# Multifractal analysis of Power Markets
# Some empirical evidence

Marina Resta[1]

[1] DIEM, sezione di Matematica Finanziaria, via Vivaldi 5, 16126, Genova, Italy
resta@economia.unige.it

**Abstract.** This work is intended to offer a comparative analysis of the statistical properties of hourly prices in the day–ahead electricity markets of several countries. Starting from the intermittent nature of typical price fluctuations in many power markets, we will provide evidence that working into a stochastic multifractal analysis framework can be of help to asses typical features of day–ahead market prices.

**Keywords**. Multifractals, Hurst Coefficient, Power Markets
**JEL classification codes: C0, C16, C5**
**02.50 , 02.70 , 05**

## 1 Introduction

In the last decade a rapidly increasing literature on stochastic models for prices of electricity grew up. The debate comes from the evidence of some featuring aspects of the electricity market, that make it an *uniquum* and a potential challenge for researchers.

Those features are at least four.

– *Non–storability.* Energy can be stored under different forms[1]. However, electricity resulting from the degradation of different forms of energy cannot be stored economically once generated [2]. This implies that prices are strongly dependent on the demand. Additionally, electricity cannot be transported from one region (country) to another one, because of existing bottlenecks, or limited transportation capacity. Hence, prices are local and can sensitively differ from country to country. Those conditions can affect the pricing of derivatives with a standard financial approach, and necessitate specific tools to be developed, since underlying products cannot be used for hedging

[1] We can quote, for instance, the water reserves stored behind the wall of a dam, or the coal stored in a coal–fired utility.

[2] In the case of a supply stack constituted only of hydropower generation as in Norway (more than 99% of hydro), it is possible to consider electricity as a storable commodity. On the other hand, in the case of a pure nuclear or thermal supply stack as in Germany (65% of thermal power, 19% of nuclear power, 8%*o* hydro-power generation and 8% in others generation), the previous remark does not hold anymore.

purposes. Previous works concerned to that problem have mainly addressed towards one of the following approaches:

(1) Modeling futures prices. This is the line of contributions [2,9,11] that have modelled the dynamics of the whole futures price curve. In this way, although the market can be considered as being complete, the information about the underlying price behaviour is completely missed.

(2) Modeling underlying price. The general approach ( [1], [4], [10]) consists in modeling prices by means of two risk factors: one capturing the short–term price dynamics, characterized by mean reversion and very high volatility, and the other factor representing long–term price behavior observed in the futures market.

- *Seasonality at different levels.* Electricity is a commodity displaying pronounced seasonal patterns: prices fluctuate cyclically in response to the variation of the demand; this, in turn, is very much influenced by the weather and other exogenous factors with various cyclical fluctuations. Since it is a deterministic feature of the price, seasonality represents useless information, and hence it should be properly removed before proceeding to any stochastic modeling of power market prices. To this aim, Pilipovic [12] assumed that the price can be splitted into two components, the underlying price, $S_t$, plus some seasonality effects, explained by a sinusoidal function:

$$P_t = S_t + \beta_A cos(2\pi(t - t_A)) + \beta_{SA} cos(4\pi(t - t_{SA})) \tag{1}$$

  where: $P_t$ is the spot price at time t; $S_t$ is the underlying spot price value; $\beta_A$ is the annual seasonality parameter; $t_A$ is the annual seasonality centering parameter (time of annual peak); $\beta_{SA}$ is the semi–annual seasonality parameter, and $t_{SA}$ is the semi–annual seasonality centering parameter (time of semi–annual peak). The main odd of this method is in the incorporation in Eq. 1 of the various types of seasonality (almost four in the case of electricity prices): daily, weekly and annual, as well as intra–daily. This latter comes from the strong change in consumption during day and night, opposing *on–peak* (from 8$a.m.$ to 8$p.m.$) and *off–peak* hours (from 8$p.m.$ to 8$a.m.$). An alternative method of data filtering can be found in [14], that uses a wavelet approach.
- *Volatility.* As highlighted by Duffie and Gray [6], prices in the energy market exhibity a volatility that is high and variable over time[3]. It is not unusual to notice annualized volatility of more than 1000%, with prices jumping from 30 Euros per MWh to more than 100 Euros per MWh. Additionally, volatility tends to increase with prices, phenomenon known under the name of *volatility clustering.* As logical consequence, electricity prices experience higher volatility in on–peak hours (period of the day displaying the highest prices) than in off-peak hours.

