Structural Changes in NICs: Some Evidences on Attractor Points

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Abstract

In this paper we use a regression and develope a kernel density based model for finding fixed points and attractors of dynamical systems to explore attractors of structural change for NICs. The results show that countries consume longer time in some structures than the others. This can be interpreted as existence of attractors that pull countries to themselves in the first stages of the development. In other words one of attractors (low level attractor) prevent countries to reach industrial structure. Awareness of this can be helpful in policymaking for transition from one structure to another. This analysis shades light on the problem that "why some countries can not get ride of traditional structure?" or bad structure phenomena.

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Key Words: Attractors, Structural changes, Fixed Points, Multidimensional Kernel Density, Regression Analysis.

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I) Introduction

Fixed points of functions is completely familiar for physicians and scientists in other branches, however finding fixed points of time series is not so straightforward and it is very important for examining behavior of dynamical systems and chaotic systems. Fixed point analysis is important not only theoretically, it also is very informative in real world applications. Fixed points of time series in real world show local equilibriums, which are interests of policy makers in economies, social scientists and the others.

In the time series, calculation of fixed point is not as easy as functions .Various works such as Aguirre and Souza (1998) for electrical circuits, Guastello (1995) in the employment and inflation, and Schreiber (1998) are good works that have introduced effective methods. Also softwares such as "Chaos Analyzer", "TISEAN" are available on the web for analyzing of fixed points. Here we use the general method of Aguirre and Souza for calculating fixed points and attractors, with little modification on the method. One dimensional and multidimensional kernel density estimation method will be introduced as alternative to other fixed point finding methods. After above debates we will provide some considerations about structural change of economies and its fixed points will be studied by means of some of methods in the literature and our method. If we can conclude that structure of economies have fixed points then we can also conclude that countries have some impediments that prevent from moving toward upper level structures. In some structures such as structures with low share for industry value added , countries are not able to increase their industry's value added share in the GDP .This structure can be treated as "Bad Structure Trap" (BST hereafter), that developing countries have to consume long time on them in the industrialization path. One of our interests here is the existence of BST and attractiveness of it, which is equivalent to finding the fixed points and examining stability of them.

The reminder of this paper is organized as follows. Section2 describes various method of estimation of fixed points and test of attractiveness, also the general nonparametric method is presents in this section. Empirical considerations and estimation of fixed points of industry value added with stability study of them are examined in section 3 .Concluding remarks and suggestions for further studies are covered in the final section.

II) Estimation of fixed points and attractors

Estimation of fixed points for time series is not so easy, because of its two important differences from similar problems in functional form .First, proper functional form is not known in advance .Second, noises are mixed with deterministic effects that may be misleading in finding correct fixed points. An obvious solution to this problem can be getting priory information or relying on theoretical considerations. In this case one can estimate given equation by OLS or MLE and solve equation with given coefficients. For example if estimated form of logistic map be in hand, it can readily be solved to give fixed points.

Fixed or equilibrium point of systems of difference equations is defined by

Definition1: Let $\mathbf{Z_t} = \mathbf{F}(\mathbf{Z_{t-1}})$, $\mathbf{Z} \in \mathbf{R^n}$,the set of values of Z that are mapped to themselves by F are fixed points.

To be more precise: $\mathbf{E} = \{\overline{\mathbf{Z}} | \overline{\mathbf{Z}} - \mathbf{F}(\overline{\mathbf{Z}}) = \mathbf{0}\}$. (1)

One method is application of Taylor approximation for finding fixed points (first or higher order polynomial regression without cross terms in lagged version model). Reasoning behind of this idea is that all of the forms of chaotic and nonchaotic equations, which are Morse type, can be approximated by Taylor expansion. For comparing this method with others, we generated data with various forms of nonlinear difference equations (Sprott 2003, pp417-428).As can be seen in Table 1, precision of this method in finding fixed points of some chaos equations is superior to other method (Aguirre and Souza).

In Table 1 deterministic equations are added by an error term that is composed of $u \sim$ i.i.d.N(0,1) and $\bar{y}$ with different magnitude relative to the dependent variable.

Table 1: Results of fixed point estimation by various methods

Equation name	equation	Theoretical fixed points	$error = 0.1*u*\bar{y}$	$error = 0.2*u*\bar{y}$	$error = 0.5*u*\bar{y}$	$error = 0.7*u*\bar{y}$	$error = u*\bar{y}$
Logistic map	$y_t = 3.92 y_{t-1}(1 - y_{t-1})$	0.7449	Estimated fixed points				
Aguirre			0.6919	0.6550	0.6123	0.6173	0.6017
polynomial			0.7213	0.6883	0.6369	0.6258	0.6209
Ricker's map	$y_t = 20 y_{t-1} \exp(-y_{t-1})$	2.995732					
Aguirre			3.29	3.28	3.20	3.14	3.10
polynomial			3.38	3.33	3.22	3.18	3.17
Cubic logistic	$y_t = 3 y_{t-1}(1 - y_{t-1}^2)$	±0.816497					
Aguirre			-.8165, +.8165	-.8165, +.8165	-.81649 +.8165	-.81646 +.8165	-.81637 +.81637
polynomial			-.8165, +.8165	-.8165, +.8165	-.8165, +.8165	-.8165, +.8165	-.8165, +.8165
Sin	$y_t = \sin(\pi y_{t-1})$	0.736485					
Aguirre			0.7398	0.6870	0.5682	0.5461	0.5161
polynomial			0.7350	0.7104	0.1516 ±0.7672i	0.5559	0.5072

For doing this:

1-Rregress time series on a polynomial of arbitrary degree.
2-Test statistical significance of the coefficients of polynomial.
3-Delet insignificant coefficient terms and let
$y_t = y_{t-1} = \cdots = y_{t-k} = \bar{y}$
4-Solve equation analytically for the fixed points.

This method can be used for detecting fixed points of system or simultaneous equations. This is a little modification on Aguirre method with some further statistical tests such as t-ratio and F test .Another interesting method which presented by Paul So et al.(1996) based on sharp changes of frequency that occur in the fixed points .For finding fixed points in this method ,variable should be transformed by the following formula.

$$\hat{y}_n = [y_{n+1} - S_n(k)]/[1 - S_n(k)] \quad (2)$$

Where $S_n(k) = (y_{n+2} - y_{n+1})/(y_{n+1} - y_n) + k(y_{n+1} - y_n),\ k \in [-1,1]$ (3)

After transformation, equation2, probability density function of $\hat{y}$ can be estimated by histogram or other nonparametric density estimation methods such as kernel density estimation. In the fixed point $\hat{\rho}(\hat{y})$ has sharp peaks due to its distribution:

$$\hat{\rho}(\hat{y}) \sim \left|\hat{y} - y^*\right|^{-1/2} \quad (4)$$

To avoid the possibility of other singularities that are not due to true fixed points, one can choose some random values for k and get average values for $\hat{y}$ .One of the problems with this method is problem of the very large and small values after the transformation relative to pre transformed data. This method is sufficiently powerful, even for small samples, but it has two shortcomings for very large experimental data. First, say for a series with 10000 numbers of observations one should calculate 10000*500 data in order to detect real fixed points instead of other singularities. It seems formidable task when we repeat it for different quantity of error term. Second, some problems arise from graphical nature of it, which depends on band width (window width) in kernel or histogram of the probability density function. Therefore, because of these problems we do not test this approach with experimental data; however it will be used in the small sample studies and estimation of fixed points of the structural changes.

Aguirre and Souza (1998) method is similar to polynomial regression but there are two important differences, we do not use clustering and our estimation results is filtered by statistical significance tests and reliability of the estimated equation at the whole .In other words, we put aside insignificant coefficients in calculation of polynomial roots of the estimated equation. Another problem with Aguirre and Souza algorithm is that results are highly sensitive to window length (L) and partially to increment between windows (delta) .We shall illustrate this by using a logistic equation:

$$y_t = 3.92 y_{t-1}(1 - y_{t-1}) \quad (5)$$

Let 0.1 be the initial point, for L=7000 and delta=1000 algorithm estimate fixed point very accurately but for L=1000 and delta=1000 or L=1000 and delta= 5,10,20,50,100,500 results are completely misleading .However, estimates are more stable than the previous ones, when we use variable z which is a linear combination of $u \sim N(0,1)$ and y ($z = y + 0.1\bar{y}u$).The results from simple polynomial regression, in this case, is 0.7241 and Aguirre and Souza's one is 0.69635 , whereas real (theoretical) fixed point is 0.7449.Therefore , polynomial regression gives a little better result than that of Aguirre and Souza algorithm. For the further analysis error term magnitude raised so that $z = y + \bar{y}u$ which Aguirre's algorithm shows instability and estimation of fixed point is nearly 0.6 in the best situation. Polynomial regression gives 0.5982 which is not better than the first one but it is stable and is independent of L and delta, in contrast to Aguirre and Souza method. As one can see from table 1, for other types of equations, polynomial regression method works as well as the Aguirre and Souza method. In small sample also polynomial regression method works quite well, for example in the case of $z = y + \bar{y}u$ and only 100 observations fixed point is calculated 0.7079 which is near to real fixed point in spite of large magnitude of error terms. For the equations of table 1 estimation of Aguirre and Souza algorithm is unstable, but in few cases it is a little accurate than polynomial regression method. In the experimental data we know correct value of fixed point and then we can choose the window length such that estimates can be as near as we want, but in the cases of real data this can not be applicable. An important point in the method of polynomial regression is the truncation of insignificant coefficient which should be done for getting accurate roots. Aguirre's method uses visual method for deletion of insignificant clusters, which is not common from statistical point of view. Although averaging fixed point of various windows increases reliability, however it cannot be substantial in low frequency data.

