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**Identification with Averaged Data and
Implications for Hedonic Regression Studies**

José A.F. Machado
João M.C. Santos Silva

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José A. F. Machado

Faculdade de Economia, Universidade Nova de Lisboa

J. M. C. Santos Silva

ISEG, Universidade Técnica de Lisboa

Abstract

In the estimation of models with averaged data, weighted least squares is often used and recommended as a way of improving the efficiency of the estimator. However, if the size of the different groups is not conditionally independent of the regressand, consistent estimation may not be possible at all. It is argued that in the case of some leading examples of averaged data regression, consistent estimation is possible using the usual weighted estimator.

Key words: Endogenous sampling; Functional form; Weighted least squares.

JEL classification code: C13.

1. INTRODUCTION

Estimation with averaged data is often used in practice and the subject is covered in many standard textbooks in econometrics (see, for example, Kmenta, 1986). Typically, in this situation interest lies on the parameters of a model at the individual level but estimation is performed using data averaged across different groups because individual data are not observed. An example of this type of situation occurs in the estimation of hedonic regression models in which the average price of a good is regressed on its characteristics. A standard reference in this area is Berndt (1991, Chapter 4), and a state of the art review of the subject can be found in the excellent survey by Triplett (2000).

Estimation with averaged data models poses problems that are akin to those encountered when using stratified samples. In both cases, the way the data is obtained generally leads to some groups or strata in the population being over or underrepresented in the sample, and care must be taken to account for these distortions when performing inference. It is well known (see, for example, Wooldridge, 1999) that if the sample is stratified as a function of conditioning variables, standard methods can be used to produce valid inference. However, inference using endogenous stratification requires more sophisticated methods. In the case of averaged data the situation is similar: if the definition of the groups across which averaging is performed is exogenous, standard estimation methods can be employed. Otherwise estimation is more difficult, or even impossible. This paper investigates the conditions under which the parameters of the individual data model can be consistently estimated using averaged data, and show that in certain circumstances it is possible to use a weighted least squares estimator (which is really an instrumental variables estimator) to obtain consistent estimates of the parameters of interest.

The rest of the paper is organized as follows. In Section 2 the problem is formally presented and different estimation strategies are studied. Section 3 discusses the implication of the results for the estimation of hedonic regressions, and Section 4 presents an illustrative example. Finally, Section 5 contains some concluding remarks.

2. ESTIMATION STRATEGIES

Consider the case in which the researcher is interested in estimating a vector of parameters whose value in the population is defined by

$$E[u\,(y, x, \beta_0)\,|x] = 0,$$

where y is the variate of interest, x is a vector of explanatory variables, β_0 is a vector of parameters and $u(\cdot, \cdot, \cdot)$ is a known function. Equivalently, β_0 solves moment conditions of the form

$$E[f\,(x)\,u\,(y, x, \beta_0)] = 0, \tag{1}$$

where $f\,(\cdot)$ is a known function. With individual data, $\{y_i,\ x_i\}_{i=1}^n$, β_0 could be consistently estimated by least squares under very general conditions on the conditional distribution of y given x, which are now assumed to hold.

Assume, however, that due to observability problems, data on $i = 1, \ldots, n$ individuals is only available in the form of averages across G groups or strata. Specifically, let S_{gi} denote an indicator variable which equals 1 if observation i belongs to group g, being 0 otherwise. The researcher observes $\bar{y}_g = \frac{1}{n_g} \sum_{i=1}^n y_i S_{gi}$, $\bar{x}_g = \frac{1}{n_g} \sum_{i=1}^n x_i S_{gi}$, and $n_g = \sum_{i=1}^n S_{gi}$, with $g = 1, \ldots G.$

Since no individual data is available, it is interesting to study under which conditions it is possible to consistently estimate the same parameters using averaged data. The splitting of the population into G groups can explicitly be taken into account by writing equation (1) as

$$\sum_{g=1}^G \pi_g E\,[f\,(x)\,u\,(y, x, \beta_0)\,|S_g = 1] = 0, \tag{2}$$

where $\pi_g = \Pr\,(S_g = 1)$. Clearly, (2) does not imply that the orthogonality condition (1) holds for every strata. It all depends on whether there is endogenous selection into the strata.

