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The Contagion Effect Between the Volatilities of the NASDAQ-100 and the IT.CAC: A Univariate and A Bivariate Switching Approach

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THE CONTAGION EFFECT BETWEEN THE VOLATILITIES OF THE NASDAQ-100 AND THE IT.CAC: A UNIVARIATE AND A BIVARIATE SWITCHING APPROACH

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Abstract. This article uses models with changes in regime and conditional variance to show the presence of co-movement between the American and the French New Technology indexes, the NASDAQ-100 and the IT.CAC respectively. For the past two years, the American and the French New Technology stock markets have been fluctuating severely, and it has been observed that the IT.CAC is considerably affected by the the NASDAQ-100. In the first part of this article, we study the volatilities of those two IT indexes, using univariate conditional variance and changes in regime models. We show that the volatilities of the two indexes have considerably increased exhibiting a certain level of correlation. We find signs of a co-movement effect between the volatilities of the NASDAQ-100 and the IT.CAC. The hypothesis of a co-movement effect is discussed in the second part of this article, using a bivariate SWARCH model to show the dependence of the high and low volatility states of the IT.CAC on the NASDAQ-100, with no intermediate simultaneous high-low volatility states.

Introduction

Since the explosion of the international speculative New Technology bubble, IT stocks have been going through a period of high instability. Investors and speculators are being careful in handling their investments in this sector, since they are not tempted by the high revenues promises of IT companies anymore, and the deceiving accounting methods of some big sized telecommunication companies, created a certain wave of fleeing from technology stocks. In fact, many New Technology companies made numerous promises of extraordinary technological progress that weren't met by proportional business to business sales, which created a big deficit in their balance on the one hand, and a high market capitalization that overvalued the true values of those companies' stocks on the other hand.

This situation lead to a high market fluctuation and very volatile IT stock prices that are affected by informations, rumors and the international New Technology markets.

To study the influence of one IT stock market in one country on another and vice-versa, we use daily data of the NASDAQ-100 and the IT.CAC, two indexes

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thoroughly scanned at the moment and exhibiting a high level of volatility and instability. We examine this increasing volatility and financial instability through the last five years using the rolling standard deviations over 21 days. Then in the first part, we model the two volatilities to reach the co-movement hypothesis that we model in the second part, to show it's extent.

In the first part we use three univariate GARCH models and two univariate models of changes in regime to show the considerable increase in volatility and to identify simultaneous periods of high and low volatilities over the two samples.

In the second part we use a bivariate SWARCH model (Edwards and Susmel, 2001) to identify the co-movement effect between the two markets and show through the model above that the American and the French New Technology markets have simultaneously increased in volatility all through the sample in question, with a high co-movement effect between the NASDAQ-100 and the IT.CAC.

Our approach is similar to studies made by Bennett and Kelleher (1988) and King and Wadhwani (1990) on the effects of the 1987 stock market crash over the financial volatility through a number of countries. Engle, Ito and Lin (1990, 1992) and Hamano, Ng and Masulis (1990) have also studied the volatility through a number of countries and identified the geographical effect of the informations' impact on the volatility through those countries. We can also cite works by Longin and Solnik (1995) and Ramchand and Susmel (1998) on time varying correlations.

The paper is organized as follows: in section I we present the data used and their corresponding descriptive statistics, and we provide information on the indexes and the way they are calculated. In section II, we model the return in percentage of the two indexes using AR(1)-GARCH(1,1), AR(1)-TARCH(1,1), regime switching and univariate SWARCH models. In section IV we use the bivariate SWARCH model. Section V is the conclusion.

1. NASDAQ-100 and IT.CAC: data description

We use two samples of daily time series of the NASDAQ-100 and the IT.CAC.

1.1. **indexes Description.**

We describe the two new technology indexes[1] following the information provided by the N.Y.S.E. and the Paris Stock Market. More information is provided about the IT.CAC than the NASDAQ-100 since the French New Technology index is less known than the American one.

1.1.1. *The NASDAQ-100.*

The NASDAQ-100 is composed of American and international non-financial companies quoted on the NASDAQ market, based on market capitalization. The index represents technology companies which include the computer sector (hardware and software), telecommunication and biotechnology. The index does not include financial and investment firms.

The NASDAQ-100 was created in January 1985 and is calculated with a methodology of capitalization weighing that considers the economic attributes of the weighted

[1]The graphs of the series can be found in appendix C.

capitalization while keeping a significant diversification. In case certain requirements are not met for the weights' distribution set *a priori*, the composition of the NASDAQ-100 is revised on a quarterly basis and the weights of the index's members are adjusted.

Certain conditions apply to the members of the index, for example, every member should have a daily exchange volume of 100,000 shares. The graph of the NASDAQ-100's historical volume of exchange is found in the appendix.[2]

1.1.2. *The IT.CAC.* The IT.CAC was created on March 29^{th} 2000 and it contains 150 French technology companies. It is not possible to define purely deterministic criteria for those companies, like the evolution of the sales turnovers, that allow the distinguishing between the members. Furthermore, the inclusion of these companies in the IT.CAC is a result of each company's decision to position itself in the New Technology sector; the impact of the companies' accounts and financial situation intervenes later on.

On December 31^{th} 1999, the target population included 113 companies with a capitalization of 360 billion euros, almost the quarter of the total capitalization of the Paris Stock Market. The market capitalization of those companies is not uniform, for example, the largest one, France Télécom is worth 134 billion euros and the smallest one is Tonna Electronique, 7 million euros. It is also quite young since half of companies joined the New Technology population in the past two years.

It is necessary to allocate weights adapted to the shares' prices of those companies because of the capitalization size diversity and the rapid growth of those companies. A traditional weight allocation based on market capitalization would lead to the domination of certain companies' shares, which would harm the diversification of the index. It is necessary then, to limit the impact of the bigger companies' capitalization in such a way that their weights in the index do not exceed a certain predetermined threshold.

The permanent and strict respect of the capitalization threshold leads to multiple adjustments that do not necessarily have a logical economic and financial significance. Therefore, the weights' revision should be relatively the least frequent possible to be credible.

The 10% limit as a maximum weight constitutes the general reference. But the limit used for the IT.CAC is 8% to master the respect of the preset limit without making multiple and frequent adjustments, while allowing it to reflect the natural evolution of the market (mergers, acquisitions, capital revision, etc). The revision of the weights takes place every month, at the end the last session of the month.

The determination of weights is done as follows:

(1) Calculation of the weight using market capitalization of each company: the weights allocated to a share is then the ratio between its market capitalization and the global market capitalization of the target population;

[2] The Paris Stock Market website(www.bourse-de-paris.fr)

(2) comparison of the highest weight with the 8% threshold:

- if it's inferior, the obtained weight is kept,
- otherwise, the excess weight is distributed over the other shares with respect to their respective capitalization proportions.

(3) repeating step 2 as long as any of the weights is greater than 8%.

When the number of shares having a limited weight of 8% is determined, the index calculation is done in the same way as for the other Paris Stock Market indexes, with a price index that does not take the dividends distribution into account and that considers the profitability index (the reinvested dividends). It is important to point out that contrary to the NASDAQ-100, there are no minimum volume exchange conditions for the IT.CAC's members, therefore it is possible to find companies with very low exchange volumes.

1.2. **Statistical Description of the Series.** The two time series of the indexes are not stationary, the details of the Dickey-Fuller test found in the appendix A, show the presence of a first order unit root I(1) in the two series and its absence for the second order I(2). The data are transformed as to have $\Delta r_t = \frac{y_t - y_{t-1}}{y_{t-1}} \times 100$[3] which eliminates the unit root in the series and stationarizes them. For convenience purposes, the studied series Δr_t will be referred to as y_t where if $y_t = 2$ it means that the corresponding index grew by 2% and in the case of a negative value this would mean that the index fell with respect to its previous value. The mean, standard deviation, asymetry, kurtosis and the Jarque-Bera normality test of the two series can be found in the appendix A.

The two series exhibit the classical characteristics of high frequency financial time series, especially the high kurtosis that leads to rejecting the normality hypothesis, and the LM test (TR^2) by Robert Engle indicates the presence of an ARCH effect.

The graphs of the rolling standard deviations over 21 days[4] in the appendix C show the "explosion" of the the two indexes' volatilities with the approach of the year 2000. This was during the formation of the New Technologies speculative bubble formation, especially Internet related shares. The volatilities of the other traditional indexes (DJ, SP, etc) remained relatively stable.

Actually, it is clear that the NASDAQ market is quite volatile with respect to the other N.Y.S.E. indexes because of the instability's evolution of sectors related to the IT sector. This evolution was highly influenced by the very rapid market capitalization of many tech companies and equally by a very brutal depreciation (after the explosion of the tech bubble) with respect to the more traditional sectors. The interest in studying this sector arises from the importance of the Information and the Immaterial Capital in it and the interest of risk managers in it, and this is because of the variety of derived financial products that it contains. The question is whether traditional parametric non-linear volatility models, like the GARCH models, can explain the temporal evolution of this sector's volatility (high and low) and

[3] The graphs of the transformed series can be found in the appendix C.

[4] Rolling standard deviation (RSD) over 21 days: $\sigma(R_t) = [253 \sum_{k=1}^{2} 1(R_{t-k} - \mu)^2/20]^{\frac{1}{2}}$, where μ is the mean of the observations over 21 days (Schwert, 2002).

whether there is co-movement between the volatility of an IT sector in one country, and the volatility of an IT sector in another country.

In the following sections, we examine the non-traditional volatility for the NASDAQ-100 and the IT.CAC. We identify periods of high volatility and the probability that they take place. Furthermore, we examine the link between the two volatilities of the two indexes.

2. Univariate Analysis

In this section, we use a number of univariate parametric models to show the presence of at least two regimes for the volatility of the studied indexes. We start by using three GARCH specifications, then we use a simple model with changes in regime, then we use a SWARCH model and finally we model the indexes using a GARCH model with changes in regime.

2.1. **GARCH, TARCH and EGARCH heteroscedastic models.** The ARCH model, introduced by Engle (1982) and generalized by Bollerslev (1986) in order to have the GARCH model is represented as follows:

$$\begin{aligned} &\epsilon_t = \sigma_t.\nu_t \;\; with \;\;\; \nu_t \sim i.i.d.(0,1) \\ &\sigma_t^2 = a_0 + \textstyle\sum_{i=1}^{q} a_i \epsilon_{t-1}^2 + \sum_{i=1}^{p} b_i \sigma_{t-i}^2. \end{aligned} \tag{2.1}$$

It models the conditional variance σ_t^2 of a time series as a linear function of the squared q lagged innovations and the p lagged conditional variances of the series. These models have been widely used to model the time variable volatility of financial time series. Those simple but powerful models in explaining time variable volatilities, have been modified several times to adapt to the financial data that has been constantly changing in their structure.

The TARCH model, introduced by Zakoian (1990) and Glosten *et al.* (1993) model the conditional variance in the following way:

$$\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i \epsilon_{t-i}^2 + \gamma \epsilon_{t-1}^2 d_{t-1} + \sum_{i=1}^{p} b_i \sigma_{t-i}^2, \tag{2.2}$$

where $d_t = 1$ if $\epsilon_t < 0$ et $d_t = 0$ otherwise. In this model, good news ($\epsilon_t < 0$) and bad news ($\epsilon_t > 0$) have different impacts over the conditional variance - good news have an impact a, whereas bad news have an impact $a + \gamma$. If $\gamma \neq 0$, the news impact is asymmetric and if $\gamma > 0$ then the leverage effect[5] exists.

[5]The leverage effect refers to the well-established relationship stock returns and both implied and realized volatility: volatility increases when stock prices fall. A standard explanation ties the phenomenon to the effect a change in market valuation of a firm's equity has on the degree of leverage in its capital structure, with an increase in leverage producing an increase in stock volatility.

The EGARCH model (exponential GARCH, introduced by Nelson (1991), has the following conditional variance:

$$\log(\sigma_t^2) = a_0 + b_1 \log(\sigma_{t-1}^2) + a_1 \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}}, \tag{2.3}$$

where $log(\sigma_t^2)$ is the log of the conditional variance. This implies that the leverage effect is exponential instead of being quadratic and that the conditional variance forecasts are surely non-negative. The presence of the leverage effect may be tested with the hypothesis $\gamma < 0$ and the impact is asymmetric if $\gamma \neq 0$. The EGARCH model that we estimate is slightly different from Nelson's (1991) model: we assume that the ϵ's follow the Normal distribution whereas Nelson (1991) assumes that they follow a generalized error distribution. Furthermore, the expression of the conditional variance that we use is also different from Nelson's:

$$\log(\sigma_t^2) = a_0 + b_1 \log(\sigma_{t-1}^2) + a_1 \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}}, \tag{2.4}$$

our estimation provides the same estimators as Nelson's with the exception of a_0 that has a scale difference of $a_1\sqrt{\frac{2}{\pi}}$ with the respect to the original model.

