defined as $\boldsymbol{\omega}_1 - r_1\boldsymbol{\omega}_3$ for milk and $\boldsymbol{\omega}_2 - r_2\boldsymbol{\omega}_3$ for meat, where $\boldsymbol{\omega}_j$ is jth column of $\boldsymbol{\Omega}$ (first column relates to milk, second to meat, third to cereals) and factors r_1 and r_2 are chosen so that the resulting arable land usage in the adjusted technologies collapses to zero.

For each technology profit is defined as average price of the output minus estimated total costs plus estimated fodder costs. Correction for fodder costs is necessary, as they are accounted separately as the costs of purchased and internally produced fodder. This also holds for fodder production and purchase, providing that the output price is zero.

The completely specified technological data as they appear in the empirical model is presented in Appendix 5.

If some technology, being unprofitable, never enters an optimal solution, but is still run by many farms, then there is a strong reason to consider this technology as a utility attribute. Thus, I include milk production into the utility function despite the fact that the reason for utility of milk production cannot be explained in this study. However, I do not do so with other production when testing the model against actual farms data, because other production is very heterogeneous. The actual value of other production intensity represents different actual technologies on different farms, so it is not possible to compose something like a utility attribute out of such intensities. In the models of case farms it seems reasonable to fix the level of other production at its actual value, as in this case the value of other production indeed represents an other production having an average on the sample structure.

5. Testing

The empirical model is not intended to ensure that the mean of the modelled profit over the data set equals the corresponding mean of observed profit. Instead, it is expected that the modelled value should be higher than actually observed. The reasons are listed below.

- Some subset of farms may not behave in accordance to the utility specified by this model.
- Another source of a positive bias of profit is excluding other production, which is unprofitable, from the utility function.

Controversially, the fact that the model considers only a subset of actual inputs does not result in biased estimates of the technology intensities and profit. When estimating Leontieff technologies we assume each fixed input in each farm to be wholly used, even if in reality there are the reserves of some fixed inputs because of other restrictions. Thus, the effect of other restrictions is captured by higher technological demands for fixed inputs.

The modelled profit should be in better accordance to the actual profit than any of the production intensities. The reason is that the marginal effect of different activities does not differ very much, so small random difference between actual and estimated technological coefficients may result in switching activities that will have a very small effect on profit. The same is true for shadow prices. So, the analysis of shadow prices should consider that their values and even the set of scarce

resources might not be identified with sufficient certainty. To draw conclusions, one has to analyse a variety of solutions.

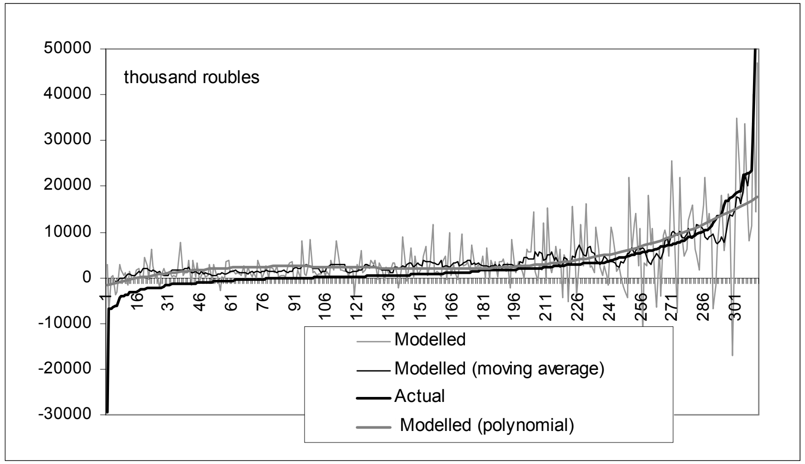
1. Modelled versus actual variables: correlation, average, variance

Variables	Correlation with actual values	Average		Variance	
		Modelled	Difference from actual	Modelled ($\times 10^6$)	% to actual
Profit, thousand roubles	0.622	4013	1266	46.7	87.6
Meat, kg×100	0.542	3813	2851	24.2	439.1
Cereals, kg×100	0.215	4416	2733	43.1	244.8
Potatoes, kg×100	0.506	2823	469	36.8	104.2

Table 1 contains the data facilitating evaluation of the model quality. This table does not include milk, vegetables and other production. The modelled milk production, which is treated as a utility attribute, is imposed to be equal to actual production. An upper bound of vegetables production is also imposed describing the amount of output at which the negative price deviation on sales volume compensates for the effect of incremental sales at the market that can be accessed by the given farm. The high correlation (0.998) between modelled and actual vegetable output is caused by the constraint aimed at quantifying vegetable market imperfections, which makes the modelled value to be no greater than the actual value. As for other production, its modelled output does not vary at all, remaining equal to zero.