Aguirre et al. (1998) method use following equation for the estimation fixed points:

$$\begin{aligned} y_t &= \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4} + \beta_5 y_{t-1}^2 + \beta_6 y_{t-2}^2 + \beta_7 y_{t-1} y_{t-2} \\ &+ \beta_8 y_{t-3} y_{t-4} + \beta_9 y_{t-1}^3 + \beta_{10} y_{t-1} y_{t-2}^2 + \beta_{11} y_{t-3}^3 + \beta_{12} y_{t-1} y_{t-2} y_{t-3} + \varepsilon \end{aligned} \quad (6)$$

The above equation is a third order Taylor expansion of a general form of a single variable difference equation with clustered variables

$$y_t = f(y_{t-1}, y_{t-2}, \ldots, y_{t-k}) \quad , L^k y \in R \quad \text{for } n \in N \quad (7)$$

This approximation is reasonable, straightforward and easily extendable for the systems of difference equations. However, as a key point about this equation in estimation fixed points, when accuracy in neighborhood of fixed points is the first priority and in the fixed points $y_t = y_{t-1} = \cdots = y_{t-k} = \bar{y}$, there is no need for cross terms in the equation and only a simple polynomial of three or four order suffices for finding fixed points. However in real world data, most observations are not points of fixed point's neighborhood .But why simple equation may work better than the (6) in some cases? The answer is in correlation of regressors in the neighborhood of fixed points:

If $y_{t-i} \to \bar{y}$ for i=0, 1,2,3,4

Then $y_{t-1}^2, y_{t-2}^2, y_{t-1} y_{t-2}, y_{t-3} y_{t-4} \to \bar{y}^2$ and $y_{t-1}^3, y_{t-3}^3, y_{t-1} y_{t-2}^2, y_{t-1} y_{t-2} y_{t-3} \to \bar{y}^3$ (8)

Therefore there are only three distinct regressors and ignoring this fact cause high multicolinearity between regressors. The consequences of multicolinearity are inefficiency of estimations and instability of estimated coefficients due to change in sample size and observations even when this change is negligible. This case is relevant for the Aguirre et al. method. (Table2).whereas the simple polynomial

regression has not this shortcoming and because of this strength of polynomial it may performs better than the equation (6).
Of course this debate can be generalized to higher order approximation of difference equations.

$$y_t = \alpha + \sum_{i_1=1}^{k} \beta_i y_{t-i_1} \sum_{i_2=1}^{k}\sum_{i_1=1}^{k} \beta_{i_1 i_2} y_{t-i_1} y_{t-i_2} + \cdots + \sum_{i_n=1}^{k} \cdots \sum_{i_2=1}^{k}\sum_{i_1=1}^{k} \beta_{i_n \cdots i_2 i_1} y_{t-i_n} \cdots y_{t-i_2} y_{t-i_1} + \varepsilon_t$$

Also in this case, in the neighborhood of fixed point, as $y_{t-i_n} \to \bar{y}$, $i = 1,2,\cdots,k$, $n \in N$ and all of terms go to $\bar{y}, \bar{y}^2, \cdots, \bar{y}^n$ respectively. Therefore, there are only n distinct regressors instead of $k + k(K+1)/2 + \cdots$ and ignoring this fact lead to colinearity .In words for finding fixed points of a time series in neighborhood of fixed point, one can use this regression

$$y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-1}^2 + \cdots + \beta_n y_{t-1}^n + \varepsilon_t$$

The above illustration leads us to proposition 1, however before that we need two following definitions:
Definiton2. The fixed point $\bar{y}$ of $y_t = f(y_{t-1}, y_{t-2},..., y_{t-p})$ is stable provided that given any ball $B(\bar{y},\varepsilon) = \{y \in R / |y - \bar{y}| < \varepsilon\}$, there is a ball $B(\bar{y},\delta) = \{y \in R / |y - \bar{y}| < \delta\}$ such that if $y \in B(\bar{y},\delta)$ then $f^t(y) \in B(\bar{y},\varepsilon)$, for $t = 0,1,2,\cdots$ (Kaslik et al.2003, p.2).

Definition3. Let $\bar{y}$ be an asymptotically stable fixed point of $y_t = f(y_{t-1})$ then, the set

$$\mathbf{S}(\bar{\mathbf{y}}) = \left\{\mathbf{y} \in \mathbf{R} \middle| \lim_{\mathbf{t}\to\infty} |\mathbf{y}_{\mathbf{t}-1} - \bar{\mathbf{y}}| = 0\right\}$$

is the basin of attraction $\bar{y}$ (Medio and Lenis 2001,pp.67-68).

Proposition1. Let $y_t = f(y_{t-1}, y_{t-2},..., y_{t-p})$ a pth order difference equation, in the basin of attraction, definition1, it can be approximated by $y_t = g(y_{t-1})$.
Corollary1. In the basin of attraction colinearity is more sever than other regions of definition of lagged regression.

Therefore correctness of Augirre and Souza method depend on distance of observation from the fixed points. Individually insignificant coefficient and high R-square show this problem. Also significance test that proposed by Aguirre and Souza cannot be statistically attractive method. Furthermore socioeconomic time series are very short and reduction of degree of freedom is a serious problem. Specification in backward method based on t-test will lead deletion of most of regressors and estimated fixed point will be incorrect.
In spite of above weakness Augirre and Souza method works very well in finding fixed points of systems .The point behind of this , is related effects of co linearity which only cause inefficiency in estimation and not biasedness. Therefore estimated coefficients in Aguirre method are unbiased in general and low t-ratios are not important for finding fixed points. We use here this method for finding fixed points and test attractness of points based on some further steps introduced in following sections.

In Aguirre and Souza method final fixed points are calculated by averaging fixed points of each subsample, however we take averages of estimated coefficients and find roots of obtained polynomial. This method can be better than when coefficients very unstable due to high multicolinearity .th is line of reasoning can illustrated by following equations

Let $x_t = (a+1)x_{t-1} - b + \varepsilon$ equation for generation a time series and subsampls $S_1, S_2, \cdots, S_p$ of this time series be available for estimation equation. For finding fixed point when a and b are known one can simply let $a\bar{x} - b = 0$ and solve it for $\bar{x}$.however this is not the case in the empirical works and one of the reasonable strategies is estimation of a and b for each of samples .then solve each estimated equation and get average of roots for finding root of $x_t = (a+1)x_{t-1} - b + \varepsilon$. (Aguirre and Souza method).our solution to this problem is averaging of coefficient instead of roots. Let estimated coefficients be denoted by $\{\hat{a}_i\}_{i=1}^p$ and $\{\hat{b}_i\}_{i=1}^p$ also suppose $\hat{a}_i = a_i + \nu_i$ and $\hat{b}_i = b_i + \eta_i$ where ν_i, η_i are white noise error terms. According to Aguirre and Souza method:

$$\hat{\bar{x}}_{a\_S} = p^{-1}\sum_{i=1}^{p}\frac{\hat{b}_i}{\hat{a}_i} = p^{-1}\sum_{i=1}^{p}\frac{b+\eta_i}{a+\nu_i} = \frac{1}{p}\frac{\sum_{j=1}^{p}(b+\eta_i)\prod_{\substack{i \\ i\neq j}}^{p}(a+\nu_i)}{\prod_{i=1}^{p}(a+\nu_i)} \tag{9}$$

Only in the case $\eta_i = 0, \nu_i = 0$, $\bar{x}$ be calculated correctly. This condition may be not met and it is stronger than $p^{-1}\sum \eta_i = 0, p^{-1}\sum \nu_i = 0$.In second method (i.e. averaging of coefficients) roots calculated as follows

$$\hat{\bar{x}} = \frac{\sum_{i=1}^{p} p^{-1}\hat{b}_i}{\sum_{i=1}^{p} p^{-1}\hat{a}_i} = \frac{\sum_{i=1}^{p} p^{-1}(b+\eta_i)}{\sum_{i=1}^{p} p^{-1}(a+\nu_i)} = \frac{b + p^{-1}\sum_{i=1}^{p}\eta_i}{a + p^{-1}\sum_{i=1}^{p}\nu_i} \tag{10}$$

The second method only needs satisfaction of the $p^{-1}\sum \eta_i = 0, p^{-1}\sum \nu_i = 0$ .In small samples with uniform distribution this condition equals to $E(\eta) = 0, E(\nu) = 0$ and in large samples this condition will be satisfied regardless of distribution in population. Therefore we use this method as a modification of Aguirre and Souza here. For polynomials with degree higher than one can write as follows:

$Q_i(x) = f(x, \hat{\beta}_i)$, $i = 1, \cdots, p$

Precision of Aguirre and Souza method depends on $\hat{\beta}_i = \beta$ whereas in coefficients averaging only needs $n^{-1}\sum \hat{\beta}_i = \beta$ condition, which more likely than the first one. Another method by Stephen Guastello (Guastello 1995) is used for detecting attractors of unemployment and inflation by exponential form of logistic equation which is derived by Laplace transformation (Guastello 1995).In this approach logistic equation in the following form should be estimated and iterations should be done for finding fixed points.