GARCH, TARCH and EGARCH models have proven to be limited in their ability to account for financial time series with a volatility that undergoes occasional regime shifts due to certain prompt events, like a stock market crash or the formation or explosion of a speculative bubble. Several models have been introduced as to include changes in regime in the processes that volatile financial time series follow.

2.2. **Constant variance models with changes in regime.** The constant variance model with changes in regime was introduced by Hamilton (1989):

$$y_t \sim \begin{cases} N(\mu_1, \sigma_1^2) & when \quad s_t = 1 \\ N(\mu_2, \sigma_2^2) & when \quad s_t = 2 \end{cases} \tag{2.5}$$

The s_t variable is an unobserved variable that can take the discrete values 1 and 2. This variable represents the state or the regime of the studied series at date t. The variable s_t follows a hidden Markov Chain.

This model allows the data in question to be drawn from two different Normal distributions, one that represents a high volatility regime and one that represents a low volatility regime. Relaxing the hypothesis of only one distribution all over the sample would allow the series to exhibit the regime change effect.

The estimation is carried out using Hamilton's iteration algorithm and which provide all of the estimators of the model without having to calculate the 2^T of the likelihood function.

The filtered probabilities are calculated as follows:

$$\Pr(s_t|\Delta r_1, ..., \Delta r_t; \theta), \tag{2.6}$$

as for the smoothed probabilities, the following expression is used:

$$\Pr(s_t|\Delta r_1, ..., \Delta r_T; \theta) \tag{2.7}$$

The filtered and the smoothed probabilities are necessary to identify periods of high and low volatilities by examining the probability of passing from one regime to another.

2.3. **The SWARCH model.** The most commonly used models for modeling financial time series are the ARCH (Engle, 1982) and GARCH (Bollerslev, 1986) models. Those two models have proven, through modeling the conditional variance as a linear or non-linear function of the lagged squared errors, their ability to provide very significant parameters most of the time, but that lack stability over time (Lamoureux and Lastrapes, 1990, 1993 and Engle and Mustafa, 1992). Hamilton and Susmel (1994) treated this problem by introducing the SWARCH model that allows the data to follow several ARCH models, with changes in regime between those models governed by a hidden Markov Chain. Furthermore, relaxing the hypothesis of having one regime in the conditional variance lowers the ARCH effect and can render the estimators of the multiple regime ARCH model less significant but more stable over time.

The SWARCH model takes into account structural changes in financial data, a SWARCH(K,q) is written as follows:

$$\begin{array}{l} y_t = c + \phi y_{t-1} + \epsilon_t \qquad \epsilon_t|\Omega_{t-1} \ \sim \ N(0, h_t) \\ \epsilon_t = \sqrt{g_{s_t}} \times \tilde{\epsilon}_t \\ \tilde{\epsilon}_t = h_t.\nu_t \\ h_t^2 = a_0 + a_1\tilde{\epsilon}_{t-1}^2 + a_2\tilde{\epsilon}_{t-2}^2 + ... + a_q\tilde{\epsilon}_{t-q}^2 + \xi.d_{t-1}.\tilde{\epsilon}_{t-1}^2, \end{array} \tag{2.8}$$

where y_t is the modeled series, the g_{s_t} are the scale changing regime parameters, and the Ω_{t-1} is the matrix of available information up to the date $t-1$; furthermore, $d_{t-1} = 1$ if $\tilde{\epsilon}_{t-1} \leq 0$ and $d_{t-1} = 0$ if $\tilde{\epsilon}_{t-1} > 0$ and ξ represents the leverage effect. The error term $\tilde{\epsilon}_t$ is multiplied by the constant $\sqrt{g_1}$ when the process is in the regime $s_t = 1$, multiplied by $\sqrt{g_2}$ when $s_t = 2$ and so on (Hamilton and Susmel, 1994). The conditional variance of ϵ_t knowing the regimes up to the date t is:

$$
\begin{aligned}
&E(\epsilon_t^2|s_t, s_{t-1}, ..., s_{t-q}, \epsilon_{t-1}, \epsilon_{t-2}, ..., \epsilon_{t-q}) \equiv \sigma_t^2(s_t, s_{t-1}, ..., s_{t-q}) \\
&= g_{s_t}\{a_0 + a_1.(\epsilon_{t-1}^2|g_{s_{t-1}}) + a_2.(\epsilon_{t-2}^2|g_{s_{t-2}}) + ... + a_q.(\epsilon_{t-q}^2|g_{s_{t-q}}) \\
&+\xi.d_{t-1}.(\epsilon_{t-1}^2|g_{s_{t-1}})\}.
\end{aligned} \tag{2.9}
$$

The transition probabilities between the regimes follow a hidden Markov chain with K states that are independent of y_t for all t, which means that the transition probabilities p_{ij} are constant. The estimation of the SWARCH model is carried out using the maximum likelihood (Hamilton, 1989 and Hamilton and Susmel, 1994) which is possible to maximize with respect to the vector or parameters $\widehat{\theta}$[6], with constraints that $g_1 = 1$, $\sum_{j=1}^{K} p_{ij} = 1$ for $i = 1, 2, ..., K$ and $0 \leq p_{ij} \leq 1$ for $i, j = 1, 2, ..., K$. The filtered and the smoothed probabilities are calculated respectively through:

$$
\begin{aligned}
&\Pr(s_t, s_{t-1}, ..., s_{t-q}|y_t, y_{t-1}, ..., y_{-3}) \\
&\Pr(s_t|y_T, y_{T-1}, ..., y_{-3}),
\end{aligned} \tag{2.10}
$$

where T is the size of the sample, using Hamilton's (1989) and Kim's (1994) algorithms.

The filtered probabilities represent the conditional probability that the regime at date t is s_t and at $t-1$ it was s_{t-1} conditionally over the observed values of y_t up to the date t. On the other hand, the smoothed probabilities represent inferences over the actual regime at date t based on the data available until T. For example, for a two regime model, the smoothed probabilities at date t are represented by a 2×1 vector which includes the estimated probabilities for the two regimes in question. This vector represents *ex-post* inferences made on the regime of the studied variable at date t, based on all the sample.

2.4. **GARCH models with changes in regime.** The changing regime GARCH model, introduced by Gray (1996b), is written in the following way:

$$
\begin{aligned}
&y_t = a_{0s_t} + a_{1s_t} y_{t-1} + \epsilon_t \\
&\epsilon_t|\Omega_{t-1} \sim N(0, h_{ts_t}), \\
&h_{ts_t} = b_{0s_t} + b_{1s_t}\epsilon_{t-1}^2 + b_{2s_t} h_{t-1s_t}.
\end{aligned} \tag{2.11}
$$

The innovative aspects of this model over existing changing regime conditional variance models lie in the inclusion of a GARCH term in the conditional variance expression and the dependence of this term's parameters on two possible regimes. This methodology is quite different from the SWARCH model discussed earlier, which undergoes changes in regime through a scale parameter and does not allow

[6] $\widehat{\theta} = \alpha,\ \phi,\ a_0,\ a_1,\ a_2,\ ...,\ a_q,\ p_{11},\ p_{12},\ ...,\ p_{kk},\ g_1,\ g_2,\ ...,\ g_k,\ \xi$ et ν

for any kind of dependence of the parameters on the different regimes.

This approach was not used by Hamilton and Susmel (1994) since they argued that the presence of a GARCH term in the changing regime model would make the model intractable, since the conditional variance at date t would depend on the entire sequence of regimes up to the date t. Gray (1996b) solved this dependence problem by aggregating the conditional variances of the two regimes at each step. This allows the variance to be conditional on the information available but that is aggregated all through the regimes. If conditional normality is assumed in each regime, the conditional variance at date t would be:

$$\begin{aligned} h_t &= E[y_t^2|\Omega_{t-1}] - E[y_t|\Omega_{t-1}]^2 \\ &= p_{1t}(\mu_{1t}^2 + h_{1t}) + (1-p_{1t})(\mu_{2t}^2 + h_{2t}) - [p_{1t}\mu_{1t} + (1-p_{1t})\mu_{2t}]^2, \end{aligned} \tag{2.12}$$

where Ω_{t-1} is the matrix of the available information up to the date $t-1$ and $\mu_{1t} = a_{0s_t} + a_{1s_t}y_{t-1}$. Furthermore, h_t does not depend on the entire sequence of regimes and can be used as the lagged conditional variance in order to construct h_{1t+1} and h_{2t+1} which follow a GARCH model:

$$\begin{aligned} &h_{it} = a_{0i} + a_{1i}\varepsilon_{t-1}^2 + b_i h_{t-1}, \\ &h_{t-1} = p_{1t-1}[\mu_{1t-1}^2 + h_{1t-1}] + (1-p_{1t-1})[\mu_{2t-1}^2 + h_{2t-1}) \\ &-[p_{1t}\mu_{1t-1} + (1-p_{1t-1})\mu_{2t-1}]^2, \end{aligned} \tag{2.13}$$

where i represents the regime (2 regimes, $i = 1, 2$). Furthermore,

$$\varepsilon_{t-1} = y_{t-1} - E[\Delta r_{t-1}|\Omega_{t-2}] = y_{t-1} - [p_{1t-1}\mu_{1t-1} + (1-p_{1t-1})\mu_{2t-1}]. \tag{2.14}$$

So Gray (1996b) assumes that:

$$h_{t-1}\left\{\varepsilon_t|\Omega_t\right\} = a_{0s_t} + a_{1s_t}\varepsilon_{t-1}^2 + b_{s_t}E_{t-2}[h_{t-2}\left\{\varepsilon_{t-1}|\Omega_{t-1}\right\}. \tag{2.15}$$

In order to construct the log-likelihood of the model above, we suppose that Δr_t undergoes changes in regime (2 regimes):

$$\begin{aligned} f(y_t|\Omega_{t-1}) &= \textstyle\sum_{i=1}^2 f(y_t, S_t = i|\Omega_{t-1}) \\ &= \textstyle\sum_{i=1}^2 f(y_t|S_t = i, \Omega_{t-1}) \Pr(S_t = i|\Omega_{t-1}) \\ &= \textstyle\sum_{i=1}^2 f(y_t|S_t = i, \Omega_{t-1}) p_{it}, \end{aligned} \tag{2.16}$$

where p_{it} is given by $\Pr(S_t = i|\Omega_{t-1})$. As a consequence, the distribution of y_t conditional on the available information can be written in the following way:

$$f(y_t|S_t = i, \Omega_{t-1}) = \frac{1}{\sqrt{2\pi h_{it}}} \exp\left\{\frac{-(y_t - a_{0i} - a_{1i} y_{t-1})^2}{2h_{it}}\right\}, i = 1, 2. \tag{2.17}$$

The calculation of the probabilities p_{it} is done in the same way as it is done by Hamilton (1994).

2.5. **The results.** In this section, we expose the results found using the different models discussed above for the NASDAQ-100 and the IT.CAC[7].

2.5.1. *Autoregressive models with heteroskedastic errors.* We start by using AR(1)-GARCH(1,1), AR(1)-TARCH(1,1) and AR(1)-EGARCH models to examine the volatilities of the two indexes NASDAQ-100 and IT.CAC. The choice of the models' order was done based on the parameter significance and the Akaike and Schwartz criterion (the values can be found in the appendix). The results presented below, show the very significant presence of an ARCH effect in the two series, as well as a significant leverage effect. The Ljung-Box statistics of the models' residuals show the absence of any correlation of those residuals and squared residuals, which means the absence of an ARCH effect.