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When the strata indicators S_g are endogenous in the sense that they are conditionally correlated with $u(y, x, \beta_0)$, that is $E[u(y, x, \beta_0)|x, S_g = 1] \neq E[u(y, x, \beta_0)|x]$,[1] there is endogenous selection into the groups. In this case it is entirely possible that, for some g, $E[f(x)u(y, x, \beta_0)|S_g = 1] \neq 0$. When the strata indicators are exogenous, (2) implies that $E[f(x)u(y, x, \beta_0)|S_g = 1] = 0$ for every group.

Under standard regularity conditions, β_0 will be identifiable by averaged data if it is possible to write (2) with the appropriate population averages as arguments. Specifically, letting $\mu_{x_g} = E[x|S_g = 1]$, $\mu_{y_g} = E[y|S_g = 1]$, identification requires that

$$\sum_{g=1}^{G} \pi_g f\left(\mu_{x_g}\right) u\left(\mu_{y_g}, \mu_{x_g}, \beta_0\right) = 0. \tag{3}$$

Whether this condition holds or not depends critically on the linearity of $u(\cdot, \cdot, \cdot)$ and on the exogeneity of the indicators S_g. Two leading cases can be distinguished depending on the nature of the indicators.

1. **Exogenous strata**

   This is the standard textbook case (e.g. Kmenta, 1986). Let $\mu_{y_g|x} = E[y|S_g = 1, x]$ and impose that $u(y, x, \beta_0) = y - x'\beta_0$. Then, exogeneity and (2) imply that $\mu_{y_g|x} = x'\beta_0$ for all g, and the law of iterated expectations yields $\mu_{y_g} = \mu_{x_g}'\beta_0$. Consequently,

   $$f\left(\mu_{x_g}\right) u(\mu_{y_g}, \mu_{y_g}, \beta_0) = f\left(\mu_{x_g}\right)(\mu_{y_g} - \mu_{x_g}'\beta_0) = 0, \ \forall g.$$

   Therefore, if there is no endogenous selectivity and $u(y, x, \beta_0)$ is linear in x and y, the parameter is identified with averaged data. Under this set of assumptions the sample analog of the moment conditions (3) is

   $$\sum_{g=1}^{G} \left[f\left(\bar{x}_g\right)\left(\bar{y}_g - \bar{x}_g\hat{\beta}\right)\right] \frac{n_g}{n} = 0$$

   and therefore β_0 can be consistently estimated by weighted linear least squares. Moreover, because it is assumed that the expectation $u(y, x, \beta_0)$ is zero in every

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[1] This condition is equivalent to $\Pr(S_g = 1|y, x) \neq \Pr(S_g = 1|x)$.

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group, the weights are not needed for consistency, and β_0 can also be consistently estimated by ordinary linear least squares on the group means.

2. **Endogenous strata**

Now, in general it will not be possible to estimate β_0. This is because averaged data always depends on the strata indicators, and it is not possible to construct moment conditions identifying β_0 using only this sort of data since all the available variables are then endogenous.[2] There is however an important special case in which the group means for the covariates coincide with the values of the covariates within the group and, in a sense, individual data on the regressors is actually available. Formally, this means that the conditional distribution of the covariates given the strata indicator is degenerate and thus $\mu_{y_g|x} = \mu_{y_g}$. Assuming that $u(y, x, \beta_0) = y - \phi(x, \beta_0)$, where $\phi(\cdot, \cdot)$ is a known function, it is possible to write

$$E\left[f\left(x\right)u\left(y, x, \beta_0\right)|S_g = 1\right] = f\left(\mu_{x_g}\right)\left(\mu_{y_g} - \phi(\mu_{x_g}, \beta_0)\right)$$

since for $S_g = 1$, $x = \mu_{x_g}$. Consequently, (2) implies that (3) is satisfied. Therefore, in spite of the endogenous sample selectivity, β_0 can be identified as long as the covariates are constant within each strata. The sample analog of (2) is now

$$\sum_{g=1}^{G}\left[f\left(x_i\right)\left(\bar{y}_g - \phi\left(x_i, \hat{\beta}\right)\right)\right]\frac{n_g}{n} = 0,$$

which shows that β_0 can be consistently estimated by weighted (possibly non-linear) least squares.[3] However, standard least squares on the averaged data will be inconsistent because the conditional expectation of $\frac{1}{n_g}u\left(y, x, \beta_0\right)S_{gi}$ is not zero due to the endogeneity of the group sizes $n_g = \sum_{i=1}^{n}S_{gi}$.