  At this time, common stochastic volatility models have revealed low explicative power: Deng [4] observes that it is difficult to implement proper stochas-

[3] This is due again to non–storability and limited transportation capacity of electricity.

tic volatility models when few historical data are available, as it happens for most European electricity markets.

– *Price spikes*. Spikes have mainly two sources. They can result from generation or transmission outages, or from sudden and unanticipated changes in the demand. Spikes are less frequent for markets with a high level of hydropower generation (Norway with 99.3 % of hydro, or Spain with 46.8%); in such markets, the reaction to an unanticipated increase in demand is quasi–instantaneous; hence, it limits spikes in price having as source an unexpected change in load conditions. However, in the countries with not extensive hydro power generation (for example Germany), price spikes are the rule more than the exception. In order to manage this *spiky* nature of prices, Deidersen et al. [3] replace Brownian motion with a positively skewed $\alpha-$stable Levy motion. In this model the price is forced back by the mean reversion after a jump, which may be not fast enough. Another solution is suggested by Geman and Roncoroni [7], who use models based on mean reversion coupled with downward jumps.

The emerging strong heterogeneity of contributions lead us to go deeper into the analysis of statistical features of power markets. Our contribution would give additional information about the way electricity prices behave, and should serve to modelers, in order to concentrate their efforts to models more *tuned* on real features of such markets. According to this ratio the work is organized as follows. **Section 2** will focus on the description of data general features. This means that, after introducing some details about their organization, we will present basic statistics of the observed markets. In **Section 3** we will deal with more specific tools for the analysis of the given time–series, namely the generalized Hurst exponent and the multifractal detrended fluctuation analysis [8] that have already proved to work proficiently with financial data ( [5],and [13]), also in presence of non–stationarity [8]. Finally, **Section 4** will end the paper with some conclusions and outlooks for future works.

## 2 Power markets: general framework

From the beginning the Electricity Power Industries (EPI) got established and developed as a natural monopoly with society. The three components resident within the EPI (i.e. Generation, Transmission and Distribution) were traditionally owned by the government or state authority. As such, public utilities were protected from competition of enterprises offering the same services. However, the utilities being vertically integrated, it was often difficult to segregate the costs incurred in generation, transmission or distribution. Additionally, the price setting was done by an external regulatory agency, often involving considerations other than economics.

In recent years, there have been widespread moves to deregulate, liberalize and privatize Electricity Power Industries across the world: vertically integrated utilities have been legally or functionally unbundled. The electricity market deregulation trend is now in full swing worldwide: in all markets, deregulation is seen as the way to increase the efficiency of already installed generation assets and hence to reduce prices for end–users. As a result, the EPI is moving from a monopoly structure to a more competitive one. Competition has been introduced both in the wholesale generation and retailing of electricity.

Focusing on the wholesale electricity markets, this ongoing process leads to organizations with several generation companies that compete to sell their electricity in a centralized pool and/or through bilateral contracts with buyers. Such organised markets typically comprise one or more of the following markets:

– *Day–ahead market.* This is the natural place where the bids are submitted. The market is cleared on the day before the actual dispatch. The day to be scheduled is divided into $n$ periods of $x$ minutes each; every bidding firm makes a price bid for every generation unit for the whole day. Commonly, in the day–ahead market either hourly contracts (for the 24 hours of the calendar day) or block contracts (i.e. a number of successive hours) are being traded. Whereas the former allows the market participants to balance their portfolio of physical contracts, the latter allows them to bring complete power plant capacities into the auction process. Block contract bidding may either be organised for a certain number of standardised blocks (dominant), or for flexible blocks.
– *Adjustment market.* The existence of this intra–day market (closing a few hours before delivery), is due to the long time span between the settling of contracts on the day–ahead market and physical delivery. It enables the participants to improve their balance of physical contracts in the short term.
– *Balancing market.* This market (also referred to as *ancillary services market*) covers the provision of a number of auxiliary services (e.g. frequency response, substitution reserve, voltage control, and reactive power support).

Figure 1 adds some explanatory remarks, presenting a general scheme of wholesale electricity market organization.

