Spot price dynamics in deregulated power markets

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Abstract. Modelling spot price behavior plays a key role in the electricity market, since this is the breeding engine for the activity in the corresponding forward and futures market: developers and generators (as well as traders) need to know how electricity prices behave, as their profitability depends on them. Additionally, credit rating agencies need to monitor the exposure of different players in the market to price fluctuations and risks. Starting from those considerations, this work is intended to offer a comparative analysis of the statistical properties of hourly prices in the day–ahead electricity markets of several countries, in order to fix some features which a good model should have to fit day–ahead prices. A number of stochastic processes will be then examined as perspective candidate to generate sample paths with explanatory power respect on the real time–series, and results will be discussed.

Keywords: spot prices, self–affinity, Hurst exponent.
JEL classification codes: C0, C16, C5

1 Introduction

The deregulation of the electricity industry starting in earlier ninety in the United States and in Scandinavian countries has traced the way to a global trend, which is in action in most European countries, yet.

Electricity markets generally manage the incorporation of suppliers'costs into the price to purchasers (who in turn interface with final customers) through a composite structure.

In particular, a first–look investigation suggests the existence of two parallel markets (where, hence two different kind of prices are fixed):

– the *spot market*, which is in practice a day–ahead market, since the system operator needs information in advance about scheduling feasibility and transmission constraints. Hourly (or half–hourly[1]) contracts with physical delivery are therein traded in the form of a once per day auction: we will hence have 24 (or 48 in the assumption of half–hour trades) different price levels fixed in advance for the next day.

[1] M–co, the New Zealand power market, for instance, is organized into three regional sections: Benmore, Hayward and Otahuhu, where the electricity price is fixed on half–hourly basis.

– the market of ancillary services, where a more composite number of tasks are performed. To our purpose it is just noteworthy to mention that this is the place where the generation–consumption balancing is adjusted, when the system regulator is noticed of defaults throughout the transmission network.

Figure 1 adds some explanatory remark to such topic, presenting a general scheme of electricity market organization.

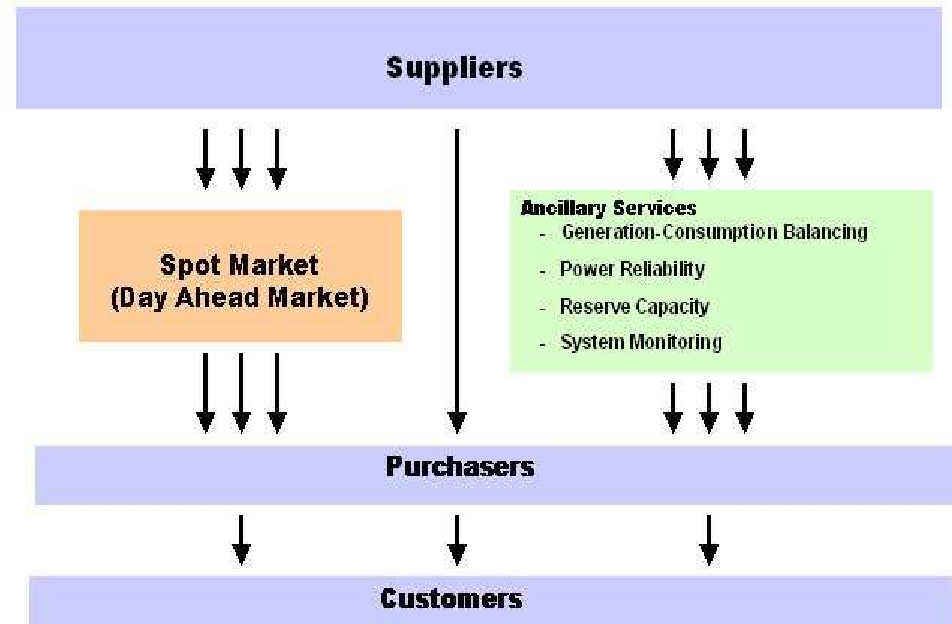

Fig. 1. Electricity market structure

Despite of the existence of such above complex structure, we will focus on the day–ahead market, studying the behaviour of the spot market of various countries.

Our choice is motivated with the residual nature of the ancillary services market, where typically random congestions of the system are resolved.

The attractiveness played by the spot market is entirely contained in the main features of the commodity therein traded:

– electricity cannot be stored, thus making nonsense of retention strategies. In other terms, it is not possible to cover the electricity demand of a single

hour with the electricity which has been produce before, or that will be made available in the next hours.
– severe weather changes may dramatically affect the load of electricity produced, thus inducing jumps into the normal level of prices.

The spiky nature of electricity market prices makes low powered the traditional approach employed in finance, which model the dynamics of price X through a diffusion-type stochastic differential equation of the form:

$$dX_t = \mu(X,t)dt + \sigma(X,t)dB_t \tag{1}$$

where $\mu(X,t)$ is the drift, $\sigma(X,t)$ is the volatility (scaling factor) and dB_t are the increments of a standard Brownian motion.

A common approach (see for instance [8], [16], or [13]) is to regard at electricity prices as mean reverting, and hence, in its simplest form, completely described by the stochastic differential equation:

$$dX_t = \alpha(\mu - X_t)dt + \sigma dB_t \tag{2}$$

α being the magnitude of the speed adjustment around the equilibrium or mean level μ.

This model, often referred to as an *arithmetic Ornstein–Uhlenbeck process*, has been generalized in [8], where the mean level is made time–dependent. Further contributions toward such direction consider a jump component random in size. ([11], [12]).

We will start from this point, and we will try to go one step further, examining the features of electricity prices in the day–ahead markets of various countries.

In particular, we will consider different alternative models as perspective candidate to simulate the behaviour of market price fluctuations: a selection scheme based on the notion of empirical scaling function, already applied in financial markets [17] will be therein use to find the best matching process.

The structure of the paper is as follows.

In section II we will discuss the main features of spot prices in the markets under examination (Alberta Pool, EEX, and OMEL). Particular emphasis will be given to the measure of correlation, and mainly to persistence (or anti–persistence) properties of the observed data.

A set of preliminary conclusions will be hence drawn:

[i] electricity power markets present some statistical features making them quite closer to *classical* financial markets;
[ii] we can then try to apply the same techniques widely used in *classical* financial market to extract relevant information also from electricity markets;

It is hence perfectly straightforward to search among an *ensemble* of candidate stochastic processes those which can be more reliable to our modelling tasks.

To such purpose, in section III we will introduce such selecting criterion in the form of the scaling function. After some theoretical remarks, we will focus on the aspects inside the estimation of the empirical scaling function of a given time–series.

In section IV, we will give a sketch of the potential of the empirical scaling function into a contest among a number of stochastic models trying to emulate the behaviour of our observed data. We will start by considering the main features of stochastic candidate processes, and hence we will present and discuss the results obtained through the generation of sample paths via Monte Carlo simulations.

