

Threshold Cointegration between Stock Returns  
: An application of STECM Models  
Cointégration à seuil entre les rentabilités  
boursières du G7 : Application des modèles  
STECM

Fredj JAWADI\* [fredj.jawadi@u-paris10.fr](mailto:fredj.jawadi@u-paris10.fr)  
( MODEM-University of Paris X-Nanterre )  
Yousra KOUBAA [yousra.koubaa@u-paris10.fr](mailto:yousra.koubaa@u-paris10.fr)  
( MODEM-University of Paris X- Nanterre)

University of Paris X- Nanterre,  
200, Avenue de la République,  
Bâtiment G, Bureau 610 A,  
92001 Nanterre Cedex France  
Phone : 01 40 97 77 90  
          01 40 97 59 75  
Fax : 01 40 97 77 84

December 2, 2004

---

\*We are grateful to Valérie Mignon for her helpful comments concerning the French version of this paper.

## Abstract

The aim of this paper is to study the efficient capital market hypothesis by using recent developments in nonlinear econometrics. In such a context, we estimate a Smooth Transition Error Correction Model (STECM). We introduce the DowJones as an explanatory variable of the dynamics of the other stock indexes. The error correction term takes into account of the structural changes that occurred progressively from both the endogenous and the DowJones series. We note that the Smooth Transition Error Correction Model, for which the dynamics of adjustment is of ESTAR type, is more adequate than the linear ECM model to represent the adjustment of the stock price to the long term equilibrium price. Estimation results reveal the nonlinearity inherent to the adjustment process. In particular, we note that the adjustment is not continuous and that the speed of convergence toward price of equilibrium is not constant but rather function of the size of the disequilibrium.

*Keywords : Efficiency, Regime-Switching Models, Threshold Cointegration, STECM.*

## Résumé

L'objet de ce travail est de tester l'hypothèse de l'efficience informationnelle des marchés financiers à long terme en ayant recours aux développements récents de l'économétrie non linéaire. A cette fin, nous estimons un modèle à correction d'erreurs à transition lisse (STECM). Nous intégrons le DowJones comme variable explicative des dynamiques des autres cours boursiers, de manière à ce que le terme à correction d'erreurs tienne compte à la fois des changements structurels relatifs à l'endogène et au DowJones. Nous constatons que le modèle STECM est plus adéquat que le modèle ECM linéaire pour représenter l'ajustement du cours boursier au prix d'équilibre de long terme. Les résultats d'estimation révèlent des non-linéarités inhérentes au processus d'ajustement du cours vers le prix d'équilibre. En particulier, nous constatons que l'ajustement n'est pas continu et que la vitesse de convergence vers le prix d'équilibre n'est pas constante mais plutôt fonction de la taille du déséquilibre.

Mots clés : STECM, ajustement non-linéaire, taille du déséquilibre.

# 1 Introduction

In a previous study ( Jawadi and Koubâa (2002)) on stock markets relating to the G7 countries, we highlighted the interest of nonlinear modelling of STAR type (Smooth Transition Autoregressive Models)<sup>1</sup>. Indeed, this kind of modelling provided an innovative and an adequate approach to fear the efficient hypothesis. Moreover, the adjustment dynamic of stock indexes (CAC40, DAX100, BCI, TSX) seems to be dependent of the DowJones one.

In order to take into account of this dependence phenomenon, we propose to simultaneously analyze the evolution of the main financial rooms of the stock indexes of the G7 and to examine the impact of the DowJones on the dynamic of the others indexes at short and long-term. But due to the presence of nonlinear dynamics in the studied series, the linear ECM (Error Corrcetion Model ) can no more constitute an adequate alternative to describe these series. In such a context, we propose to estimate for each series a STECM model (Switching Transition Error Correction Model) for which the adjustment is of ESTAR type (exponential STAR)<sup>2</sup>. The nonlinearity will be introduced through a process of threshold cointegration in an ECM model.

The present work will be proceed as follows. The second section will be dedicated to a theoretical recall of efficient hypothesis in the weak sense. The third section will propose a theoretical formulation of the STECM model. The section four will conclude.

## 2 Efficiency and Cointegration

### 2.1 The efficient capital market hypothesis

Fama (1965) shows that a market is efficient if all available information is instantaneously reflected in stock prices. In this sense, it is impossible to forecast future price variations since the knew or anticipated events are already integrated in the present price. It is current to assimilate this hypothesis to the random walk of prices : the expectation of the future price is today's price. So, it is impossible to forecast future returns<sup>3</sup> from past returns.