### 2.1 Data

We consider data from six different markets: the Canadian Alberta Power Pool, the Austrian EXAA, the French Powernext, the German EEX, the Dutch APEX[4], and the Spanish OMEL. As it can be seen from Table 1 which summarizes their main features, the data are heterogeneous for a number of reasons. Firstly, data have different lengths, since the markets began operative at different times. Additionally, their internal structure, although sharing similarities with that in Fig. 1, is conditioned by the type of prevalent generation plants (hydropower, nuclear, eolic to cite some). This means that we are considering data that could potentially exhibit different dynamics, since they come from markets characterized by different levels of maturity.

[4] Since 23 June 2004 APEX has turned its name into APX; however, in order to avoid confusions with Alberta Pool (AP), we will maintain the original (old) denomination.

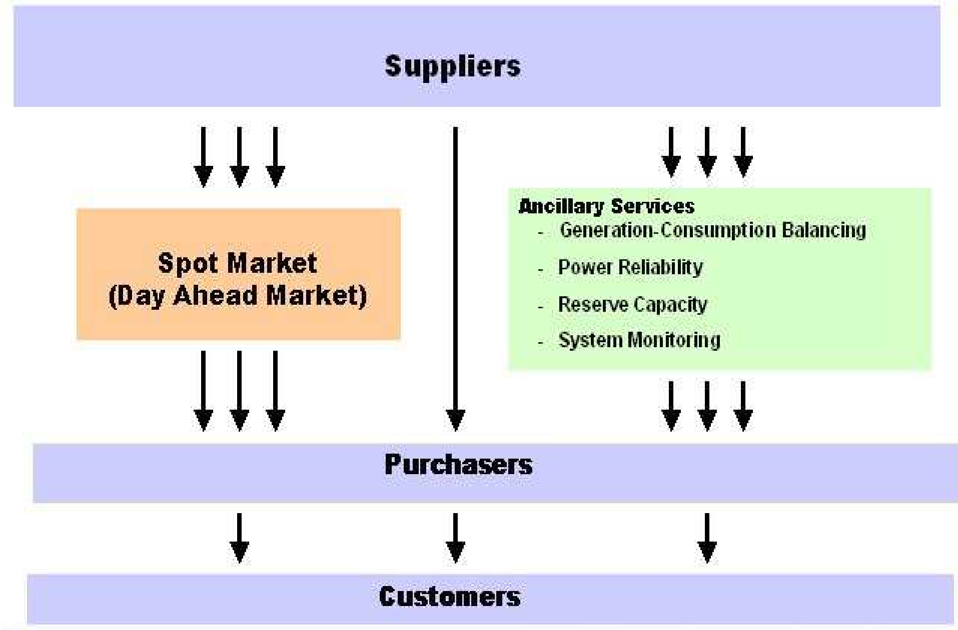



**Fig. 1.** The typical structure of Electricity market.

| Country | Time Frame | Sample size | ID–Tag | Length |
|---|---|---|---|---|
| Alberta | 01/01/1997 – 06/16/2004 | 2721 × 24 | AP | 2721 |
| Austria | 03/24/2002 – 06/16/2004 | 816 × 24 | EXAA | 816 |
| France | 11/26/2001 – 06/16/2004 | 1017 × 24 | PN | 993 |
| Germany | 06/16/2000 – 06/16/2004 | 1460 × 24 | EEX | 1460 |
| Netherlands | 06/23/1999 – 06/16/2004 | 1689 × 24 | APEX | 1689 |
| Spain | 01/01/1998 – 06/16/2004 | 2351 × 24 | OMEL | 2351 |

**Table 1.** General features of data under examination

Note that the availability of 24 series of data for each market is typical of power markets, since hourly prices are generally trade as a separate commodity for each hour $h$, $(h = 1, \ldots, 24)$. This leads to have a multivariate set of data for each market. Once moved from the sequence of price levels $\{P_i^{(h)}\}$ to that of corresponding price changes $\{P_i^{(h)}\}, (h = 1, \ldots, 24)$:

$$X(t)^{(h)} = log(P(t+1)^{(h)}/P(t)^{(h)}) \tag{2}$$

we have averaged at daily scale:

$$MI(t) = \frac{1}{N}\sum_{h=1}^{N} X(t)^{(h)} \tag{3}$$

where $N = 24$.