$$y_t = \alpha y_{t-1} \exp(-\beta y_{t-1}) + \gamma \quad (9)$$

In addition to the above methods spectral density and frequency analysis can be a practical method especially when even fixed point attractors or attractors in general are main interest. We will use univariate and multivariate kernel density estimation method for finding attractors here.
For examining attractiveness of fixed points in Aguirre and Souza method one can use the following condition:

Let $x_{t+1} = f(x_t) \quad t = 0,1,2,\cdots$ (10)

Be a nonlinear first order autonomous difference equation with fixed points satisfying $\bar{x} = f(\bar{x})$, and let y denote x_{t+1} and x denote x_t, then:

$$y = f(x) \quad (11)$$

Expanding (8) around a fixed point $(\bar{x}, \bar{x})$ gives:

$$y - \bar{x} = f'(\bar{x})(x - \bar{x}) \quad (12)$$

There are three possibilities for the $f'(\bar{x})$ (Shone1997, p.78)

$|f'(\bar{x})| < 1$, then $\bar{x}$ is an attractor

$|f'(\bar{x})| > 1$, then $\bar{x}$ is an repellor

$|f'(\bar{x})| = 1$, then $\bar{x}$ is not attractor or repellor

Therefore we can summarize it as following theorem

Theorem1: Let $x_{t+1} = f(x_t) \quad t = 0,1,2,\cdots$,a first order difference equation defined on R and $\bar{x}$ is a fixed point of it then $\bar{x}$ is attractor iff $|f'(\bar{x})| < 1$.

The regression form of the difference equation for industry value added share is specified as follows:

$$x_t = \alpha + \beta_1 x_{t-1} + \beta_2 x_{t-1}^2 + \beta_3 x_{t-1}^3 + \varepsilon_t \quad (13)$$

Third order expression is used for raising the approximation precision. When $x_t = f(x_{t-1})$ linearized .Also according to (9) kind of fixed point is depends on $|f'(\bar{x})|$: $\hat{x}_t = \hat{\alpha} + \hat{\beta}_1 x_{t-1} + \hat{\beta}_2 x_{t-1}^2 + \hat{\beta}_3 x_{t-1}^3$

$$y = f(\bar{x}) = \hat{\alpha} + \hat{\beta}_1 \bar{x} + \hat{\beta}_2 \bar{x}^2 + \hat{\beta}_3 \bar{x}^3 \tag{14}$$

$$\frac{dy}{d\bar{x}} = f'(\bar{x}) = \hat{\beta}_1 + 2\hat{\beta}_2 \bar{x} + 3\hat{\beta}_3 \bar{x}^2 \tag{15}$$

The results confirm existence of bad structure and attracting nature of low level structures in the LDCs. However, for generalization we should use al of that variables that are related to structures of economies .Hence we should expand the framework to include variables such as MVA ,AVA ,GDPPER , CAPEXP, and FDIGCF .These are some other variables as proxies of different aspect of structural changes .Because of close relation between this macroeconomic variables and endogenity of them ,they will be modeled in a VAR form. However Chenery and Syrquin (1975,1989) believe that per capita GDP is independent variable others (structure variable) are dependent to each other, although Granger causality test don't support this idea and there is two side causality between them.

In a VAR form one can write:

$$\mathbf{Z_t} = \mathbf{F}(\mathbf{Z_{t-1}}, \mathbf{Z_{t-2}}, \cdots, \mathbf{Z_{t-p}}, \boldsymbol{\Gamma}) + \boldsymbol{\nu}_t \quad \mathbf{t} = 1,2,\cdots,\mathbf{T} \ , \tag{16}$$

$$\mathrm{E}(\nu_t) = 0 \quad \mathrm{E}(\nu_t \nu_s') = \begin{cases} \Omega \text{ if } t=s \\ 0 \text{ if } t\neq s \end{cases} \tag{17}$$

Where Z=[IVA,MVA,AVA,GDPPER,CAPEXP,FDIGCF]', $\boldsymbol{\Gamma}$ is a vector of parameters that should be estimated and $\boldsymbol{\nu}_\mathbf{t}$ is a vector of error terms which satisfying classical assumptions .Models are nonlinear systems of pth order difference equations which have linear approximation.

Estimation methods for nonlinear simultaneous equations are NL2S, BNL2S, MNL2S and NLLI, which are debated in Amemiya (1985, pp.245-265).To find fixed points of systems of equation we should estimate it for unknown parameters and then solve it. Hence we involve with nonlinearity in two stages, first in the estimation of nonlinear VAR system (NLVAR hereafter) and second in finding fixed points. However fortunately, that kind of nonlinearity which is important in econometrics, but it is not so important in dynamical system and the nonlinearity in variables that is important in dynamical systems is not important for econometric analysis. Furthermore in dynamical systems nonlinearity can be transformed to a linear one easily. Therefore our equation can be linearized as third order Taylor expansion:

$$\begin{aligned} \mathbf{Z_t} &= \boldsymbol{\Phi}_1 \mathbf{Z_{t-1}} + \boldsymbol{\Phi}_2 \mathbf{Z_{t-2}} + \cdots + \boldsymbol{\Phi}_\mathbf{p} \mathbf{Z_{t-p}} + \boldsymbol{\Psi}_1 \mathbf{Z}^2_\mathbf{t-1} + \boldsymbol{\Psi}_2 \mathbf{Z}^2_\mathbf{t-2} + \cdots + \boldsymbol{\Psi}_\mathbf{p} \mathbf{Z}^2_\mathbf{t-p} \\ &+ \boldsymbol{\Xi}_1 \mathbf{Z}^3_\mathbf{t-1} + \boldsymbol{\Xi}_2 \mathbf{Z}^3_\mathbf{t-2} + \cdots + \boldsymbol{\Xi}_\mathbf{p} \mathbf{Z}^3_\mathbf{t-p} + \boldsymbol{\nu}_\mathbf{t} \end{aligned} \tag{18}$$

Although approximation with higher order lags such as (21), is generally more precise than the AR(1), however due to small number of observations we can't use it. In addition small number observations, in the neighborhood of fixed points, models with higher lags cannot be better than low lag models. To illustrate, let $\bar{\mathbf{Z}}$ be a fixed point of system and $\mathbf{Z_{t-1}} = \mathbf{Z_{t-2}} = \cdots = \mathbf{Z_{t-p}} = \bar{\mathbf{Z}}$,In the neighborhood of $\bar{\mathbf{Z}}$, as $\mathbf{Z_{t-i}} \to \bar{\mathbf{Z}}$ an AR(p) model is equivalent to AR(1).

Poposition2. Let $\mathbf{Z}^{(p)}_\mathbf{t} = \mathbf{F}(\mathbf{Z_{t-1}}, \mathbf{Z_{t-2}}, \cdots, \mathbf{Z_{t-p}})$ and $\mathbf{Z}^{(1)}_\mathbf{t} = \mathbf{F}(\mathbf{Z_{t-1}})$ be systems of pth and first order difference equations respectively, for $\mathbf{Z_{t-i}} \in \mathbf{S}(\bar{\mathbf{Z}})$, $i = 1,\cdots,p$ where S is basin of attraction for the first system, $\left\|\mathbf{Z}_{\mathbf{t}}^{(\mathbf{p})}-\mathbf{Z}_{\mathbf{t}}^{(1)}\right\|<\delta$ such that δ can be an infinite small positive number.

$$S(\overline{Z})=\left\{Z_{t-i}\in R^{n}\left|\lim_{t\to\infty}\right\|Z_{t-i}-\overline{Z}\|=0\right\}$$

Corollary2. According to proposition 2 in the domain of attraction of an AR(p), it can be approximated by an AR(1).

Corollary3. Based on proposition 2 and corollary 2 colinearity in domain of attraction is more severe than the other regions.

There is six variable with three terms for each variable and for p=3, the system will have (6) (6) (3) (3) =324parameters which can't be estimated with 40 number of observations. Therefore we confine the model to first lag and only Z= (IVA GDP AVA).

$$\mathbf{Z_t}=\mathbf{\Phi_1 Z_{t-1}}+\mathbf{\Psi_1 Z_{t-1}^2}+\mathbf{\Xi_1 Z_{t-1}^3}+\boldsymbol{\nu}_\mathbf{t} \tag{19}$$

Which is NLVAR (1).Estimation of model is almost straightforward .However, after estimation solving of the model is not easy in general. After solving the system with Newton method we examine attractness of fixed points with following theorem:

Let $\hat{\mathbf{Z}}_{\mathbf{t}}=\hat{\mathbf{\Phi}}_1\mathbf{Z}_{\mathbf{t}-1}+\hat{\mathbf{\Psi}}_1\mathbf{Z}_{\mathbf{t}-1}^2+\hat{\mathbf{\Xi}}_1\mathbf{Z}_{\mathbf{t}-1}^3 \quad , \quad \hat{\mathbf{Z}}_{\mathbf{t}}=\mathbf{F}(\hat{\mathbf{\Phi}}_1,\hat{\mathbf{\Psi}}_1,\hat{\mathbf{\Xi}},\mathbf{Z}_{\mathbf{t}-1})$ (20)

be a first order nonlinear autonomous system of difference equation.