To test the weak-form efficiency hypothesis, we verify if it is possible to forecast future returns from past returns. In other words, we test the existence of

---

<sup>1</sup>Results of linearity tests show that the nonlinear dynamic is only verified for the following series : CAC40, DAX100, BCI, and TSX, ( Jawadi and Koubaa (2002)).

<sup>2</sup>The adjustment dynamics of the DowJones has been represented by an ESTAR model.

<sup>3</sup>Let  $R_t$  be the return of an asset at the time  $t$  :  $R_t = \frac{P_{t+1} - P_t + D_t}{P_t}$ ,

Where :  $P_t$  is the stock price at  $t$  and  $D_t$  is the dividend at  $t$ .

Nevertheless, the relative variations of prices are generally assimilated to returns. This result is due to the fact that the ratio ( dividends /prices ) is generally considered as negligible compared to the relative variations of prices.

a serial interrelationship in the series of returns. The rejection of this hypothesis makes it possible to retain efficiency in the weak sense<sup>4</sup>.

Formally, let  $P_t$  be the price of a financial asset at the time  $t$ ,  $P_{t-1}$  the price at  $(t-1)$ . The hypothesis of the random walk of prices ( in logarithm) is given by the following relation :

$$\log(P_t) = \log(P_{t-1}) + \varepsilon_t \quad (1)$$

Where :  $\varepsilon_t$  is a white noise.

Let  $R_t$  be the return of the asset at the moment  $t$ , then we will have :

$$R_t \sim \log(P_t) - \log(P_{t-1}) = \varepsilon_t \quad (2)$$

Thus, Returns follow a white noise.

## 2.2 Reconciliation of Concepts of Efficiency and Cointegration

The cointegration concept has been introduced by Granger (1981), developed by Granger (1986), Engle and Granger (1987) and Johansen and Juselius (1990). It stipulates that some variable undergo some short-term disruptions, but while possessing the long-term same properties, can tie between them stable relations which converge towards an equilibrium of long term.

According to the concept of cointegration, let  $x_t$  and  $y_t$  be two variables non stationary in level but stationary while differentiating them  $d$  times. In the long term, if it is possible to find a linear combination ( $z_t$ ) between these two variable which is stationary :

$$z_t = x_t - ay_t \quad (3)$$

then  $x_t$  and  $y_t$  are said to be cointegrated.

The variable  $z_t$  constitutes the residual of the cointegration relation. It measures the variation between the variable  $x_t$  and its equilibrium value ( $y_t$ ) at the moment  $t$ . The stationnarity of  $z_t$  and the existence of a stable economic relation imply that it is possible to anticipate the evolution of the dynamics of  $x_t$  while knowing that of  $y_t$ . In this context, it is possible to test the weak-form efficiency hypothesis.

However on an efficient market, prices of two assets, noted  $x_t$  and  $y_t$ , can not evolve together in the long-term. It won't be therefore possible to forecast

---

<sup>4</sup>According to Fama (1970 and 1991), even though autocorrelations are statistically significant in the short-term, they are not significant according to an economic view point. Moreover, Fama (1991) notes that, in spite of their statistical significativity, these autocorrelations remain close to zero. In front of this state of fact, several authors such as Summers (1986), Fama and French (1988a), are tied to test the hypothesis of efficiency in the weak sense at long horizons.

the price of the asset  $x_t$  from that of  $y_t$ . Moreover, since the price of an asset on an efficient market in the weak sense integrates all the available information, the price of  $x_t$  at  $t$  must not permit to forecast the future variations of the price of  $y_t$ . Consequently, the cointegration is incompatible with efficiency (Campbell and Shiller (1987)).

On the contrary, if price and its fundamental value are not cointegrated (resp. cointegrated)<sup>5</sup>, there will be a durable spread (resp. a null spread) between the course and its equilibrium value which can be interpreted as a source of inefficiency (resp. efficiency) (Fontaine 1990).

Campbell and Shiller (1987) tested the cointegration between dividends and prices. They showed that these two variables are cointegrated and accepted the efficiency hypothesis because of the existence of a long-term stable relation between the price and its fundamental value. On the other hand, Fontaine (1990) showed that stock price evolutions are divergent and concluded to the inefficiency of the financial market.