In other words, we have chosen to assume a mean indicator as *proxy* of the behaviour of the whole 24 hours market. We are perfectly aware of the traps inside this approach (see also [13]), but at the same time, we think that it should be useful to specify the features of the observed markets, almost at a first approximation level. The labels in use are reported in Table 2, while basic statistics for MI indexs are given in Table 3.

| Country | Time Frame | Index ID–Tag | Length |
|---|---|---|---|
| Alberta | 01/01/1997 – 06/16/2004 | $MI_{AP}$ | 2721 |
| Austria | 03/24/2002 – 06/16/2004 | $MI_{EXAA}$ | 816 |
| France | 11/26/2001 – 06/16/2004 | $MI_{PN}$ | 933 |
| Germany | 06/16/2000 – 06/16/2004 | $MI_{EEX}$ | 1460 |
| Netherlands | 06/23/1999 – 06/16/2004 | $MI_{APEX}$ | 1689 |
| Spain | 01/01/1998 – 06/16/2004 | $MI_{OMEL}$ | 2351 |

**Table 2.** Mean index for each observed market

| Stats | $MI_{AP}$ | $MI_{APEX}$ | $MI_{EEX}$ | $MI_{EXAA}$ | $MI_{PN}$ | $MI_{OMEL}$ |
|---|---|---|---|---|---|---|
| Mean | 0.000186 | 0.000134 | 0.000383 | 0.000889 | 2.985505 | $3.56E-05$ |
| Median | $-0.00159$ | $-0.021070$ | $-0.040327$ | $-0.034960$ | 2.796000 | $-0.026402$ |
| Max | 0.642661 | 3.538937 | 2.369608 | 1.516261 | 10.87400 | 1.698445 |
| Min | $-0.559136$ | $-2.672086$ | $-2.275751$ | $-1.150805$ | 0.177000 | $-2.140389$ |
| Std.Dev. | 0.114793 | 0.473413 | 0.356327 | 0.357191 | 1.040563 | 0.350336 |
| Sk | 0.146376 | 0.629779 | 0.788246 | 0.920958 | 1.946115 | 0.405740 |
| Ku | 6.003130 | 9.235085 | 7.711840 | 4.141578 | 11.48714 | 6.851964 |
| J–B | 1032.222 | 2847.567 | 1501.778 | 159.6591 | 7228.756 | 656.6485 |
| Pr. | 0 | 0 | 0 | 0 | 0 | 0 |

**Table 3.** Common statistics on MIs indexes. Here $Sk$ and $Ku$ are used as abbreviations for Skewness and Kurtosis respectively. J–B is the acronym for Jarque–Bera, and $Pr.$ is the corresponding probability to accept the null hypothesis of Normality in the J–B test.

Common statistics are typical of skewed time–series. The kurtosis is well accomplishing with non–Normal data. This assertion is also supported by the results of Jarque–Bera test, that reject the Null Hypothesis of Normality in all the observed cases.

Finally, looking at the autocorrelation plots (Figure 2), seasonality patterns are evident. In order to reduce seasonality, we have removed linear trends via a least–squared fitting, and then filtered the data using Inverse Fast Fourier transform method.