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[2] In an instrumental variables interpretation that will be provided below, this means that there are no valid instruments available.

[3] This estimator can be interpreted as an instrumental variables estimator with instruments defined by $f\left(x_i\right)n_g/n$. It is also worth noting that this estimator is numerically equal to the one that would be obtained by running a least squares regression of y_i on x_i, if individual data were available.

Whether the selection into the groups is endogenous or not is an empirical question, which should be checked in each application. A formal statistical test for the hypothesis that the strata indicators are exogenous can be performed by comparing weighted and ordinary least squares estimators. This test is reminiscent of the general specification test proposed by White (1980a), but the appropriate weights to be used here are the group sizes, which are not functions of the regressors as required by White's test. In the formulation of White (1980a), the test comparing weighted and ordinary least squares estimators was proposed as an Hausman (1978) type test. However, in this context, it is not clear which estimator is more efficient, and therefore it is not straightforward to apply Hausman's results. Alternatively, the test can be formulated as a conditional moments test (Newey, 1985) which, if the grouped data model is homoskedastic, can easily be performed as an omitted variables test, as explained in Godfrey (1988, pp.155-157). This test can also be made robust to heteroskedasticity using the method suggested by Wooldridge (1991).

3. IMPLICATIONS FOR HEDONIC REGRESSIONS

The estimation of hedonic regressions is typically performed by regressing the average price of a good (or its logarithm) on a set of its characteristics. The number of observations in each group depends on the quantity sold of each of the products considered.[4] Basic economics suggests that quantities sold depend on prices, and therefore hedonic regressions are an obvious example of a situation in which the size of the groups across which averaging is performed can be endogenous, in the sense of being correlated with unobserved characteristics of the product that determine its price. In case the group sizes are indeed endogenous, the results of the previous section show that consistent estimation of the parameters in an hedonic regression requires essentially two conditions: i) the

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[4]In case the sample available for estimation is not a random sample of the population but is stratified as a function of the price of the product, the situation would be even more complex.

varieties of the product must be homogeneous in the sense that the characteristics are identical within groups; ii) weighted least squares has to be used.

There has been some controversy in the literature on the need for weighted least squares when estimating models with averaged data. The standard textbook approach to this problem (e.g. Kmenta, 1986) recommends the use of weighted least squares to improve efficiency, assuming that at the individual level the errors are independent and homoskedastic. However, Dickens (1990) noticed that if the errors are correlated within groups, weighting can lead to noticeable losses in efficiency. This discussion assumes that the group sizes are exogenous. If that is not the case, the need for weighted least squares is not just a question of efficiency, but, more fundamentally, a matter of consistency. Therefore, weighted least squares should be used, regardless of the effect it has on the efficiency of the estimator.

Although the econometrics of hedonic regressions has been the subject of a vast literature, the serious consequences of the possible endogeneity of the group sizes seems to have escaped the attention of most researchers working in this field. In fact, although it has long been noticed that it would be desirable to weight observations by the corresponding market shares (see Griliches, 1971), no precise econometric justification has been given for this and most studies have not followed this approach.[5] The problem was however noticed from an economic rather than econometric point of view by Brown (2000).

Because data on sales is needed to test for the endogeneity of the group sizes, and to correct its consequences, hedonic regressions are more data demanding that it has been assumed so far. This makes alternative approaches to the problem of quality change relatively more attractive. In particular, accounting for endogenous group sizes makes hedonic regression as demanding as the discrete choice approach proposed by Trajtenberg (1990).