Lilti (1994) studied the sense of causality between dividends and prices in order to determine if it is possible to forecast dividends from prices and the past dividends. He concluded to the absence of cointegration between prices and dividends but his interpretation is different from that of Fontaine insofar as he considers that the absence of cointegration relation is a source of inefficiency on the financial market.

Nevertheless, if the efficiency hypothesis is tested for a long time against a linear dependence, few works tried to verify this hypothesis against an alternative of nonlinear type. To this end, we propose to examine the dynamics of long-term adjustment of the price toward the equilibrium by testing the efficiency hypothesis against a nonlinear model of STECM type.

### **3 Threshold Cointegration : Estimation of a STECM model**

#### **3.1 Generalities**

Threshold cointegration has been the object of several works such as those of Anderson (1997), Balke and Fomby (1997), Escibano (1997), Michael, Peel and Taylor (1997), Van Dijk and Franses (1997), Dufrenot and Mignon (1999), Swanson(1999), Van Dijk, Franses and Lucas (2000) as well as Rothman, Van Dijk and Franses(2001), Dufrenot and Mignon (2002). These works tried to apply models of threshold cointegration on different markets in order to take account of the asymmetric dynamics in the existing relations between variables. Escibano (1997) was interested in the nonlinear adjustment mechanism in the demand for money of United Kingdom over the period 1878-1970. Swanson(1999), Rothman, Van Dijk and Franses(2001) showed that the threshold model seemed adequate to translate the existing relations between the production and the demand

---

<sup>5</sup>i.e. Residuals of the relation between prices and dividends are non stationary (resp. stationary).

for money. As for Anderson (1997), Michael, Peel and Taylor (1997), Van Dijk and Franses and Lucas (2000), they showed that the alternative of threshold cointegration permits to represent the financial asset dynamics in presence of transaction costs. In particular, Anderson (1997) supposes that the economic agent is indifferent facing the different types of assets and to their corresponding market. The deviation of the price compared to its equilibrium value creates opportunities of arbitrage that makes offer prices toward the equilibrium. Nevertheless, frictions on the market imply that the adjustment of these deviations can be asymmetric.

Moreover, the asymmetry can be explained either by the fact that agents don't react in the same way following shocks of different nature and sign, or by the fact that in presence of transaction and information costs, agents hesitate in their choice; they act on the market only when the hoped profit exceeds the costs. In this sense, the STECM model constitutes an adequate specification since the degree of error correction is a function of the sign and the size of the variation compared to equilibrium.

In other terms, asymmetrical effects resulting from the positive and negative deviations can be modeled while keeping the logistical transition function in which the error correction term noted,  $x_{t-d}$ , ( $d = 1, \dots, D$ ), where  $d$  is the delay parameter, constitutes the transition variable<sup>6</sup>. In this case, the phenomenon of Mean Reversion of  $x_t$  toward zero is much more important and the speed of adjustment is high as  $x_{t-d}$  is increasing<sup>7</sup>. As for the asymmetrical behaviors in the extent of the variation compared to equilibrium, they can be absorbed by an exponential transition function with  $s_t = x_{t-d}$ . In this second case, the return of  $x_t$  toward a stationary state is done with an adjustment term passing from  $\rho_1 + \rho_2$  to  $\rho_1$  as  $x_{t-d}$  increases.

### 3.2 Representation of ESTECM model

We refer to the work of Michael, Peel and Taylor (1997) to specify a STECM model for which its dynamics of adjustment is of ESTAR type. The model is given by :

$$\begin{aligned} \Delta Y_t = & \theta_0 + \sum_{i=1}^{p_1} \theta_{1i} \Delta Y_{t-i} + \sum_{j=1}^{p_2} \theta_{2j} \Delta W_{t-j} + \rho_1 x_{t-1} \\ & + \left( \theta'_0 + \sum_{i=1}^{p_1} \theta'_{1i} \Delta Y_{t-i} + \sum_{j=1}^{p_2} \theta'_{2j} \Delta W_{t-j} + \rho_2 x_{t-1} \right) \times (1 - \exp\{-\gamma(x_{t-d} - c)\}^2) + \mu_t \end{aligned} \quad (4)$$

Where :  $Y_t$  designates the stock price in logarithm,

$Y_{t-i}$  designates the endogenous lagged,  $i=1, \dots, p_1$ ,

$W_{t-j}$  designates the lagged explanatory variable,  $j = 1, \dots, p_2$ ,

<sup>6</sup> See Van Dijk, Franses and Teräsvirta (2001), p.11.