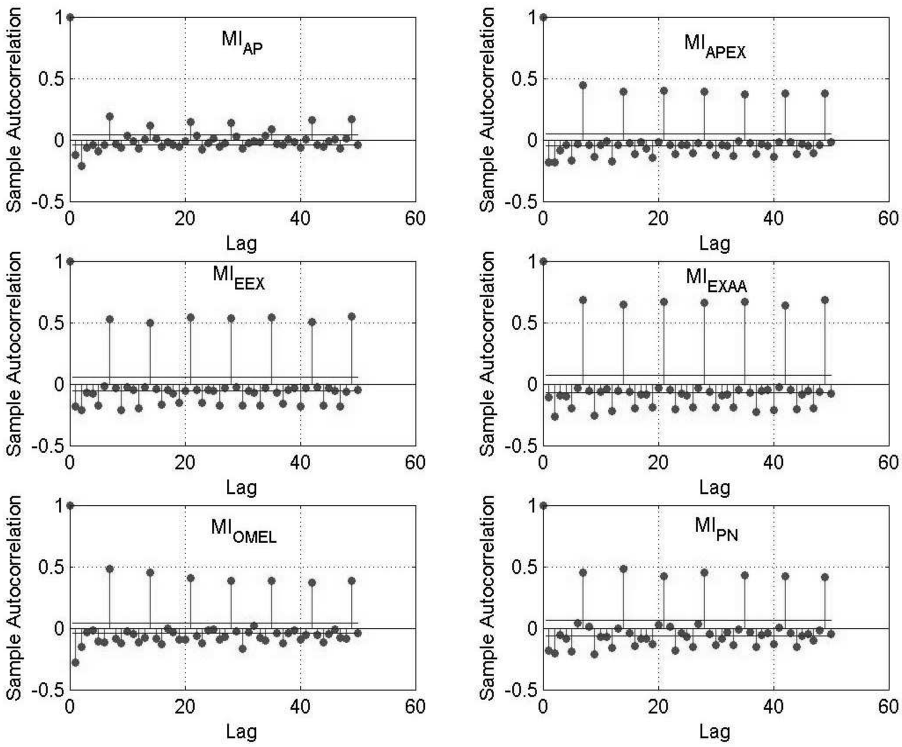



**Fig. 2.** Autocorrelation structure of mean indexes before de-seasonaling. The lag is expressed in days. Note typical peaks at lags multiple of 7 days.

# 3 Case study

## 3.1 Methodology

We basically refer to two tools.

– We analyze the q–order moments of the distribution of the Mean Indexes (MIs):

$$K_q(\Delta t) = \frac{E[|MIs(t+\Delta t) - MIs(t)|^q]}{E[|MIs(t)|^q]} \tag{4}$$

which has proved to give a good characterization of the statistical evolution of stochastic variables. We have then derived the generalized Hurst Exponent from the scaling behaviour of $K_q$, assumed that: $K_q(\Delta t) \approx (\frac{\Delta t}{v})^{qH(q)}$.

– Additionally, we use the multifractal detrended analysis method, as described in [8], that has proved to get affordable results also in presence of non–stationarity:

$$F^2(s,\nu) = \frac{1}{s}\sum_{i=1}^{s}\{IMIs[N-(\nu-N_s)s+i] - y_\nu(i)\}^2 \tag{5}$$

for $\nu = N_s{+}1, \ldots, 2N_s$, where $IMIs$ is the integrated series of MIs, $N_s$ is the sample size, and $y_\nu(i)$ is the fitting polynomial in the segment $\nu$. Although the fitting procedure can be performed with higher order polynomials, we have used only linear polynomial fitting.

### 3.2 Discussion of results

In order to evaluate the q–order moments of MIs indexes, we have used $\Delta t$ varying between $v = 1$ and 28 days. The behaviour of $q$ versus $qH(q)$ is shown in Figure 3.

One can immediately note the non–linearity of $qH(q)$ with respect of $q$ in the cases of mean indexes of the Alberta Pool ($MI_{AP}$), and of the Spanish market ($MI_{OMEL}$). This could be interpreted as a possible deviation of those indexes from the behaviour accomplishing with classical additive models.

We have also calculated the value of the generalized Hurst exponent both when $q = 1$, and $q = 2$, with results that are reported in Table 4.

| Market Index | Generalised Hurst Exponent | |
|---|---|---|
| | $q = 1$ | $q = 2$ |
| $MI_{AP}$ | 0.2693 | 0.2295 |
| $MI_{APEX}$ | 0.1379 | 0.1713 |
| $MI_{EEX}$ | 0.2787 | 0.0915 |
| $MI_{EXAA}$ | 0.1819 | 0.0933 |
| $MI_{OMEL}$ | 0.2184 | 0.2065 |
| $MI_{PN}$ | 0.2532 | 0.1300 |

**Table 4.** The Generalised Hurst Exponent is evaluated for $q = 1$ and $q = 2$.

It is possible to observe that the values maintain significantly beneath 0.5, in accordance with the anti–persistence of spot prices.

