Transport contract optimization under information asymmetry: an example

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ABSTRACT : The present paper shows why information asymmetry and bivariate stochastic demand and spot price induce different behaviours and economic inefficiency in a carrier – shipper relationship. An example is offered of a single period, single echelon, shipper-carrier transport model where demand addressed to the shipper and the spot transport price, two exogenous stochastic variables, follow a bivariate exponential probability distribution function. We evaluate the objective functions of the carrier and shipper over one period reiterated with a mix of long-term and short-term procurement strategies under five scenarios of information sharing. Some clues as to ways of solving for other types of bivariates are provided.

Keywords: Supply chain management; transport; coordination; information sharing; bivariate exponential distribution.

INTRODUCTION

It is the purpose of this paper to present a new model able to take in both the aspects of transport as a supply chain member and as a service type of industry with its particularities.

LITERATURE REVIEW

Supply chain performance depends critically on how its members coordinate their decisions and it is hard to imagine coordination without some form of information sharing, as Fangruo Chen remarked in Chen (2002). In the supply chain management literature, transport service providers as suppliers are not usually individualised as such. One line of literature research focuses on efficient planning of routes, networks, warehouse location etc. In the other line of literature, supply chain efficiency can be increased by coordination, truth-inducing mechanisms, contractual engineering and information sharing (see Chen's review of 2002 and 1998, 2001a, 2001b, Chen and Yu 2001a, Chen and Yu 2001b, Anupindi and Bassok 1998, Porteus and Whang 1991, Lee and Whang 2000, Cachon and Larivière 1999, Zhao 2002). However, since their supplier definition entails back-logging of orders and inventory management, not all results apply to carriers or shippers. Ertogral et al. (1998) bridges both lines of thought: a single model integrates production and transportation planning, taking into account transport costs and schedules. This approach does not take into account the impact of imperfect information and decentralised decisions. Neither does it take into consideration the eventual over or under utilisation of the transport capacity involved.

The present paper follows on the tracks of Wu et al. (2002) which had modelled contracting arrangements between a Seller and a Buyer, when the deliverable product is a non-stockable product or service produced from a non-scalable capital-intensive production facility. A direct inspiration is also Gavirneni et al (1999) for capacitated suppliers and Li and Kouvelis (1999) for flexible contracts in the face of price uncertainty. The Seller and Buyer can negotiate a bilateral contract in advance and still negotiate "on the day" additional product at a reference "Spot" market price. The contract involves a fixed "reservation" price and an execution price, variable upon the actual demand. Optimal bidding, contracting parameters and procurement strategy is proven. The model considers that the buyer works under a WTP (willingness To Pay) function as per the standard tools found in Marketing Science. This enables the authors to find an equilibrium using the standard demand curve and to enunciate the optimal contracting policy when the spot price follows any probability density function.

Spinler and Huchzermeier (2003) propose a variation of the preceding model by using options in lieu of future and spot market contracts to increase capacity utilisation in the presence of state-contingent demand. They show that such a strategy effectively is Pareto improving things for both the seller of the option (transport supplier) and the buyer (the shipper). To circumvent the liquidity problem of transport as a non-standardised service, the model assumes that options will be traded on electronic marketplaces. However, as Grieger (2003) reported, carriers and shippers may be wary to trade with partners of unknown quality and customer service levels.

The assumption that the Buyer is able to reduce his demand when the spot price is too high may not reflect effectively normal practice as has been evidenced in spot markets for energy when prices have been recorded to exceed any economical level several times. Transport, as a service, which otherwise falls neatly in the description of the goods or services that may be included in the applications of Wu et al (2002), is not easily a service that an industrial firm may do without because the spot price has temporarily exceeded economical levels.

This paper is organised as follows. In the next section we describe the model involving one single tier in the supply chain: the contractual relationship between one shipper as client and one carrier as transport supplier. In the second section we describe the information asymmetries that both shipper and carrier may face through five scenarios of behaviour: in the first, base scenario, the information is common to both, decisions are centrally coordinated. In the second scenario, both carrier and shipper enjoy common information and stick to the letter of the contract but may privilege their particular interest when warranted. In the third scenario, the carrier retains information from the shipper. In the fourth, both shipper and carrier hide information from each other. In the fifth, the shipper retains private information on her received demand. We enunciate the necessary objective functions and compare results across different scenarios. In the third section, we solve for the optimal contract characteristics according to an instance of a bivariate exponential distribution function involving demand addressed to the shipper and spot market price for capacity. Finally, we draw conclusions from the results.

TRANSPORT MARKET MODEL

As in Wu et al. (2002), a model involving a Carrier C selling transport services to a Shipper S is presented. The contract involves setting a base capacity that has to be provided for the long term at a pre agreed price. Failing to provide this capacity entails a penalty to be paid by the carrier. Additional capacity may called up when the Shipper faces extra demand. This additional capacity is also provided at a pre agreed price and failure to meet the engagement entails payment of a penalty on the Carrier's part. So that the shipper does not over estimate the capacity she needs, she ahs to pay the Carrier a penalty for unused capacity under the base commitment in the contract. "On the day", the Shipper can cover transport needs in an alternative market, called here the "spot" market. In difference to the energy market as described in Wu et al (2002), the capacity is not sold at a "reservation" price but strictly as needed. The penalty levied on the unscrupulous Shipper who bids for more capacity then she will actually need is there to enforce coordination and signalling of demand information to the Carrier.

2.3 Opportunistic behaviour

Opportunistic behaviour occurs when either S or C can escape from their contractual engagements without incurring retaliation from the other party. All retaliation depends upon verifying opportunistic behaviour, which bears a cost. We will focus in this paper on certain pieces of information which can make a significant impact on the cost functions of either party. We centre our attention here on two particularly sensitive pieces of information.

2.3.1 Transport capacity of carrier

The first piece of information is the size of the transport capacity the carrier owns or otherwise controls. Ex-ante the shipper verifies the available capacity of the carrier and the carrier must convey all necessary information so that the shipper can be assured that the required capacity exists. Thereafter, no further control is undertaken by the shipper. So, in the course of the life of the contract, this information is no longer observable. Only when the contract comes up for renewal can the shipper use records of past shipments to assess the capacity of the carrier. This situation escapes our model.

2.3.2 Available cargo to be shipped

The second piece of information involves the size of the available transport requirements of the shipper: C cannot verify that the orders handed him by S represent her entire need. This information is also neither observable directly nor verifiable without cost to C. S may contract added capacity with other carriers whenever its suits her financially.