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[5] As an example of this, it is interesting to note that Triplett (2000) does not mention the need to use weights in the estimation of hedonic regressions with averaged data, and that Berndt (1991) recommends its use just as a way of reducing possible heteroskedasticity problems.

It should be pointed out that in many applications the bias caused by the endogeneity of the group sizes is likely to be small, which explains why the results from weighted least squares often do not differ much from those obtained without weights. In fact, the reason for the inconsistency of the standard least squares estimator in presence of endogenous selection into the groups is that $u(y, x, \beta_0)$ is not mean independent of S_g, conditionally on x. Therefore, by using a large set of conditioning variables, as is often done in hedonic regression studies, it is possible to reduce the size of the bias. That is, by explicitly including in the model most of the characteristics affecting prices, the potential for conditional correlation between $u(y, x, \beta_0)$ and S_g is reduced.[6]

A final point is worth noting. The discussion in Section 2 shows that when averaged data is used, identification of the parameters of the individual data equation is only possible under certain restrictions on the form of $u(y, x, \beta_0)$. In particular, $u(y, x, \beta_0)$ has to be linear on the arguments that are not constant within groups. Therefore, the types of functional forms that can be used in hedonic regressions depend on the kind of data available. For example, the usual practice (see Triplett, 2000, and the references therein) of estimating models with different transformations of the dependent variable and then choose a preferred specification on the basis of some specification test or goodness-of-fit criterion can only be recommended when using individual data.

4. AN ILLUSTRATION

In this section, the data set studied in Chow's (1967) pioneering work on computer pricing is used for purely illustrative purposes. This data set is given in Berndt (1990), where an extended discussion of the original study can be found. Chow (1967) estimated various models in which the dependent variable is the monthly rental of general-purpose digital computers, and the regressors are multiplication time, memory size, and access

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[6]Notice that finding that the hedonic regression with average data has a high R^2 does not imply that a large proportion of the variation of y_i is explained by x_i because the R^2 of averaged data regressions is typically inflated (see Cramer, 1964).

time. All variables are in logarithms. Apart from these variables, the number of new installations of each computer in each year is also available. For data sources and further details on the definition of the variables and on the computer market in the early sixties see Chow (1967) and Berndt (1990).

The purpose of this section is solely to illustrate the importance of using weights in averaged data models. Therefore, no attempt is made of improving the model specification used by Chow (1967), which is also adopted here. In particular, attention is focused on the equation pooling data from 1960 to 1965, which is given at the bottom of Table 1 in Chow (1967). This is a standard hedonic regression in which the dummy variable method is used to construct the price index, controlling for quality changes. The model is estimated by ordinary least squares (OLS) and the results are replicated in Table 1 below. This table also contains the results obtained estimating the models by weighted least squares (WLS), using the number of new installations of each computer in each year as weights.

As can be seen in Table 1, using weights makes a reasonable difference in the economic interpretation of the results. In particular, two points are worth noting. First, the coefficient on multiplication time changes considerably as a result of using weights, becoming positive. Although the correlation coefficient between the multiplication time and rental is negative, even when using weights, the effect of this regressor is positive when conditioning on the remaining regressors and using weights. This change of signs is not entirely surprising because the equation estimated by OLS chiefly measures the effect of the regressors on the prices set by the producers, while the WLS results also take into account the valuation of the characteristics by the consumers. Therefore, the two equations estimate two different sets of parameters which would only be identical if the computers were priced according to the consumers perceptions of the value of their characteristics.[7] A second point that is worth noting is that the estimates of the dummy variables also change considerably by using weights, implying a much slower reduction in

[7]Brown (2000) also reports different signs in the estimates obtained with and without accounting for sales volume.

the price index. This result is not unexpected as the WLS regression only accounts for the gains in quality that are effectively passed on to the consumers, as it is explained in Brown (2000). In fact, in this data set most of the sales are accounted for by inexpensive computers in which the quality improvements may be less noticeable than in the top of the range models. Therefore, the vast majority of consumers will only benefit from quality improvements introduced in a relatively small segment of the market.[8]