Theorem2: let $\mathbf{F}:\mathbf{R}^{\mathbf{n}}\to\mathbf{R}^{\mathbf{n}}$ and X_0 is a fixed point of the system, then X_0 is an attractor, iff sum of absolute values of elements of Jacobian matrix evaluated in X_0 be less than one.

To be more specific $\sum_{\mathbf{j}=1}^{\mathbf{n}}\left|\frac{\partial\mathbf{f}_{\mathbf{i}}}{\partial\mathbf{x}_{\mathbf{j}}}\right|_{\mathbf{X}_0}<1 \;,\mathbf{i}=1,2,\cdots,\mathbf{n}$ is the condition of attractness. In our case this condition for VAR (1) can be stated as:

$$\begin{bmatrix} \hat{IVA}_{t-1} \\ \hat{GDP}_{t-1} \\ \hat{AVA}_{t-1} \end{bmatrix}=\begin{bmatrix} \hat{\varphi}_{11} & \hat{\varphi}_{12} & \hat{\varphi}_{13} \\ \hat{\varphi}_{21} & \hat{\varphi}_{22} & \hat{\varphi}_{23} \\ \hat{\varphi}_{31} & \hat{\varphi}_{32} & \hat{\varphi}_{33} \end{bmatrix}\begin{bmatrix} \hat{IVA}_{t-1} \\ \hat{GDP}_{t-1} \\ \hat{AVA}_{t-1} \end{bmatrix}+\begin{bmatrix} \hat{\psi}_{11} & \hat{\psi}_{12} & \hat{\psi}_{13} \\ \hat{\psi}_{21} & \hat{\psi}_{22} & \hat{\psi}_{23} \\ \hat{\psi}_{31} & \hat{\psi}_{32} & \hat{\psi}_{33} \end{bmatrix}\begin{bmatrix} \hat{IVA}_{t-1}^2 \\ \hat{GDP}_{t-1}^2 \\ \hat{AVA}_{t-1}^2 \end{bmatrix}$$
$$+\begin{bmatrix} \hat{\xi}_{11} & \hat{\xi}_{12} & \hat{\xi}_{13} \\ \hat{\xi}_{21} & \hat{\xi}_{22} & \hat{\xi}_{23} \\ \hat{\xi}_{31} & \hat{\xi}_{32} & \hat{\xi}_{33} \end{bmatrix}\begin{bmatrix} \hat{IVA}_{t-1}^3 \\ \hat{GDP}_{t-1}^3 \\ \hat{AVA}_{t-1}^3 \end{bmatrix} \tag{21}$$

From the above theorem:

$$\left|\frac{\partial I\hat{V}A}{\partial IVA}\right|_{\bar{z}} + \left|\frac{\partial I\hat{V}A}{\partial GDP}\right|_{\bar{z}} + \left|\frac{\partial I\hat{V}A}{\partial AVA}\right|_{\bar{z}} < 1$$

$$\left|\frac{\partial G\hat{D}P}{\partial IVA}\right|_{\bar{z}} + \left|\frac{\partial G\hat{D}P}{\partial G\hat{D}P}\right|_{\bar{z}} + \left|\frac{\partial G\hat{D}P}{\partial AVA}\right|_{\bar{z}} < 1 \qquad (22)$$

$$\left|\frac{\partial A\hat{V}A}{\partial IVA}\right|_{\bar{z}} + \left|\frac{\partial A\hat{V}A}{\partial GDP}\right|_{\bar{z}} + \left|\frac{\partial A\hat{V}A}{\partial AVA}\right|_{\bar{z}} < 1$$

Which it is

$$\left|\hat{\varphi}_{i1} + 2\hat{\psi}_{i1}\mathbf{I\overline{V}A} + 3\xi_{i1}\mathbf{I\overline{V}A}^{2}\right| + \left|\hat{\varphi}_{i2} + 2\psi_{i2}\mathbf{G\overline{D}P} + 3\xi_{i2}\mathbf{G\overline{D}P}^{2}\right|$$
$$+ \left|\hat{\varphi}_{i3} + 2\hat{\psi}_{i3}\mathbf{A\overline{V}A} + 3\hat{\xi}_{i3}\mathbf{A\overline{V}A}^{2}\right| < 1 \quad , i = 1,2,3 \qquad (23)$$

Because of problems with specification and estimation in parametric models, use nonparametric approaches such as kernel density can be a valuable method. With this method attractors can be detected without finding fixed points, although some attractors may be a fixed point. Following graphs show attractors for logistic mape with r=2.5 and r=3.52 respectively (initial point x0=0.1).

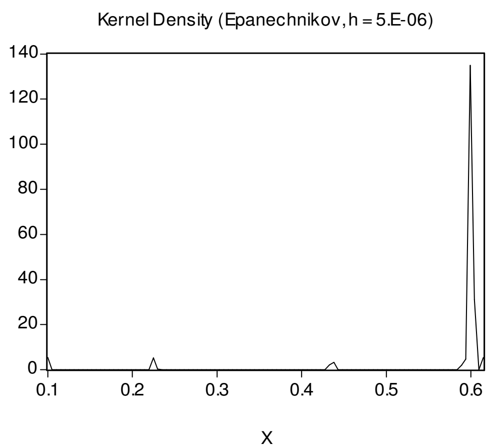


Figure 1: Kernel Density for logistic map with r=2.5 and 0.1 as initial point

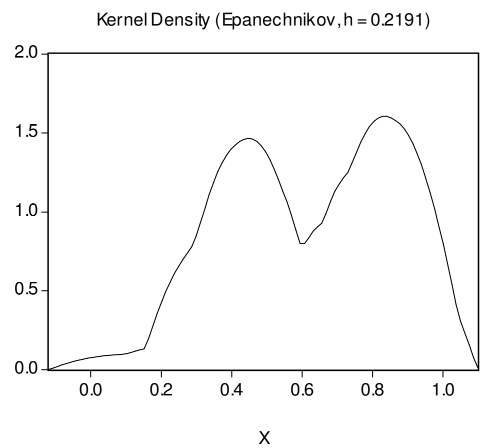


Figure 2: Kernel Density for logistic with r=3.52 and 0.1 as initial point

This method also is useful for finding bifurcation values. None of picks in the second graphs are fixed points however they are attractors. This is true for other maps in table 1.

Multidimensional kernel density estimation method is counterpart for one dimensional case in finding attractors.

$$\hat{\mathbf{f}}\,(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h_1 \cdots h_q}\kappa\left(\frac{x_1 - X_{i1}}{h_1}, \ldots, \frac{x_q - X_{iq}}{h_q}\right)$$

Where, X_{iq} is (iq)th element of random vector $\mathbf{X} = (X_1, \ldots, X_q)^T$, h_q an element of bandwidth vector $h_q = (h_1, \ldots, h_q)^T$, n is number of observations and κ Stands for kernel function in q dimension. One easiest form for $\boldsymbol{\kappa}(\mathbf{u}) = \kappa(u_1, \ldots, u_q)$ is multiplicative kernel $\boldsymbol{\kappa}(\mathbf{u}) = K(u_1) \cdot \ldots \cdot K(u_q)$ and

$$\hat{\mathbf{f}}(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} \prod_{j}^{q} h_j^{-1} K\left(\frac{x_j - X_{ij}}{h_j} \right)$$

We estimate multidimensional kernel for Henon map:

$$\begin{cases} x_t = 1 - 1.4x_{t-1}^2 + 0.3y_{t-1} \\ y_t = x_{t-1} \end{cases}$$

Bandwidth selection is done by following formula

$$\tilde{h}_j = \left(\frac{4}{d+2} \right)^{1/(d+4)} n^{-1/(d+4)} \hat{\sigma}_j$$

Where $\tilde{h}_j$ stands for bandwidth for jth dimension, d dimension of system, n number of observations and $\hat{\sigma}_j$ estimation of standard deviation. In Henon map dimension is two, $\hat{\sigma}_1 = 0.7231$, $\hat{\sigma}_2 = 0.7231$ for 1000 observations and $(0,0)$ as an initial point. Therefore

$$\tilde{h}_1 = \left(\frac{4}{2+2} \right)^{1/(2+4)} (1000)^{-1/(2+4)} (0.7231) = 0.2287$$

$$\tilde{h}_2 = \left(\frac{4}{2+2} \right)^{1/(2+4)} (1000)^{-1/(2+4)} (0.7231) = 0.2287$$

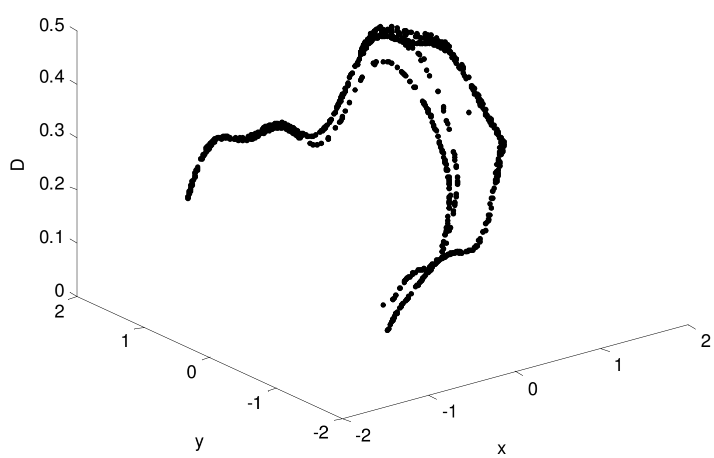


Figure 3: Attractor set of Henon map with bandwidths

$(\tilde{h}_1, \tilde{h}_2) = (0.2287, 0.2287)$ **and evaluated at pairwise points**