2.7 Demand and capacity characteristics

2.7.1 Stochastic variables

State of nature is represented using two exogenous variables: P is the market price for immediate transport. This price ranges from VC, the variable cost common to all carriers in the universe of carriers available to S, to infinity. The demand that the shipper meets is an exogenous, stochastic variables Q. $\Omega(P,Q)$ is the probability plane containing the possible realisations of the tuples of transport spot price and demand addressed to shipper S. $F_q(.)$is the continuously differentiable, invertible and monotonous cumulative distribution function of demand addressed to S. $f_q(.)$ (mean μ_q, standard deviation σ_q) is the density functions of $F_q(.)$. $F_p(.)$ is the continuously differentiable, invertible and monotonous cumulative distribution function of the spot market price and $f_p(.)$ its density function (mean μ_p and standard deviation σ_p). Let be the correlation factor between $F_q(.)$ and $F_p(.)$.

We call F(.) the bivariate continuously differentiable, invertible and monotonous cumulative distribution function of both demand Q addressed to shipper S and spot price P. $f(.)$ is the density function of F(.) with mean μ, standard deviation σ and correlation coefficient ρ.

The shipper S knows ex-ante the mean μ_q and standard deviation σ_q of the cumulative distribution function of demand. The demand has to be satisfied in full at each period.

All other production costs of S are ignored.

The total capacity of C is W. C has a variable cost per unit transported VC and a fixed cost F_c. No assumption is made regarding W. F_c is a function of this capacity W.

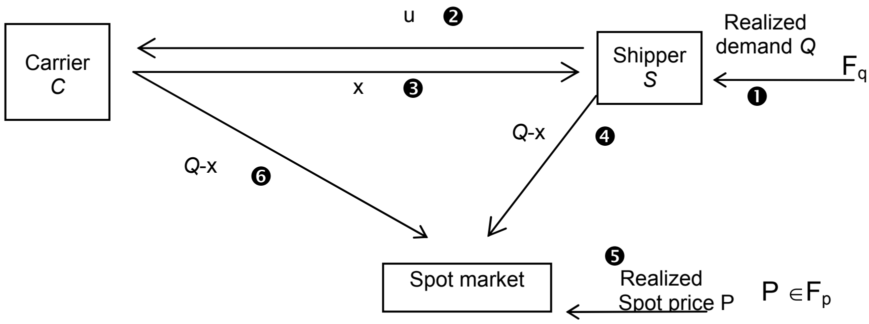

Fig. 1: Capacity allocation

In figure 1, u is the demand that S chooses to allot to C.

We list below the variables and parameters:

C Carrier
S Shipper
c Base capacity contracted
q Contract price for the base capacity q
q_a Additional capacity that S can call upon from C specified in contract
p_a Price for additional capacity qa, specified in contract
θ_s Penalty paid by S to C for not complying with contract specifications
θ_c Penalty paid by C to S for not complying with contract specifications
$F_q(.)$ Cumulative distribution function of demand Q
$F_p(.)$ Cumulative distribution function of spot price P
$f_q(.)$ Marginal probability distribution function of Q
$f_p(.)$ Marginal probability distribution function of P
F(.) Bivariate cumulative distribution function of demand Q and price P
$f(.)$ Bivariate probability distribution function of demand Q and price P
ρ Correlation coefficient between P and Q
μ_q,σ_q Mean and standard deviation of Q
μ_p,σ_p Mean and standard deviation of P
VC Variable cost faced by C when transporting
F_c Fixed cost of C
u Decision variable of S: what share of her demand to allocate to C
x Decision variable of C. What share of capacity to allocate to S

2.8 Objective functions

2.8.1. Regionalizing the probability space

We divide the probability space Ω into regions so as to facilitate the discussion regarding the best decisions by S and C (fig. 2):

$$\begin{aligned}
&\Omega 1(Q,P)=\{Q:0\le Q\le q;VC\le P\}\\
&\Omega 2(Q,P)=\{Q:q<Q;P:VC\le P\le p_a-\theta_s\}\\
&\Omega 3(Q,P)=\{Q:q<Q\le q+q_a;P:p_a-\theta_s<P\le p_a+\theta_c\}\\
&\Omega 4(Q,P)=\{Q:q<Q\le q+q_a;P:p_a+\theta_c<P\}\\
&\Omega 5(Q,P)=\{Q:q+q_a<Q\le W;VC\le P\le p_a-\theta_s\}\\
&\Omega 6(Q,P)=\{Q:q+q_a<Q\le W;p_a-\theta_s<P\le p_a+\theta_c\}\\
&\Omega 7(Q,P)=\{Q:q+q_a<Q\le W;p_a+\theta_c<P\}\\
&\Omega 8(Q,P)=\{Q:W<Q;VC\le P\}\\
&\Omega 9(Q,P)=\{Q:W<Q;p_a-\theta_s<P\le p_a+\theta_c\}\\
&\Omega 10(Q,P)=\{Q:W<Q;p_a+\theta_c<P\}
\end{aligned} \tag{1.1}$$

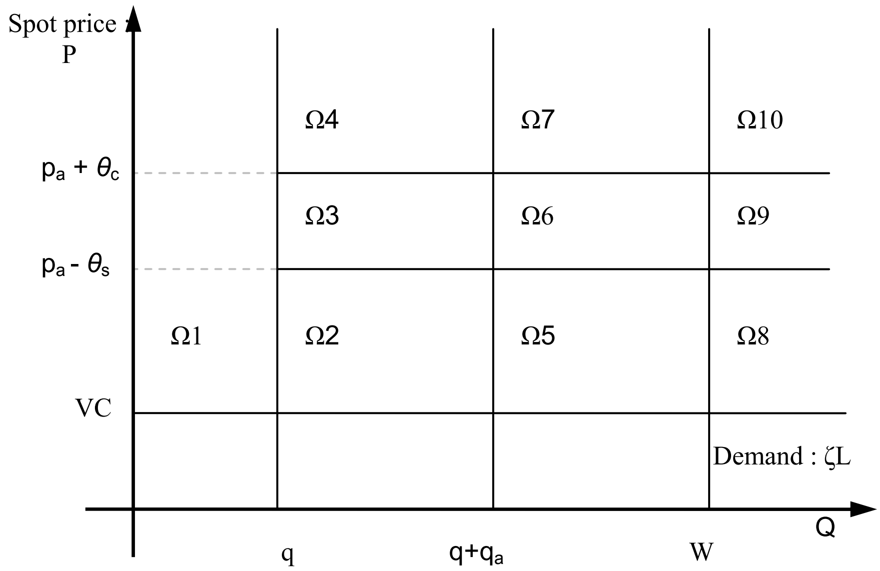

Fig. 2: Probability spaces for spot price and demand addressed to S

We assume that for spot prices under VC, the carrier C will not carry cargo.