Finally, section V will end this note, giving some conclusions and outlooks for future works.

2 Data analysis

Prices in the day–ahead market of different countries have been taken into account.

Table 1 summarizes the main features of the data under examination: the labels used to refer to them are indicated together with the temporal frame considered, and the total length of sample paths.

Table 1. Day–ahead markets considered in the study.

Country	Label	Observation Period	Sample size	Index Name	Length
Alberta	AP	01/01/1997 – 06/10/2003	2352×24 matrix	MI_{AP}	2351
Germany	EEX	06/16/2000 – 10/15/2003	1093×24 matrix	MI_{EEX}	1092
Spain	OMEL	01/01/1998 – 10/15/2003	1990×24 matrix	MI_{OMEL}	1989

The choice of such data is firstly motivated by their public availability; additionally we have considered markets which can exhibit sufficiently long records paths, in order to give greater robustness and consistency to our analysis.

For each market we have then operated as follows:

- We have worked with detrended data, i.e. we have removed the sample mean and hence, after a least–squares fitting, linear trends have been eliminated. Additionally, data have been filtered using classical Fourier transform methods.
- We have hence moved from the sequence of price levels $\{p_i^{(h)}\}$ to that of corresponding price changes $\{p_i^{(h)}\}$:

  $$X(t)^{(h)} = log(p(t+1)^{(h)}/p(t)^{(h)}) \tag{3}$$

  where $h = 1, ..., 24$.

– The results obtained in the previous step have been averaged at a daily scale, hence obtaining a synthesis indicator MI for each market, whose single observations are as follows:

$$MI(t) = \frac{1}{N} \sum_{h=1}^{N} X(t)^{(h)} \tag{4}$$

where $N = 24$. In this way we got the sequence $\{MI(t)\}$ which have used in our study (see Table 1 once again for some further details).

Our purpose is primarily to investigate whether or not the quantitative approach typical of financial economics can be applied *tout court* to electricity markets.

To such aim, this section will concentrate on the search for such stylized facts which are claimed to be typical of financial markets and make the gaussianity assumption of log–returns not reliable to modelling tasks.

We start with common statistics, which are reported in Table 2 for MIs indexes, and in Tables 10– 12 in the Appendix A for each hourly market[2].

Table 2. Common statistics on MIs indexes. For the Jarque–Beran test H0 is assumed to be the normality of data.

Statistics	MI_{AP}	MI_{EEX}	MI_{OMEL}
Minimum	−0.5594	5.5909	2.0472
Maximum	0.6425	−2.7644	−1.0599
Range	1.2018	2.8265	0.9873
Median	−0.0038	−0.0129	−0.0045
Mean	0.0000	0.0000	0.0000
Std.Dev	0.1234	0.2710	0.1011
Variance	0.0152	0.07343	0.0102
Skewness	0.1374	0.2887	0.4725
Kurtosis	2.2065	34.2048	25.5772
Jarque–Beran	REJECT $H0$	REJECT $H0$	REJECT $H0$

We have also performed usual normality tests (Kolmogorov–Smirnoff type see Table 3), and some more qualitative studies (qq–plots reported in Figure 2).

[2] For sake of clarity, and readability of this note, from now on the results for each hourly markets will be reported in Appendix A.

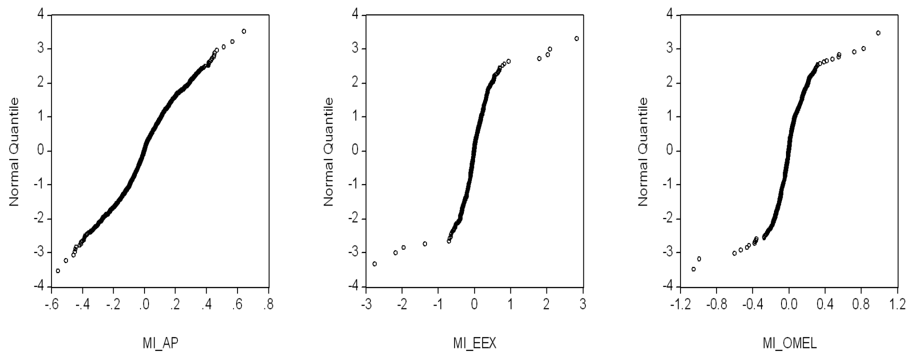

Fig. 2. QQ–plots for MI indexes

Table 3. Normality tests. L:Lilliefors; CVM:Cramer–Von Mises; W:Watson; AnD:Anderson–Darling. Pr is the associated p–value.

	L		CVM		W		AnD	
	Val	Pr	Val	Pr	Val	Pr	Val	Pr
MI_{AP}	0.117708	0	6.826083	0	6.810103	0	40.17197	0
MI_{EEX}	0.061684	0	3.969573	0	3.951013	0	21.90298	0
MI_{OMEL}	0.127144	0	11.77841	0	11.64072	0	64.74497	0

The results obtained put clearly into evidence that the assumption of normality of log–returns is not sustainable, both for hourly markets and for their representative synthesis indexes.

This is true for each examined market: the departures from normality are particularly evident looking at the results of Jarque–Beran test provided for each given time–series (see also Table 13 in the Appendix A).

Additional evidence is provided by the analysis of the sample moments (of order ≥ 2) as function of the time lag k, which is given in Figure 3. the common interpretation suggests that if the theoretical moment is finite, then the sample moment should fluctuate within a defined region centered on its theoretical limit. In the case where the true value is infinite, the sample moment should either diverge as a function of sample size, or exhibit erratic behaviour and large fluctuations.