AR(1)	GARCH(1,1)		GARCH-L(1,1)*		EGARCH(1,1)	
	NAS	IT	NAS	IT	NAS	IT
μ	0,16	0,06	0,04	-0,03	0,04	-0,02
	(0,06)	(0,08)	(0,07)	(0,08)	(0,07)	(0,08)
	NAS	IT	NAS	IT	NAS	IT
ϕ	-0,06	0,14	-0,04	0,16	-0,04	0,15
	(0,03)	(0,03)	(0,03)	(0,03)	(0,03)	(0,03)
	NAS	IT	NAS	IT	NAS	IT
a_0	0,20	0,04	0,27	0,09	-0,04	-0,13
	(0,06)	(0,02)	(0,06)	(0,03)	(0,03)	(0,03)
	NAS	IT	NAS	IT	NAS	IT
a_1	0,12	0,10	0,007	0,06	0,17	0,22
	(0,02)	(0,02)	(0,01)	(0,01)	(0,03)	(0,03)
	NAS	IT	NAS	IT	NAS	IT
b_1	0,86	0,89	0,87	0,88	0,96	0,97
	(0,02)	(0,02)	(0,02)	(0,02)	(0,00)	(0,00)
			NAS	IT	NAS	IT
γ	-	-	0,19	0,11	-0,12	-0,06
	-	-	(0,03)	(0,03)	(0,02)	(0,02)
J-B	8,37	12,70	3,05	20,90	4,39	19,60
Pr.	0,02	0,00	0,22	0,00	0,11	0,00
Q_{10}	6,59	7,73	9,03	8,35	8,17	9,16
Pr.	0,68	0,56	0,43	0,50	0,52	0,42

TABLE 1. Estimators for the 3 GARCH models used

[7]The choice of the most fitting model in each section as well as the likelihood functions, the AIC and Schwartz criteria of the results of each specification used, are indicated by a star in the appendix B.

where Q_{10} is the Box-Pierce test of order 10 of the normalized residuals and the (.) represent the standard deviations of the estimated parameters. The sum of the α_1 and the β_1 of the AR(1)-GARCH(1,1) model is close to 1 which indicates that the conditional variance is quite persistent over time. Lamoureux and Lastrapes (1990) and Hamilton and Susmel (1994) argued that the high persistence of the conditional variance is an indicator of a regime change effect present in the process explaining the variance.

2.5.2. *The simple regime change model.* We estimate a simple model with changes in regime (Hamilton, 1989) in order to put into evidence the presence of two distributions that each of the indexes might have (equation 2.5), where p_{11} is the probability that the model is in regime 1 and p_{22} is the probability that the model is in regime 2.

	NASDAQ100		IT.CAC	
μ_1	0,1630	(0,0799)	-0,0589	(0,0767)
μ_2	-0,2203	(0,2057)	0,0358	(0,1755)
σ_1	2,0044	(0,0654)	1,6115	(0,0570)
σ_2	3,8762	(0,1167)	3,4517	(0,1161)
P	0,9860	(0,0059)	0,9872	(0,0062)
Q	0,9772	(0,0096)	0,9825	(0,0083)

TABLE 2. Estimators for the simple regime change model

The estimators of the parameters ($\mu_1, \mu_2, \sigma_1, \sigma_2, p_{11} = P$ and $p_{22} = Q$) and their standard deviations in Table 2 above are all significant which means that the two indexes follow a mixture of two distinct distributions each with a probability of passing from one distribution to another depending on the series volatility (high and low). The low volatility variance of the IT.CAC is twice lower than its variance for the high volatility state.

The graphs of the *ex-ante* probabilities, the filtered probabilities, the conditional standard deviations, and the the conditional means of the two indexes show a change from a low volatility regime to a high volatility regime from the end of 1999 till the end of 2000, the year of the New Technology stock market crash. The smoothed probabilities corresponding to the regime changes of the two indexes seem to vary simultaneously and we find that the the regime change for both of the series take place simultaneously also.

The graphs of the *ex-ante* probabilities and the conditional means and standard deviations (see appendix) confirm what the smoothed probabilities show and at the same dates. In July 2000, the graphs show some irregular fluctuations compared to the rest of the sample and that is because the two indexes increased in value before getting back to the constant decrease until today.

2.5.3. *The univariate SWARCH model.* We estimate different specifications of the SWARCH model with $q = 1$ to 3 ARCH terms and $K = 2$ to 3 states, with innovations that follow a Student-t distribution and a Normal distribution and with the presence and the absence of a Leverage effect ξ. With $K = 4$ states, the model does not converge because the Hessian matrix is nearly singular.

The likelihood test and the Akaike and Schwartz criteria reject the Student-t distribution in favor of the Normal distribution, furthermore, the ARCH parameters and the Leverage effect parameter, ξ are all significant with the exception of the second ARCH parameter of the IT.CAC. It is clear, then, that the SWARCH effect is significant for both series and the smoothed probabilities in the appendix show the volatilities' evolution for both of them and indicate the presence of 3 regimes: a low volatility regime, a medium volatility regime and a high volatility regime.

The first regime appears in the first place at the beginning of the sample, where both of the indexes had very low volatilities in comparison with the rest of the samples. This regime shows the state of the indexes before the high increase of New Technology stock prices and the formation of the speculative bubble. The other two regimes show the high and medium volatility states of the indexes during the high fluctuation periods that took place later on afte the year 2000. It can be clearly seen on the graphs of the two smoothed probabilities $P(s_t = 2)$ and $P(s_t = 3)$ that they move simultaneously over the same period of time, which leads us to make the hypothesis of a co-movement between the two indexes and possibly a contagion effect if this co-movement is one-sided.

On the other hand, we notice that the ARCH effect decreased in the conditional variance and we find that it is less persistent. This result is similar to what Hamilton and Susmel (1994) and Klaassen (2001) found in their financial modeling of heteroscedastic models with changes in regime. But as a counterpart, we obtain considerable information about the volatility states and their nature (low, medium or high) through the smoothed probabilities.

AR(1)SWARCH-L(3,2)	NAS100	IT.CAC
c	0,147 (0.001)	0,027 (0,031)
ϕ	-0,046 (0.036)	0,145 (0,069)
a_0	2,084 (0.043)	1,428 (0,026)
a_1	0,032 (0,002)	0,073 (0,031)
a_2	0,102 (0.049)	5,9e-011 (0,112)
g_1	2,218 (0.055)	2,520 (0,002)
g_2	7,173 (0.421)	8,234 (0,011)
γ	0.161 (0.132)	0.052 (0,020)
J-B	317	267
Pr.	0,001	0,001
Q_{10}	139	238
Pr.	0,002	0,001

TABLE 3. Estimators for the SWARCH model

2.5.4. *The GARCH model with changes in regime.* We obtain the GARCH model with changes in regime by relaxing the hypothesis of constant variances within the regimes. We obtain then two conditional variances that correspond to each of the regimes in question, where each of the conditional variance equations has its own set of parameters. Table 4 below shows the results obtained by estimating an AR(1)-SWGARCH(2,1,1) model for the NASDAQ-100 and the IT.CAC.

The parameters are significant which reveals the presence of two regimes for the volatility. Earlier in this paper, we find that the Jarque-Bera test rejects the hypothesis of Normally distributed errors, whereas the SWGARCH model that we estimate provides residuals that follow the Normal distribution and eliminates any autocorrelation of the squared residuals. Table 1 in the appendix shows (following the likelihood test and the Akaike and Schwartz criteria) that the SWGARCH is the most convenient model for the data in question.

The limitation of this model with respect to the 3 states SWARCH model is that it is not possible to include three regimes in it since it becomes intractable due to the large number of parameters to estimate and the nearly singular Hessian matrix. The 3 states SWARCH model provides more information and flexibility with respect to the number of regimes that the volatility can have, whereas the SWGARCH model better explains the conditional variance because of the presence of a GARCH term in its expression.

It is clear that the parameters of the SWGARCH model (Table 4) satisfy the stationarity condition, $b_{1i} + b_{2i} < 1$. Furthermore, as it was found with the SWARCH model earlier, the smoothed probabilities obtained with the SWGARCH model appear to move simultaneously all along the sample, which confirms again the co-movement hypothesis made earlier with the SWARCH model; the two indexes are simultaneously in a low or high volatility regimes.

	NAS-100	IT.CAC
a_{01}	0,963 (0,264)	0,904 (0,292)
a_{02}	-0,496 (0,246)	0,007 (0,062)
a_{11}	-0,258 (0,105)	0,211 (0,118)
a_{12}	0,014 (0,067)	0,123 (0,035)
b_{01}	0,000 (0,000)	0,016 (0,252)
b_{11}	0,000 (0,000)	0,000 (0,000)
b_{21}	0,540 (0,121)	0,990 (0,034)
b_{02}	0,000 (0,000)	0,035 (0.027)
b_{12}	0,135 (0,049)	0,093 (0,018)
b_{22}	0,731 (0,124)	0,899 (0,019)
P	0,696 (0,132)	0,980 (0,012)
Q	0,801 (0,067)	0,999 (0,000)
J-B	3,239	2,355
Pr.	0,197	0,308
Q_{10}	5,660	12,462
Pr.	0,843	0,255

Table 4. Estimators for the SWGARCH model

3. Bivariate Analysis: The Bivariate AR(1)-SWARCH(1,1) Model

In this section, we model a bivariate specification of the SWARCH model to investigate the co-movement hypothesis made earlier and possibly the presence of a contagion effect between the volatilities of the two indexes.

3.1. **The model and the results.**

Edwards and Susmel (2001) study in their article a group of countries of Latin America and Asia on two levels: the first is to analyze the possible increase of financial instability over the past years by using stock market returns for those countries and on the second level to see if this increase in volatility coincides through the countries in question.

They treat the subject by using univariate and bivariate SWARCH models and they show that volatility increased over time and that this increase coincide in those countries; they use daily stock market returns.

Their work is a bivariate extension of the univariate SWARCH model introduced by Hamilton and Susmel (1994). This model allowed them to examine the co-movements of the countries' volatilities. This type of modeling is up to date because it studies the financial contagion through countries. In fact, the existence of a statistically significant one-way volatility co-movement shows the presence of a contagion effect. In particular, the simultaneous increase of the conditional variance of the financial variables in question, can have very important implications with respect to the interpretation of the contagion effect through countries, and the detection points of regime changes of the volatility through time.

Since multivariate SWARCH models are very difficult to estimate due to the big number of parameters in their equations, Edwards and Susmel (2001) use a bivariate SWARCH model which means that they study two countries at a time. They show evidence of a contagion effect between the couples of countries that they study, through the dependence of the volatilities through the countries in question. Furthermore, they find that the correlation increases 2 to 4 times during periods of high volatility, in comparison with periods of low volatility.

The SWARCH model is expressed as follows:

$$\mathbf{y}_t = \mathbf{A} + \mathbf{B}\mathbf{y}_{t-1} + \varepsilon_t|\mathbf{\Omega}_{t-1} \sim N(0, \mathbf{H}_t), \tag{3.1}$$

where $\mathbf{y}_t = [y_t^{nas}, y_t^{cac}]$ is the 2×1 vector containing the series in question and $\varepsilon_t = [\varepsilon_t^{nas}, \varepsilon_t^{cac}]$ is the 2×1 vector containing the Normally distributed errors having a conditional covariance matrix that varies with time. Each of the diagonal elements of this matrix follows a SWARCH model. The $\mathbf{H}_t$ is a 2×2 matrix and the diagonal elements follow the SWARCH model given by equation (2.8). In the bivariate case, the transition and the smoothed probabilities are calculated in the same way as with the univariate model and the four cases that the transition probabilities represent are:

(1) probability that variables 1 and 2 are both in a low volatility state,
(2) probability that variable 1 is in a high volatility state and variable 2 is in a low volatility state,
(3) probability that variable 1 is in a low volatility state and variable 2 is in a high volatility state,
(4) probability that both variable are in a high volatility state.

Edwards and Susmel (2001) show evidence of a contagion effect between the countries in question and they find significant probabilities of the 2 and 3 kind above. This means that there is a certain latency before the contagion effect reaches the receptor country. They in fact find that there is not a country that has a big influence on another among the couples of countries that they study.

The NASDAQ-100 index which represents the New Technology market in the U.S.A. and which introduced the idea of having a New Technology sector, is the largest internationally. This leads to its great influence over New Technology sectors and markets in the countries of the rest of the world. Those countries became receptor countries with respect to the American IT market in the sense that events that take place in the American New Technology sector, have great influence on them. For example, the recent crisis that arose due fraudulent accounting methods of Enron and Worldcom in the U.S.A., reached the French market and raised doubts conerning accounting methods adopted by the French IT companies, especially Vivendi Unniversal.