2.8.2. Carrier objective function

In our setting, carrier C has just one customer: S (fig. 1). If the capacity required to carry the realised demand from S does not reach total capacity, the excess capacity is lost for all intents and purposes, impacting the carrier's profitability

The objective function of the carrier is to increase revenue and profits. His ex post decision variables are the capacity he allots to S: x is the allotted capacity to S. $W-x$ is the wasted capacity. We consider that the fixed costs of supporting the necessary assets are specific, sunk and that the carrier does not have the choice to withdraw from the allocation game with S. We therefore neglect all considerations as to fixed costs of C. His profit function can thus be written by using the terms of the contract.

We restate here all the contract characteristics as defined above:

$W \geq x$	Total transport capacity of C (fleet capacity)
$0 < q + q_a \leq W$	contracted capacity plus negotiated additional capacity has to be less than total capacity
$0 \leq \theta_s < c,$ $0 \leq \theta_c < c$	penalties paid by shipper or carrier are less than the contract price
$0 \leq q_a < q,$	additional capacity is not higher than the base capacity contracted
$c \leq p_a$	price for additional capacity is higher than the base capacity price
$0 \leq u \leq Q$	u is a capacity, decision variable of the shipper is at most equal to total demand received by shipper.
$0 \leq x \leq u$	x, decision variable of the carrier is at most equal to the effective capacity that the shipper asks him to provide.
$VC \leq P,$	the spot price for transport capacity cannot take values less than the variable cost, assumed to be the same for the whole transport sector (the shipper will not find a carrier who will carry cargo under this price).

The profit function is conditional upon the allocation by S and the spot market price:

$$\pi\left(x \mid u, \Omega_i\right) = R_i\left(x \mid \Omega i\right) + P\left(Q - x\right) - VCx \qquad (1.2)$$

where R_i is a revenue function, conditional upon the demand u addressed by S and the spot market price, of the form:

$$R_i\left(x \mid u, \Omega i\right) = \begin{cases} xc - \left(\min\left(u, q\right) - x\right)\theta_c + \left(q - u\right)\theta_s : 0 \leq x < q \\ qc + \left(x - q\right) p_a \qquad : q \leq x \leq q + q_a \\ qc + \left(x - q\right) p_a + \left(x - q - q_a\right) P : q + q_a < x \leq W \end{cases} \qquad (1.3)$$

One possible graph of such a function is represented in figure 3.

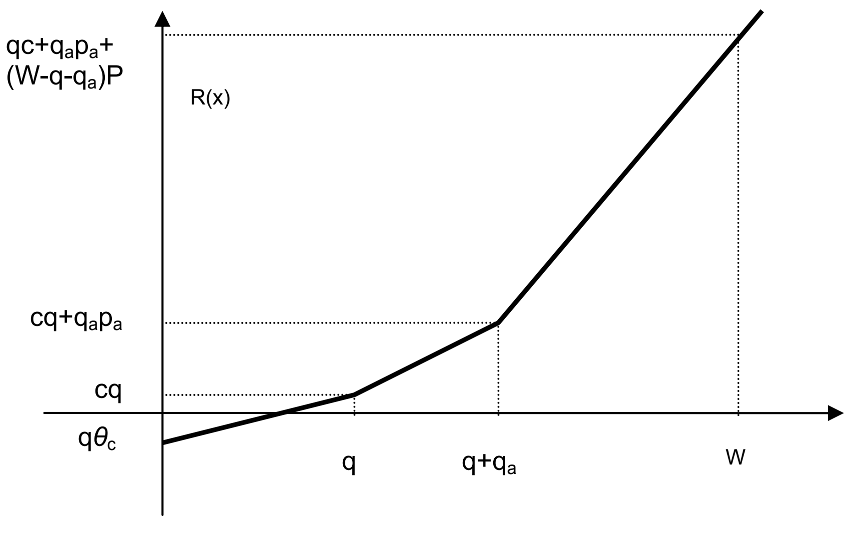

Fig 3: Behaviour of $R_i(x)$where u = W

2.8.3. Shipper objective function

Shipper S produces and sells a product that requires transportation. She, as Stackelberg leader, must decide whether to allocate her necessity to her chosen contractual carrier at the ex-ante contractual price or to the spot market at the going spot market price. She decides to allocate u quantity to carrier C.

The decision variable u can take all values between 0 and total received demand Q (see fig. 4). Whatever transport necessity is not being allocated to C will be offered to the spot market at the going spot price P. The function is conditional upon the response S receives from C, which is represented by x(u). By investigation, we see also that S has an opportunity to reduce transport cost by diverting cargo to the spot market when conditions of the spot price relative to the contract parameters warrant it. Let us call O the shipper's objective function and characterize it as such in each region Ωi:

$$O_i\left(u \mid x,\Omega i\right)=\begin{cases} cx(u)+[q-u]^{+}\theta_s+\left(\min(q,u)-x(u)\right)\theta_c+\left(Q-x(u)\right)P: 0\leq x(u)\leq q \\ cq+\left(x(u)-q\right)p_a-\left[u-x(u)\right]^{+}\theta_c+[Q-u]^{+}\theta_s+\left(Q-x(u)\right)P: q<x(u)\leq q+q_a \\ cq+\min\left(\left(x(u)-q\right),q_a\right)p_a-\min\left(\left[u-x(u)\right]^{+},q_a\right)\theta_c+ \\ \min\left([Q-u]^{+},q_a\right)\theta_s+\left(Q-x(u)\right)P: q_L+q_a<x(u) \end{cases} \quad (1.4)$$

A possible graph of the shipper's cost function is shown in Figure 4.

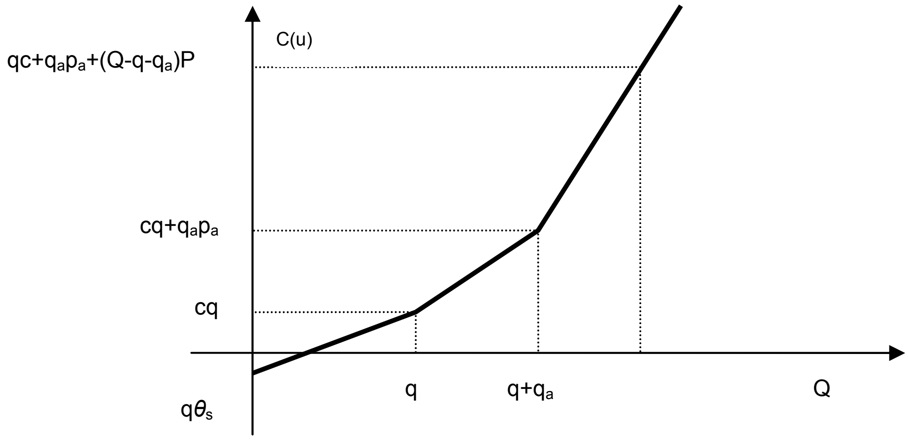

Fig.4: Behaviour of $O_i(u)$

2.8.4. Defining optimal decisions according to demand and spot price

In each region of probability space, the optimal decisions by each player vary. Let us call $R_{\Omega i}$ and $C_{\Omega i}$ the revenue and cost functions over each separate region identified by its number i $i\in\{1,2,...,10\}$. The profit function of C that has to be maximised depends upon the regions of probability space and can be written:

$$\pi_i\left(x \mid u,\Omega i\right)=R_i\left(x \mid u,\Omega i\right)-VCQ \quad (1.5)$$

$$\text{s.t.}:\begin{cases} x \le W \\ 0 \le x \le u \\ 0 \le u \le Q \\ VC \le P \\ 0 \le \theta_c \le c \le p_a \\ 0 \le \theta_s \le c \\ 0 \le q_a \le q \end{cases}$$