Given the purpose of the model, the fact that the modelled and actual averages significantly differ is not really harmful, since the researcher who is interested in the modelled values can easily correct them for the difference. As for the values of shadow prices, it is not possible to calculate the differences between actual and modelled values. Thus, their values are unlikely to be interpretable unless we study them in the context of the specially defined case farms. However, the Boolean variable indicating whether the shadow price is zero or not is interpretable in the context of testing the model against actual farms.

Figure 1 facilitates comparison of modelled versus actual values of profit. It displays the series of farms in the data set arranged by actual profit. A bold black line represents actual values. Other three lines relate to modelled values: thin grey line displays the modelled values themselves, thin black line presents the moving average of the modelled values and the thick grey line indicates the result of 3rd power polynomial smoothing of the modelled values.



X axis value is a rank of farm on actual profit

Figure 1. Profit: modelled versus actual values

According to the diagram, the model overestimates the profit for the least profitable farms, fits the data of the farms having average profit very well and fits a little less well the data of farms having very high profit. A brief analysis has brought to light the following reasons of the observed imperfections:

- ♦ The model is not able to capture the occurrences of accidental losses that cause the value of profit to be extremely low;
- ♦ The utility of some least profitable farms cannot be represented as a linear combination of the utility attributes considered by this model;
- ♦ The most profitable farms make extensive use the technologies that differ greatly from average technologies;
- ♦ The higher the profit the smaller part of it results from sales of agricultural outputs, being replaced by the profit from financial and resale operations.

The data of Appendix 6 characterises the local properties of the farm-specific profit functions and shadow prices of the fixed inputs. The scarcity of *fodder* is most widely spread. However, only 56 farms of 311 (18.01%) can benefit from purchased fodder (that is indicated by unitary shadow price). Hence, the fodder price is too high to facilitate production growth in the regional agriculture. If the policy makers are interested in growth they may also be interested in subsidising the fodder price.

The second scarcest resource in terms of scarcity spread is *operating capital* and the third scarcest is the *sources of costs repay* that correspond to right hand side of total costs constraint. As the nature of these constraints is very similar in the sense that a monetary inflow softens them both, it is interesting to count the farms that have both constraints unbound. The model outlined 30 such farms, that is 9.65% of our data set. This is less than the share of farms that are not constrained in fodder, but, unlike the case of fodder, the value of shadow prices of both operating capital and source of costs repay is not constrained by these resources market price. So, the financial market does not regulate the scarcity of these resources. If they had access to short-term financial market, 163 farms could benefit from buying credit at 57.5% rate (an average of the year 1999 Central Bank

rate)[8]. Of them, 159 could relax the operating capital constraint and 4 could relax the expenses constraint. At 86.25% rate (that is 1.5 times higher that used above) the corresponding figures are 81, 78 and 3 (extremely high shadow prices caused by penalty values are excluded). There are 8 (7 and 1) farms that can even benefit from short-term credit at 115%, that is a doubled Central Bank rate. These figures suggest that the financial imperfections are an essential factor hampering agricultural production in this region. However, this aspect should be addressed in more details considering seasonality of agricultural production[9].

Only 98 farms (31.51%) face *machinery* shortages. If we couple them with those having non-zero shadow prices of *buildings*, we come only to 155 farms (49.84%) that can benefit from long-term financing at zero rate (assuming that the long-term financing is intended for investments in buildings and machinery). However, as all the non-zero shadow prices of buildings indicate an effect of a penalty for additional milk production, suggesting that the milk production technology on these farms is more efficient compared to average technology, it is not clear whether or not the buildings are indeed the scarce resource in these farms. In general, the non-zero economically grounded demand for long-term credit in the studied set of farms is less common than the demand for short-term financing.