Interestingly and contrary to the results found by Edwards and Susmel (2001), we find that the probabilities of kind 1 and 4 are very significant, whereas probabilities of kind 2 and 3 are much less significant. This means that the American and the French markets are simultaneously in a low or high volatility states and that the there is no latency of the contagion effect; the contagion is transmitted almost immediately. This result is compatible with the studied sector, the IT sector where information is transmitted instantly, unlike traditional sectors.

Smoothed probabilities of kinds 1 and 4 in the appendix, show the significance of the contagion effect of the NASDAQ-100 over the IT.CAC and we can clearly see that both indexes are in high volatility states during the formation of the New Technology speculative bubble. This effect is also clear on the graphs of the rolling 21 day standard deviations (see appendix). It is clear that the volatility increased considerably and simultaneously in the two IT markets and the sample correlation coefficient between the two standard deviation is as high as 0.76 (76%).

The estimators of the bivariate SWARCH model in Table 5 below are significant which confirms the co-movement hypothesis made earlier in the univariate section:

	Bi-SWARCH(1,1)	
c^{nas}	-0,042	(0,0299)
c^{cac}	-0,003	(0,002)
ϕ^{nas}	-0,059	(0,003)
ϕ^{cac}	0,156	(0,022)
a_0^{nas}	5,307	(2,165)
a_0^{cac}	2,719	(1,273)
a_1^{nas}	0,023	(0,012)
a_1^{cac}	0,003	(0,001)

Table 5. Estimators for the Bi-SWARCH model

4. Conclusion

In this article we examined the volatilities of the American and the French New Technology indexes, the NASDAQ-100 and the IT.CAC. We show on the one hand that the volatilities of the two indexes have considerable risen because of the New Technology speculative bubble, and on the other hand, we showed that there is co-movement between the volatilities of those two indexes by using a bivariate SWARCH model.

Furthermore, we find that the contagion effect between the two series is minimal which lead to the conclusion that the the indexes are simultaneously in a low volatility state or in a high volatility state. Classical conditional volatility models, like GARCH models for example are unable to provide such informations, even if they model the volatility of financial time series in a good way.

Following the empirical results that we find, we can make a number of conclusions:

- including regimes in volatile financial time series models is very important, even a constant variance changing regime model can give results as good as a conditional variance models.
- in the case of volatile financial time series, there is evidence of heteroskedasticity in the regimes, so allowing for an ARCH or a GARCH effect is important in changing regime models.
- The GARCH effect which is absent in the SWARCH model is important in case the series studied exhibit a GARCH effect.

Several extensions are possible, for example, a SWGARCH model which works in the case of 3 or 4 states, another possibility is to develop a bivariate SWGARCH model.

References

1. Akaike, H., 1976, "Canonical correlation analysis of time series analysis and the use of an information criterion", dans : Raman K. Mehra and Dimitri G. Lainiotis, eds, System identification : Advances and case studies (Academic Press, New York, NY).
2. Ang, A. and G. Bekaert (1998), "Regime Switches in Interest Rates," NBER Working Paper No. 6508.
3. Archer G. and Titterington D., (1995), "Parameter estimation for hidden Markov chains", Technical report, University of Glasgow, Department of Statistics.
4. Baillie, R. T. and R. P. DeGennaro, (1990), "Stock returns and volatility", Journal of Financial and Quantitative Analysis 25, 203-214.
5. Bates, D. S., (1991), "The carsh of '87 : Was it expected ? The evidence from option markets", Journal of Finance 46, 1009-1044.
6. Bera, A. K., E. Bubnys and H. Park, (1988), "Conditional heteroskedasticity in the markets model and efficient estimates of betas", Financial Review 23, 201-214.
7. Black, F., (1976), "Studies of stock market volatility changes", 1976 Proceedings of the American Statistical Association, Business and Economic Statistics Section, 177-181.
8. Bollerslev, Tim, (1987), "A conditionally heteroskedastic time series model for speculative prices and rates of return", Review of Economics and Statistics 69, 542-547.
9. Bollerslev, T., Ray Y. Chou and Kenneth F. Kroner, (1992), "Arch modeling in Finance : A review of the theory and empirical evidence", Journal of Econometrics, 52, 5-59.
10. Cai, J. (1994), "A Markov Model of Unconditional Variance in ARCH," Journal of Business and Economic Statistics, 12, 309-316.
11. Campbell, J. Y., A. W. Lo and A. C. MacKinlay, (1997), The Econometrics of Financial Markets, Princeton University Press.
12. Cecconi, M., Giampiero M. Gallo and M. J. Lombardi, (2002), "GARCH-based volatility forecasts for market volatility indexes", working paper, Università degli Studi di Firenze, Dipartimento di Statistica.
13. Chaudhuri, K. and F. Klaassen (2000), "Have East Asian Stock Markets Calmed Down? Evidence from a Regime Switching Model," mimeo, University of Amsterdam.
14. Edwards, S. and R. Susmel, (2001), "Volatility dependence and contagion in emerging equity markets", Working paper, IASE Seminar in Buenos Aires.
15. Engle, R. F. and T. Bollerslev (1986), "Modeling the Persistence of Conditional Variances", Econometric Reviews, 5, 1-50.
16. Engle, R. F.and C. Mustafa, (1992), "implied ARCH models from Options Prices", Journal of Econometrics, 52, 289-311.
17. Filardo, A.J. (1994), "Business-cycle phases and their transitional dynamics," Journal of Business and Economic Statistics, 12, 299-308.
18. Francq, C. and J.-M. Zakoian, (1999), "Linear-Representation Based Estimation of Switching-Regime GARCH Models", Institut National de la Statistique and des Etudes Economiques papier no. 9957.
19. Franses, P. H. and D. van Dijk, (2000), Non-Linear Time Series Models in Empirical Finance, Cambridge University Press.
20. Gable, J., S. van Norden and R. Vigfusson, (1995), "Analytical derivatives for Markov switching models", Working paper, Bank of Canada.
21. Glosten, L.R., R. Jagannathan, and D. Runkle (1993) On the Relation between the Expected Value and the Volatility of the Normal Excess Return on Stocks, Journal of Finance, 48, 1779-1801.
22. Gray, S.F. (1996a), "Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process", Journal of Financial Economics, 42, 27-62.
23. Gray, S. F. (1996b), "An Analysis of Conditional Regime-Switching Models", Working Paper, Fuqua School of Business, Duke University.
24. Hamilton, J. D. (1988), "Rational Expectations Econometric Analysis of Changes in Regime," Journal of Economic Dynamics and Control, 12, 385-423.
25. Hamilton, J. D. (1989), "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle", Econometrica, 57, 357-384.
26. Hamilton, J.D. and R. Susmel (1994), "Autoregressive Conditional Heteroskedasticity and Changes in Regime", Journal of Econometrics, 64, 307-333.

27. Hamilton, J. D. (1996), "Specification Testing in Markov-Switching Time Series Models," Journal of Econometrics, 70, 127-157.
28. Hamilton, J. D. and G. Lin, (1996), "Stock market volatility and the business cycle", UCSD Economics Discussion Papers, no. 96-18, University of California at San Diego, Department of Economics.
29. Hamilton, J. D., (1994), Time Series Analysis, Princeton University Press.
30. Kim, C.-J. and C. R. Nelson, (1999), State-Space Models with Regime Switching, MIT Press.
31. Klaassen, F. (2002), "Improving GARCH Volatility Forecasts with Regime-Switching GARCH," Empirical Economics, 27, 363-394
32. Krolzig, H.-M., (1997), Markov-Switching Vector Autoregressions, Springer.
33. Lamoureux, C. G. and W.. D. Lastrapes, (1990), "Persistence in Variance, Structural Change, and the GARCH Model," Journal of Business and Economic Statistics, 8, 225-234.
34. Lamoureux, C. G. and W. D. Lastrapes, (1993), "Forecasting stock return variance: toward an understanding of stochastic implied volatilities", Review of Financial Studies, 5, 293, 326.
35. Nelson, D. B. (1991) "Conditional Heteroskedasticity in Asset Returns: A New Approach," Econometrica, 59, 347–370.
36. Nelson, C. R., R. Startz and C. M. Turner, (1989), "A Markov model of heteroskedasticity, risk, and learning in the stock market", Journal of Financial Economics 25, 3-22.
37. Pagan, A.R. and G.W. Schwert (1990), "Alternative Models for Conditional Stock Volatility," Journal of Econometrics, 45, 267-290.
38. Ramchand, L. and R. Susmel (1998), "Volatility and cross correlation across major stock markets," Journal of Empirical Finance, Vol.5, 397-416.
39. Schaller, H. and S. van Norden, (1994), "Speculative behaviour, regime-switching, and stock market crashes", Working paper, Bank of Canada.
40. Schwert, G. W., (2002), "Stock volatility in the new millennium: how wacky is NASDAQ?", Journal of Monetary Economics 49, 3-26.
41. Schwert, G. W., (1989), "Why does StockMarket Volatility Change over Time?" Journal of Finance, 44, 1115-1153.
42. So, M., K. Lam and W. Li (1998), "A Stochastic Volatility Model With Markov Switching," Journal of Business and Economic Statistics, 16, 244-53.
43. Susmel, R. (2000), "Switching Volatility in International Equity Markets," International Journal of Economics and Finance.
44. Teräasvirta, T. (1994), "Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models", Journal of The American Statistical Association, 89, 208–218.
45. Timmermann, A. (2000), "Moments of Markov Switching Models," Journal of Econometrics, 96, 75-111.
46. van Norden, S. and R. Vigfusson, (1996a), "Can regime-switching tests detect bublles?", Working paper, Bank of Canada.
47. van Norden, S. and R. Vigfusson, (1996b), "Regime-switching models: a guide to the Bank of Canada Gauss precedures", Working paper, Bank of Canada.
48. Zakoian, J.M. (1990) Threshold Heteroskedastic Models, manuscript, CREST, INSEE, Paris.
49. Zakoian, J.–M. (1994), "Threshold Heteroskedastic Models", Journal of Economic Dynamics and Control, 18, 931-955.

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Appendix A. Descriptive statistics of the two series

Descriptive statistics for the IT.CAC
Sample: 04/01/1999 14/08/2002
Number of observations: 1271

IT.CAC	Mean	Median	Max.	Min.	Stand. Dev.	Asym.	Kurtosis	Obs.
[0, 1000)	817.8103	794.54	996.84	633.88	115.5408	0.116798	1.490647	71
[1000, 2000)	1333.74	1285.245	1959.6	1000.06	246.9123	0.99548	3.067775	538
[2000, 3000)	2502.235	2467.305	2998.86	2002.9	304.9252	0.166785	1.729722	182
[3000, 4000)	3271.471	3188.4	3921.83	3003.18	232.9316	1.00056	3.008213	116
[4000, 5000)	4155.1	4135.59	4363.65	4015.65	116.0502	0.501824	1.901583	9
Total	1799.028	1384.785	4363.65	633.88	831.8377	0.865439	2.617248	916

Table 6. Descriptive statistics for the IT.CAC

Augmented Dickey-Fuller test for the IT.CAC				
ADF stat.	-0.85744		1% Critical value	-3.4402
			5% Critical value	-2.8651
			10% Critical value	-2.5687
Variable	Coeff.	Stand. Dev.	t-stat.	Prob.
CAC(-1)	-0.00188	0.002194	-0.85744	0.3914
D(CAC(-1))	0.19976	0.033255	6.006877	0
D(CAC(-2))	-0.05428	0.03389	-1.60172	0.1096
D(CAC(-3))	-0.02844	0.033879	-0.83941	0.4015
D(CAC(-4))	0.008253	0.033256	0.248155	0.8041
C	2.964034	4.357022	0.680289	0.4965

Table 7. Augmented Dickey-Fuller test for the IT.CAC

ADF test for the IT.CAC after first differences				
ADF stat.	-13.1269		1% Critical value	-3.4402
			5% Critical value	-2.8651
			10% Critical value	-2.5687
Variable	Coeff.	Stand. dev.	t-stat.	Prob.
D(CAC(-1))	-0.86368	0.065795	-13.1269	0
D(CAC(-1),2)	0.062478	0.059078	1.057553	0.2905
D(CAC(-2),2)	0.00726	0.051266	0.141624	0.8874
D(CAC(-3),2)	-0.02103	0.042614	-0.49342	0.6218
D(CAC(-4),2)	-0.01743	0.03325	-0.5241	0.6003
C	-0.40448	1.819434	-0.22231	0.8241