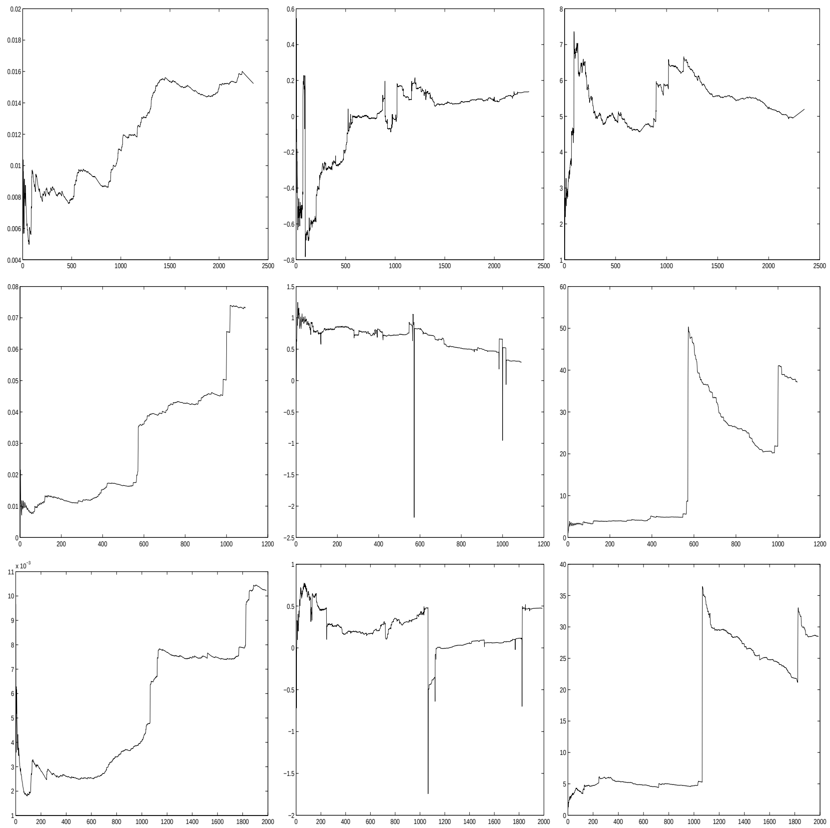

Fig. 3. Behaviour of empirical moments: from top to bottom: 2nd to 4th moments behaviour varying the lag k from 2 to the maximum length of data.

It is possible to see that both MI_{AP} and MI_{OMEL} exhibit increasing volatility which on the other hand is more stable for the MI_{EEX} index; additionally, while MI_{AP} shows stable 3rd and 4th order empirical moments, in the remaining indexes the behaviour of corresponding moments seem to be more erratic.

Finally, we have given particular emphasis to the study of correlation measures.

Results are documented for various kinds of measures:

– classical autocorrelation:

$$C_1(k) = corr(X(t), X(t+k)) \tag{5}$$

– autocorrelation of the squared returns:

$$C_2(k) = corr(|X(t)|^2, |X(t+k)|^2) \tag{6}$$

For this measure, we have been mainly interested to the search for autocorrelation functions remaining positive and decaying slowly (with significant positive values over several days, sometimes weeks): since this gives evidence of what is sometimes called the *ARCH effect*. However this property is model–independent and it is typical of time–series which exhibit volatility clustering.

– correlation of the logarithm of absolute returns:

$$C_0(k) = corr(ln|X(t)|, ln|X(t+k)|) \tag{7}$$

This measure can play an important role, since in a recent work [3] has shown that slow decaying $C_0(k)$ functions could signal the evidence of multifractality of the data.

– leverage effect given by the correlation of returns with subsequent squared returns:

$$L(k) = corr(|X(t)|^2, |X(t+k)|) \tag{8}$$

This measure can be helpful to give additional information about the volatility of data.

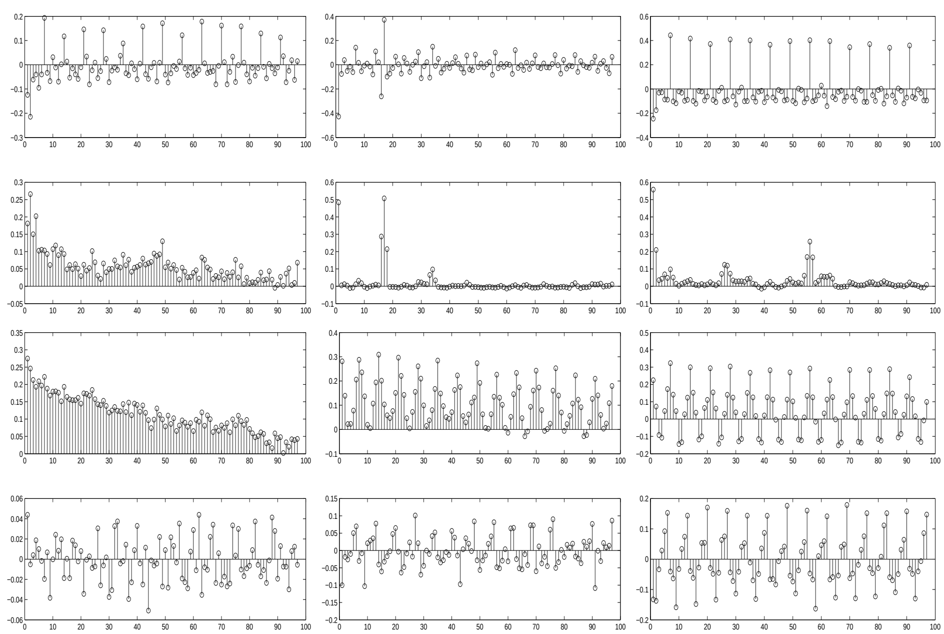

Fig. 4. Correlation measures: from top to bottom: $C_1(k), C_2(k), C_0(k)$, and $L_(k)$ as function of the lag amplitude k.

The analysis of results in Figure 4 suggests a number of observations:

(a) The detrending task needs probably to be refined, since the weekly effect is still high in all the observed markets;
(b) The ARCH effect is evident in AP and EEX markets, and it is less significant in OMEL;
(c) MI_{AP} index exhibits a slow decaying $C_0(k)$ function which should be deeply investigated to control if it is accomplishing with the assumption of its multifractality.

Before proceeding further, we have hence performed an additional filtering on (already detrended and deseasonalized) data with classical Fourier transform methods, in order to remove the residual cyclical effects evidenced form autocorrelation analysis.

Hence, further remarks have come from rescaled range analysis (RSA) of the observed data, whose results are given in Table 4.

Both classical Hurst coefficient estimates (H value) and empirical corrections are provided[3].

Table 4. Rescaled Range Analysis on MIs indexes

	MI_{AP}	MI_{EEX}	MI_{OMEL}
MIN LAG	4	4	4
MAX Lag	2048	1024	1024
H VALUE	0.2423	0.4293	0.3356
A–L VALUE	0.5063	0.4805	0.4805
mA–L VALUE	0.4748	0.4456	0.4456
E[R/S]	0.4546	0.4233	0.4233
DIFF. R/S–E[R/S]	−0.2128	0.006	−0.0801

[3] Shorcuts: A–L VALUE, mA–LVALUE, and E[R/S] stand, respectively for Anis and Lloyd, modified Anis and Lloid and empirical correction estimates. Such values has been computed as follows:

$$A - L_n = \frac{\Gamma(0.5(n-1))}{\sqrt{\pi}\Gamma(0.5n)} \sum_{r=1}^{n-1} \sqrt{\frac{n-r}{r}} \tag{9}$$

$$mA - L_n = \frac{\sum_{r=1}^{n-1} \sqrt{\frac{n-r}{r}}}{\sqrt{n\frac{\pi}{2}}} \tag{10}$$

$$E[R/S_n] = \frac{(n-0.5)\sum_{r=1}^{n-1} \sqrt{\frac{n-r}{r}}}{n\sqrt{n\frac{\pi}{2}}} \tag{11}$$