Only 19.9% of farms can use their *land* wholly. This is a sad result of the previous years of reform that left the farms with a great shortage of assets that hampers efficient land use. Finally, 92.9% of farms appear to hire more workers than they need, because:

- ♦ They are interested in having control over land share holders[10];
- ♦ Very low wages in the agricultural sector do not strongly motivate farm managers to discharge workers;
- ♦ The farms experience pressure from local authorities to keep workers employed in order to soften the employment problems in the rural area.

All these results conform to the theoretical expectations about the resources scarcity and to the results reported by Swinnen & Gow (1997), Epstein & Tillack (1999), Macours & Swinnen (1999), Svetlov (2001a).

The conclusion about the utility attributes is that the depreciation should very commonly be interpreted as an attribute of farm's utility. This is in line with the reasoning presented in Uzun (1999). 224 farms (72.03%) behave as if they wish to pay some profit in favour of wages, social costs or both. Specifically wages are included in the utility function of 67.2% of farms. Thus, the initial hypothesis that the wages and social costs should be included in farms' utility function has been definitely supported.

Appendix **!Синтаксическая ошибка, !** suggests that the model over-estimates the profit for many low-profitable farms. The most likely reasons are:

- ♦ The model is not able to capture accidental losses;
- ♦ The true utility of such farms might include attributes that are not considered in the model.

6. Case farms utility

As it was mentioned in Section 4, we study 6 case farms representing each of 6 most widely used production patterns, which are specified in Appendix 2. In addition, for each case farm

[8] In these calculations we assume the rate of operating capital turnover 1.721, that equals to an average on the whole data set.

[9] Under Russian climate, which is mostly continental, the influence of seasonality on agricultural production is greater than in Europe and the USA.

[10] Svetlov (2000) provide useful information about agricultural land ownership in Russia that explains this abnormal situation. In short, during land reform in 1993 collective and state workers (both present and former) were granted land shares representing the right to land on the farm that employed (or had employed) them.

we obtain two solutions. In one of them we consider other production as a utility attribute. In another the other production technology is unconstrained.

To complete case farm specifications, we assume in addition to what was said in Introduction that a case farm:

- ♦ Has an average amount of resources on a subset of farms having the same production pattern;
- ♦ Has an average level of utility attributes within the same subset;
- ♦ Does not have components in its utility function except for profit, depreciation, wages and social costs.

Thus, the right hand side values of the corresponding linear programmes are set according to Appendix 3. The model has been solved for both a fully specified utility function and for a utility function that does not include other production. The latter specification allows us to evaluate the effect of omitting other production while model testing, the results being compared to the former specification results to give insight into the robustness of the model.

The causes of biased profit estimation specified in the previous section do not affect the profit and shadow prices estimated for the case farms. However, the estimated shadow prices are valid only if the assumption is valid that the case farms' utility functions do not include components except those explicitly specified in the model.

The modelled production programmes are presented in Appendix 7. In all the 12 solutions the vector of incremental outputs $\mathbf{b}_2$ is $\mathbf{0}$, meaning that the case farm's resources are sufficient for running average technologies and achieving an average (on the given production pattern) utility level.

Appendix 8 presents the estimated local properties of a utility function for each case farm. According to the model, the farms do not distinguish depreciation and profit, except for the farms that do not produce cereals. That is in full accordance with my research hypothesis.

The role of wages and social costs is variable. In some cases the Pareto optima chosen by the farms are such that the wages value is preferred to the value of profit, in other cases the objective of increasing wages does not influence farm behaviour at all, being replaced by the objective of increasing social expenses. Although the estimation of π_2 and π_3 (assuming that $\boldsymbol{\pi} = (\pi_i)$) is not robust due to their high dependence on the right hand side values of the linear programme, which are not certain, the hypothesis that the studied farms consider workers' incomes while making decisions about production is firmly supported by the model.

The interpretation of π_4 and π_5 follows the reasoning given in Section 3. These shadow prices indicate a hidden part of utility (measured in units of profit) assigned to each unit of milk and other production, respectively. This study does not address the nature of this utility (whether this is a hidden part of profit, wages, social costs etc.). The values suggest that milk price delivers to a farm 68.3-86.0% of the overall economic effect of milk production, depending on the production pattern. The residual part of the effect is delivered by an unobserved sources of utility. As for other production, its share is 46.5-53.8%.