Table 8. ADF test for the IT.CAC after first differences

Desciptive statistics for the NASDAQ-100
Sample: 1/10/1997 14/08/2002
Number of observations: 1271

NASDAQ-100	Mean	Median	Max.	Min.	Stand.-dev.	Asym.	Kurtosis	Obs.
[0, 1000)	964.8405	975.67	998.46	857.08	34.60176	-1.34478	4.122783	41
[1000, 2000)	1429.74	1391.11	1999.04	1000.7	267.344	0.387757	2.169127	663
[2000, 3000)	2356.226	2330.025	2999.78	2000.18	249.616	0.611176	2.591512	266
[3000, 4000)	3572.002	3622.1	3998.26	3003.49	271.7814	-0.40952	2.129734	207
[4000, 5000)	4303.725	4264.045	4704.73	4030.26	203.6289	0.373506	1.944481	40
Total	2075.42	1751.11	4704.73	857.08	947.1657	0.88532	2.632854	1217

TABLE 9. Desciptive statistics for the NASDAQ-100

ADF test for the NASDAQ-100				
ADF stat.	-0.94996		1% Critical value	-3.4402
			5% Critical value	-2.8651
			10% Critical value	-2.5687
Variable	Coeff.	Stand.-dev.	t-stat.	Prob.
NASDAQ100(-1)	-0.002	0.002106	-0.94996	0.3423
D(NASDAQ100(-1))	-0.06763	0.028815	-2.34689	0.0191
D(NASDAQ100(-2))	-0.07568	0.028873	-2.62102	0.0089
D(NASDAQ100(-3))	-0.02285	0.028872	-0.79148	0.4288
D(NASDAQ100(-4))	0.015973	0.028809	0.554464	0.5794
C	3.898322	4.809552	0.810538	0.4178

TABLE 10. ADF test for the NASDAQ-100

ADF test for the IT.CAC after first differences				
Statistique ADF	-16.662		1% Critical value	-3.4402
			5% Critical value	-2.8651
			10% Critical value	-2.5687
Variable	Coeff.	Stand.-dev.	t-stat.	Prob.
D(NASDAQ100(-1))	-1.1883	0.071318	-16.662	0
D(NASDAQ100(-1),2)	0.119851	0.063241	1.89514	0.0583
D(NASDAQ100(-2),2)	0.042312	0.053566	0.789907	0.4297
D(NASDAQ100(-3),2)	0.016188	0.042173	0.38385	0.7012
D(NASDAQ100(-4),2)	0.029214	0.028807	1.014155	0.3107
C	-0.27092	1.989252	-0.13619	0.8917

TABLE 11. ADF test for the NASDAQ-100 after first differences

Appendix B. Estimation Results

Specification	Number of param.	Log-like.	AIC	Schwarz
IT.CAC (Sample=913 observations):				
AR(1)-ARCH(2)	5	-2105	-2111	-2113
AR(1)-ARCH-L(1)	5	-2110	-2115	-2118
AR(1)-EGARCH(1)	5	-2113	-2119	-2121
AR(1)-GARCH(1,1)	5	-2040	-2045	-2048
AR(1)-GARCH-L(1,1)*	6	-2031	-2037	-2039
AR(1)-EGARCH(1,1)	6	-2033	-2039	-2041
Simple regime change	4	-2059	-2055	-2064
AR(1)-SWARCH(2,1)-Gauss.	7	-2084	-2091	-2094
AR(1)-SWARCH-L(2,1)-Gauss.	8	-2080	-2088	-2092
AR(1)-SWARCH(2,2)-Gauss.	8	-2075	-2083	-2087
AR(1)-SWARCH-L(2,2)-Gauss.	9	-2070	-2079	-2083
AR(1)-SWARCH(2,3)-Gauss.	9	-2055	-2064	-2068
AR(1)-SWARCH-L(2,3)-Gauss.	10	-2050	-2060	-2065
AR(1)-SWARCH-L(3,1)-Gauss.	11	-2025	-2036	-2041
AR(1)-SWARCH(3,1)-Gauss.	10	-2071	-2081	-2085
AR(1)-SWARCH(3,2)-Gauss.	11	-2019	-2030	-2035
AR(1)-SWARCH-L(3,2)-Gauss.*	12	-2012	-2024	-2029
AR(1)-SWARCH-L(3,2)-Student	13	-2025	-2038	-2044
AR(1)-SWARCH(3,3)-Gauss.	13	-2023	-2036	-2042
AR(1)SWGARCH(1,1)-Gauss.*	12	-1991	-2003	-2008
NASDAQ-100 (Sample=1213 observations):				
AR(1)-ARCH(2)	5	-2938	-2944	-2946
AR(1)-ARCH-L(1)	5	-2961	-2967	-2969
AR(1)-EGARCH(1)	5	-2968	-2974	-2977
AR(1)-GARCH(1,1)	5	-2884	-2889	-2892
AR(1)-GARCH-L(1,1)*	6	-2862	-2868	-2871
AR(1)-EGARCH(1,1)	6	-2865	-2872	-2875
Simple regime change	4	-2899	-2895	-2905
AR(1)-SWARCH(2,1)-Gauss.	7	-2954	-2961	-2965
AR(1)-SWARCH-L(2,1)-Gauss.	8	-2938	-2946	-2950
AR(1)-SWARCH(2,2)-Gauss.	8	-2921	-2929	-2933
AR(1)-SWARCH-L(2,2)-Gauss.	9	-2916	-2925	-2930
AR(1)-SWARCH(2,3)-Gauss.	9	-2915	-2924	-2929
AR(1)-SWARCH-L(2,2)-Student-t	10	-2917	-2927	-2932
AR(1)-SWARCH-L(2,3)-Gauss.	10	-2908	-2918	-2923
AR(1)-SWARCH(3,1)-Gauss.	10	-2895	-2905	-2910
AR(1)-SWARCH-L(3,1)-Gauss.	11	-2870	-2881	-2887
AR(1)-SWARCH(3,2)-Gauss.	11	-2867	-2878	-2884
AR(1)-SWARCH-L(3,2)-Gauss.*	12	-2863	-2875	-2882
AR(1)-SWGARCH(1,1)-Gauss.*	12	-2862	-2874	-2879

Table 12. Estimation results

Note: The number of parameters for the GARCH estimations does not include the initial variance estimator $\hat{\sigma}^2$. The number of paramters for the SWARCH-L(3,2) does not include the transition probabilities p_{ij}. The second column indicates the maximum value of the log-likelihood function. Twice the difference between two maximum likelihood values of two different specifications is distributed as χ^2 with degrees of freedom equal to the difference between the number of parameters of the two specifications. AIC is calculated as L-k, where k is the number of parameters in column 1. Schwartz is calculated as L-(k/2).ln(T), where T is the size of the sample.

Appendix C. Figures

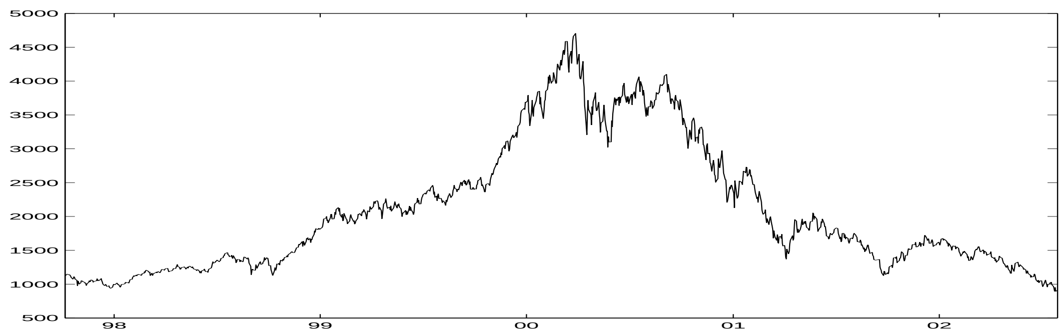

Figure 1. NASDAQ-100

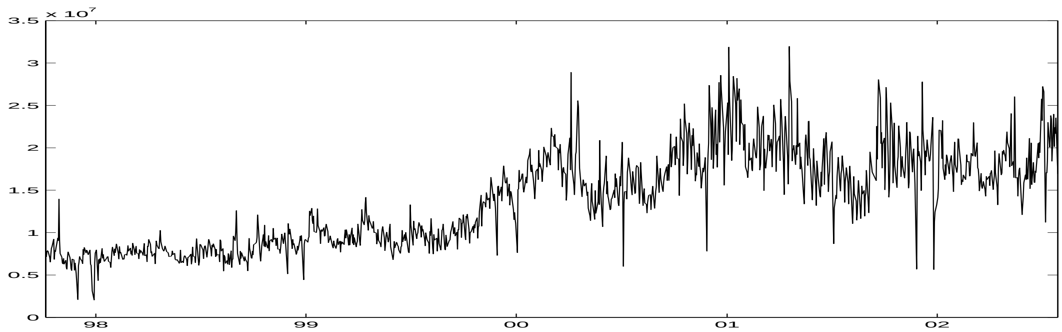

Figure 2. NASDAQ-100 Exchange Volumes

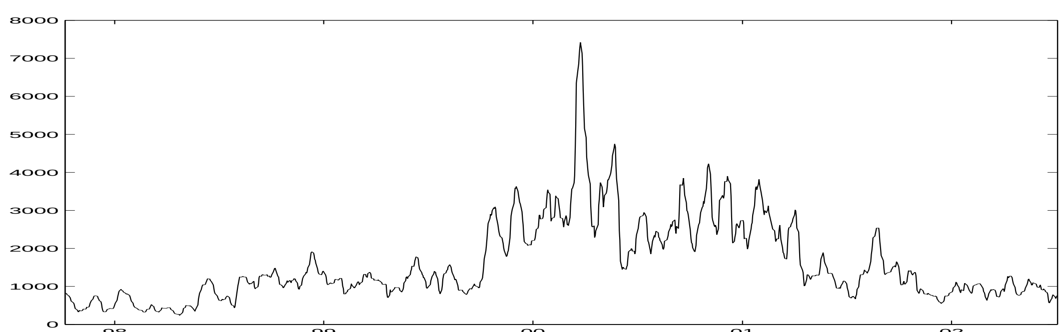

Figure 3. NASDAQ-100 21-day Rolling Standard Deviations (RSD)