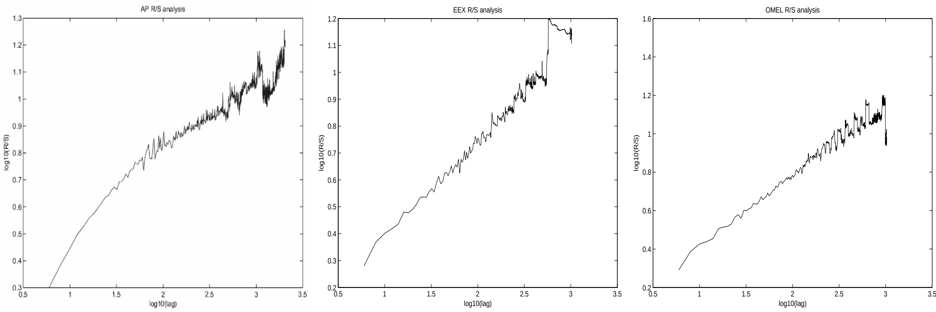

Fig. 5. R/S plots for MI indexes

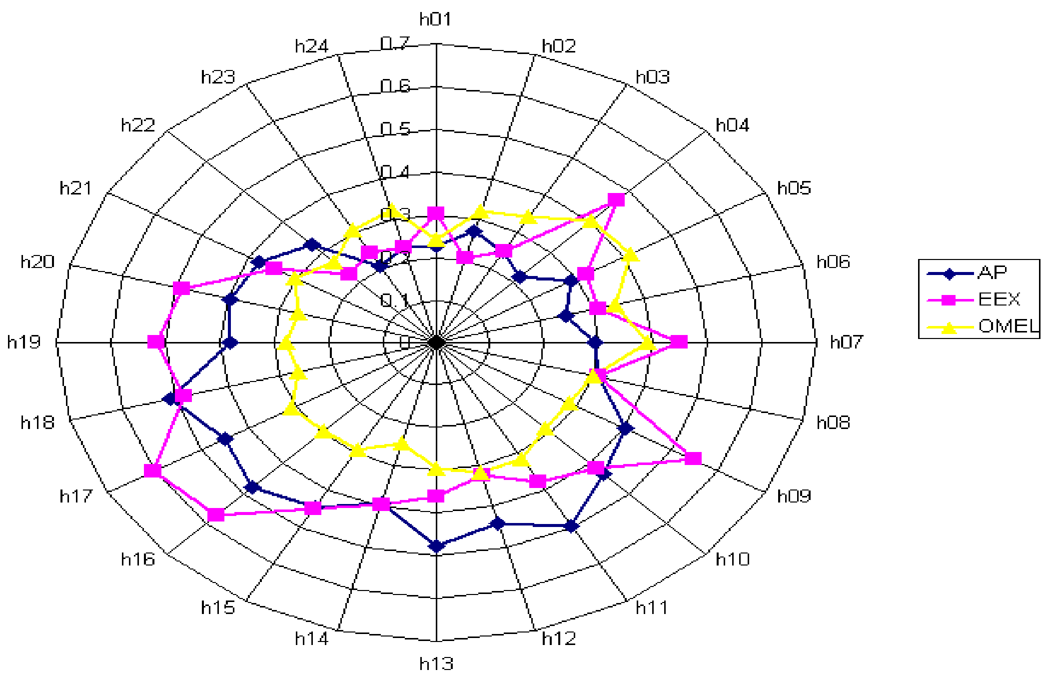

Fig. 6. R/S plots for AP, EEX and OMEL markets

Such results can be compared to those obtained on single hourly markets which are plotted in Figure 6, and analytically given in Table 14 of Appendix A. It is noteworthy to observe that, despite of the results given by the R/S on MI_{EEX}, the corresponding hourly markets are in some cases clearly persistent. For the remaining markets, on the other hand, we record the same anti–persistence features already seen on the corresponding indexes.

A first set of conclusions may be hence drawn at this point:

[i] electricity power markets presents some statistical features making them quite closer to *classical* financial markets;
[ii] the synthesis indexes that we have analysed exhibit in two cases (MI_{AP} and MI_{OMEL}) anti–persistence properties which should well accomplish with mean–reverting stochastic models. However, the result on MI_{EEX}, closer to those reliable with the assumption of normality, arises some perplexities and needs further investigation, since in the other tests the null hypothesis of gaussianity has been strongly rejected.
[iii] the different behaviour of EEX market on a side, and OMEL and Alberta Pool on the other side, lead us to state that, in order to give a proper representation of price movements, more than a single candidate model should be taken into account.

It is hence perfectly straightforward to search among an *ensemble* of candidate stochastic processes those which can be more reliable to our prefixed tasks.

Starting from this point we will introduce in the next section a selecting criterion helping to choose the best candidate process to fit the data, which has already given satisfying results when applied on financial markets.

3 A classification of stochastic processes by means of the scaling function

In a recent work [17] the authors provided a justification of the scaling function, derived from the empirical function of moments, and offered a powerful tool to compare and classify realizations from *ad hoc* processes in the simulation of the behaviour of the observed variable.

3.1 Self–affine and multifractal processes

We will now some brief remarks on the theoretical framework where the notion of scaling function (i.e. the function that will be referred to as $\tau(q)$ since now on), has been developed.

Let us assume $\{X(t)\}$ to be a stochastic process. Hence, $\{X(t)\}$ is properly self–affine (self–similar) with scaling coefficient H if the following equality in distribution holds:

$$\{X(lt_1), X(lt_2), , X(lt_k)\} \simeq \{c^H X(Lt_1), c^H X(Lt_2), ..., c^H X(Lt_k)\} \tag{12}$$

with $H > 0$, $c = l/L$, $t_1, t_2, , t_k \geq 0$, for every l, L, with $L > l$, that is:

$$P_l(X) = P_L[(l/L)^{-H} X] \tag{13}$$

where $P_L(X)$ and $P_l(X)$ are, respectively, the probability density functions at the temporal scales L and l.

In the same way, a stochastic process is said to be auto–affine if its moments (when existent) rescale according to a power law, that is:

$$M(q,t) = t^{Hq}[M(1,t)]^q \tag{14}$$

where $M(q,t)$ is the q–th order general moment at the time scale t. The brownian motion, the fractional brownian motion, as well as stable processes are consistent examples of proper self–affine processes, whose definition directly comes from Eq. 13.

Eq. 14 can be extended to more general cases in the following way:

$$M(q,t) = c(q)t^{\tau(q)+1} \tag{15}$$

where the functions $c(q)$ and $\tau(q)$ are defined on the real space.