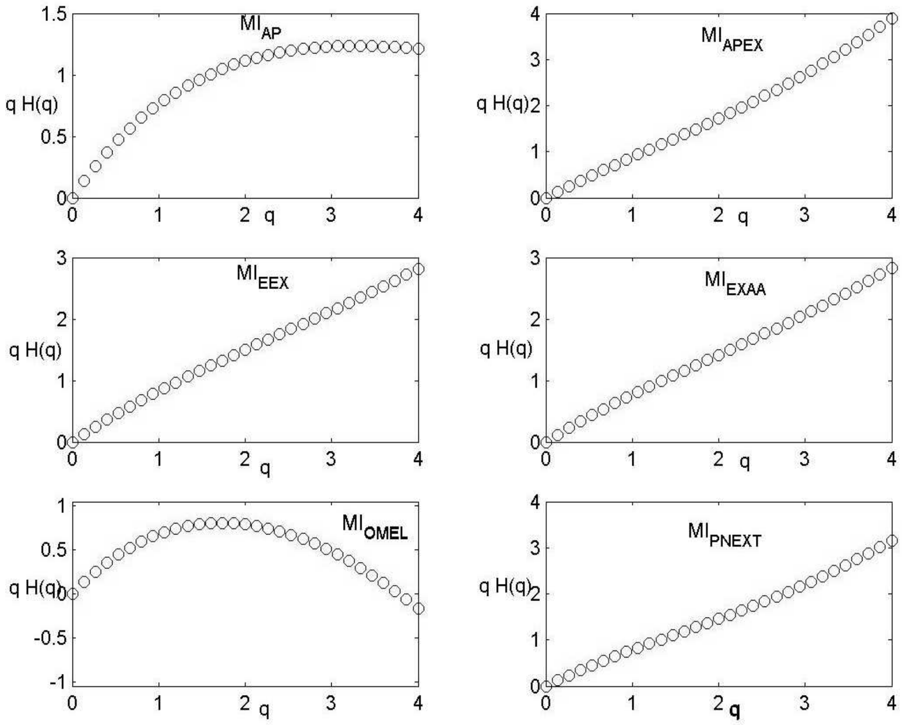


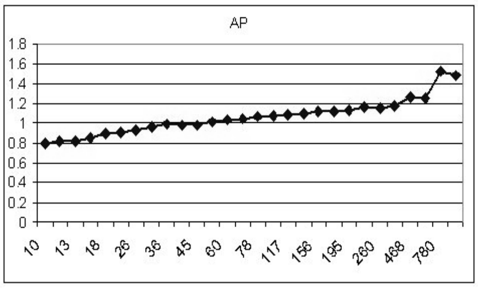

**Fig. 3.** Behaviour of $q$ vs. $qH(q)$. Note the variety of shapes in the markets under examination.

Results for the multifractal detrended fluctuations analysis are provided in Figure 4. In all the cases we observe that $F^2(s,\nu)$ is linearly dependent in $s$. However, the results are to be interpreted carefully, since we are managing relatively short time–series.

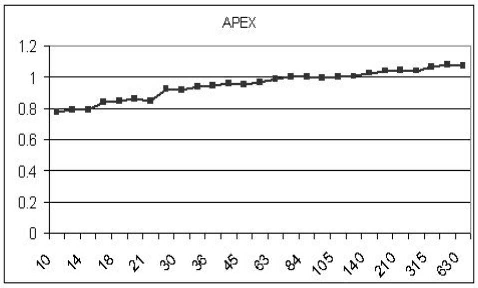
## 4 Conclusions

We have analysed features of spot (day–ahead) market prices in different countries. This empirical study has been performed by means of methods of multifractal analysis: the generalized Hurst exponent, and the multifractal detrended analysis.

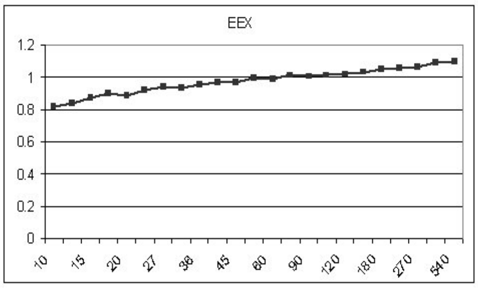
The results obtained seem reasonably confirm that modelling efforts should concentrate towards models that are able to incorporate multiscaling features.