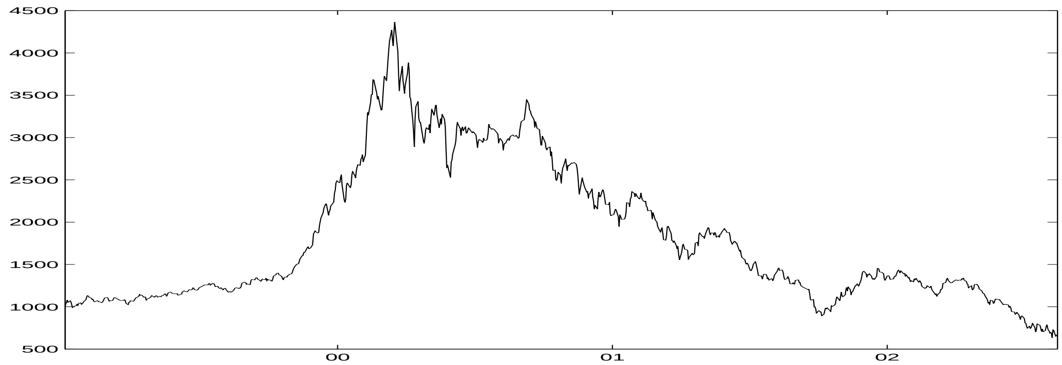

Figure 4. IT.CAC

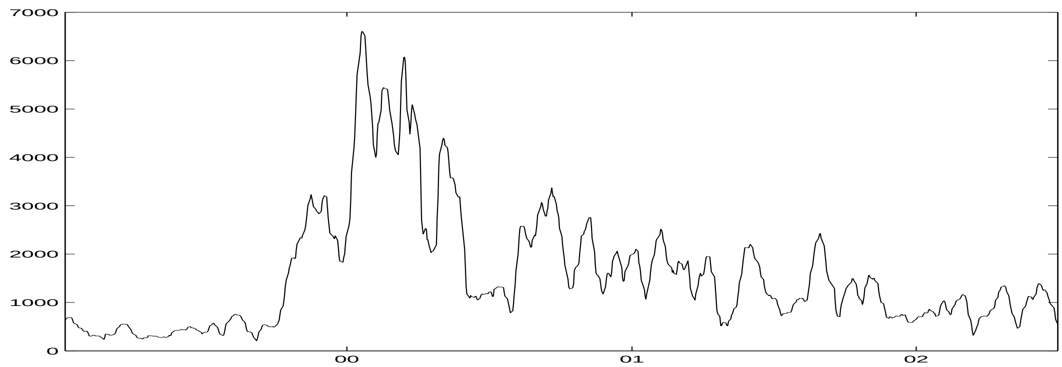

Figure 5. RSD IT.CAC

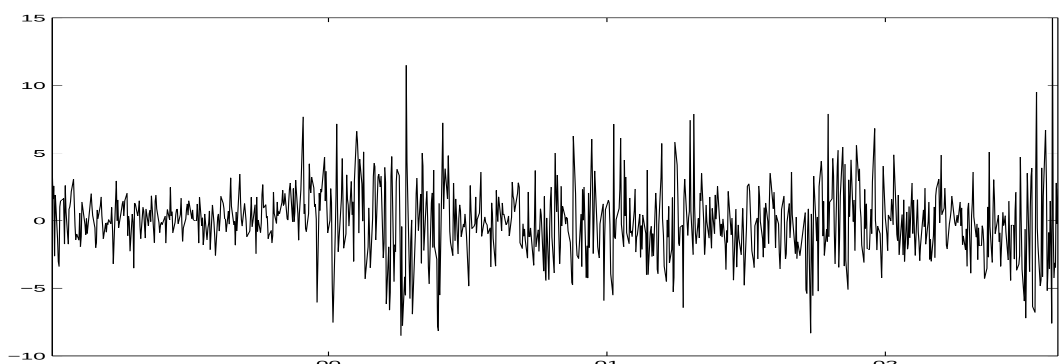

Figure 6. D(%)IT.CAC

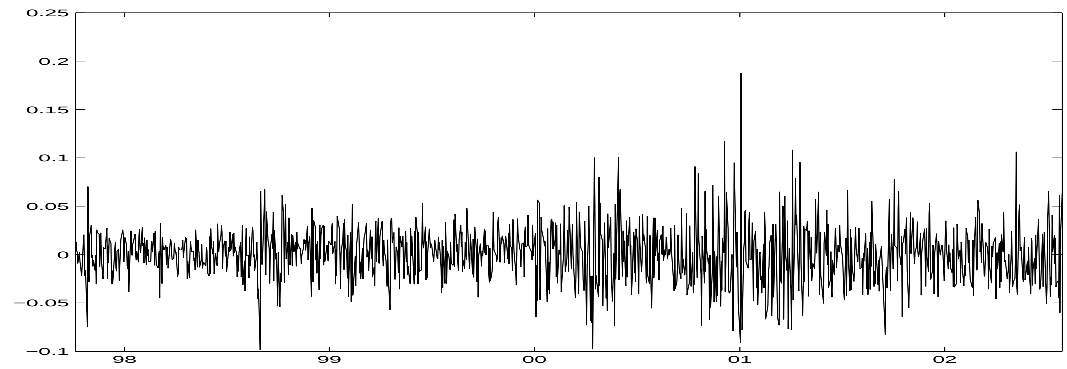

Figure 7. D(%)NASDAQ-100

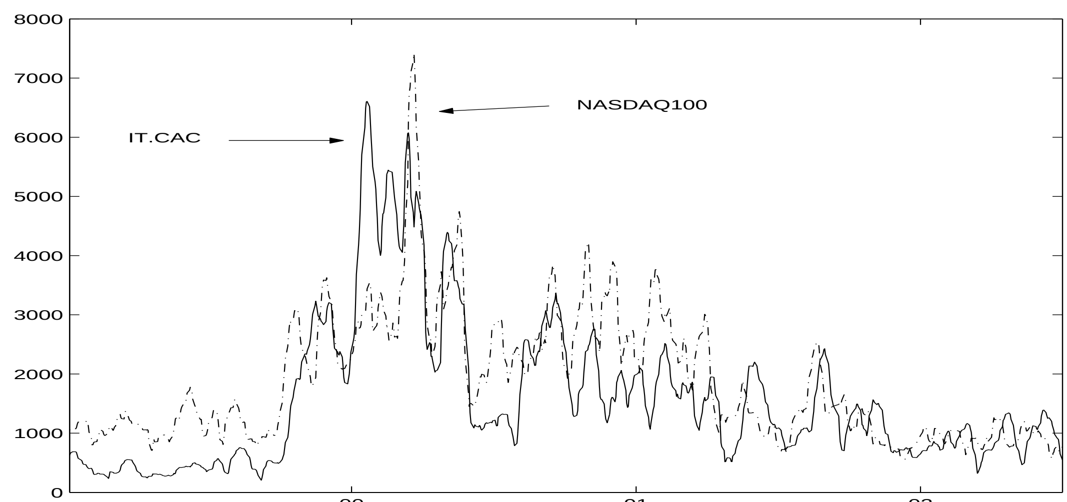

Figure 8. RSD IT.CAC and NASDAQ-100

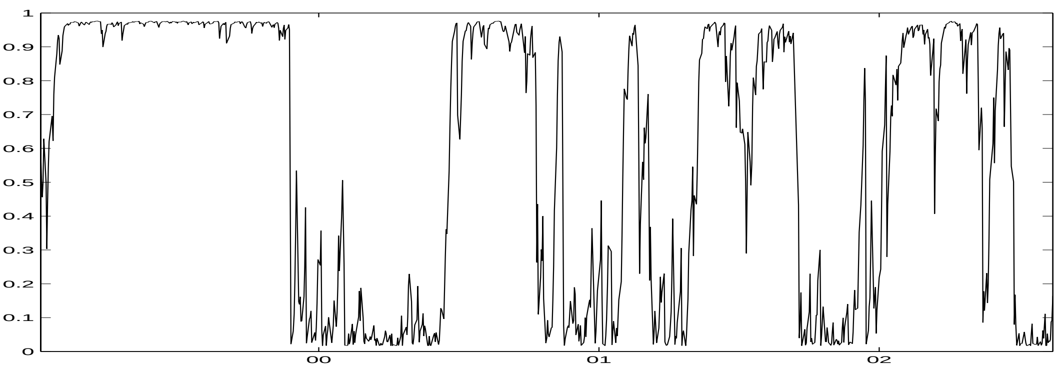

FIGURE 9. IT.CAC $P(s_t = 1)$ ex-ante for constant variance switching model

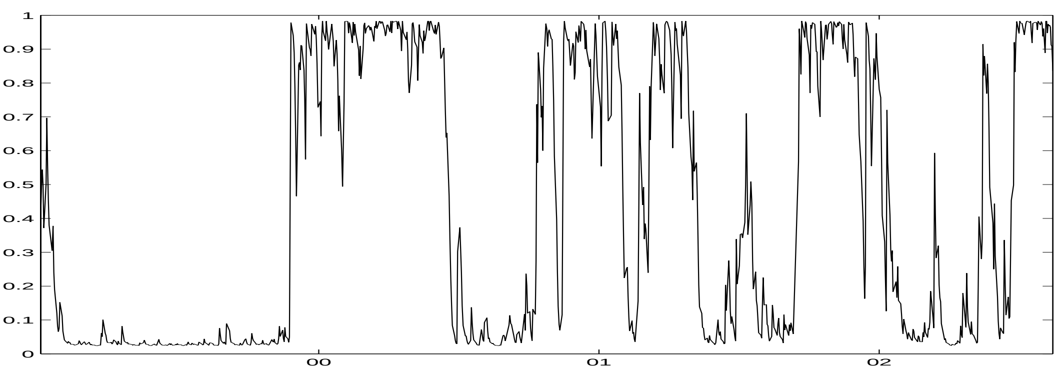

FIGURE 10. IT.CAC $P(s_t = 2)$ ex-ante for constant variance switching model

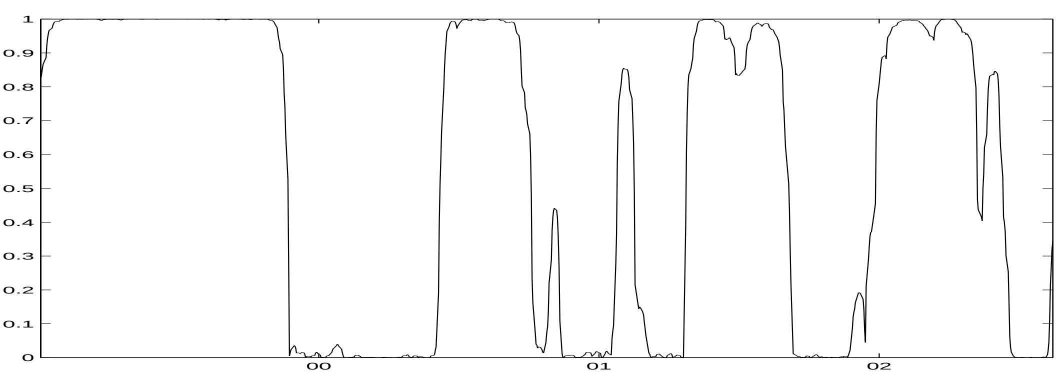

FIGURE 11. IT.CAC $P(s_t = 1)$ smoothed probabilities for constant variance switching model

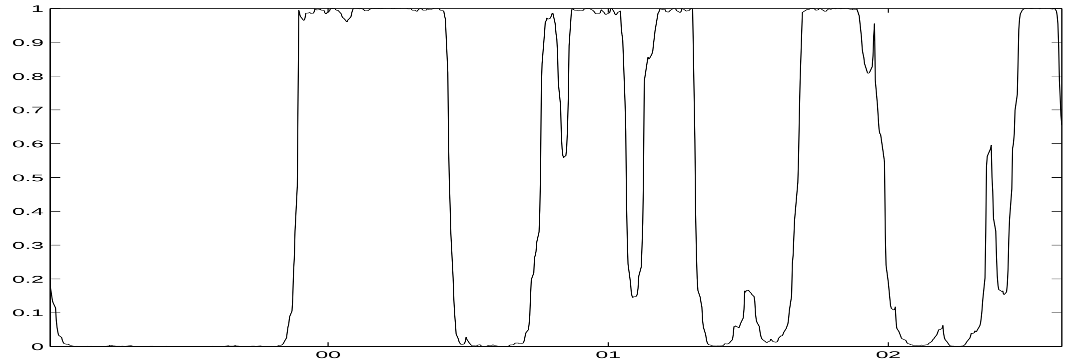

FIGURE 12. IT.CAC $P(s_t = 2)$ smoothed probabilities for constant variance switching model

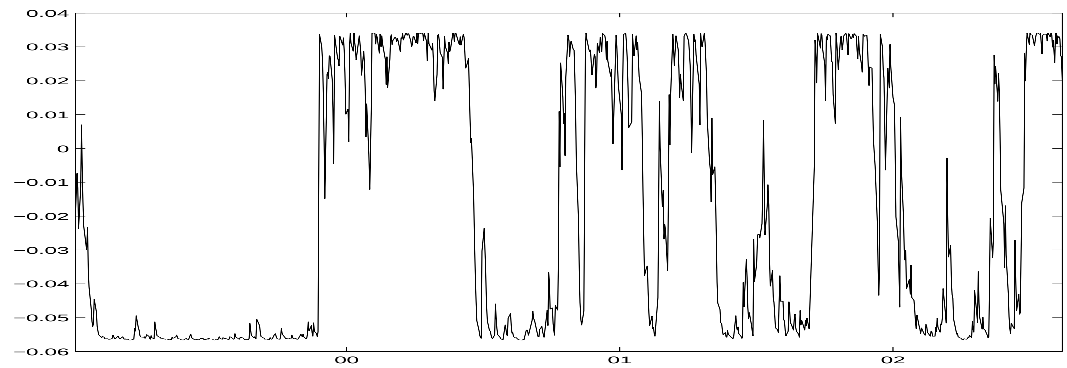

FIGURE 13. IT.CAC Conditional Means for constant variance switching model

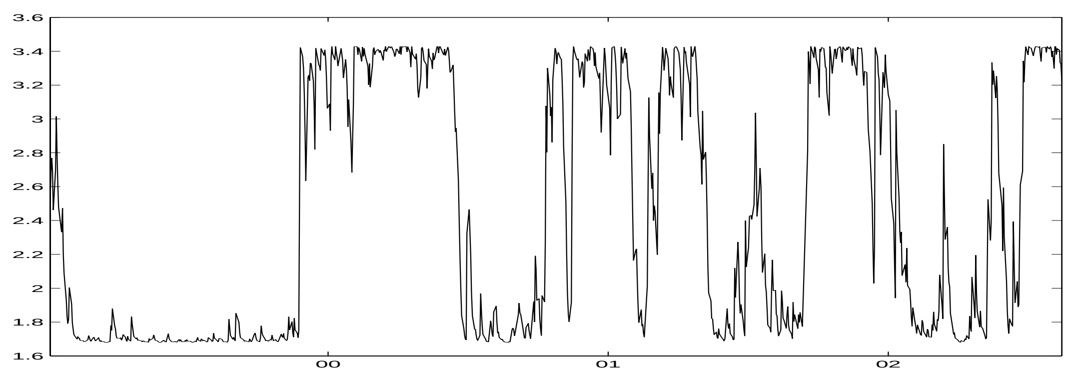

FIGURE 14. IT.CAC Conditional Standard Deviations for constant variance switching model