The function τ is the *scaling function* of the process: in the case of monofractals we will have: $\tau(q) = Hq - 1$; in the case of multifractal processes $\tau(q)$ will be a nonlinear concave function [4], with $\tau(0) = -1$.

[4] More precisely, Schmitt, Schertzer and Lovejoy [19] speak about the *bilinear* form of $\tau(q)$ in the case of Lévy stable processes, since they get the following estimation: $\tau(q) = Hq - 1$, when $q < 1/H$, and $\tau(q) = 1$ for $q \geq 1$, where: $H = 1/\alpha$, and $0 < \alpha \leq 2$ is the Lévy index.

3.2 The empirical approach

We will now introduce a selecting criterion, which is based on the classification of processes through their scaling function.

It could be argued that the representation of a process via its scaling function is properly possible when a number of requirements hold, namely:

[a] stationarity of increments;
[b] process self–affinity;
[c] process multifractality [5];
[d] log–linearity of the partition function, with respect to time increments, when the q parameter is varied along the half–positive real axes.

Conversely, the processes we will refer to will be mostly non satisfying either requirements $[b]$ or $[c]$.

However, in all the examined cases the scaling function can be always reconstructed from raw data, through an empirical procedure, assumed a number of conditions hold, like we will specify in this note.

We will compare the shape of $\tau(q)$ drawn from raw data to that empirically derived from sample paths generated by different stochastic processes.

The empirical evidence will suggest us some preliminary remarks:

(i) in many cases of econometric interest, the $\tau(q)$ can be properly modelled through a quadratic function, and
(ii) such parabolic shape is generally a straightforward approximation of the empirical function $\tau(q)$, also when the observed processes do not fulfill one or more requirements set in $[a]$–$[d]$.

The value of $\tau(q)$ can be estimated according to a two–step procedure which will be described on following:

(1) evaluation of the empirical partition function:

$$S_q(\Delta t) = \sum_{i=1}^{N} (|X(i\Delta t + \Delta t) - X(i\Delta t)|)^q \qquad (16)$$

where Δt is the time scale. Here we have chosen $q \in [0.3; 5.5]$ with step 0.3, and Δt varying according to powers of 2, from 2 to 1024 (2048 in the case of AP).

(2) if it is possible to give a linear approximation to $log_2[S_q(\Delta t)]$ versus $log_2(\Delta t)$, the estimation of $\tau(q)$ comes in an easy way, by regressing $log_2[S_q(\Delta t)]$ onto $log_2(\Delta t)$.

The results for real data are presented in Figure 7.

[5] The notion is here intended in the *strict sense*, as multifractality in the continuum of all possible scales.

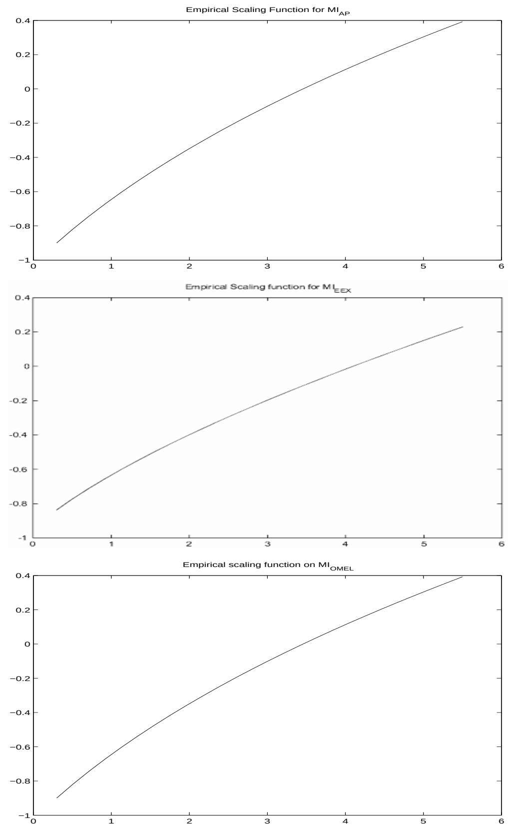

Fig. 7. Empirical scaling function for the three market indexes.

3.3 Simulation Features

The procedure described above has been applied on a group of data–sets obtained through Monte–Carlo simulations, relying on different types of generating processes, that will be therein briefly presented.

Table 5 summarizes the features of the performed simulations.

Since a number of words have already been spent on classical Ornstein–Uhlenbeck type processes, we will directly focus on the others newly introduced:

Gamma Ornstein–Uhlenbeck type processes. The sequence of realizations (increments) $X = \{X(t)\}_{t=1}^{29627}$ of the stochastic process have been obtained through the transformation:

$$X(t) = \varepsilon(t)\sigma^2(t) \tag{17}$$

Table 5. Simulation features.

Process	Shortcut	Model Type
Ornstein–Uhlenbeck	OU	Stoch. Volatility Model
Gamma Ornstein–Uhlenbeck	OU–Gamma	Stoch. Volatility Model
Fractional Brownian Motion	fBm	Log–returns Stoch. Model
Multifractal Random Walk	MFRW	Log–returns Stoch. Model

where ε is a zero mean normally distributed random variable, and $\sigma^2(t)$ comes from:

$$\sigma^2(t) = e^{\lambda t}\sigma^2(0) + \int_0^t e^{-\lambda(t-s)} dz(\lambda s) \tag{18}$$

We have replaced $\sigma^2(t)$ with the series representation suggested in Barndorff–Nielsen and Shephard (2001) [4], that gives equality in law:

$$\int_0^t e^{-\lambda(t-s)} dz(\lambda s) = \alpha^{-1} e^{\lambda t} \sum_{i=1}^{N(1)} log(c_i^{-1}) e^{-\lambda t r_i} \tag{19}$$

where: $c_1 < c_2 < \ldots < c_{N(1)}$ are the arrivals on a Poisson process with intensity $\nu\lambda t$; $N(1)$ is the related number of events until time 1, r_i is a uniformly distributed random variable, and $\sigma^2(0)$ is a random variable with distribution $\Gamma(\nu, \alpha)$.

Throughout the simulations we have used the estimated parameters values: $\nu = 7$, $\lambda = 0.04$, and $\alpha = 8.5$ in the case of MI_{AP}; $\nu = 7$, $\lambda = 0.08$, and $\alpha = 8.5$ in the case of MI_{EEX}, and $\nu = 14$, $\lambda = 0.01$, and $\alpha = 8.5$ in the case of MI_{OMEL}.

Fractional Brownian Motion (*fBm*). We refer to the model introduced by Mandelbrot and Wallis [14]:

$$X(t) = W_H(t) - W_H(t-1) \tag{20}$$

where

$$W_H(t) = CV_H^{1/2} \int_{-\infty}^{+\infty} \gamma_H (|t-s|^{H-\frac{1}{2}} I_{(-\infty,t)}(s) - |s|^{H\frac{1}{2}} I_{(-\infty,0)}(s)) dB(s) \tag{21}$$

H being the Holder exponent, with $W_H(0) = 0$, $V_H = (2H+1)sin(\pi H)$, and $\gamma_H = \frac{1}{\Gamma(H+1/2)}$.