<sup>7</sup> The speed of adjustment belongs to the interval  $[\rho_1, \rho_1 + \rho_2]$ , where  $\rho_1, \rho_2$  designate respectively the adjustment term in the first and the second regime.

$x_{t-1}$  designates the error correction term,  
 $x_{t-d}$  indicates the transition variable,  
 $\rho_1, \rho_2$  designate respectively the adjustment term in the first and the second regime.,  
 $\theta_{kij}, \theta'_{kij}$  represent the autoregressive parameters in the first and in the second regime, with  $k=1,2$ .  
 $\mu_t \rightsquigarrow N(0, \sigma_\mu^2)$ .

Note that stages of specification and estimation of the STECM model are similar to those of the STAR model. The only difference concerns the definition of the transition variable, which is a lagged endogenous variable in the STAR model, whereas it is defined by the error correction term in the STECM model ( $x_{t-d}$ ).

Our empirical study concerns the stock indexes of the G7 countries (DowJones, CAC40, DAX100, BCI, TSX, FTSE100, Nikkei225). The series have been collected from the DATASTREAM data base. The graphics of prices and stock returns are given in the appendix 1.

In order to take into account of the nonstationnarity of stock indexes, prices have been transformed in logarithmic first differences. Series of returns are stationary, as it is illustrated by the results of unit root tests reported in appendix 2.

The third appendix provides usual descriptive statistics on the stock returns sets. The normality hypothesis is rejected and stock returns appear to be leptokurtic and asymmetric. This can be the sign of the presence of nonlinearities in the evolution process of these returns and/or in the adjustment of the price toward the equilibrium.

Previously to tests of linearity<sup>8</sup>, we examine the stationarity of the variations compared to the equilibrium,  $x_t$ , by applying the test of the Trace of Johansen (1988).

---

<sup>8</sup>The application of linearity tests against a STAR type model of (Jawadi and Koubaa (2002)) shows that Nikkei225 and FTSE100 don't present any trace of nonlinearity on average. The estimation of the STECM model will be carried on sets of (Dowjones, CAC40, BCI, TSX, DAX100).

The results of this test are presented in the following table :

**Table 1**

**Results of cointegration test ( Test of Johansen )**

Equation	Stat LR	Critical Value 5%	Critical Value 1%	p	Decision
CAC40 DJ	23.2138	12.53	16.31	1	cointegrated system
DAX100 DJ	6.2283	15.41	20.04	2	system not cointegrated
TSX DJ	25.3817	15.41	20.04	4	cointegrated system
BCI DJ	8.1881	12.53	16.3	2	system not cointegrated

The results in table 1 indicate that only the CAC40 and the TSX are cointegrated with DowJones. These results imply that the model seems to have a certain long-term validity for the CAC40 and the TSX <sup>9</sup>.

The application of linearity tests on the error correction term for all the possible values of d shows that the linearity was strongly rejected. These tests led us otherwise to retain the following values of d (see table 2).

**Table 2**

**Choice of the transition variable**

	CAC40	TSX
$\hat{d}$	3	1
$X_{t-d}$	$X_{t-3}$	$X_{t-1}$

Dumas(1992) and Flood, Taylor (1996) estimated a model of threshold cointegration of ESTAR type, and have put in evidence the nonlinear behavior of deviations of the exchange rate compared to the PPA. They also indicated that the major parameters are  $\rho_1$  and  $\rho_2$  . In the same context, Michael, Peel and Taylor (1997) suggested that more the gap to the PPA is big, more the tendency to come back to the equilibrium is strong. In this sense, even though one considers that  $\rho_1 \geq 0$ , the relations  $\rho_2 < 0$  and  $(\rho_1 + \rho_2) < 0$  must be respected to validate the nonlinear cointegration process. This means that returns could have

---

<sup>9</sup>It is worth noting that the stock prices are really explained by fundamental determinants. Nevertheless, we propose, within the framework of this work, a simplifying assumption which consists in introducing the DowJones as an explanatory variable of the other indexes in the long run.

an unit root ( $\rho_1 = 1$ ) or an exploding behavior for the small deviations while for the important gaps the process would be characterized by a phenomenon of Mean Reversion. The adjustment term of the linear ECM must belong to the interval  $[\rho_1 + \rho_2, \rho_1]$ .