The imposed $\pi_5 = 0$ does not significantly influence other components of $\boldsymbol{\pi}$, except for the farms that do not produce cereals. That emphasises the robustness of the estimated utility functions: considering different Pareto optima, in which other production is not produced, results in either the same or similar estimated preference of different utility attributes.

2. Modelled profit and utility of the case farms (in thousand roubles of profit)

Production patterns (according to Appendix 2)	Other production is a utility attribute	Other production is not a utility attribute
I	2442 / **6984** = *0.35*	3617 / **6984** = *0.52*
II	1890 / **5729** = *0.33*	2673 / **5729** = *0.47*
III	5293 / **21463** = *0.25*	8414 / **22048** = *0.38*
IV	407 / **8357** = *0.05*	1747 / **8357** = *0.21*
V	6390 / **21622** = *0.30*	8774 / **15292** = *0.57*
VI	–272 / **13202** = *–0.02*	1597 / **13212** = *0.12*

Meaning of values: profit/**utility**=*share of profit in utility*

Case farms profit and utility are characterised in Table 2. The profit never exceeds 2/3 of overall utility. When other production is considered as a utility attribute, the greatest share of profit in utility is 0.35. So, the profit does not dominate other utility attributes. However, its share is the greatest among all the utility attributes. Exceptions are the case farm VI having negative profit and case farm IV where the depreciation exceeds profit. Both exceptions exist in case of fully specified utility function (including other production).

In Appendix 9 there are the shadow prices of the resources. They yield the same conclusions as those obtained while model testing, but no one of the case farms can benefit from the subsidised fodder prices, unless the subsidies would cover more than 2/3 of price. Lower subsidies would not affect the production decisions of the case farms.

The shadow prices of maximal vegetable production constraint are defined only for patterns III and V. They are 0.152 and 0.137 when considering other production as a utility attribute or 0.137 and 0.136 otherwise. These values allow us to calculate the influence of vegetables supply on their price. According to the meaning of this constraint, a farm does not sell more vegetables than it actually does, as having increased the vegetables supply by 100 kg, it will gain 152 roubles because of a higher quantity and loose 152 roubles because of lower prices (case of farm III and full utility function). Thus, the price is expected to decrease by 152 / 18548 = 0.0082 roubles per 100 kg of sold vegetables (18548 is an optimal vegetables output in kg×100). The elasticity of vegetables price with respect to supply is –0.461 that leads to price agreements and induces the oligopoly on this market. The estimation of the same values on the farm V data (full utility function) gives 0.0045 roubles per 100 kg and an elasticity –0.415.

7. Conclusions and discussion

The study presented in this paper gives an example of approaching econometric problems by means of a specially formulated linear program. This approach allows us to study the utility function properties of case large-scale Russian farms located in the Moscow Region having average technological capacity and to measure marginal efficiency of fixed inputs. The conclusions are summarised below.

1. The case of large-scale agricultural farms in the Moscow Region has displayed the ability of the LP approach to facilitate research tasks related to a firm behaviour modelling. The results of this study show that the conclusions that are drawn up by means of the LP econometric model are in line with theoretical expectations and with the results reported by other researchers.

2. The study has supported the hypothesis that the utility of case farms and many real farms in the Moscow Region includes depreciation, wages and social costs. Milk production and

other production are sources of hidden utility. However, considering only these utility attributes does not consistently explain the behaviour of many farms having relatively low profits.

3. The results of modelling case farms show that the hypothesis about the negative impact of a lack of operating capital on the volume of agricultural production on the studied farms is correct. Increasing turnover assets is the most common way of increasing the amount and profitability of agricultural production. It is also desirable from the point of view of farm utility functions.

4. In the present situation the existing system of short-term agricultural production financing does not sufficiently facilitate the demands of agricultural production. The development of regulations should be considered aimed at avoiding existing impediments to providing short-term credits to agricultural farms. In particular, local authorities should support hedging the risks of banks by means of co-operation with commercial insurance services. Another direction of policy regulations improvement is that the level of budget support of a farm should depend on how much care this farm takes of increasing its operating capital.

5. This study has provided evidence that Russian farms (the Moscow Region farms in this case) hire extra workers, as social motivation prevails over economic motivation when decisions about hiring and discharge are made.