We now derive below the decisions taken by both players:

Ωi	**u_i^***	**x_{Li}^***	**$\pi_{\Omega i}$**	**$O_{\Omega i}$**
Ω1	Q_L	Q_L	$Qc+(q-Q)\theta_s-VCQ$	$Qc+(q-Q)\theta_s$
Ω2	q_L	q_L	$qc+(Q-q)(P+\theta_s)-VCQ$	$qc+(Q-q)(P+\theta_s)$
Ω3	Q_L	Q_L	$qc+(Q-q)p_{as}-VCQ$	$qc+(Q-q)p_a$
Ω4	Q_L	q_L	$qc+(Q-q)(P-\theta_c)-VCQ$	$qc+(Q-q)(P-\theta_c)$
Ω5	q_L	q_L	$qc+q_a\theta_s+(Q-q)P-VCQ$	$qc+q_a\theta_s+(Q-q)P$
Ω6	Q_L	Q_L	$qc+q_a p_a+(Q-q-q_a)P-VCQ$	$qc+q_a p_a+(Q-q-q_a)P$
Ω7	Q_L	q_L	$qc+q_a\theta_c+(Q-q)P-VCQ$	$qc+q_a\theta_c+(Q-q)P$
Ω8	q_L	q_L	$qc+q_a\theta_s+(W-q)P-VCW$	$qc+q_a\theta_s+(Q-q)P$
Ω9	Q_L	Q_L	$qc+q_a p_a+(W-q-q_a)P-VCW$	$qc+q_a p_a+(Q-q-q_a)P$
Ω10	Q_L	q_L	$qc+q_a\theta_c+(W-q)P-VCW$	$qc+q_a\theta_c+(Q-q)P$

Table 1: Regions of probability space with relevant optimal decision and objective expression

2.8.5. Expected cost and variance of transport cost

Given that we now have defined the costs to the shipper over all regions of the probability space, we can define her expected cost as a function of the received demand Q and spot price P using the notation introduced in 2.3.

$$E\left(O\left(u^*,x^*\right)\right)=\int_{VC}^{\infty}\int_{0}^{\infty}O\left(u^*,x^*\right)f\left(Q,P\right)dQdP \tag{1.6}$$

When we open up this equation among the different regions we have:

$$
\begin{aligned}
E\left(O\left(u^*,x^*\right)\right) = & \iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQdP + \\
& \iint_{\Omega 2}\left(qc-\left(Q-q\right)\left(P-\theta_s\right)\right)f\left(Q,P\right)dQdP + \\
& \iint_{\Omega 3}\left(qc+\left(Q-q\right)p_a\right)f\left(Q,P\right)dQdP + \\
& \iint_{\Omega 4}\left(qc+\left(Q-q\right)\left(P-\theta_c\right)\right)f\left(Q,P\right)dQdP + \\
& \iint_{\Omega 5\cup\Omega 8}\left(qc+\left(Q-q\right)\left(P-\theta_s\right)+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP + \\
& \iint_{\Omega 6\cup\Omega 9}\left(qc+q_a p_a+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP + \\
& \iint_{\Omega 7\cup\Omega 10}\left(qc+q_a\theta_c+\left(Q-q\right)P\right)f\left(Q,P\right)dQdP
\end{aligned}
\tag{1.7}
$$

The regions 5 and 8, 6 and 9 and 7 and 10 share the same objective function (they only change for carrier C).

INFORMATION SCENARIO ANALYSIS

We can now start modelling how each actor behaves according to the information he holds privately or that is common to both and see analytically the impact on the objective functions of C and S. In the first scenario, the information about the realized demands for the shippers is common knowledge to both shipper and carrier and decisions are centrally taken to maximize supply chain profits. In all scenarios, the spot market price for carrying that particular cargo at that particular period is revealed to both. In the second scenario, both cargo and carrier capacity are known but each will take advantage of the spot market price when this proves more attractive. In the third scenario, the capacity of C is unknown to S. In the fourth scenario, C's capacity is unknown to S and S's demand is unknown to C. In the fifth scenario, the carrier's capacity is known to S but S's demand is not known to the carrier.

We put a superscript index for each scenario on the carrier profit, shipper cost and standard deviation functions (e.g. $\pi_C^1; O^1; \sigma^1; R^1$ for scenario 1).

3.1. *Scenario 1: Centralised decision-making, perfect information:*

The carrier and shipper share information truthfully, and are coordinated by a single centralized decision maker. According to the observed demands and spot price, shipper S allocates the maximum of the realized demand to C and C allocates the maximum of his capacity to satisfy S.

$$u = Q, \quad x = \min\left(W, Q\right) \tag{1.8}$$

The conditional expected cost and expected profit as a function of the received demand Q subject to P come to:

$$E\left(O^1\left(u^{1*},x^{1*}\right)\right)=\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQdP+\iint_{\Omega 2\cup\Omega 3\cup\Omega 4}\left(qc+\left(Q-q\right)p_a\right)f\left(Q,P\right)dQdP+\iint_{\Omega 5\cup\Omega 6\cup\Omega 7\cup\Omega 8\cup\Omega 9\cup\Omega 10}\left(qc+q_a p_a+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP \tag{1.9}$$

$$E\left(\pi^1\left(x^{1*},u^{1*}\right)\right)=\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQdP+\iint_{\Omega 2\cup\Omega 3\cup\Omega 4}\left(qc+\left(Q-q\right)p_a\right)f\left(Q,P\right)dQdP+\iint_{\Omega 5\cup\Omega 6\cup\Omega 7}\left(qc+q_a p_a+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP+\iint_{\Omega 8\cup\Omega 9\cup\Omega 10}\left(qc+q_a p_a+\left(W-q-q_a\right)P\right)f\left(Q,P\right)dQdP \tag{1.10}$$

To maximise the overall profit to the echelon, we subtract the expected cost from the expected profit:

$$M^1=E\left(\pi^1\left(x^{1*},u^{1*}\right)\right)-E\left(C^1\left(u^{1*},x^{1*}\right)\right)=\iint_{\Omega 8\cup\Omega 9\cup\Omega 10}\left(W-Q\right)Pf\left(Q,P\right)dQdP \tag{1.11}$$

This expression is negative but nearest to 0 when W is large. Since W is a constraint that is not easily nor quickly lifted, the ROI of added capacity compared to the increased net cost to the echelon given the bivariate pdf of the spot and demand received are to be taken into account.