After having presented conditions of validity of the ESTECM model<sup>10</sup>, we can proceed to the stage of estimation of the model. The estimation is achieved in two stages. In a first stage, we estimated a linear ECM whose results are reported in the table 3.

**Table 3**  
**Results of Estimation of a linear ECM**

Variable	$\Delta Y_{tCAC40}$		$\Delta Y_{tTSX}$	
	Coef	T-st	Coef	T-st
<i>Const</i>	0.0002	1.14	0.002	1.84**
$\Delta Y_{t-1}$	0.027	1.75**	0.127	7.52*
$\Delta Y_{t-2}$	-0.0193	-1.25	-0.027	-1.65**
$\Delta Y_{t-3}$	-0.0206	-1.34	0.026	1.62**
$\Delta Z_{t-1}$	0.0125	0.65	-0.0139	-0.99
$\Delta Z_{t-2}$	0.05	2.58*	-0.0204	-1.46
$\Delta Z_{t-3}$	0.4186	21.54*	-0.0133	-0.95
$x_{t-1}$	-0.002	-2.48*	-0.006	-4.51*
ARCH(4)	445.1		82.88	
$\bar{R}^2$	0.11		0.02	
DW	2.18		1.99	

(\*), (\*\*): significativity at the 5\% level, at the 10\% level.

In a second stage, we kept the estimated parameters as an initial parameters, being given values of  $\gamma$  and  $c$ , and we have estimated the STECM model ( see table 4).

---

<sup>10</sup>The exponential transition function makes it possible to model the adjustment of the price towards its equilibrium value according to the extent of the variations of the price compared to its equilibrium price.

**Table 4**  
**Results of Estimation of a nonlinear ECM :**

Variable	$\Delta Y_{tCAC40}$		$\Delta Y_{tTSX}$	
	Coef	T-st	Coef	T-st
$\theta_0$	-0.0817	-1.88**	-0.014	-1.18
$\rho_1$	1.1838	1.92**	0.266	1.02
$\theta_{11}$	-0.8868	-1.5	0.421	3.01*
$\theta_{12}$	-1.074	-1.76**	-0.06	-0.57
$\theta_{13}$	-0.168	-1.3	0.072	0.74
$\theta_{21}$	0.831	1.52	0.095	1.06
$\theta_{22}$	1.35	2.26*	-0.1641	-1.87**
$\theta_{23}$	0.276	1.59	0.205	1.81**
$\theta_0$	0.082	1.88**	0.0146	1.2
$\rho_2$	-1.2186	-1.93**	-0.275	-1.95**
$\theta_{11}$	0.907	1.53	-0.306	-2.16*
$\theta_{12}$	1.053	1.73**	0.035	0.32
$\theta_{13}$	0.15	1.15	-0.0486	-0.48
$\theta_{21}$	-0.814	-1.48	-0.128	-1.25
$\theta_{22}$	-1.309	-2.19*	0.153	1.64**
$\theta_{23}$	0.144	0.802	-0.226	-1.97*
$\gamma$	7.79	1.99*	1.345	1.93**
c	0.071	52.6*	0.047	25.12*
ARCH(4)	461.86		100.68	
$\bar{R}^2$	0.128		0.06	
DW	2.17		1.99	

(\*), (\*\*): significativity at the 5\% level, at the 10\% level.

Delays of the ECM model have been determined on the basis of criterias of information (AIC, BIC).

From table 4, we can affirm that the considered model present good statistical properties and implies a strongly nonlinear adjustment of the variation of the stock price compared to DowJones towards the equilibrium. In other words, variables ((CAC40, DowJones), (TSX, DowJones)) undergo some short-term disruptions, but while having the same properties of long term, can tie between them steady relations which converge toward an equilibrium of which the dynamics of adjustment is of ESTAR type.

Thus, the analysis suggests that the smoothing parameter is significant for the two sets, the transition is faster for the CAC40. The threshold is positive and significant. This permits us to keep the ESTAR model.