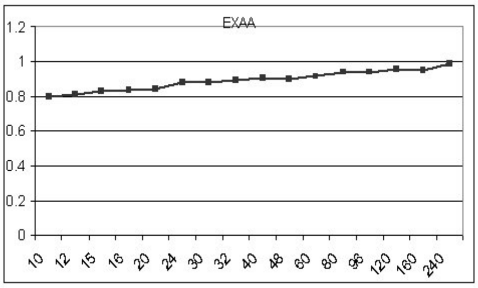

(a)



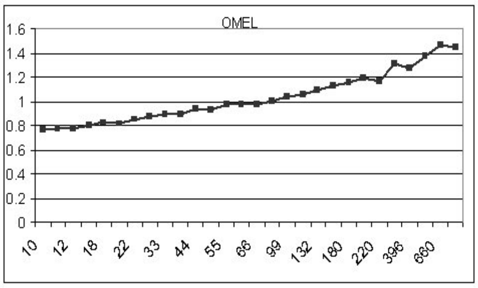

(b)



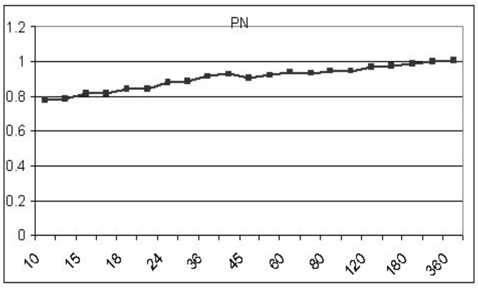

(c)




(d)




(e)




(f)

**Fig. 4.** Multifractal detrended analysis in the Alberta Pool (AP), Amsterdam Power Exchange (APEX), German (EEX), Austrian (EXAA), Spanish (OMEL), and French (PN) markets. Behaviour of the corresponding daily mean indexes. Here $F^2(s, \nu)$ is shown versus the scale $s$.

This is especially true for those markets that have achieved a good level of maturity, since they have been operative for much years.

Our conclusion, however, is merely a *stylised fact*: further inspections are needed, which can be obtained, for instance, by analyzing the stability of resuls for every hourly market, or by considering subsets of different lengths of the given data.

## References


1. Barz, G. L., *Stochastic financial models for electricity derivatives*, Dissertation, Standford University, 1999.
2. Clewlow, L., and C. Strickland, *Energy Derivatives- Pricing and Risk Management*, Lacima Publications, London, 2000
3. Deidersen, J., and S. Truck, *Energy price dynamics- quantitative studies and stochastic processes*, University of Karlsruhe, Tech. Report TR–ISMW–12/2002.
4. Deng, S., *Stochastic models of energy commodity prices and their applications: mean reversion with jumps and spikes.*, Working Paper, University of California Energy Institute, 2000.
5. Di Matteo, T., Aste, T., and M. M. Dacorogna, *Scaling behaviors in differently developed markets*, Physica A, 324:183–188, 2003.
6. Duffie, D. and S. Gray, *Volatility in energy prices, Managing energy price risk.*, Risk Publications, chapter 2, 1995
7. Geman, H., and A. Roncoroni, *A class of marked point processes for modelling electricity prices.* ESSEC Graduate Business School preprint, 2002.
8. Kantelhardt, J. W., Zschiegner, S. A., Koscielny–Bunde, E., Bunde, A., Havlin, S., and H. E. Stanley, *Multifractal Detrended Fluctuation Analysis of Nonstationary Time Series*, Physica A, 316: 1–4, December 2002
9. Koekebakker, S., and F. Ollmar, *Forward curve dynamics in the Nordic electricity market.*, Preprint, Norwegian School of Economics and Business Administration, 2001.
10. Lucia, J. J., and E. S. Schwartz, *Electricity prices and power derivatives: Evidence from the Nordic Power Exchange*, Review of Derivatives Research 5: 5–50, 2002.
11. Manoliu, M., and S. Tompaidis, *Energy Futures Prices: Term Structure Models with Kalman Filter Estimation.*, Applied Mathematical Finance, 9, 21–43, 2002.
12. Pilipovic, D., *Energy Risk: Valuing and Managing Energy Derivatives*, McGraw-Hill, New York,1998.
13. Resta, M., *Some issues on quantitative tools applied in the analysis of financial and deregulated power markets*, PhD Dissertation Thesis, University of Genova, Italy, 2004.
14. Simonsen, I., *Measuring anti–correlations in the nordic electricity spot market by wavelets*, Physica A, 322:597, 2003.