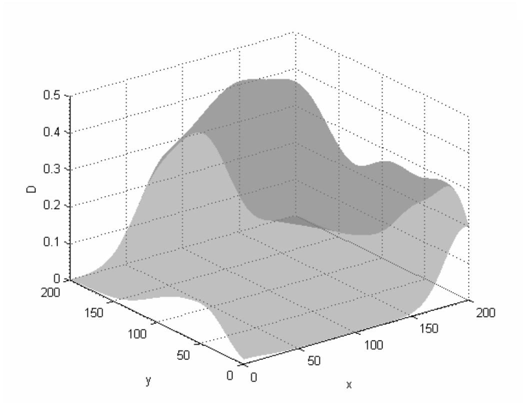


Figure 4: Density of Henon map with bandwidths
$(\tilde{h}_1, \tilde{h}_2) = (0.2287, 0.2287)$

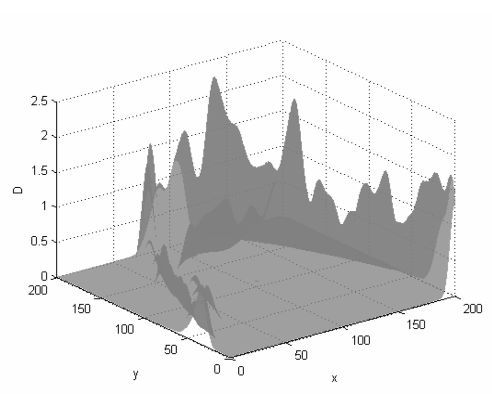


Figure 5: Density of Henon map with bandwidths
$(\tilde{h}_1, \tilde{h}_2) = (0.05, 0.05)$

III) Structural Changes' Fixed Points and Attractors

Structural change in economic literature and economic development is referred to relative importance of sectors in the contribution to GDP and share of sectors from total employment. It should be noted that structural change is a multidimensional phenomenon and is not restricted to merely industrialization. Economists such as Kuznets (Kuznets 1959, 1966),Chenery and Syrquin(Chenery and Syrquin 1975,1989) among the others give a brief definition of structural change. However almost all of the structural analysts believe that change of industry's share in the GDP, Industrialization is at the center of structural change. Therefore industry value added as percent of GDP is the variable which is of our interest here. Finding fixed points of this series can tell us about proper actions in policy making. If one concludes that industry share has fixed points and these fixed points are attractors then she can prescribe that countries consume much time in the transition of some stages and policy makers should pay attention to this in the development plans.
Data source for the industrial value added for newly industrialized countries (NICs hereafter) is World Bank CD (WDI 2004) which repots 575 series for 225 countries over 43 years. This source in spite of its full coverage has some mistakes that may affect results and it should be considered in the interpretations and suggestions. For the Hong Kong data is very short and cannot be analyzed efficiently therefore it deleted from NICs list in this study.

Table2:Fixed points of structural change

INDONESIA	Aguirre(32,1)	9.1458	39.435	51.822
	Aguirre(37,1)	11.127	41.688	41.688
	Aguirre(37,2)	11.059	41.828	41.828
	Coef.Aver(37,1)	10.974	42.793	42.793

KOREA (S.)	Aguirre(32,1)	27.131	29.297	43.920
	Aguirre(37,1)	28.228	28.228	43.218
	Aguirre(37,2)	28.416	28.416	43.198
	Coef.Aver(37,1)	25.449	25.449	43.371
MALAYSIA	Aguirre(32,1)	25.812	40.649	58.286
	Aguirre(37,1)	24.455	39.985	39.985
	Aguirre(37,2)	24.641	40.141	40.141
	Coef.Aver(37,1)	22.348	41.313	43.313
SINGAPOUR	Aguirre(32,1)	18.810	34.675	44.112
	Aguirre(37,1)	20.479	34.659	46.060
	Aguirre(37,2)	32.141		
	Coef.Aver(37,1)	11.558	11.558	35.057
THAILAND	Aguirre(32,1)	28.368	28.368	39.992
	Aguirre(37,1)	27.168	27.168	41.475
	Aguirre(37,2)	27.144	27.144	41.513
	Coef.Aver(37,1)	27.758	27.758	40.209

Finding of fixed points is conducted by each of three methods. Results presented in table3 shows fixed points for five NICs countries. For Indonesia Aguirre algorithm gives three fixed points (11.127, and 41.668) which it's range is a little wider than the ours.(12.911,40.1271).The latter 40.1271 is real part of polynomial roots which drives the behavior of system. Also it gives only two fixed points that is more consistent than three fixed points. The reasoning in favor of two fixed points instead of the three fixed points arises from S-shape dynamism for the structural changes due to upper (100%) and lower (0%) asymptotes .In other words it is supposed that structural changes to be bimodal and economies consume more time in low level structures," Bad Structure", as well as high level structures (high percent of industry value added share) relative to mid periods. Higher readings for fixed points cannot be so reasonable, when economies finally have to converge to long run path and may de industrialized after some periods.

Results of estimation of the fixed points according to our modification (coefficients averaging instead of roots averaging) gives fourth row in above table. Substitution of fixed points value in derivatives of each equation gives us the following table.

Table3: absolute values of derivatives of Structural Changes difference equation

Country	1st Fixed point	2nd Fixed point	3rd Fixed point
INDONESIA	2.3513	**0.7456**	2.5899
	1.0122	**1.0122**	1.9272
	1.0101	**1.0101**	1.9863
KOREA	**0.5063**	**0.9720**	**0.9098**
	0.4973	**0.8624**	**0.8624**
	0.5017	**0.8409**	**0.8410**
MALAYSIA	1.6652	**0.7098**	2.0077
	1.0202	**1.0202**	1.6469
	1.0099	**1.0099**	1.7405
SINGAPORE	**0.8003**	**0.6064**	**1.0013**
	0.9659	1.0425	2.5356

	0.7177	1.5199	2.9195
THAILAND	**0.4267**	**0.9998**	**0.9998**
	0.4538	**0.9608**	**0.9608**
	0.4704	**0.9604**	**0.9604**

So's method lead to three fixed points with density estimation with histogram (graphs1-4) and gives two points by means of kernel density estimation before and after "k" averaging.

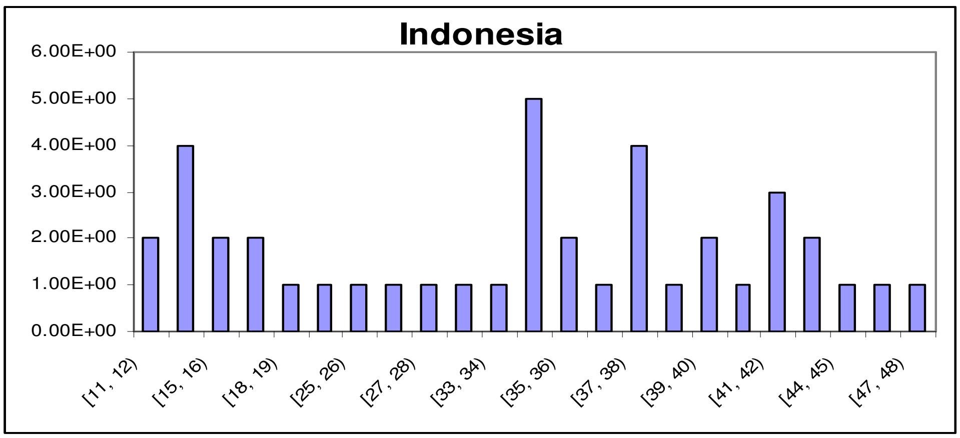
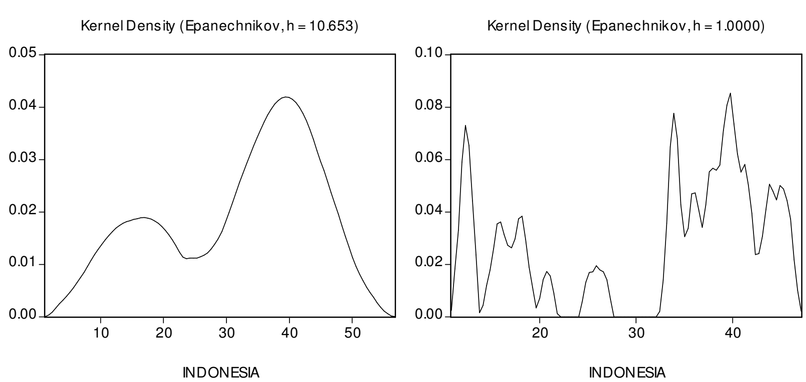
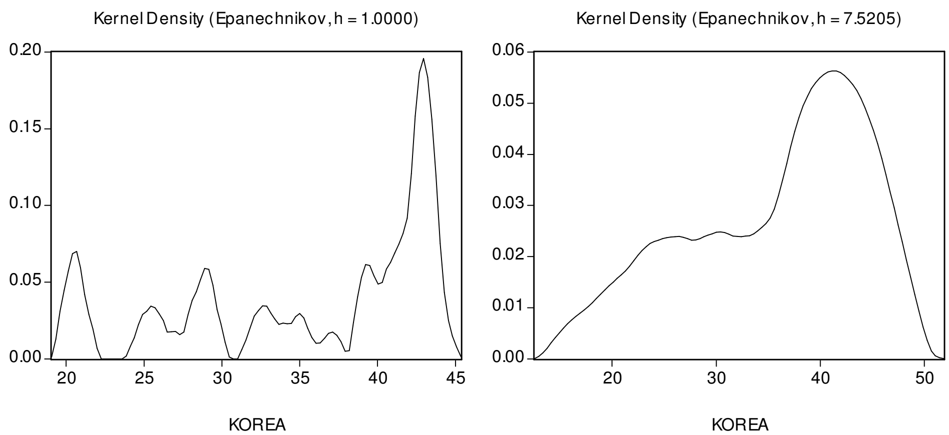