The term of adjustment of the first regime ( $\rho_1$ ) is positive for the two indexes. It is significant and superior to unity only for the CAC40, which can be explained by the fact that the dynamics of the CAC40 in the first regime diverges from those of the DowJones. The CAC40 has an exploding behavior in the first regime. But, as soon as one passes a certain threshold, the variation of the CAC40 in relation to the DowJones tends toward a stationary state.

Conditions of validity of the ESTECM model are verified for the two sets, the adjustment term ( $\rho_2$ ) in the second regime is negative and significant, the sum of the two strengths of recall ( $\rho_1 + \rho_2$ ) is also negative and significant. The adjustment term of the linear ECM belongs to the interval  $[\rho_1 + \rho_2, \rho_1]$ . Consequently, we keep the model of threshold cointegration to reproduce the cyclic movements characterizing the adjustment process.

A possible interpretation of the presence of nonlinearities is based on the presence of information and transaction costs. Indeed, the economic agent on the national market is better informed, gets information more quickly and pays less transaction costs as he was placed on the international market. Moreover, the operator's behavior can be determined according to other variables such as the relative degree of confidence, the level of development of the country, the degree of risk aversion, the non anticipated information and the role of surprise effects.

Nevertheless, the optimal choice of the agent remains in most cases bound to an arbitrage between expected gain and assumed costs. It is the reason for which, the agent has tendency to stand on the national market so much that his profit is optimal. But as soon as he anticipates a more elevated gain while varying his wallet (international wallet), he reached to the international market.

Such a result is not easily compatible with the weak-form of the efficient capital market hypothesis. Indeed, the existence of a cointegration relation between ((CAC40, DowJones), (TSX, DowJones)) shows that it would be possible to anticipate the evolution of the CAC40 and the TSX while knowing variations of the DowJones. Besides, the existence of such a relation of dependence between prices goes in opposition to the weak efficiency hypothesis since the price of an asset can be forecasted from another asset.

## 4 Conclusion

The efficiency hypothesis has been feared here facing a nonlinear type of dependence. Results of estimation of STECM models have some implications relating to this hypothesis. Indeed, our results put forward the existence of nonlinearity whose dynamics is given by an ESTAR model for which parameters testify a slow nonlinear adjustment toward the equilibrium. A possible extension will be to introduce other variables as the interest rate, inflation rate, dividends, information and transaction costs and the macro-economic factors as determinants of the stock price dynamics and to try to improve the stock price predictability both at short and long-term.

## References

- [1] Anderson H.M. (1997), “ Transaction Costs and Nonlinear Adjustment Towards Equilibrium in The US Treasury Bill Markets”, *Oxford Bulletin of Economics and Statistics*, Vol.59, pp.465-484.
- [2] Balke N. S et Fomby T.B. (1997), “ Threshold Cointegration”, *International Economic Review*, Vol.38, pp.627-646.
- [3] Brock W.A. Dechert W.D et Scheinkman J.A. (1987), “ A Test for Independence based on the correlation Dimension”, *Working Paper*, University of Wisconsin.
- [4] Campbell J.Y. et Shiller R.J. ( 1987), “ Cointegration and Tests for Present Value Models ”, *Journal of Political Economy*, Vol. 95, n°5,pp. 1062-1088.
- [5] Chan K.S et Tong H. (1996), “ On estimating Threshold in Autoregressive Models”, *Journal of Times Series Analysis* Vol. 7,pp. 179-190.
- [6] Davies R.B. (1977), “ Hypothesis Testing when a Nuisance Parameter is Present only under the Alternative”, *Biometrika*, Vol. 64,pp. 247-254.
- [7] Dijk D.V. and Franses P.H. (2000), “ Nonlinear Error-Correction Models for Interest Rates in the Netherlands”, Cambridge : *Cambridge University Press*, pp. 203-227.
- [8] Dufrénot G. and Mignon V. (2001), “ La cointégration non linéaire: une note méthodologique”, *Working Paper*, N°13, Université de Paris X- Nanterre, à paraître dans *Economie et Prévision*.
- [9] Dufrénot G. and Mignon V. (2002), *Recent Developments in Nonlinear Cointegration with Applications in Macroeconomics and Finance*, Kluwer Academic Publishers.