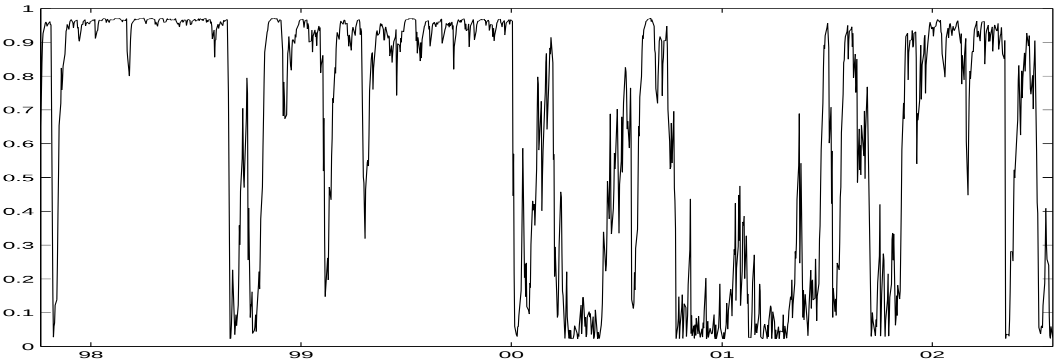

FIGURE 15. NASDAQ-100 $P(s_t = 1)$ ex-ante for constant variance switching model

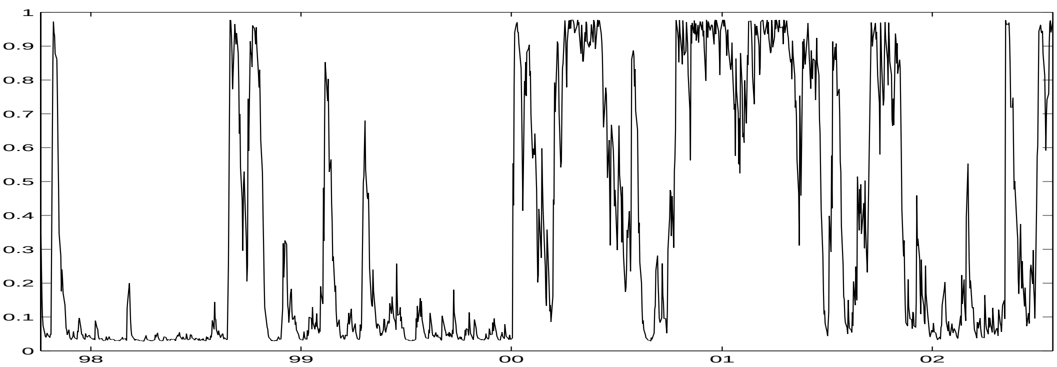

FIGURE 16. NASDAQ-100 $P(s_t = 2)$ ex-ante for constant variance switching model

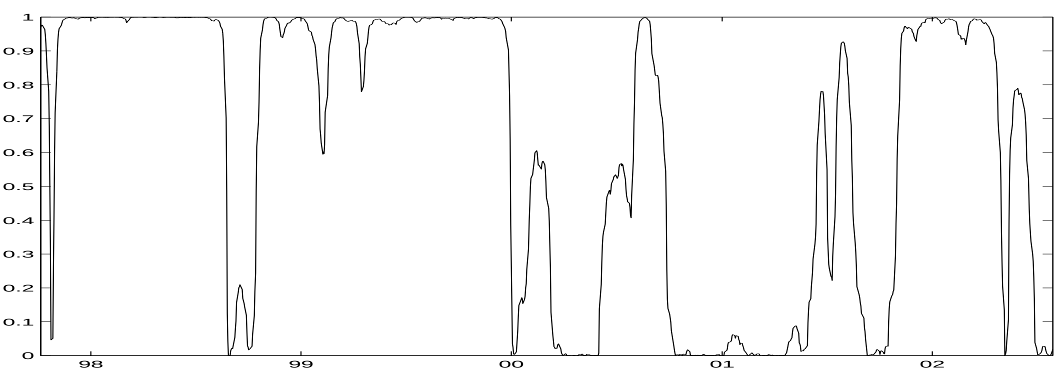

FIGURE 17. NASDAQ-100 $P(s_t = 1)$ smoothed probabilities for constant variance switching model

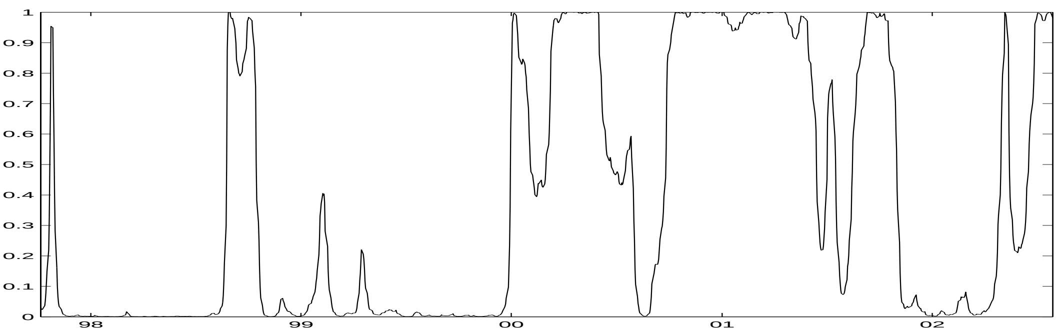

Figure 18. NASDAQ-100 $P(s_t = 2)$ smoothed probabilities for constant variance switching model

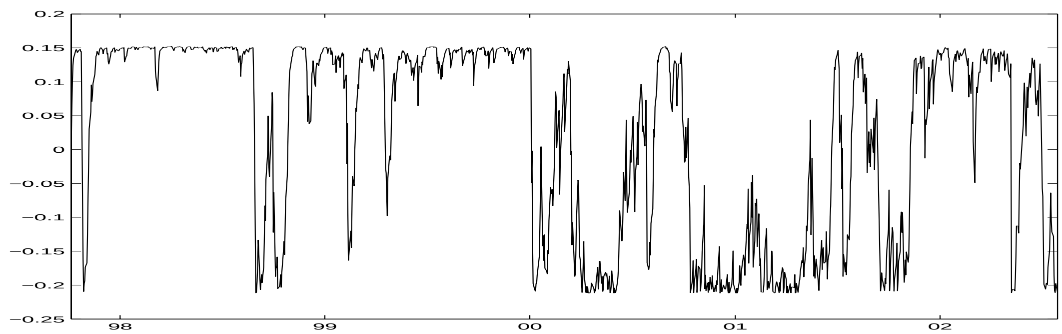

Figure 19. NASDAQ-100 Conditional Means for constant variance switching model

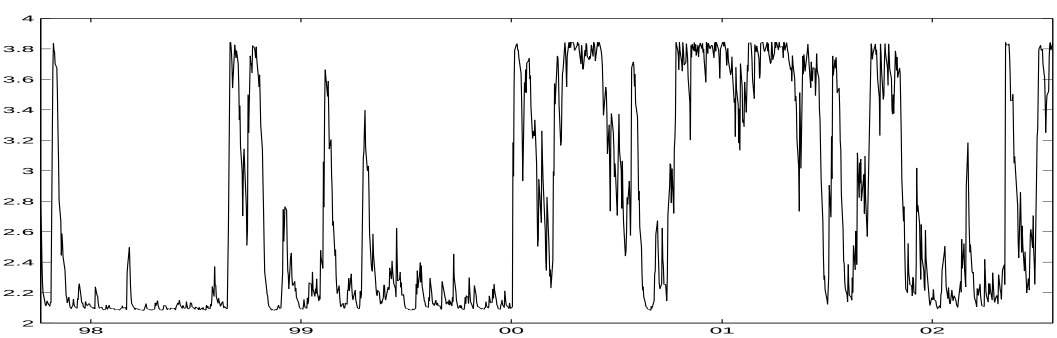

Figure 20. NASDAQ-100 Conditional Standard Deviations for constant variance switching model

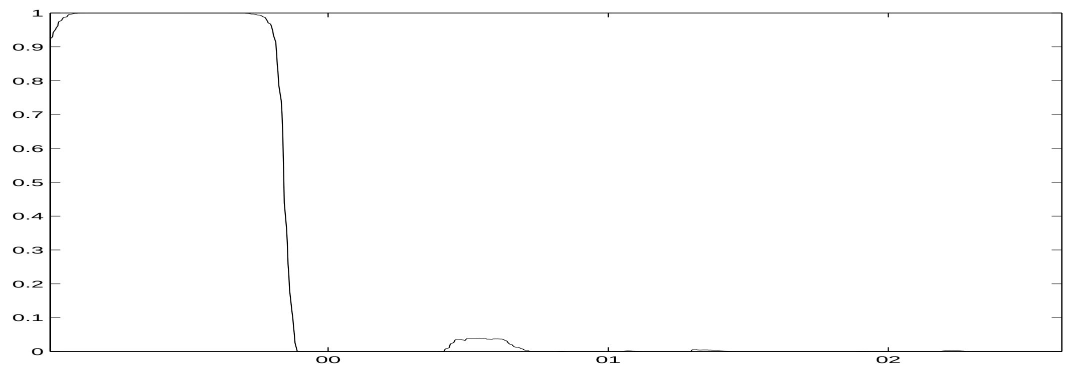

FIGURE 21. IT.CAC $P(s_t = 1)$ smoothed probabilities for univariate AR(1)-SWARCH-L(3,1)

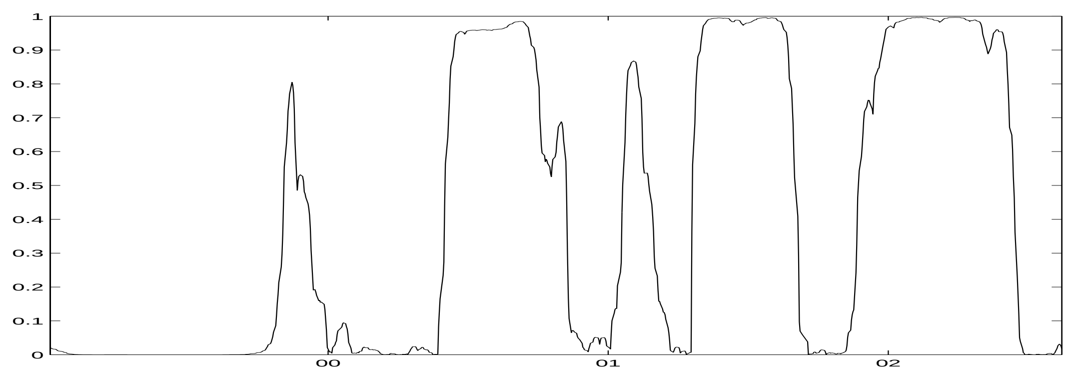

FIGURE 22. IT.CAC $P(s_t = 2)$ smoothed probabilities for univariate AR(1)-SWARCH-L(3,1)