Figure 6: Histogram Approximation for Structural Changes' Density



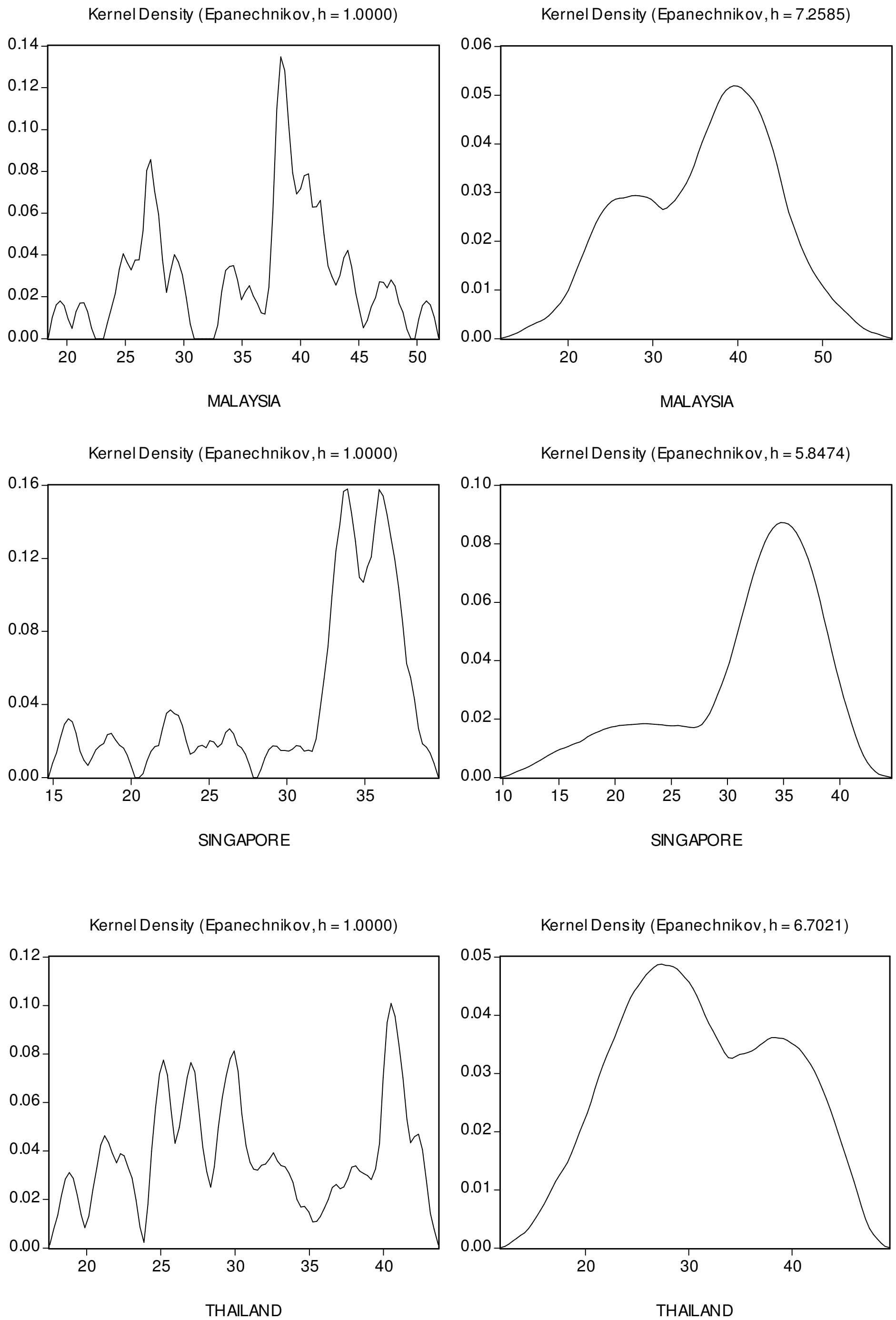
Figures 7-9: Kernel Densities for Indonesia with Various Bandwidth

Histogram method for density estimation shows three intervals [15, 16], [34, 35] and [37, 38] which is consistent with our results. However kernel density estimations gives two points and there isn't so much difference for pre and post transformation kernels. Also as noted above bandwidth is important in both the histogram and kernel estimation. Wider bandwidth results in smooth density and imprecise location of fixed points, therefore we choose small bandwidth and only graphs for h=1(bandwidth size) is reported here. (Further results available from the first author upon request).For the Indonesia kernel estimation numerical output (table3) shows 12.77 and 34.275 as fixed points. Korea structure data has two fixed points 27.51 and 43.469(Aguirre algorithm), 23.17 and 42.96(polynomial regression) and, 20.6 and 42.7(So's method).the two latter estimates are very near to each other than Aguirre one. Graphical view can be seen in the following kernel estimation graphs .We don't present histograms for Korea and three other countries when kernel is very precise than histogram and other density estimation methods (Pagan and Ullah1999).



Figures 10 and 11: Kernel Densities for South Korea with Various Bandwidths

Industry value added share for Malaysia in some years is constant and denominators of equation (2) will be zero, therefore So et al. transformation can't be applied. However graphs of Indonesia show that in some cases transformation is not so



Figures 12-17: Kernel Densities for Other countries Various Bandwidth

important. We use pre transformation data for density estimation and compare it with other estimation methods results to get better view on fixed points. In case of Malaysia Aguirre algorithm is very sensitive window length for example for the length of 35 and 30 fixed points vary at least 10 percent .So et al. method estimate fixed points value 27.5 and 38.7 which is very reliable than the Aguirre's algorithm output. Polynomial regression method estimate two fixed points at 26.98 and 42.25 that in estimation of low level fixed point of structure is similar to the So et al. method estimation. Structure of Singapore has some different fixed points when one uses different bandwidth in Aguirre algorithm. These fixed points are inconclusive without consideration of outputs of other methods. Based on So's method only one fixed point is detected which is in 34.075 percent. It should be noted that 33.45 and 34.7 also are acceptable for fixed points. Fixed points with polynomial regression method is 10.9 and 35.491 which are consistent with Aguirre's in low level fixed point and So's estimation in the high level fixed point .Comparison of methods in the case of Singapore shows power of polynomial regression method. the fixed points of polynomial regression is more reliable than the further more ,as one can see from the results of these countries there is only one maximum value for the density of series and therefore one fixed point can be detected if we make strict criteria (the highest value only in density). Thailand industry value added as Malaysia don't accept transformation (1) therefore pre transformation data is used for density estimation which gives 25.17 and 29.89 as low level fixed points and , 38.74 and 39.33 as high level fixed points. The last two values are sufficiently close to each other which can be treated as one fixed point. (also one can use average value ~ 40 as fixed point).the low level fixed points are different to some extent and there other values in neighbor of them which can be treated as fixed point with a little lower likelihood.(Table).Aguirre algorithm in the case of Thailand gives very stable results. Fixed points are presented in table 2 for different assumptions. The results of polynomial regression in the high level fixed point is very clear to the So's method (39.66 vs. 39.33) and in low level is very close to the Aguirre's one. Two low levels fixed points of So's method is sufficiently wide to be consistent with the both of polynomial regression and Aguirre algorithm.

After finding fixed points which addressed above we should determine kind of fixed point (attractor, repllors or limit cycle) here we interested to attractors only, therefore we use the iteration method which is used by Gaustello in extracting attractors of inflation and unemployment for US data. In the Guastello approach is supposed that exponential form of logistic equation is proper equation generating the time series. But we don't confine us to this assumption and we will use general form which is polynomial regression. In this formulation one should take care that don't use only significant coefficients. It should be noted that So method can't be used for this end because of nonparametric nature of it that don't give any estimated equation. Also Aguirre et al. algorithm aggregates estimates and equations are not available for detecting the kind of attractors. Therefore polynomial regression method will be used here for determining the kind of fixed points. The iteration equation can be stated as follows:

$$y_t = \hat{\alpha} + \hat{\beta}_1 y_{t-1} + \hat{\beta}_2 y_{t-1}^2 + \beta_3 y_{t-1}^3 \tag{24}$$

Results show that for Indonesia fixed points aren't attractors instead they are repllors .However in the case of Malaysia 42.30309 is an attractor so that there is no matter iteration stated from which fixed point. The Korea has only one attractor 43.287 which is not insensitive to starting point of iteration and it is almost equal to high level fixed point 42.9596.it should be noted that in the iteration estimated equation don't contain AR and MA term's coefficients. The reason for this is that AR and MA terms are entered for explanation of some variation in error term which is regular partly .therefore deterministic part of equation apart of residuals should be used for finding fixed points. Attractor of Singapore is 35.49125 which is very close to high level fixed point obtained by polynomial regression method. The interesting point about Singapore attractor is that it is sensitive to starting point and for starting values less than low level fixed point don't converge to the attractor. This is the meaning of bad structure which prevent from industrialization. Very high values as starting point say 70 and higher, also cause divergence in the iteration such that system don't reach to attractor in any iteration. For the Korea and Thailand attractors are 43.287 and 39.56011 respectively .As So et al. (1996, p.4708) it is possible one fixed point don't be on the attractor.