- [10] Dumas B. (1992), “ Dynamic Equilibrium and the Real Exchange Rates in a Spatially Separated World”, *Review of Financial Studies*, Vol. 2,pp. 153-180.
- [11] Eitrheim O. and Teräsvirta T. (1996), “ Testing the Adequaty of Smooth Transition Autoregressive Models”, *Journal of Econometrics*, Vol. 74,pp. 59-75.
- [12] Engle R.F. (1982), “ Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation”, *Econometrica*, Vol. 50, n°4, pp. 987-1007.
- [13] Engle R.F. and Granger C.W.J.(1987), “ Cointegration and Error Correction: Representation, Estimation and Testing ”, *Econometrica*, Vol. 55, n°2, pp. 251-276.
- [14] Escribano A.(1997), “ Nonlinear Error-Correction : The case of Money Demand in the U-K (1878 – 1970) ”, *Working Paper*, N°5,96-55, Universidad Carlos III De Madrid.
- [15] Escribano A. and Jordă (1999), “ Improving Testing and Specification of Smooth Transition Regression Models”, In Rothman P. (ed), *Nonlinear Times Series Analysis of Economic and Financial Data*, Boston: kluwer, pp. 298-319.
- [16] Fama E.F. (1965), “ The Behavior of Stock Market Prices ”, *Journal of Business*, Vol. 38, n°1, pp. 31-105.
- [17] Fama E.F. (1970), “ Efficient Capital Markets : A Review of Theory and Empirical Work”, *Journal of Finance*, Vol. 25, n°2, pp. 383-417.
- [18] Fama E.F. (1990), “ Efficient Capital Markets : II”, *Journal of Finance*, Vol. XLVI, n°5, pp. 1575-1617.
- [19] Granger C.W.J. (1991), “ Developpements in the Nonlinear Analysis of Economics Series”, *Scandinavian Journal of Economics*, Vol. 93, n°2, pp. 263-276.
- [20] Granger C.W.J. and Teräsvirta T. (1993), *Modelling Nonlinear Economic RelationShips*, Oxford University Press.
- [21] Granger C.W.J. and Teräsvirta T. (1999), “ A Simple Nonlinear Time Model with Misleading Linear Proprieties”, *Economics Letters*, N°62,pp.161-165.
- [22] Jawadi and Koubâa (2002), “ Dynamique non linéaire sur les marchés boursiers du G7 : Une application des modèles STAR, *Working Paper*, MODEM, Université de paris 10 - Nanterre.
- [23] Johansen S. (1988), “Statistical Analysis of Cointegration Vectors”, *Journal of Economic Dynamics and Control*, Vol. 12, pp.231-254.

- [24] Johansen S. (1991), “ Estimation and Hypothesis Testing of Cointegration Vector in Gaussian Vector Autoregressive Models”, *Econometrica*, Vol. 59, pp.1551-1580.
- [25] Lin C-F.J. and Teräsvirta T. (1994), “ Testing The Constancy of Regression Parameters Against Continuous Structural Change”, *Journal of Econometrics*, Vol. 62, pp.201-228.
- [26] Liu Y. (2001), “ Modelling Mortgage Rate Changes with a Smooth Transition Error Correction Model”, *Working Paper*, N°23, Bank of Canada.
- [27] Ljung G. and Box G.E. (1978), “ On a Measure of Lack of Fit in Time Series Models”, *Biometrika*, Vol. 65, pp.297-303.
- [28] Lundbergh S. and Teräsvirta T. (1998), “ Modelling Economic High-Frequency Time Series with STAR-GARCH Models”, *Working Paper Series in Economics and Finance*, N°291, Stockholm School of Economics.
- [29] Luukkonen R., Saikkonen P. and Teräsvirta T. (1988a), “ Testing Linearity Against Smooth Transition Autoregressive Models”, *Biometrika*, Vol. 75, N°3, pp.491 – 499.
- [30] Luukkonen R., Saikkonen P. and Teräsvirta T. (1988b), “ Testing Linearity in Univariate Time Series Models”, *Scandinavian Journal of Statistic*, Vol. 15, pp.161 – 175.
- [31] Luukkonen R. and Teräsvirta T. (1991) “ Testing Linearity of Economic Time Series Against Cyclical Assymetry”, *Annales d’Economie et de Statistiques*, N°20 / 21, pp.125 – 142.
- [32] Luukkonen R. and Saikkonen P.(1988), “ Lagrange Multiplier Tests for Testing Nonlinearity in Time Series Models”, *Scandinavian Journal of Statistic*, Vol. 15, pp.65 – 68.
- [33] Michael P., Nobay A.R. and Peel D.A. (1997), “ Transactions Costs and Nonlinear Adjustment in the Real Exchange Rates : An Empirical Investigation”, *Journal of Political Economy* Vol. 105, pp. 862-879.
- [34] Michael P., Peel D.A. and Taylor M.P (1997), “ Ajustement non linéaire vers le taux de change d’équilibre à long terme : le modèle monétaire revisité”, *Revue Economique*, Vol. 48, N°3, pp. 653-659.
- [35] Rothman P., Dijk D.V. and Franses P.H. (2001), “ A Multivariate STAR Analysis of the Relationships Between Money and Output”, *Macroeconomic Dynamics*, To appear.
- [36] Teräsvirta T. (1994a), “ Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models”, *Journal of the American Statistical Association*, Vol. 89, pp. 208-218.