Sample paths have been generated through the simulation method of the Cholesky decomposition, as indicated in [9].

Multifractal Random Walk. Recently introduced by Bacry et. al [3], it simulates log–returns according to the:

$$X_{\Delta t}(t) = \sum_{k=1}^{t/\Delta t} \varepsilon_{\Delta t}(k) exp\{\omega_{\Delta t}(k)\} \tag{22}$$

where $X(0) = 0$, $t = k\Delta t$, $\varepsilon_{\Delta t}(k)$ and $\omega_{\Delta t}(k)$ are independent gaussian variables. Additionally, $\varepsilon_{\Delta t}(k)$ is normally distributed with zero mean and variance $\sigma^2 \Delta t$, with $\varepsilon_{\Delta t}(k_i)$ independent from $\varepsilon_{\Delta t}(k_j)$, for each i, j, and $\omega_{\Delta t}(k)$ is a normal multivariate random variable, with expected value:

$$E[\omega_{\Delta t}(k)] = -\lambda^2 ln(\frac{L}{\Delta t}) \tag{23}$$

and covariance function:

$$Cov[\omega_{\Delta t}(k_1), \omega_{\Delta t}(k_2)] = \lambda^2 ln \rho_{\Delta t}(|k_1 - k_2|) \tag{24}$$

where:

$$\rho_{\Delta t}(k) = \frac{L}{(|k|+1)\Delta t} \tag{25}$$

Our simulation assumes the following estimated values for the parameters: $\lambda = 0.03$, $\Delta t = 2$, and $L = 2048$, for MI_{AP}; $\lambda = 0.03$, $\Delta t = 2$, and $L = 2048$, for MI_{EEX}, $\lambda = 0.03$, $\Delta t = 2$, and $L = 2048$, for MI_{OMEL}.

Since the procedure is computationally very expensive, we have generated sample paths no longer than 2048 observations.

3.4 Discussion of the results

For each generated time–series, the values of $\tau(q)$ have been regressed on q. Second order polynomial fitting has been performed.

Tables 6- 8 shows the coefficients for second order fitting polynomial, for both empirical data, and series generated by candidate processes.

Table 6. Second order polynomial coefficients for MI_{AP}

Shortcut	Coefficients		
	2nd Order	1st Order	0th Order
MI_{AP}	−0.0198	0.2441	−0.993
OU	−0.0178	0.7232	−1.393
OU–Gamma	−0.0184	0.2234	−1.403
fBm	−0.0073	3.5922	−1.482
MFRW	−0.0315	0.7146	−1.397

Table 7. Second order polynomial coefficients for MI_{EEX}

Shortcut	Coefficients		
	2nd Order	1st Order	0th Order
MI_{EEX}	−0.0126	0.2731	−0.900
OU	−0.0088	0.3542	−1.793
OU–Gamma	−0.0095	0.0328	−1.032
fBm	−0.0173	0.2232	−1.329
MFRW	−0.0016	0.4961	−1.007

Table 8. Second order polynomial coefficients for MI_{OMEL}

Shortcut	Coefficients		
	2nd Order	1st Order	0th Order
MI_{OMEL}	−0.0189	0.3518	−0.984
OU	−0.0043	0.5681	−1.093
OU–Gamma	−0.0024	0.0213	−1.121
fBm	−0.0011	4.1532	−1.492
MFRW	−0.0006	0.6912	−0.905

The best results have been provided by the OU–type processes, in the case of MI_{AP}, and by the fBm, in the case of MI_{EEX}: both two exhibiting coefficients for second order term very close to the corresponding ones on real data. The case of MI_{OMEL} is quite particular, since all the candidate processes have performed poorly. Possible explanations could be searched in the need for more proper estimated parameters. However, it could be also plausible than different processes, others than those considered should be taken into account.

A more remarkable contribution, adding strength to the previous assertions, however has come by observing the sensitivity of the function $\tau(q)$ to variations in the q level, evaluated as:

$$r(q) = -\frac{\tau''(q)}{\tau'(q)} \tag{26}$$

where $\tau''(q)$, and $\tau'(q)$ are, respectively, second and first order derivative of $\tau(q)$.

In our case, since $\tau(q)$ has approximated by $\widehat{\tau}(q) = \hat{a}q^2 + \hat{b}q + \hat{k}$, this has led to:

$$r(q) = -\frac{2\hat{a}}{2\hat{a}q + \hat{b}} \tag{27}$$

where $\hat{a}, \hat{b}, \hat{k}$ are, respectively, second, first and zero–th order term coefficients in the variable q.

We have evaluated $r(q)$ for the different candidate processes, and the results on $r(q)$ confirm those previously evidenced on $\tau(q)$ for MI_{AP} and MI_{EEX}, and indicate the classical OU–type as the more closer process (among those under examination) to observed data.

In particular, the Euclidean distance between $r(q)$ trajectories for the observed data and for each candidate process have been evaluated: the results are reported in Table 9.

Table 9. Normalized values of the distance between $r(q)$ on MIs indexes and simulated sample paths generated by candidate processes

ID	MI_{AP}	MI_{EEX}	MI_{OMEL}
OU	0.4605	0.1940	0.2734
OU–GAMMA	0.01	0.5578	0.5500
fBm	1	0.1523	0.4016
MFRW	0.1397	0.2225	0.3903

4 Conclusions

In this work we have focused on empirical aspects inside the behaviour of a number of electricity spot price markets.

Our aim has been two–fold:

(a) testing the power of the quantitative approach usually applied into *classical* financial markets.
(b) make a scan on some candidate processes which may operate as good substitute of the real underlying one. This task has appeared important to the extent of having analitically tractable tools to model the market behaviour.

We have taken into account data on the Alberta Pool, the German EEX market and the Spanish Omel.

Our choice has been motivated by the availability of data (which are freely downloadable at the corresponding national system operator homepage), as well as by reasonings connected with the need to manage with *robust* markets, i.e. with markets whose activity has been consolidated over a period of almost three (or more) years.

Before studying them, such data has been variously transformed, in order to take off seasonality components as well as possible.

Hence, three synthesis indexes (one for each market) have been obtained by daily averaging the returns.

The performed tests have shown that such indexes spare with common financial times–series a number of features (the non–gaussianity, for instance), but also maintain some properties which are typical of electricity markets (an incredibly resisting seasonal –weekly– component).

It is at the same time clear that hoping to have a common model to explain the behaviour of a multitude of electricity markets is just an utopia. Our analysis has given evidence that although two markets exhibit strong anti–persistence features (Alberta, and Omel), the EEX seems to be more erratic than expected.