3.2. Scenario 2: Common information but distinct profit centres

$\forall P \mid P < p_a - \theta_c$, the shipper reduces her cost by paying the penalty θ_s agreed upon in the ex ante contract to the carrier for the cargo that is being diverted to the spot market above base capacity q.

$\forall P \mid P > p_a + \theta_s$, the carrier increases his profit by refusing the offered cargo from S, paying a penalty θ_c and selling this capacity at the spot price. S has to do the same. The division of the probability region is the one represented in figure 1. We get the following profit and cost functions:

$$
\begin{aligned}
E\left(O^2\left(u^{2*},x^{2*}\right)\right) = &\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 2}\left(qc+\left(Q-q\right)\left(P+\theta_s\right)\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 3}\left(qc+\left(Q-q\right)p_a\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 4}\left(qc+\left(Q-q\right)\left(P-\theta_c\right)\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 5\cup\Omega 8}\left(qc+q_a\left(P+\theta_s\right)+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 6\cup\Omega 9}\left(qc+q_a p_{a+}\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 7\cup\Omega 10}\left(qc+q_a\left(P-\theta_c\right)+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP
\end{aligned}
\tag{1.12}
$$

$$
\begin{aligned}
E\left(\pi^2\left(u^{2*},x^{2*}\right)\right) = &\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 2}\left(qc+\left(Q-q\right)\left(P+\theta_s\right)\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 3}\left(qc+\left(Q-q\right)p_a\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 4}\left(qc+\left(Q-q\right)\left(P-\theta_c\right)\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 5}\left(qc+q_a\left(P+\theta_s\right)+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 6}\left(qc+q_a p_{a+}\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 7}\left(qc+q_a\left(P-\theta_c\right)+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 8}\left(qc+q_a\left(P+\theta_s\right)+\left(W-q-q_a\right)P\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 9}\left(qc+q_a p_{a+}\left(W-q-q_a\right)P\right)f\left(Q,P\right)dQdP+\\
&\iint_{\Omega 10}\left(qc+q_a\left(P-\theta_c\right)+\left(W-q-q_a\right)P\right)f\left(Q,P\right)dQdP
\end{aligned}
\tag{1.13}
$$

3.3. Scenario 3: Asymmetric information favouring carrier

C has private information on W, the transport capacity. Ex post, S cannot verify the existence or size of the additional capacity S has promised in the contract.

So C has an opportunity to deviate when P is higher than p_a. If C deviates, the demand in excess of q by S has to be offered to the spot market. So the cost increases for S. C has been modelled to take that same amount from the spot market at the spot price so as to make it easier to compare performance and rent transfer between both players in the conclusions. We are conscious that this is a simplification that underestimates the transaction costs incurred by C in finding this available cargo in the spot market. The exact demand Q of S is here assumed observable by both S and C. We have a new drawing of the region boundaries (fig. 5).

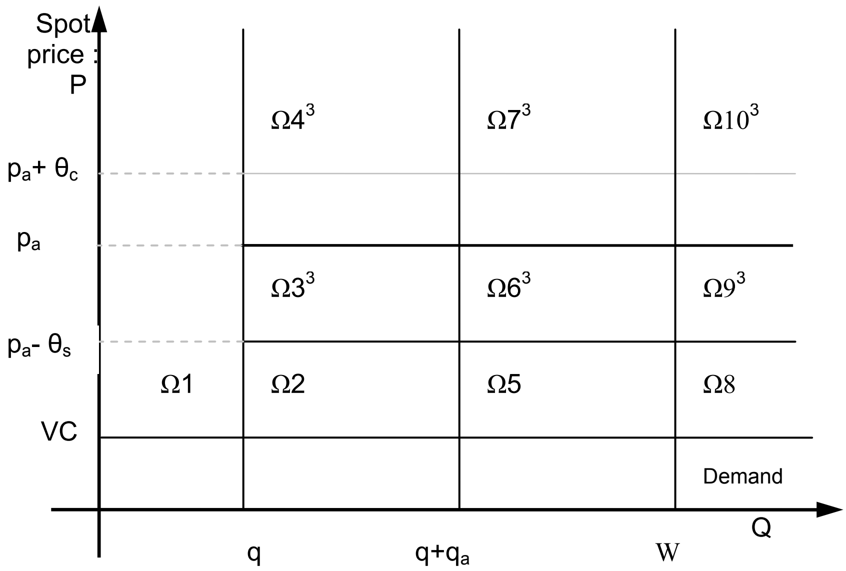

Fig. 5: Probability regions in scenario 3

This leads to the following cost function for S:

$$
\begin{aligned}
E\left(O^2\left(u^{2*},x^{2*}\right)\right) = &\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right) f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 2}\left(qc+\left(Q-q\right)\left(P+\theta_s\right)\right) f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 3}\left(qc+\left(Q-q\right)p_a\right) f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 4}\left(qc+\left(Q-q\right)\left(P-\theta_c\right)\right) f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 5\cup\Omega 8}\left(qc+q_a\left(P+\theta_s\right)+\left(Q-q-q_a\right)P\right) f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 6\cup\Omega 9}\left(qc+q_a p_{a+}\left(Q-q-q_a\right)P\right) f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 7\cup\Omega 10}\left(qc+q_a\left(P-\theta_c\right)+\left(Q-q-q_a\right)P\right) f\left(Q,P\right)dQdP
\end{aligned}
\tag{1.12}
$$

and to the following expected cost function:

$$
\begin{aligned}
E\left(O^3\left(u^{3*},x^{3*}\right)\right) = &\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 2\cup\Omega 3^3}\left(qc+\left(Q-q\right)p_a\right)f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 4^3} qc+\left(Q-q\right)Pf\left(Q,P\right)dQdP + \\
&\iint_{\Omega 5\cup\Omega 6^3\cup\Omega 8\cup\Omega 9^3}\left(qc+q_a p_a+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 7^3\cup\Omega 10^3}\left(qc+\left(Q-q\right)P\right)f\left(Q,P\right)dQdP
\end{aligned}
\quad .(1.13)
$$

In the same way, the expected profit to the carrier is:

$$
\begin{aligned}
E\left(\pi^3\left(x^{3*},u^{3*}\right)\right) = &\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 2\cup\Omega 3^3}\left(qc+\left(Q-q\right)p_a\right)f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 4^3}\left(qc+\left(Q-q\right)P\right)f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 5\cup\Omega 6^3}\left(qc+q_a p_a+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 8\cup\Omega 9^3}\left(qc+q_a p_a+\left(W-q-q_a\right)P\right)f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 7^3}\left(qc+\left(Q-q\right)P\right)f\left(Q,P\right)dQdP + \\
&\iint_{\Omega 10^3} qc+\left(W-q\right)Pf\left(Q,P\right)dQdP
\end{aligned}
\quad .(1.14)
$$

The overall profit to the echelon still is:

$$
\begin{aligned}
M^3 &= E\left(\pi^3\left(x^{3*},u^{3*}\right)\right)-E\left(C^3\left(u^{3*},x^{3*}\right)\right) \\
&= \iint_{\Omega 8\cup\Omega 9\cup\Omega 10}\left(\left(W-Q\right)P\right)f\left(Q,P\right)dQdP\ .
\end{aligned}
\quad (1.15)
$$

But, unlike in scenario 1 the profit is not distributed in the same manner between shipper and carrier. The overall profit is not a function of the contract parameters.