Guastello method tackles the problem directly. In other words it doesn't need to find fixed points .standardization of the data in this method is very proper, therefore before estimation the (4) we transform the data by:

$$Z = (x - \min(x)) / stdv(x) \qquad (25)$$

After transformation two kind of exponential form of logistic map can be used. The first one is the (4) and the second one is the same except than bifurcation parameters absence:

$$y_t = \exp(-\beta y_{t-1}) + \gamma \qquad (26)$$

Nonlinear Least Squares (NLS) is the relevant estimation method for the above equations. We used Eviews software for the estimation of the models which results are available up on request. However summery of results is presented in table 4.Results for countries Indonesia, Korea and Singapore show 41.046, 43.47 and 35.49 respectively which are surprisingly close to estimates of previous method(43.47 vs.43.29 for Korea and 35.49 vs.35.4903 in the case of Singapore) , except Indonesia which hadn't any attractors in the polynomial regression for the Thailand and Malaysia when equations with bifurcation term are used Lyapunov exponent isn't significant, therefore we put aside these two countries and results of polynomial regression can used for them. Number of iteration in this method is considerably higher than the polynomial regression and insensitive to inclusion of intercept in the iteration procedures. We used equations without intercept for iteration and results for the above countries which attractors are 37.859, 38.674 and 31.622.they are different from the results of polynomial regression and Guastello methods with intercept.

Estimation results VAR models for Z=[IVA,MVA,AVA,GDPPER, CAPEXP,FDIGCF]' leads us systems which don't converge in solving iterations .The main reason is many terms in each of equations even after deleting insignificant variables. Reduction dimension of variables space to 3 with Z= [IVA, AVA, GDPPER]' makes problem easer to solve for some countries; however obtained fixed points are not attractors. Some of reasons for this may be little observations which make estimations imprecise, convergence problem of nonlinear algorithms and weakness of algorithms in finding all solutions of nonlinear systems. Results three variates restricted VAR which is estimated by SUR method and fixed points are presented in appendices. A short report of fixed points and Jacobian matrix of them in is presented in following table4.

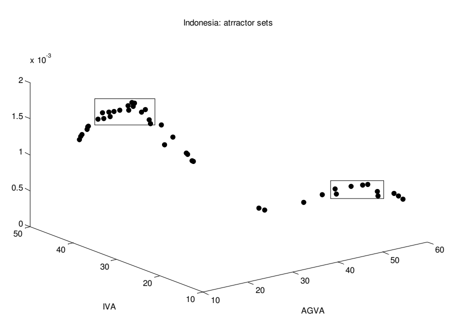
As one can see from table 4 none of fixed points are attractor because of sum absolute value of rows in Jacobian matrices is greater than one.

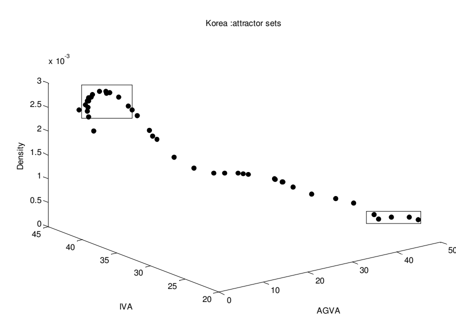
Table 4 :Fixed points and Jacobian matrix for examination of existence of attractors

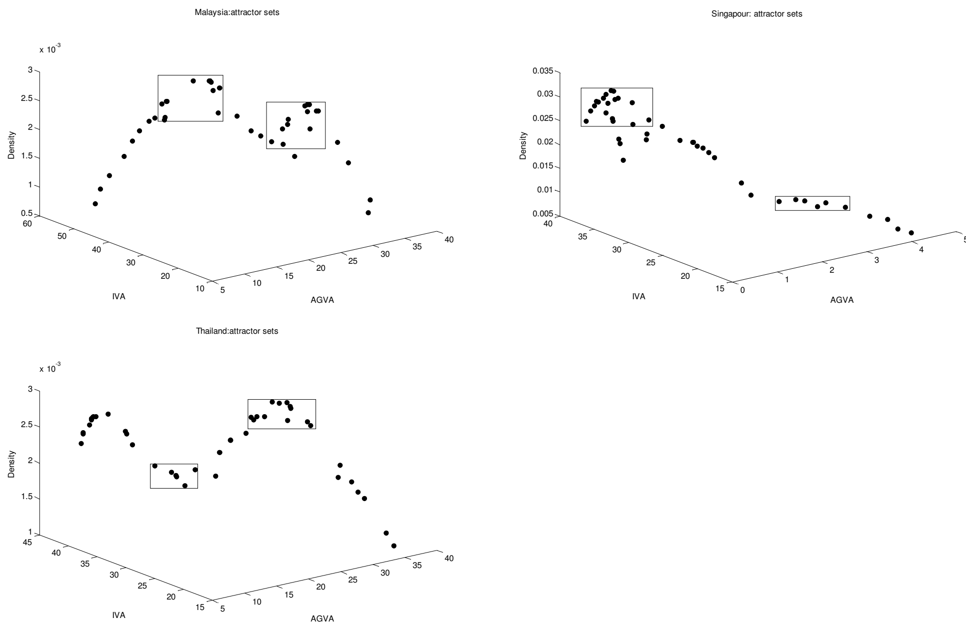
	Fixed point			Jacobian matrix		
Country	AVA	GDPPER	IVA	derivative respect to AVA	derivative respect to GDPPER	derivative respect to IVA
INDONESIA	18.051	1039.535	43.321	0.5372	-0.0042	0.8119
				-1.4316	0.4492	-10.1025
				0.1405	0.0308	0.6063
KOREA (S.)	43.946	948.177	18.948373	0.0000	-2.3376	-0.0001
				-36.2069	-30.8143	1.2353
				0.3391	1.1745	0.0041
MALAYSIA	26.134095	1678.7658	33.096812	0.5976	0.0011	-0.3583
				29.7076	1.1862	0.0000
				0.5081	0.0076	0.6099
SINGAPOUR	3.959182	2420.9518	16.818765	-1.1382	-0.0004	-0.2458
				- 832.5852	1.1818	0.0000
				0.0000	0.0035	1.0387
THAILAND	**No fixed point**					

Therefore, although in single equation form industry value added has fixed points that some of them are attractors in systems of equations fixed points are not detected for Thailand and are not attractors in other cases.

Some other methods such as spectrum of Lyapunov exponents (Anishchenko et al., 1998) can be useful in detecting quasihyperbolic attractors and quasiattractors. However we use multidimensional kernel density estimation as final solution to this problem.
As before structural variables IVA and AGVA for NICs are analyzed by kernel density to detect attractors of structural changes. Optimal bandwidth is selected by Silverman formula.







Figures 18-22: Kernel Densities for NICs with Optimal Bandwidth

As one can see from above figures both of Indonesia and South Korea have law and high levels of structural attractors. In figures attractors denoted by boxes (High and low level attractors).this method shows that although one cannot find deterministic attractors, but some points can be treated as stochastic attractors. Density estimation point help us find attractors set instead of one ,two or three distinct point ,which also appear more reasonable in real world. Consistency with results of single equation (polynomial) attractor, one variable density attractors and two variable density attractor points is interesting in this study.this can be cheked from tables

IV) Conclusion and Suggestions

Attractors of structural change can help development planers in targeting and driving structural change in optimal path such that with less resource and in shorter time structural change and industrialization can be take place. In the all of NICs at least one attractor can be detected and for Singapore there are one attractor and one repellor. The low level fixed point of Malaysia which is a reppler conveys bad structure trap. However for other countries existence of only one attractor is supported by empirical data. Short data period and unavailability of high qualified data are among the main reasons for failure in finding bad structure trap for NICs.Also NICs relatively fast transition can cause missing the effects of bad structure trap. Further more to information about attractors of structural change existence of fixed points confirms some equilibrium which economies of NICs experience in their transition period. Fixed point's values were confirmed with various methods which are very similar. Results of Aguirre et al. method is very close to simple polynomial regression proposed in this paper, but the second one is better than the firs in some cases. Comparative analysis for methods of finding fixed points shows their relative power and differences in some cases are negligible. However generality, simplicity in use and little assumption criterion make polynomial regression proffered to the others. In other words Aguirre et al. method is sensitive to window length and delta .Also this method with large number of parameters will be inefficient in the case of system equation for finding fixed points of multivariate time series models. So et al. method can be improved by using kernel density estimation method instead of histogram, but it is sensitive to bandwidth size as smoothing parameter. Also histogram method has similar problem as kernel namely classification .Finally Guastello method give only one attractor which may be different from fixed point.