6. High fodder price is a limiting factor for agricultural production. As the internal production of fodder is little less expensive than purchase, the question arises whether the existing system of fodder supply is economically efficient.

7. Improved supply of long-term credit is expected to have less impact on the economic situation at the studied farm compared with short-term credit. Many farms have an excess of fixed assets caused by a lack of assets in turnover.

8. A relatively high elasticity of vegetables price with respect to their supply motivates the suppliers to price and quota agreements. Although vegetables production is the most efficient branch of rural economy in the Moscow Region, its contribution in the agricultural production growth is limited by oligopoly. Increasing vegetables market capacity should be among the aims of regional agrarian policy. The possible means are promotion of investments in vegetables processing and preserving, expanding vegetables export opportunities, inspiration of competition in the retail vegetables markets.

The model parameters (**A** and **C**) can be estimated even on relatively short data sets. If least squares procedure does not yield reliable results due to a lack of data, it is possible to use generalised maximal entropy approach (Golan et al., 1996) in order to obtain a robust estimation of the required parameters.

The above formulated idea leads to some criticism to my approach to the production patterns analysis. One may suggest that I use the subset of farms having a given production pattern to approach Leontieff technologies used by this particular subset, as they may differ from average Leontieff technologies defined for the whole set. To some extent it is true, so such an approach is definitely worth trying. However, I do not think that it would be much more fruitful than that used by me. Indeed, in such subsets the correlation between certain technology intensities is expected to be higher than in the whole set, as the real technologies are not pure Leontieff technologies. In the less heterogeneous subsets of farms having the same production pattern the fact that both milk and meat production depend on the scale of animal husbandry may make the intensity of the corresponding Leontieff technologies highly correlated. That would result in a rather uncertain estimation of their parameters.

On the other hand, the generalised maximum entropy approach allows us to obtain an uncertain but robust estimation of the Leontieff technologies properties for any subset of farms and even for each farm individually. That may be very useful when there are the research tasks requiring special attention to separate farms.

The most promising extensions of the presented study are depicted below.

1. Using non-linear mathematical programmes is a natural extension of the approach used in this study. In many cases the LP approach appears to be too restrictive: the research tasks concerning risk analysis are a good example. In the case of this particular study the limitations of LP formalism are not restrictive.

2. As the model concentrates on the agricultural production decisions, for both theoretical and practical reasons it would be desirable to re-estimate the model using the data of costs and profit of *agricultural production activities* only. I expect that such a model would better fit the actual data.

3. The seasonality should be considered in order to precisely approach the negative effect of operating capital shortages and to draw precise conclusions about opportunities to diminish this effect.

4. Using panel data, it is possible to further improve the quality of the Leontieff technologies parameters estimation. It is also possible to estimate farm specific effects for profit and technology intensities. That allows us to improve an explanatory power of the model. However, it is not possible to estimate farm specific effects for shadow prices, as their actual values are not observable.

5. It may be promising to assume the technologies to be von Neumann instead of Leontieff. Using von Neumann technologies, which are closer to real technologies, provides an obvious opportunity allowing us to improve the quality of the model. However, it is not clear to me whether an efficient estimation procedure can be developed to approach von Neumann technologies without specially organised statistical observations.

6. The data on fertilisers and fuel usage, if available, might allow us to consider the impact of their prices on the decisions made in the studied farms. In addition, it could make the model more precise.

7. Deeper analysis is needed to identify a wider variety of utility attributes to be introduced in the model.

8. Approaching other Pareto optima by means of fixing profit at its actual level and maximising some other attributes of utility allows us to study variance of shadow prices and utility function within the variety of possible Pareto optima and to test the model for robustness.

9. Agricultural production is risky. Thus, considering risks in the model and approaching risk aversion by means of the general reciprocity theorem is a promising extension of this study. The results can be validated by means of classical risk approaching techniques summarised in Hardaker et al. (1997).

10. One of the important improvements of this study is developing a statistical measure of reliability of the estimated utility function coefficients, shadow prices and elasticity values. However, this problem may be too complicated concerning its mathematical content.