Table 1: Ordinary and weighted least squares results

	OLS		WLS	
	$\hat{\beta}$	s.e.	$\hat{\beta}$	s.e.
Intercept	−0.10446	0.31494	−1.21871	0.05178
Multiplication time	−0.06537	0.02841	0.02564	0.00434
Memory size	0.57933	0.03539	0.65519	0.00580
Access time	−0.14060	0.02933	−0.18032	0.00491
Dummy 1961	−0.13980	0.16647	−0.19064	0.02413
Dummy 1962	−0.48911	0.17377	−0.41695	0.03836
Dummy 1963	−0.59385	0.16610	−0.21098	0.02058
Dummy 1964	−0.92482	0.16630	−0.74276	0.02509
Dummy 1965	−1.16317	0.16611	−0.73652	0.02218
Effective sample size	$G = 82$		$n = 3126$	
Estimation period	$1960 - 1965$			

The previous discussion shows that the use of weights leads to conclusions that are significantly different in economic terms. In order to check for the statistic significance of these differences, the test for endogenous strata discussed in Section 2 was performed. Since White's (1980b) homoskedasticity test gives no evidence of heteroskedasticity, the

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[8]Of course, the use of unweighted regression does not necessarily leads to biases in this direction. The bias will depend on the characteristics of the market and on the distribution of sales across the price range.

test comparing OLS and WLS estimates was performed as a simple variable addition test (Godfrey, 1988). The results of the two specification tests are displayed in Table 2.

As can be seen from the results in Table 2, at the usual 5 per cent level the hypothesis of identical coefficients would not be rejected. However, the opposite conclusion is reached if the less conservative 10 per cent level is used. In view of the significantly different economic results found, and knowing that the OLS estimator can be biased for the parameters of interest, whereas the WLS estimator is always consistent, is seems prudent to conclude in favour of using the WLS results.

Table 2: Specification tests

Alternative hypothesis	Test statistic	Asymptotic distribution	p-value
Heteroskedasticity	26.5885	$\chi^2(29)$	0.5939
Endogenous strata	15.0591	$\chi^2(9)$	0.0893

5. CONCLUDING REMARKS

This paper addressed the problem of obtaining a consistent estimator for the parameters of an individual data model when the observations available for estimation are averages across groups of individuals. When the mechanism selecting individual observations into the groups is endogenous, in the sense of depending on unobservable individual characteristics affecting the dependent variable of the regression, estimation can only be performed if the observed individual characteristics are fixed within each group. In this case, an appropriate weighted least squares is consistent, even if the model is non-linear in the parameters and regressors. Notice that in presence of endogenous selection into groups the use of weights is not just a question of efficiency, as it is often suggested, but, more importantly, it is required to achieve consistency. Hedonic regression models are a leading case in which averaged data and endogenous selection into the groups is likely

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to be found. Fortunately, hedonic regression studies typically meet the requirements for consistent estimation by weighted least squares.

Although the main type of applications considered in the paper refers to the estimation of hedonic regression models, there are other empirical studies in which the results obtained here are of interest. An example is the estimation of wage equations with data averaged across firms. In this case, it is likely that the number of workers in each firm is related to the wage paid and therefore consistent estimation of the parameters of the equation at the individual level is only possible under the restrictive conditions mentioned above. A related example are some of the wage curves estimated by Blanchflower and Oswald (1994). For example, Table 4.26 in Blanchflower and Oswald (1994, p.168) reports the regression results for different models in which "all variables, including the dependent variable, are measured as the mean of all observations in a year/region cell". The dependent variable of these models "is the log of the average annual wage in each cell" and the regressors include, for example, unemployment rates, experience, years of schooling and time and industry dummies. If the number of workers in each group is not conditionally independent of the wage, estimation of the parameters of the equation at the individual level is not possible since the regressors are not constant within groups. Moreover, even if the group sizes are exogenous, this inconsistency will persist because of the nonlinear transformation of the dependent variable. Therefore, it is not surprising that when confronting the results obtained using individual data with those of the averaged data model the authors conclude that using averaged data "reduces the coefficient on unemployment by approximately one half" (Blanchflower and Oswald, 1994, p.169).

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