As additional remark, volatility clustering phenomena have been clearly confirmed only on one market over the three under examination.

To make more light on our working scenario, we have then taken into account different candidate processes which are generally employed in financial markets modelling, and we have explored their potential in a sort of contest to find the best matching process to the real one underlying our data.

To such aim, we have employed a technique that we have already successfully experimented on financial data, based on the empirical estimation of the scaling function $\tau(q)$ of a set of data.

After providing a brief theoretical introduction to that concept, we have given some results based on Monte Carlo simulations.

Special attention has been given to the sensitivity of $\tau(q)$ to variations in the levels of q.

In this way we have been able to provide some earlier results which seem to give foundation, for instance, to the mono–scaling nature of the EEX market, whose behaviour is well fixed by a fractional brownian motion, while confirm that classical Ornstein–Uhlenbeck processes or conceptual evolutions of theirs (Gamma Ornstein–Uhlenbeck processes) can give satisfying results on some markets (namely on Alberta Pool, and Omel).

This obviously cannot put the final sentence on the debate on how electricity markets work: in a ideal wishing list, a huge number of tasks still remains to be completed, but we trust that tools like the empirical scaling function might offer an helpful criterion in the choice of good stochastic models of market behaviour.

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Appendix A: Common statistics on markets log–returns

Table 10. Common statistics on AP

	Min	Max	Range	Median	Mean	Standard Dev.	Variance	Skewness	Kurtosis
h01	−1.2079	1.1597	2.3676	−0.0001	0.00	0.2023	0.0409	−0.101672517	4.648545794
h02	−1.136	1.1184	2.2544	−0.0001	0.00	0.2157	0.0465	−0.084256507	3.17914448
h03	−1.0121	1.0162	2.0283	−0.0001	0.00	0.2171	0.0471	−0.03993425	2.795604124
h04	−1.0122	1.09	2.1022	−0.0001	0.00	0.2168	0.0470	−0.001557863	2.818767719
h05	−2.7231	2.7222	5.4453	−0.0002	0.00	0.2355	0.0555	−0.070873377	16.71510741
h06	−1.1003	1.0898	2.1901	−0.0001	0.00	0.2137	0.0457	−0.017608364	3.195616226
h07	−2.6769	2.7224	5.3993	−0.0002	0.00	0.2437	0.0594	−0.02367297	17.23116352
h08	−1.0515	1.424	2.4755	−0.0025	0.00	0.2399	0.0575	0.365617273	4.285621909
h09	−1.2129	1.8842	3.0971	−0.0033	0.00	0.2357	0.0555	0.419064024	5.833321036
h10	−1.0246	1.2431	2.2677	−0.0017	0.00	0.2218	0.0492	0.225425525	5.099191926
h11	−1.1858	1.0925	2.2783	−0.0014	0.00	0.2331	0.0543	0.115794746	5.680323715
h12	−1.4451	1.4105	2.8556	−0.0010	0.00	0.2337	0.0546	0.080931459	6.693938405
h13	−1.5187	1.4081	2.9268	−0.0006	0.00	0.2490	0.0620	0.023091932	7.732356233
h14	−1.7876	1.4087	3.1963	−0.0003	0.00	0.2465	0.0608	−0.055756631	7.150489709
h15	−1.4225	1.412	2.8345	−0.0008	0.00	0.2466	0.0608	0.060091745	7.493411332
h16	−1.4219	1.4095	2.8314	−0.0008	0.00	0.2454	0.0602	−0.043642748	7.775098975
h17	−1.3508	1.2359	2.5867	−0.0005	0.00	0.2620	0.0686	−0.067991388	5.166109329
h18	−1.4131	1.46	2.8731	−0.0003	0.00	0.2688	0.0723	0.031055678	4.66094141
h19	−1.188	1.053	2.241	−0.0003	0.00	0.2276	0.0518	−0.035123326	4.990485748
h20	−1.115	1.1793	2.2943	−0.0002	0.00	0.2201	0.0485	−0.092175017	6.625314112
h21	−1.0722	1.4359	2.5081	−0.0003	0.00	0.2072	0.0429	0.265156377	6.899792744
h22	−1.2547	1.2033	2.458	−0.0002	0.00	0.1955	0.0382	−0.059762463	7.242185117
h23	−1.0438	1.0724	2.1162	0.0000	0.00	0.1686	0.0284	−0.094963045	6.464455362
h24	−1.1235	1.206	2.3295	−0.0003	0.00	0.1909	0.0365	0.14454406	4.945904165

Table 11. Common statistics on EEX

	Min	Max	Range	Median	Mean	Standard Dev.	Variance	Skewness	Kurtosis
h01	1.17227	−0.61859	0.55368	0.0024	0.00	0.1511	0.0228	−0.015458788	1.3695
h02	3.7995	−1.6037	2.1958	0.0011	0.00	0.2090	0.0437	0.730438439	15.5515
h03	15.9081	−7.7921	8.116	0.0011	0.00	0.4241	0.1798	0.778761643	226.9540
h04	15.7537	−7.7027	8.051	0.0011	0.00	0.6467	0.4182	0.368424527	99.2877
h05	15.7776	−7.8014	7.9762	−0.0007	0.00	0.4625	0.2139	0.353338753	158.0180
h06	15.8675	−7.8733	7.9942	−0.0006	0.00	0.5542	0.3072	−0.056413887	103.0399
h07	16.0955	−8.1448	7.9507	−0.0098	0.00	0.7308	0.5341	0.168483736	56.6773
h08	16.4855	−8.3349	8.1506	−0.0200	0.00	0.6760	0.4570	0.201590296	72.0424
h09	16.6001	−8.3169	8.2832	−0.0215	0.00	0.5951	0.3541	0.592107689	105.0359
h10	3.7581	−1.5223	2.2358	−0.0135	0.00	0.2705	0.0732	0.528737658	6.4576
h11	2.34282	−0.97762	1.3652	−0.0104	0.00	0.2323	0.0540	0.390360121	2.6546
h12	2.6986	−1.1456	1.553	−0.0156	0.00	0.2807	0.0788	0.336054975	3.2462
h13	2.09794	−0.92284	1.1751	−0.0151	0.00	0.2230	0.0497	0.325586668	2.4099
h14	2.324	−1.0432	1.2808	−0.0240	0.00	0.2406	0.0579	0.431899361	2.3954
h15	4.0397	−1.8819	2.1578	−0.0263	0.00	0.2672	0.0714	0.446594855	9.7373
h16	14.6471	−7.4385	7.2086	−0.0220	0.00	0.4724	0.2231	0.13414236	153.6887
h17	12.2084	−6.0168	6.1916	−0.0194	0.00	0.3494	0.1221	0.538125179	170.7336
h18	16.8401	−8.8398	8.0003	−0.0140	0.00	0.4952	0.2452	−1.255793185	196.7287
h19	3.5966	−1.8022	1.7944	−0.0068	0.00	0.2117	0.0448	−0.279483881	15.6102
h20	2.6199	−1.1877	1.4322	−0.0014	0.00	0.1783	0.0318	0.24336806	10.3372
h21	1.9178	−0.8681	1.0497	−0.0011	0.00	0.1608	0.0258	0.063245274	3.7637
h22	1.83341	−0.84015	0.99326	−0.0011	0.00	0.1444	0.0208	0.143156658	3.4571
h23	1.35094	−0.58422	0.76672	−0.0014	0.00	0.1284	0.0165	0.073599168	2.3966
h24	1.32198	−0.55161	0.77037	0.0013	0.00	0.1339	0.0179	0.147345033	2.5159