3.4. Scenario 4: Private information

In this scenario, C has private information on W, S has private information on the demand Q: so both have an option to behave opportunistically according to the spot price P. Each sticks to q, basic capacity contracted for. In this last scenario, the menu of prices is unenforceable. The carrier's penalty is unenforceable either, rendering it pointless. For any spot price either higher or lower than the menu price p_a according to the additional capacity necessary, either the shipper or the carrier decides to go to the spot market. The other party, for lack of knowledge of capacity or cargo, cannot ask for nor receive any compensation. This means that we have a redrawing of the regions' boundaries into barely 5 regions (fig. 6). Our expected cost and profit functions become:

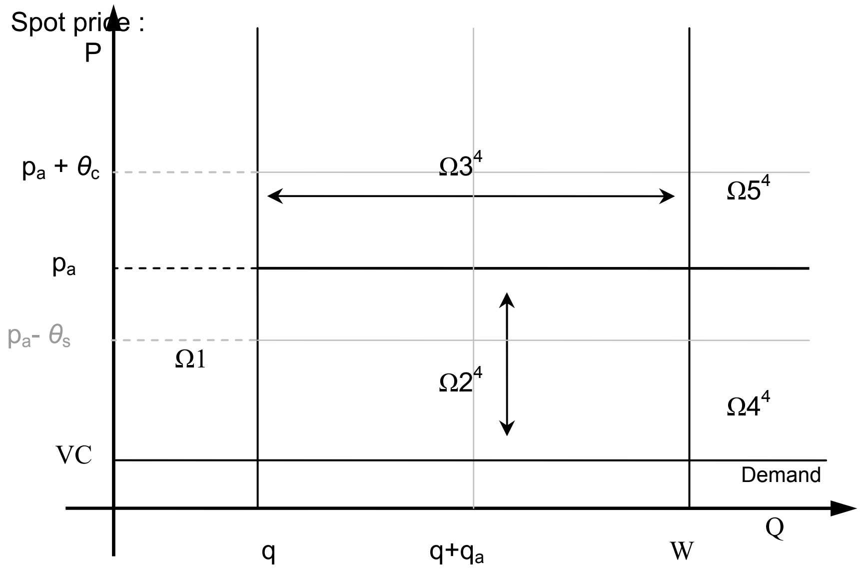

Fig. 6: Probability regions for scenario 4

$$O^4\left(u^{4*},x^{4*}\right)=\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQP+ \iint_{\Omega 2^4\cup\Omega 3^4\cup\Omega 4^4\cup\Omega 5^4}\left(qc+\left(Q-q\right)P\right)f\left(Q,P\right)dQdP+ \tag{1.16}$$

Expected profit:

$$E\left(\pi^4\left(x^{4*},u^{4*}\right)\right)=\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQP+ \iint_{\Omega 2^4\cup\Omega 3^4}\left(qc+\left(Q-q\right)P\right)f\left(Q,P\right)dQdP+ \iint_{\Omega 4^4\cup\Omega 5^4}\left(qc+\left(W-q\right)P\right)f\left(Q,P\right)dQdP \tag{1.17}$$

3.5. Scenario 5: Asymmetric information favouring the shipper

In this scenario, the shipper knows the capacity of the carrier and the carrier is not aware of the exact demand received by the shipper. The carrier cannot shirk his contractual engagements but the shipper can contract additional capacity from the spot market when the spot price is lower than the menu of prices for additional capacity. She would not have to pay penalty to the carrier since the carrier is unaware of the extra cargo to ship.

Unhappily for the simplicity of our model, we have to consider different limits to the areas where such behaviour takes place. As compared to the mapping of overall probability space in scenario 1, the lower regions are larger (fig. 7):

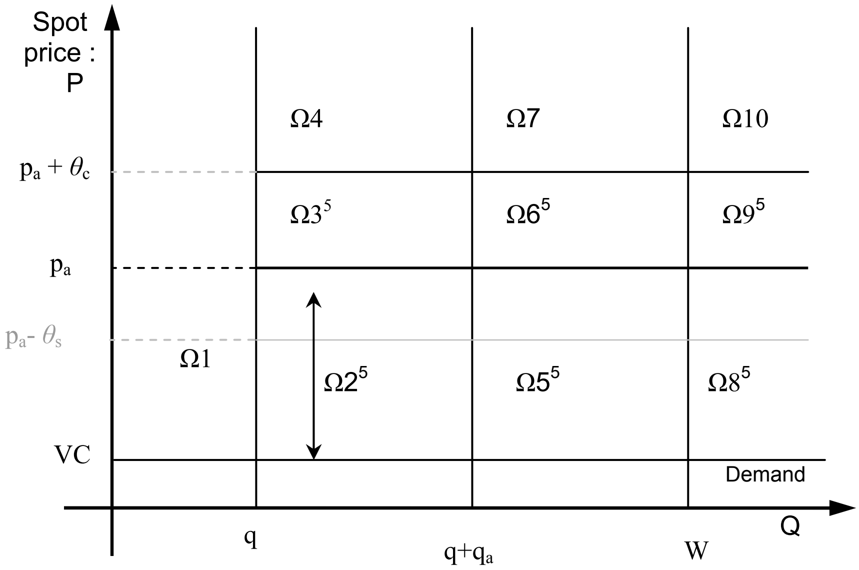

Fig. 7: Probability regions for scenario 5

These equations can be contracted into the following cost function for S:

$$\begin{aligned} O^5\left(u^{5*},x^{5*}\right) = &\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQdP + \\ &\iint_{\Omega 2^5\cup\Omega 5^5\cup\Omega 8^5}\left(qc+\left(Q-q\right)P\right)f\left(Q,P\right)dQdP + \\ &\iint_{\Omega 3^5\cup\Omega 4\cup\Omega 6^5\cup\Omega 7}\left(qc+\left(Q-q\right)p_a\right)f\left(Q,P\right)dQdP + \\ &\iint_{\Omega 9^5\cup\Omega 10}\left(qc+q_a p_a+\left(Q-q-q_a\right)P\right)f\left(Q,P\right)dQdP \end{aligned} \tag{1.18}$$