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Appendix

1. Properties of the data set

Variable	Min*	Max	Mean value	Standard deviation ($\times10^6$)	Variance ratio, %
Sales					
Milk, kg×100	58	148346	17785	336.0	103.1
Meat, kg×100	6	26299	962	5.5	244.0
Cereals, kg×100	2	40559	1682	17.6	249.3
Potatoes, kg×100	5	45109	2354	35.3	252.5
Vegetables, kg×100	1	201373	5320	419.1	384.8
Other production**, thousand roubles	8	31215	1636	9.1	183.8
Fixed inputs					
Arable land, ha	330	9131	2598	2.1	55.1
Hayfields and pastures, ha	5	4739	706	0.4	88.5
Fodder, thousand roubles	5	31625	4917	23.3	98.2
Operating capital at the beginning of year, thousand roubles:					
buildings	1	127952	19214	436.9	108.8
machinery	5	57580	9611	75.7	90.5
Total costs, thousand roubles	18	127071	15520	254.3	102.7
Average annual number of workers	1	1275	236	0.00307	74.3
Utility attributes, thousand roubles					
Depreciation	7	10284	1472	2.2	101.6
Wages	9	19920	3017	7.7	92.0
Social costs	3	6043	942	0.7	91.5
Profit	-29413	63959	2748	53.3	265.7

* Excluding zeroes

** Other production is all marketable production except milk, meat (poultry included), cereals, potatoes and vegetables.

2. Definition of production patterns

Leontieff technologies	Production patterns					
	I	II	III	IV	V	VI
Milk						
Meat						
Cereals				X	X	X
Potatoes		X		X		
Vegetables	X	X		X		X
Other production*						
Number of farms	94	76	55	31	17	15

* Other production is all marketable production except milk, meat (poultry included), cereals, potatoes and vegetables.

X indicates that the technology is not available in the production patterns.

Another 9 production patterns appearing in the data set are represented by 23 farms (7.37% of the data set)

3. Average resource usage by farms following different production patterns

Resources, utility attributes and outputs	Production patterns					
	I	II	III	IV	V	VI
Arable land, ha	2879	2873	2946	1644	1545	1836
Hayfields and pastures, ha	764	620	832	473	1058	693
Buildings, thousand roubles	19457	12991	27023	16313	26810	20886
Machinery, thousand roubles	10331	7454	13727	5781	12191	10013
Workers	236	185	361	153	340	201
Operating capital, thousand roubles	8897	6777	14416	6646	11771	8979
Total costs, thousand roubles	15433	12969	22808	10239	21528	15981
Depreciation, thousand roubles	1483	1000	2472	1026	1622	1961
Wages, thousand roubles	2880	2341	4684	2145	4037	3263
Social costs, thousand roubles	905	712	1452	652	1327	1064
Milk, kg×100	20253	15337	21304	10479	22862	21499
Meat, kg×100	1084	1191	560	1291	731	983
Cereals, kg×100	2907	1846	1740	0	0	0
Potatoes, kg×100	2806	0	5886	0	4209	3517
Vegetables, kg×100	0	0	18548	0	30194	0
Other production, thousand roubles*	1358	912	3454	1217	2317	1681
Fodder, thousand roubles	5596	4189	5793	3845	5843	5555
Profit, thousand roubles	2631	1072	5353	1053	7676	2580

* Other production is all marketable production except milk, meat (poultry included), cereals, potatoes and vegetables.

4. Leontieff technologies: estimated technological coefficients

(in units of resource/utility attribute per unit of technology intensity)

Resources and utility attributes	Technologies					
	Milk, kg×100	Meat, kg×100	Cereals, kg×100	Potatoes, kg×100	Vegetables, kg×100	Other production, thousand roubles*
Arable land, ha	0.067 (13.1)	0.143 (3.2)	0.114 (4.3)	0.061 (3.4)	0**	0
Buildings, thousand roubles	0.505 (7.3)	0	0	0	0	2.303 (3.3)
Machinery, thousand roubles	0.262 (12.4)	0	0.251 (2.5)	0.476 (5.8)	*0.026* (*1.0*)	0.615 (3.4)
Workers	0.0061 (17.6)	0.0140 (4.9)	0.0037 (2.2)	0.0058 (4.3)	0.0019 (4.5)	0.0199 (6.6)
Fodder, thousand roubles	0.176 (40.7)	0.912 (24.2)	0	0	0	0.411 (12.9)
Operating capital, thousand roubles	0.253 (16.7)	0.907 (7.3)	0.196 (2.7)	0.164 (2.8)	0.095 (5.1)	0.908 (7.0)
Total costs, thousand roubles	0.476 (19.6)	1.439 (7.2)	*0.037* (*0.3*)	0.293 (3.1)	0.135 (4.5)	1.755 (8.4)
Depreciation, thousand roubles	0.036 (10.3)	*0.026* (*0.9*)	*0.0086* (*0.5*)	0.052 (3.9)	0	0.236 (7.9)
Wages, thousand roubles	0.087 (20.7)	0.299 (8.7)	*0.031* (*1.5*)	0.068 (4.2)	0.031 (6.0)	0.262 (7.2)
Social costs, thousand roubles	0.0272 (19.6)	0.0954 (8.4)	*0.0094* (*1.4*)	0.0205 (3.8)	0.0086 (5.1)	0.0826 (6.9)
Average price, roubles	417.8	1387.0	173.4	376.4	330.0	1000.0