Table 12. Common statistics on OMEL

	Min	Max	Range	Median	Mean	Standard Dev.	Variance	Skewness	Kurtosis
h01	0.84869	−0.40257	0.44612	−0.0001	0.00	0.0959	0.0092	0.043380625	2.1799
h02	1.50443	−0.79263	0.7118	0.0001	0.00	0.0882	0.0078	−0.157141809	7.5017
h03	4.8459	−2.5167	2.3292	−0.0002	0.00	0.1083	0.0117	−1.611321345	259.4892
h04	5.4728	−2.9415	2.5313	−0.0002	0.00	0.1130	0.0128	−3.218413	357.2647
h05	5.63	−3.2271	2.4029	−0.0001	0.00	0.1201	0.0144	−5.850864102	345.1164
h06	6.0968	−3.2043	2.8925	−0.0001	0.00	0.1486	0.0221	−1.674051026	258.4552
h07	6.318	−3.197	3.121	−0.0002	0.00	0.1496	0.0224	−0.147067137	222.7117
h08	6.5978	−3.172	3.4258	−0.0029	0.00	0.2629	0.0691	1.084261714	78.9709
h09	6.8602	−3.2245	3.6357	−0.0069	0.00	0.3098	0.0959	1.189427988	60.8370
h10	4.5264	−2.1152	2.4112	−0.0079	0.00	0.1991	0.0396	1.07631559	21.4850
h11	2.528	−1.1135	1.4145	−0.0050	0.00	0.1606	0.0258	0.762071665	9.4902
h12	1.80006	−0.78566	1.0144	−0.0036	0.00	0.1429	0.0204	0.690292163	4.9015
h13	1.52951	−0.66402	0.86549	−0.0038	0.00	0.1345	0.0181	0.594268125	3.5452
h14	1.75794	−0.99601	0.76193	−0.0035	0.00	0.1234	0.0152	0.33752055	5.4473
h15	2.7042	−1.4453	1.2589	−0.0033	0.00	0.1215	0.0148	0.096055267	19.2392
h16	2.7187	−1.5023	1.2164	−0.0037	0.00	0.1370	0.0188	−0.21961333	17.2605
h17	5.1978	−2.8758	2.322	−0.0041	0.00	0.1663	0.0277	−0.974274735	66.7315
h18	2.4019	−1.3691	1.0328	−0.0064	0.00	0.1522	0.0232	0.150155214	9.0802
h19	1.06647	−0.48392	0.58255	−0.0049	0.00	0.1274	0.0162	0.58892751	2.1852
h20	0.99724	−0.49595	0.50129	−0.0029	0.00	0.1172	0.0137	0.530370231	2.1831
h21	1.04393	−0.54044	0.50349	−0.0008	0.00	0.0956	0.0091	0.101517972	2.9807
h22	2.04721	−1.0587	0.98851	−0.0002	0.00	0.0921	0.0085	−0.05850077	19.3223
h23	2.04726	−1.0587	0.98856	−0.0001	0.00	0.0961	0.0092	−0.25413515	20.8390
h24	1.92943	−0.9896	0.93983	−0.0002	0.00	0.0957	0.0092	0.040582795	15.1284

Table 13. Jarque–Beran statistics on hourly data

Market ID	AP		EEX		OMEL	
	JB	Pr	JB	Pr	JB	Pr
h01	2106.558	0	83.72986	0	391.3921	0
h02	985.3183	0	10953.49	0	4644.955	0
h03	760.3338	0	2316038	0	5553171	0
h04	772.2314	0	443201.9	0	10527665	0
h05	27217.58	0	1122692	0	9831945	0
h06	993.0126	0	477336.7	0	5508953	0
h07	28925.02	0	144386.7	0	4089788	0
h08	1837.439	0	233292	0	514592.3	0
h09	3379.426	0	496254.7	0	305556.3	0
h10	2549.894	0	1926.404	0	38414.13	0
h11	3145.464	0	343.4343	0	7610.463	0
h12	4364.888	0	493.1466	0	2133.17	0
h13	5822.516	0	279.4883	0	1149.528	0
h14	4980.597	0	290.1344	0	2482.427	0
h15	5468.096	0	4297.692	0	30509.46	0
h16	5887.047	0	1061942	0	24575.95	0
h17	2598.963	0	1310576	0	367534.7	0
h18	2114.712	0	1740256	0	6800.568	0
h19	2424.103	0	10967.02	0	508.1429	0
h20	4276.582	0	4803.669	0	485.4346	0
h21	4660.179	0	635.635	0	734.5668	0
h22	5108.015	0	538.0007	0	30777.04	0
h23	4071.918	0	257.1315	0	35815.53	0
h24	2387.038	0	286.6097	0	18864.58	0

Table 14. Estimated Rescaled Range Coefficient on hourly markets

	AP	EEX	OMEL
h01	0.2247	0.30566	0.24226
h02	0.20583	0.20428	0.31739
h03	0.20966	0.25319	0.33972
h04	0.21204	0.47133	0.4052
h05	0.19758	0.3229	0.41435
h06	0.21231	0.30955	0.34206
h07	0.22511	0.4614	0.38913
h08	0.19649	0.31364	0.2979
h09	0.17832	0.54638	0.28145
h10	0.1627	0.4161	0.28529
h11	0.12284	0.37793	0.31365
h12	0.14979	0.32529	0.31282
h13	0.13897	0.36312	0.29536
h14	0.16527	0.39115	0.24441
h15	0.1537	0.44888	0.28889
h16	0.12224	0.57453	0.29603
h17	0.14695	0.60302	0.30659
h18	0.11435	0.47877	0.26504
h19	0.18732	0.51849	0.27701
h20	0.1836	0.48053	0.26326
h21	0.21351	0.34463	0.30246
h22	0.24304	0.22679	0.26684
h23	0.28357	0.25259	0.30333
h24	0.25721	0.23409	0.31936