Expected profit:

$$\begin{aligned} E\left(\pi^5\left(x^{5*},u^{5*}\right)\right) = &\iint_{\Omega 1}\left(Qc+\left(q-Q\right)\theta_s\right)f\left(Q,P\right)dQdP + \\ &\iint_{\Omega 2^5\cup\Omega 5^5}\left(qc+\left(Q-q\right)P\right)f\left(Q,P\right)dQdP + \\ &\iint_{\Omega 8^5}\left(qc+\left(W-q\right)P\right)f\left(Q,P\right)dQdP + \\ &\iint_{\Omega 3^5\cup\Omega 4^5\cup\Omega 6^5\cup\Omega 7}\left(qc+\left(Q-q\right)p_a\right)f\left(Q,P\right)dQdP + \\ &\iint_{\Omega 9^5\cup\Omega 10}\left(qc+q_a p_a+\left(W-q-q_a\right)P\right)f\left(Q,P\right)dQdP \end{aligned} \tag{1.19}$$

3.6. Comparison between scenarios

3.6.1. Comparison between scenario 1 and 2

The difference between these scenarios is between one profit centre and decentralised profit centres. The differences occur only when P is either too low or too high:

$$E\left(O^4\left(x^{4*},u^{4*}\right)-O^1\left(x^{1*},u^{1*}\right)\right)=\iint\limits_{\Omega 2\cup\Omega 3\cup\Omega 4}\left(\left(Q-q\right)\left(P-p_a\right)\right)f\left(Q,P\right)dQdP+\iint\limits_{\Omega 5\cup\Omega 6\cup\Omega 7\cup\Omega 8\cup\Omega 9\cup\Omega 10}\left(q_a\left(P-p_a\right)\right)f\left(Q,P\right)dQdP \quad (1.22)$$

The higher both penalties and the lower the extra contracted capacity are, the lower the difference. However, it is clear that the difference is not null, meaning that the variance is higher and proportional to the penalties laid out in the ex ante contract. The rent transfer between either will be minimal if $p_a = \mu_p$, if $f_P(.)$ is symmetric around the mean and $\theta_c = \theta_s$.

3.6.2. Comparison between scenario 1 and 3

There is a transfer of rent from S to C when C can deviate from truthful behaviour by hiding the exact capacity he has at his disposal and withhold extra capacity from S to sell it to the spot market at a higher price.

The conditional expected cost of the difference in information is written:

$$\begin{aligned}E\left(O^3\left(u^{3*},x^{3*}\right)-O^1\left(u^{1*},x^{1*}\right)\right)&=\iint\limits_{\Omega 4^3}\left(\left(Q-q\right)\left(P-p_a\right)\right)f\left(Q,P\right)dQdP+\\&\iint\limits_{\Omega 7^3\cup\Omega 10^3}\left(q_a\left(P-p_a\right)\right)f\left(Q,P\right)dQdP \quad (1.20)\\&=E\left(\pi^3\left(x^{3*},u^{3*}\right)-\pi^1\left(x^{1*},u^{1*}\right)\right)\end{aligned}$$

Hence the difference in overall profit for both:

$$\begin{aligned}M_{1-3}&=E\left(\pi^3\left(x_L^{3*},u^{2*}\right)-\pi^1\left(x_L^{1*},u^{1*}\right)\right)-E\left(O^3\left(x_L^{3*},u^{3*}\right)-O^1\left(x_L^{1*},u^{1*}\right)\right) \quad (1.21)\\&=0\end{aligned}$$

because the difference between both scenarios happen within the overall limit of the fleet capacity W. The carrier wishes to maximise the difference in profit between scenario 2 and 1, whereas the shipper wishes to minimize the same expression.

The important conclusion is that the difference in cost between scenario 3 and 1 is positive, so the expected variance is also positive. The variance of the transport cost to S increases with the variances of the component pdf: Q and P affected by the values given to the contractual parameters.

3.6.3. Comparison between scenario 1 and 4

The conditional expectation of this difference subject to P and Q can be written as:

$$E\left(O^4\left(x^{4*},u^{4*}\right)-O^1\left(x^{1*},u^{1*}\right)\right)=\iint_{\Omega2\cup\Omega3\cup\Omega4}\left(\left(Q-q\right)\left(P-p_a\right)\right)f\left(Q,P\right)dQdP+\iint_{\Omega5\cup\Omega6\cup\Omega7\cup\Omega8\cup\Omega9\cup\Omega10}\left(q_a\left(P-p_a\right)\right)f\left(Q,P\right)dQdP \tag{1.22}$$

Following the same reasoning, we can write the conditional expectation of the difference, subject to P and Q, of the profit to the carrier as:

$$E\left(\pi^4\left(x^{4*},u^{4*}\right)-\pi^1\left(x^{1*},u^{1*}\right)\right)=\iint_{\Omega2\cup\Omega3\cup\Omega4}\left(\left(Q-q\right)\left(P-p_a\right)\right)f\left(Q,P\right)dQdP+\iint_{\Omega5\cup\Omega6\cup\Omega7}\left(q_a\left(P-p_a\right)\right)f\left(Q,P\right)dQdP \tag{1.23}$$

Above the total capacity of the fleet of C (areas Ω8, Ω9, Ω10), the carrier cannot make any profit. These indications give guidance to the way the contractual parameters have to be negotiated by the shipper and the carrier so that if the information conditions are not given, at least the differences between both scenarios can be minimized for both the shipper and carrier. This means that we have to maximize the expected profit less the expected cost:

$$M_{1-4}=E\left(\pi^4\left(x^{4*},u^{4*}\right)-\pi^1\left(x^{1*},u^{1*}\right)\right)-E\left(O^4\left(x^{4*},u^{4*}\right)-O^1\left(x^{1*},u^{1*}\right)\right) \tag{1.24}$$

This difference effectively means looking for:

$$Max\left(M_{1-4}\right)=\iint_{\Omega8\cup\Omega9\cup\Omega10}\left(q_a\left(P-p_a\right)\right)f\left(Q,P\right)dQdP \tag{1.25}$$

M_{1-4} can be written as:

$$M_{1-4}=q_a\iint_{\Omega8\cup\Omega9\cup\Omega10}Pf\left(Q,P\right)dQdP-q_a p_a\iint_{\Omega8\cup\Omega9\cup\Omega10}f\left(Q,P\right)dQdP \tag{1.26}$$

To get the maximum, we have to optimize our decision variables which are the contract parameters: qa and pa. Evidently, M_{1-4} increases when qa increases and pa decreases. However, both are not unlinked as they have to satisfy both C and S. We must have a price for additional capacity more interesting than the base price c, which means that the "slopes" of each are linked by: $\frac{c}{q}<\frac{p_a}{q_a}$. The limit is for the carrier to accept an equal ratio between $\frac{c}{q}$ and $\frac{p_a}{q_a}$.