The zero values are imposed on the base of both theoretical reasons and results of preliminary estimations.

The values in brackets are *t*-values.

The values in *italic* are not certain at 10% level. However, they are used in the model, as the estimated values are positive in accordance with a theoretical expectation. Before being used in the model, such values were additionally tested by means of case studies based on several actual farms data. The tests have shown that the estimated value belongs to the variety of actual values in the studied cases.

The average prices are not estimated. They are taken at their average level on 407 farms. The source is the regional agriculture and food department of the Moscow Region.

* Other production is all marketable production except milk, meat (poultry included), cereals, potatoes and vegetables.

** The demand for land by vegetables production is assumed to be zero, as the available data does not allow us to separate field and glasshouses production. So, we assume that the vegetables production is either a glasshouse production or requires a "very small" land area. This decision brings an extra uncertainty to the model.

5. Technological coefficients of the empirical model

(in units of resource/utility attribute per unit of technology intensity)

	Milk, kg×100	Meat, kg×100	Cereals, kg×100	Potatoes, kg×100	Vegetables, kg×100	Other production, thousand roubles*	Purchased fodder	Fodder production
Arable land, ha	–	–	0.114	0.061	–	–	–	1.000
Buildings, thousand roubles	0.505	–	–	–	–	2.303	–	–
Machinery, thousand roubles	0.115	–	0.251	0.476	0.026	0.615	–	2.201
Workers	0.0040	0.0094	0.0037	0.0058	0.0019	0.0199	–	0.0322
Fodder, thousand roubles	0.176	0.912	–	–	–	0.411	–1.000	–5.000
Operating capital, thousand roubles	0.1381	0.663	0.196	0.164	0.095	0.908	–	1.716
Total costs, thousand roubles	0.455	1.394	0.037	0.293	0.135	1.755	–	0.321
Depreciation, thousand roubles	0.0307	0.0152	0.0086	0.0519	–	0.2358	–	0.0757
Wages, thousand roubles	0.0697	0.2607	0.031	0.068	0.031	0.262	–	0.271
Social costs, thousand roubles	0.0217	0.0836	0.0094	0.0205	0.0086	0.0826	–	0.0824
Profit, thousand roubles	0.037	0.688	0.137	0.084	0.195	–0.345	–1.000	–0.321

The conversion rates of profit into other utility attributes are 0.72 for wages and 1 for depreciation and social costs. The value 0.72 captures average extra taxation of funds spent for wages.

The upper matrix outlined by a bold line is **A** according to (7). The lower one is **C**.

* Other production is all marketable production except milk, meat (poultry included), cereals, potatoes and vegetables.

6. Model testing: overview of shadow prices

(in thousand roubles per unit of constraint)

Constraints	% of non-zero shadow prices	Non-zero shadow prices*		
		Minimal	Average	Maximal
Arable land, ha	19.9	0.04	2.75	5.16
Machinery, thousand roubles	30.5	0.01	0.53	2.34
Workers	7.1	1.11	41.17	77.63
Fodder, thousand roubles	93.3	0.01	0.45	1.00
Operating capital, thousand roubles	57.7	0.12	0.86	1.53
Total costs, thousand roubles	45.7	0.03	0.58	16.24
Depreciation, thousand roubles	86.8	1.00	0.97	0.08
Wages, thousand roubles	67.2	1.38	1.27	0.01
Social costs, thousand roubles	64.3	1.00	0.99	0.24
Minimal milk output, kg×100	97.1	7.38	0.19	0.04
Maximal vegetables output, kg×100	26.0	0.10	0.15	0.22

* Negative values are multiplied by –1.