- [37] Teräsvirta T. Dijk D.V. and Franses P.H. (2001), “ Smooth Transition Autoregressive Models- A Survey of Recents Developpements”, SSE/ EFI, *Working Paper Series in Economics and Finance*, N°380.
- [38] Tsay J.M. (1998), “ Testing and Modelling Multivariate Threshold Models”, *Journal of the American Statistical Association*, Vol. 47, pp. 5-46.
- [39] Wooldridge J.M. (1991), “ On the application of Robust Regression-Based Diagnostics to Models of Conditional Means and Conditional Variance”, *Journal of Econometrics*, Vol. 47, pp.5 – 46.

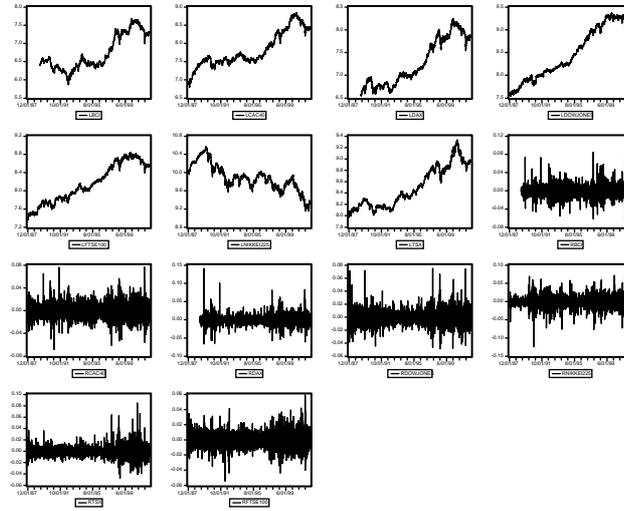


Figure 1:

**Appendix 1 :**  
**Stock prices and of stock returns**

**Appendix 2:**  
Unit root tests on stock returns

Serie	ADF Test			PP Test		
	Model	p	Stat ADF	Model	p	Stat.PP
DowJones	2*	1	-44.68	2	7	-60.64
CAC40	2	0	-58.69	2	7	-58.64
DAX100	3**	5	-24.4	3	7	-57.5
TSX	3	0	-54.47	3	7	-54.38
BCI	3	3	-26.89	3	7	-52.11
Nikkei225	3	5	-28.31	3	7	-73.5
FTSE100	2	7	-22.38	3	7	-57.91

**Appendix 3 :**

**Descriptive Statistics on stock Returns**

<u>Serie</u>	<u>Average</u>	<u><math>\sigma</math></u>	<u>Max</u>	<u>Min</u>	<u>Skew</u>	<u>Kur</u>	<u>J-B</u>
<b>DowJones</b>	-0.0004	0.0097	0.0745	-0.0486	0.5912	9.0083	5849.85
<b>CAC40</b>	-0.0004	0.0123	0.0068	-0.0681	-0.2072	5.5535	1044.85
<b>DAX100</b>	-0.0004	0.01229	0.1405	-0.0665	0.8785	12.8254	14080.64
<b>BCI</b>	-0.0003	0.0127	0.085	-0.0637	0.446	6.4714	1815.56
<b>TSX</b>	-0.0002	0.0084	0.085	-0.047	0.8204	11.9957	13110.03
<b>Nikkei225</b>	-0.0002	0.0122	0.1614	-0.1243	0.1651	15.3319	33683.26
<b>FTSE100</b>	-0.0003	0.0095	0.0589	-0.0544	0.1196	5.1508	732.12