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Appendix A: proofs

Proof of Proposition1. According to definition of asymptotic stable fixed point and basin of attraction, with any initial state in ball $B(\bar{y}, r)$;
$|y_t - y_{t-i}| < r$ for $i = 1 \cdots p$ and $f^p(y) \rightarrow \bar{y}$ then $y_t \sim y_{t-i}$ for $i = 1 \cdots p$ and $y_t = f(y_{t-1}, y_{t-2}, ..., y_{t-p}) \sim y_t = g(y_{t-1})$ although their degree can be different.

Proof of Theorem1.let $\bar{x}$ be fixed point of $x_{t+1} = f(x_t)$ $t = 0,1,2,\cdots$, and according to definition of derivation $f'(x) = \lim_{x \rightarrow \bar{x}} (f(x) - f(\bar{x}))/(x - \bar{x})$, α is chosen between $f'(x)$ and 1.For x in an interval I around $\bar{x}$,

$0 \leq (f(x) - f(\bar{x}))/(x - \bar{x}) \leq \alpha$

In the interval I we then have:

$$|f(x) - f(\bar{x})| \leq \alpha |x - \bar{x}|$$

Let us now start the iteration with a number x_0, belongs to I. according to definition of fixed point $f(\bar{x}) = \bar{x}$, then:

$$|x_1 - \bar{x}| = |f(x_0 - f(\bar{x}))| \leq \alpha |x_0 - \bar{x}|$$

It is clear that x_1 is closer to $\bar{x}$ than x_0. With more iterations

$$|x_2 - \bar{x}| \leq \alpha^2 |x_0 - \bar{x}|$$

For n steps

$$|x_n - \bar{x}| \leq \alpha^n |x_0 - \bar{x}|$$

Remembering that $0 < \alpha < 1$

$\lim_{n \to \infty} \alpha^n |x_0 - \bar{x}| = 0$ Therefore:

$$\lim_{n \to \infty} |x_n - \bar{x}| = 0$$

This shows that $\bar{x}$ is an attractor point and $x_x \to \bar{x}$ when $n \to \infty$.

Appendix B: Results

Table 3:Fixed points estimation by method of kernel density estimation

Indonesia		Korea		Malaysia		Singapore		Thailand	
industry value added share	Density value	industry value added share	Density value	industry value added share	Density value	industry value added share	Density value	industry value added share	Density value
11.8500	0.0514	20.0700	0.0516	19.4000	0.0174	15.7000	0.0286	18.5200	0.0275
12.5490	**0.0712**	**20.5650**	**0.0716**	20.0410	0.0103	16.1690	0.0307	19.0130	0.0302
13.2480	0.0348	21.0610	0.0496	20.6820	0.0114	16.6390	0.0175	19.5060	0.0160
13.9480	0.0000	21.5560	0.0265	21.3240	0.0174	17.1080	0.0063	19.9990	0.0075
14.6470	0.0146	22.0510	0.0013	21.9650	0.0090	17.5780	0.0127	20.4910	0.0263
15.3460	0.0284	22.5470	0.0000	22.6060	0.0000	18.0470	0.0174	**20.9840**	**0.0433**
16.0450	0.0333	23.0420	0.0000	23.2470	0.0009	18.5160	0.0237	**21.4770**	**0.0429**
16.7440	0.0259	23.5370	0.0000	23.8890	0.0155	18.9860	0.0204	21.9700	0.0332
17.4430	0.0306	24.0320	0.0063	24.5300	0.0352	19.4550	0.0163	22.4630	0.0390
18.1430	**0.0402**	24.5280	0.0175	25.1710	0.0356	19.9240	0.0083	22.9560	0.0313
18.8420	0.0200	25.0230	0.0306	25.8120	0.0377	20.3940	0.0000	23.4490	0.0141
19.5410	0.0048	25.5180	0.0347	26.4530	0.0461	20.8630	0.0052	23.9410	0.0011

20.2400	0.0122	26.0140	0.0289	**27.0950**	**0.0857**	21.3330	0.0151	24.4340	0.0432
20.9390	0.0171	26.5090	0.0163	**27.7360**	**0.0624**	21.8020	0.0173	**24.9270**	**0.0719**
21.6390	0.0049	27.0040	0.0181	28.3770	0.0233	**22.2710**	**0.0349**	**25.4200**	**0.0725**
22.3380	0.0000	27.5000	0.0171	29.0180	0.0373	**22.7410**	**0.0344**	25.9130	0.0428
23.0370	0.0000	27.9950	0.0355	29.6600	0.0333	23.2100	0.0303	26.4060	0.0562
23.7360	0.0000	28.4900	0.0459	30.3010	0.0154	23.6800	0.0165	**26.8990**	**0.0739**
24.4350	0.0061	**28.9860**	**0.0600**	30.9420	0.0000	24.1490	0.0153	**27.3910**	**0.0682**
25.1340	0.0172	29.4810	0.0444	31.5830	0.0000	24.6180	0.0172	27.8840	0.0390
25.8340	0.0200	29.9760	0.0220	32.2240	0.0000	25.0880	0.0200	28.3770	0.0246
26.5330	0.0174	30.4710	0.0000	32.8660	0.0050	25.5570	0.0164	28.8700	0.0498
27.2320	0.0091	30.9670	0.0000	33.5070	0.0304	26.0270	0.0236	**29.3630**	**0.0690**
27.9310	0.0000	31.4620	0.0106	34.1480	0.0342	26.4960	0.0248	**29.8560**	**0.0820**
28.6300	0.0000	31.9570	0.0250	34.7890	0.0220	26.9650	0.0170	**30.3490**	**0.0621**
29.3300	0.0000	32.4530	0.0315	35.4310	0.0248	27.4350	0.0104	30.8410	0.0374
30.0290	0.0000	32.9480	0.0335	36.0720	0.0174	27.9040	0.0000	31.3340	0.0312
30.7280	0.0000	33.4430	0.0252	36.7130	0.0107	28.3730	0.0055	31.8270	0.0345
31.4270	0.0000	33.9390	0.0238	37.3540	0.0264	28.8430	0.0152	32.3200	0.0362
32.1260	0.0000	34.4340	0.0214	37.9960	0.1115	29.3120	0.0172	32.8130	0.0351
32.8260	0.0131	34.9290	0.0298	38.6370	0.1288	29.7820	0.0143	33.3060	0.0340
33.5250	**0.0645**	35.4240	0.0226	**39.2780**	**0.0841**	30.2510	0.0139	33.7990	0.0290
34.2240	**0.0728**	35.9200	0.0118	**39.9190**	**0.0720**	30.7200	0.0174	34.2910	0.0176
34.9230	0.0304	36.4150	0.0104	**40.5600**	**0.0800**	31.1900	0.0133	34.7840	0.0167
35.6220	0.0441	36.9100	0.0171	**41.2020**	**0.0581**	31.6590	0.0137	35.2770	0.0099
36.3210	0.0442	37.4060	0.0152	**41.8430**	**0.0607**	32.1290	0.0371	35.7700	0.0134
37.0210	0.0373	37.9010	0.0048	42.4840	0.0310	32.5980	0.0671	36.2630	0.0186
37.7200	0.0587	38.3960	0.0204	43.1250	0.0246	33.0670	0.1182	36.7560	0.0265
38.4190	0.0560	38.8920	0.0479	43.7670	0.0399	**33.5370**	**0.1489**	37.2490	0.0234
39.1180	**0.0753**	**39.3870**	**0.0627**	44.4080	0.0336	**34.0060**	**0.1522**	37.7410	0.0313
39.8170	**0.0865**	39.8820	0.0498	45.0490	0.0134	**34.4760**	**0.1214**	38.2340	0.0312
40.5170	**0.0619**	40.3780	0.0527	45.6900	0.0071	34.9450	0.1071	38.7270	0.0303
41.2160	0.0594	40.8730	0.0624	46.3310	0.0172	35.4140	0.1199	39.2200	0.0282
41.9150	0.0428	41.3680	0.0751	46.9730	0.0275	**35.8840**	**0.1556**	39.7130	0.0425
42.6140	0.0221	41.8630	0.0894	47.6140	0.0266	**36.3530**	**0.1462**	**40.2060**	**0.0916**
43.3130	0.0345	**42.3590**	**0.1511**	48.2550	0.0212	**36.8220**	**0.1242**	**40.6990**	**0.0974**
44.0120	0.0516	**42.8540**	**0.1944**	48.8960	0.0104	37.2920	0.0937	**41.1910**	**0.0741**
44.7120	0.0493	**43.3490**	**0.1697**	49.5380	0.0000	37.7610	0.0595	41.6840	0.0448
45.4110	0.0449	43.8450	0.1029	50.1790	0.0103	38.2310	0.0397	42.1770	0.0473
46.1100	0.0333	44.3400	0.0387	50.8200	0.0174	38.7000	0.0174	42.6700	0.0385

List of exact modes for NICs

INDONESIA	KOREA	MALAYSIA	SINGAPORE	THAILAND
39.776	42.9	38.423	33.795	40.48
39.632	41.5	27.08	35.967	29.907

33.942	41.352	40.549	36.205	25.18
39.29	20.558	39.872	35.162	27.078
12.362	40.794	40.067	22.85	42.253