The shadow price of buildings differs from zero in 28.5% of farms. It only happens when the amount of buildings is not compatible with minimal milk output. The corresponding non-zero shadow price is not interpretable as it depends on the corresponding attribute of penalties vector **ψ** in (7).

7. Production programmes of the case farms

Production patterns (according to Appendix 2)	Milk	Meat	Cereals	Potatoes	Vegetables	Other production*	Fodder (internal production)
	kg×100					thousand roubles	
Other production is a utility attribute							
I	20469	859	9963	6157	×	1359	805
II	15337	2821	1423	×	×	912	979
III	21304	2061	13798	2709	18548	3454	1208
IV	10479	2218	×	×	×	1217	759
V	22862	747	×	5715	30194	2317	876
VI	21499	83	×	9891	×	1681	742
Other production is not a utility attribute							
I	20469	1819	12255	6413	×	0	868
II	15337	3515	2853	×	×	0	1031
III	21304	6941	9924	0	18548	0	1815
IV	10479	3711	×	×	×	0	931
V	22862	3649	×	5421	30194	0	1215
VI	21499	1803	×	11587	×	0	918

The table presents outputs (sales) of milk, meat, cereals, potatoes, vegetables and other production and gross production of fodder.

The crossed cells correspond to outputs that are not produced under the given production pattern.

According to all optimal solutions, the case farms do not purchase fodder.

* Other production is all marketable production except milk, meat (poultry included), cereals, potatoes and vegetables.

8. Utility functions of the case farms

Notations: ω_0 is profit after taxation, ω_1 is depreciation, ω_2 is wages, ω_3 is social costs, ω_4 is milk production and ω_5 is other production.

Pattern	Utility function ($\boldsymbol{\pi' \omega}$)
a) Other production is a utility attribute	
I	$\omega_0 + \omega_1 + 0.241\omega_3 + 0.081\omega_4 + 0.865\omega_5$
II	$\omega_0 + \omega_1 + 0.431\omega_2 + 0.068\omega_4 + 0.858\omega_5$
III	$\omega_0 + \omega_1 + 1.380\omega_2 + \omega_3 + 0.124\omega_4 + 0.911\omega_5$
IV	$\omega_0 + \omega_1 + 1.380\omega_2 + \omega_3 + 0.188\omega_4 + 1.101\omega_5$
V	$\omega_0 + 0.758\omega_1 + 1.380\omega_2 + \omega_3 + 0.194\omega_4 + 1.151\omega_5$
VI	$\omega_0 + \omega_1 + 1.380\omega_2 + \omega_3 + 0.189\omega_4 + 1.117\omega_5$
b) Other production is not a utility attribute	
I	$\omega_0 + \omega_1 + 0.241\omega_3 + 0.081\omega_4$
II	$\omega_0 + \omega_1 + 0.431\omega_2 + 0.068\omega_4$
III	$\omega_0 + \omega_1 + 1.380\omega_2 + \omega_3 + 0.152\omega_4$
IV	$\omega_0 + \omega_1 + 1.380\omega_2 + \omega_3 + 0.188\omega_4$
V	$\omega_0 + \omega_1 + \omega_3 + 0.156\omega_4$
VI	$\omega_0 + \omega_1 + 1.380\omega_2 + \omega_3 + 0.190\omega_4$

9. Case farms: shadow prices of resources

thousand roubles per unit of resource

Production patterns (according to Appendix 2)	Arable land, hectares	Machinery, thousand roubles	Fodder, thousand roubles	Operating capital, thousand roubles	Total costs, thousand roubles
Other production is a utility attribute					
I		0.064	0.304	0.673	
II		0.064	0.304	0.812	
III	0.960		0.304	0.366	0.449
IV			0.010		0.816
V			0.014		0.811
VI			0.034		0.772
Other production is not a utility attribute					
I			0.304	0.673	
II			0.304	0.812	
III	1.532		0.304		0.623
IV			0.010		0.816
V	0.150		0.095		0.503
VI		0.028	0.022		0.808

Workers are the limiting factor of farm's utility only for the farm VI under utility function including other production. The corresponding shadow price is 4.163 thousand roubles per worker.