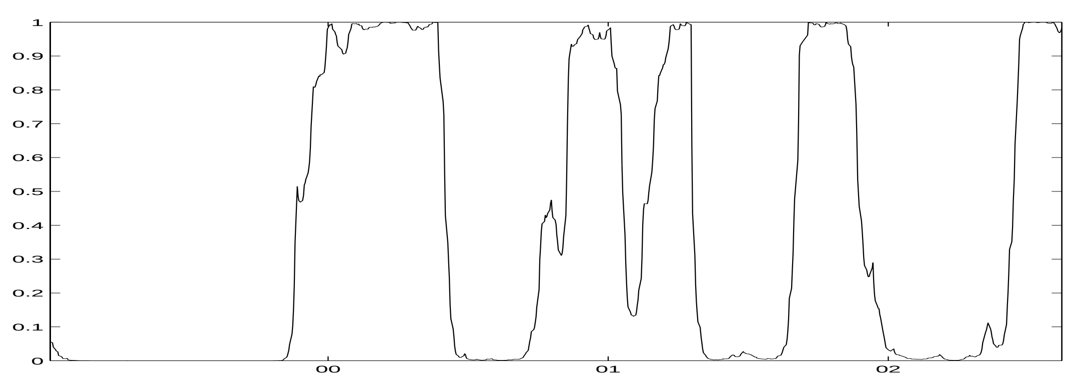

FIGURE 23. IT.CAC $P(s_t = 3)$ smoothed probabilities for univariate AR(1)-SWARCH-L(3,1)

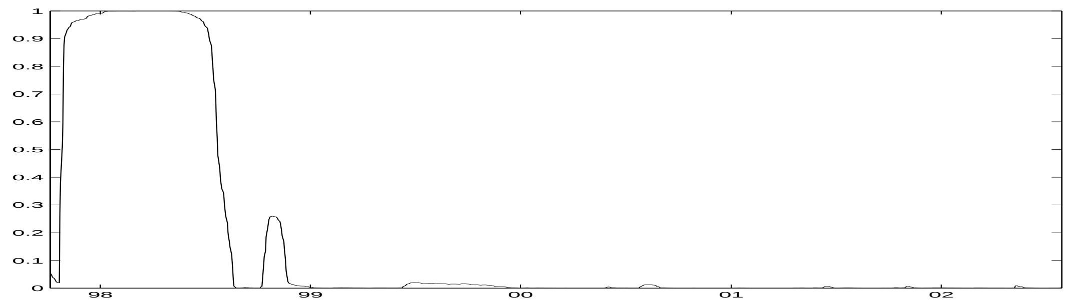

FIGURE 24. NASDAQ-100 $P(s_t = 1)$ smoothed probabilities for univariate AR(1)-SWARCH-L(3,2)

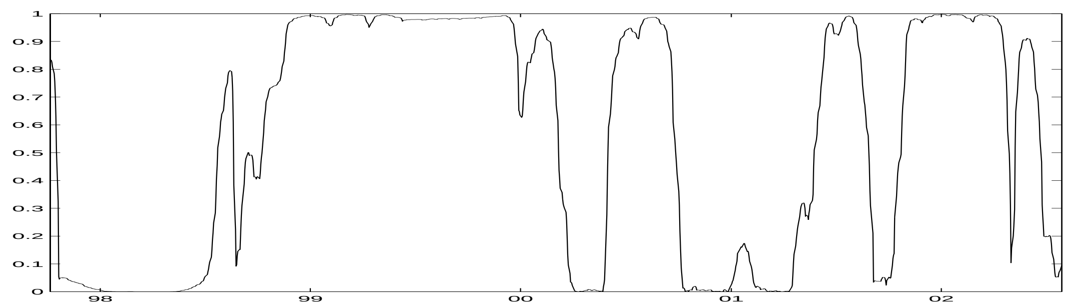

FIGURE 25. NASDAQ-100 $P(s_t = 2)$ smoothed probabilities for univariate AR(1)-SWARCH-L(3,2)

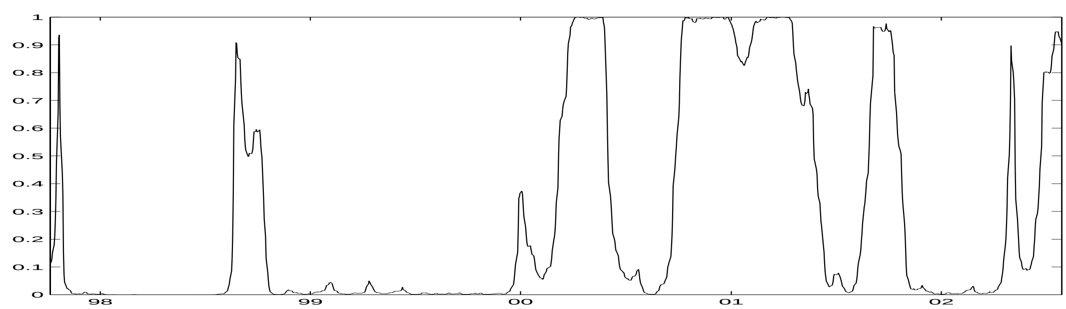

FIGURE 26. NASDAQ-100 $P(s_t = 3)$ smoothed probabilities for univariate AR(1)-SWARCH-L(3,2)

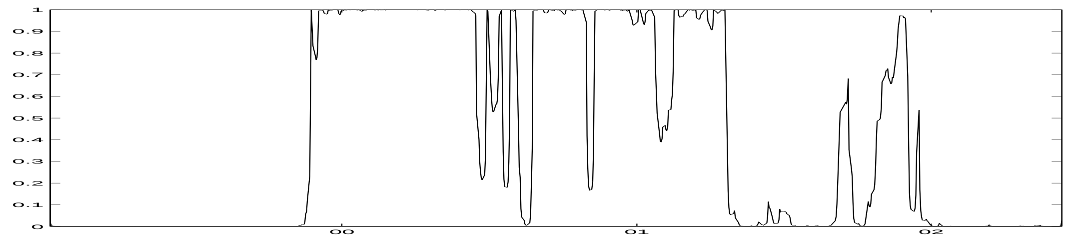

FIGURE 27. IT.CAC $P(s_t = 1)$ smoothed probabilities for univariate SWGARCH(2,1,1)

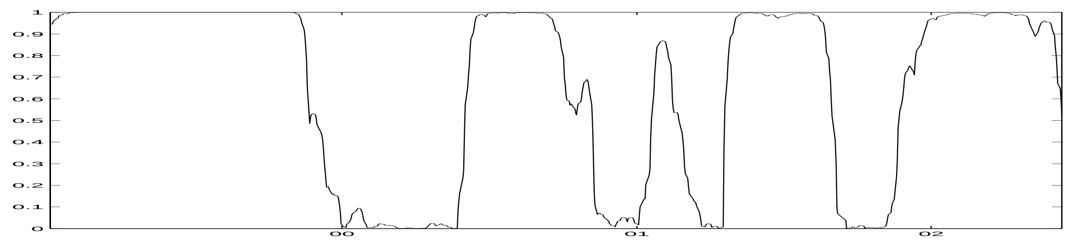

FIGURE 28. IT.CAC $P(s_t = 2)$ smoothed probabilities for univariate SWGARCH(2,1,1)

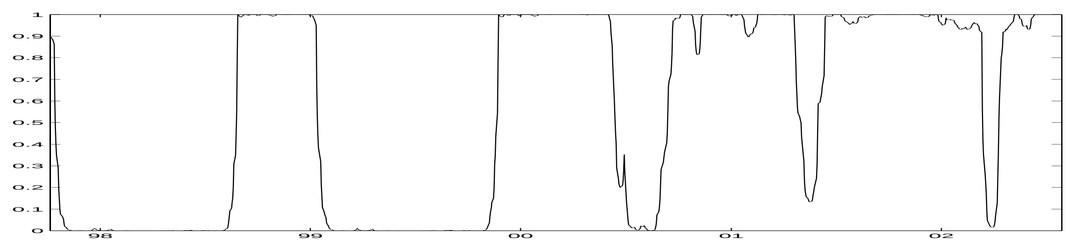

FIGURE 29. NASDAQ-100 $P(s_t = 1)$ smoothed probabilities for univariate SWGARCH(2,1,1)

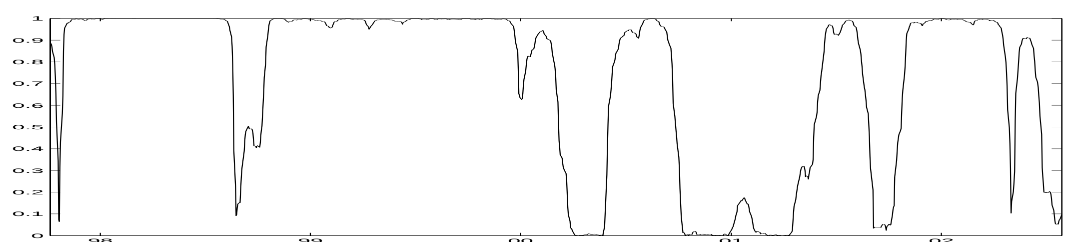

FIGURE 30. NASDAQ-100 $P(s_t = 2)$ smoothed probabilities for univariate SWGARCH(2,1,1)

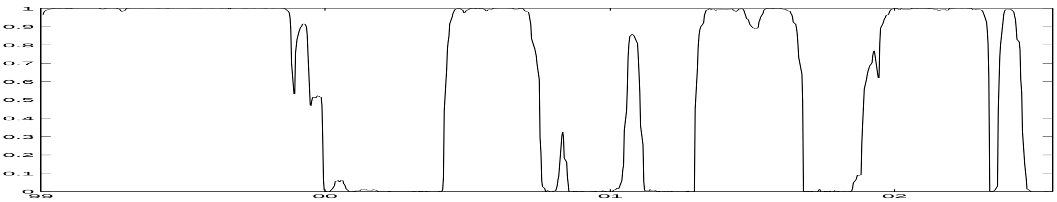

Figure 31. Smoothed probabilities for low-low volatilities for bivariate SWARCH(1,1)

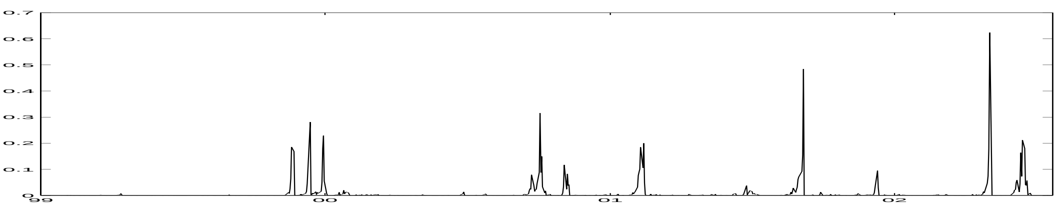

Figure 32. Smoothed probabilities for low-high volatilities for bivariate SWARCH(1,1)

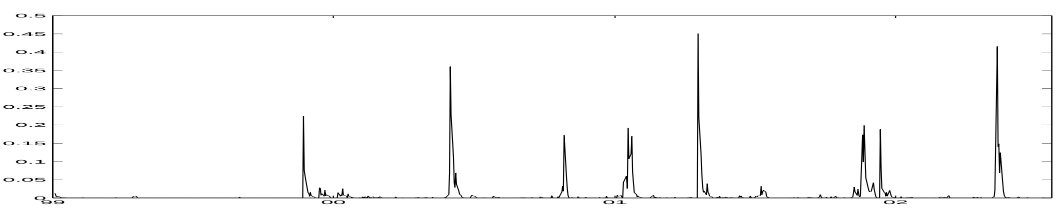

Figure 33. Smoothed probabilities for high-low volatilities for bivariate SWARCH(1,1)

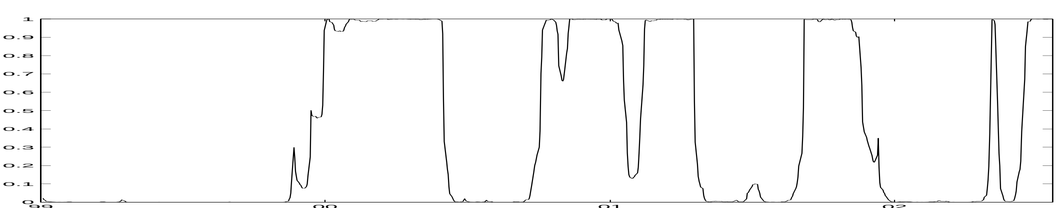

Figure 34. Smoothed probabilities for high-high volatilities for bivariate SWARCH(1,1)