3.6.4. Comparison between scenario 1 and 5

The difference is not null in only 3 sub-regions of probability space: $\Omega2^5$, $\Omega5^5$, $\Omega8^5$. By inspection, the difference in expected costs becomes:

$$E\left(C^5\left(x_L^{5*},u^{5*}\right)-C^1\left(x_L^{1*},u^{1*}\right)\right)=\iint_{\Omega2^5}\left(\left(Q_L-q_L\right)\left(P_s-p_{La}\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\iint_{\Omega5^5\cup\Omega8^5}\left(q_{La}\left(P_s-p_{La}\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s \tag{1.27}$$

The difference in expected profits for C is symmetric:

$$E\left(\pi^5\left(x^{5*},u^{5*}\right)-\pi^1\left(x^{1*},u^{1*}\right)\right)=\iint_{\Omega 2^5}\left(\left(Q-q\right)\left(P-p_a\right)\right)f\left(Q,P\right)dQdP+ \iint_{\Omega 5^5\cup\Omega 8^5}\left(q_a\left(P-p_a\right)\right)f\left(Q,P\right)dQdP \quad (1.28)$$

The conclusion regarding variance drawn in the previous comparisons also applies.

INSTANCE USING AN EXPONENTIAL BIVARIATE DISTRIBUTION

We first note that all the above objective functions and differences between scenarios can be expressed in a general way as:

$$A=\iint_{\Omega}\Psi_{\Omega}\left(x,y\right)f\left(x,y,\lambda_1,\lambda_2,\rho\right)dxdy \quad (1.29)$$

Further, all our above objective functions can be expressed like this objective function ψ as:

$$\Psi\left(x,y\right)=\alpha x+\beta y+\gamma xy+\delta\,, \quad (1.30)$$

which leads to:

$$A=\alpha A_1+\beta A_2+\gamma A_3+\delta A_0\,, \quad (1.31)$$

with:

$$\begin{aligned}
A_0&=\iint_{\Omega}f\left(x,y,\lambda_1,\lambda_2,\rho\right)dxdy,\\
A_1&=\iint_{\Omega}xf\left(x,y,\lambda_1,\lambda_2,\rho\right)dxdy\\
A_2&=\iint_{\Omega}yf\left(x,y,\lambda_1,\lambda_2,\rho\right)dxdy\\
A_3&=\iint_{\Omega}xyf\left(x,y,\lambda_1,\lambda_2,\rho\right)dxdy
\end{aligned}. \quad (1.32)$$

4.1. Downton’s bivariate exponential function

In this section we will give the results of the analysis of how the contract parameters would be negotiated by each party. The calculations are relegated to the annexes.

As mentioned above, we will now discuss the optimal parameters in the special case when the spot price is correlated with demand, using a **positive** correlation between both variables.

We study stochastic processes for demand and for the spot price.

One of the most important bivariate distributions in reliability theory is the bivariate exponential. One of the most authoritative reviews on bivariate distributions can be found in Kotz et al. (2000). In this paper, we are interested in **Downton’s bivariate exponential distribution** with probability density function (pdf):

$$f(x,y;\lambda_1,\lambda_2,\rho)=\frac{\lambda_1\lambda_2}{1-\rho}e^{-\frac{\lambda_1 x+\lambda_2 y}{1-\rho}}I_0\left(\frac{2(\rho\lambda_1\lambda_2 xy)^{1/2}}{1-\rho}\right), \quad (1.33)$$

where x, y, λ_1, $\lambda_2 > 0$, $0 \le \rho < 1$, and $I_0(z)=\sum_{k=0}^{\infty}\frac{(z/2)^{2k}}{k!^2}$ is the modified Bessel function of the first kind of order zero.

The marginal probability density functions are written:

$$f_q(x)=\begin{cases}\lambda_1 e^{-\lambda_1 x} & ,x>0\\ 0 & ,x\le 0\end{cases}$$
$$f_p(y)=\begin{cases}\lambda_2 e^{-\lambda_2 y} & ,y>0\\ 0 & ,y\le 0\end{cases} \quad (1.34)$$

We refer the reader to **Annex I** for details of the calculations to transform the double integral with which we have to deal into expressions using the Bessel functions and a specially defined K-function but no integrals.

Applying to the optimization of contract parameters under different information criteria, we can come to different solutions.

4.2. Scenario 3-1:

Here as, earlier mentioned, the shipper has to reduce the cases where the carrier fails her. Let us call $EC_{3\text{-}1}$ the expression coming from (1.20) the shipper S has to minimize:

$$EC_{3-1}=\iint_{\Omega 4^3}\left((Q-q)(P-p_a)\right)f(Q,P)\,dQdP+\iint_{\Omega 7^3\cup\Omega 10^3}\left(q_a(P-p_a)\right)f(Q,P)\,dQdP \quad (1.35)$$

Let us list the decision variables that have to be optimized:

q : base capacity contracted. Has to bear some relation with the average demand received by S.

q_a : additional capacity contracted in QF clause

p_a : price of additional capacity, set in QF clause

$\Omega 4^3 = [p_a,\infty[\times[q,q+q_a]$: boundaries are a function of contract parameters

$W \to a$: capacity limit of carrier C. Not a decision variable.

$\Omega 7^3 \cup \Omega 10^3 = [p_a,\infty[\times[\ q\ +\ q_a,\infty[$: boundaries are a function of contract

Optimization has to be done over the variables q, q_a and p_a.

The analytical expressions of the first and second differentials of the expression according to these variables were not tractable. We set up a comprehensive algorithm to calculate the result of this expression for all possible values of the decision variables having set λ and μ to 1. Because of (2.3) and(2.22), this entails the following:

$$\lambda=1\Rightarrow\lambda_1=(1-\rho) \quad (1.36)$$

The marginal mean for P is set in this way at $\mu_p = \frac{1}{1-\rho}$. The same is true for the marginal mean of Q: $\mu_q = \frac{1}{1-\rho}$. We have explored numerically the space defined by the decision variables. The results show that for a large variety of values $EC_{3\text{-}1}$ is worth basically zero. Meaning that for a large range of values for the contract, both information sharing scenarios generate the same cost to the shipper. The sharpest values happen near the low values for the contract parameters reflecting the heightening cost for the shipper when the contract parameters are set at too low values compared to the mean demand observed and mean price on the spot market (Fig. 8)

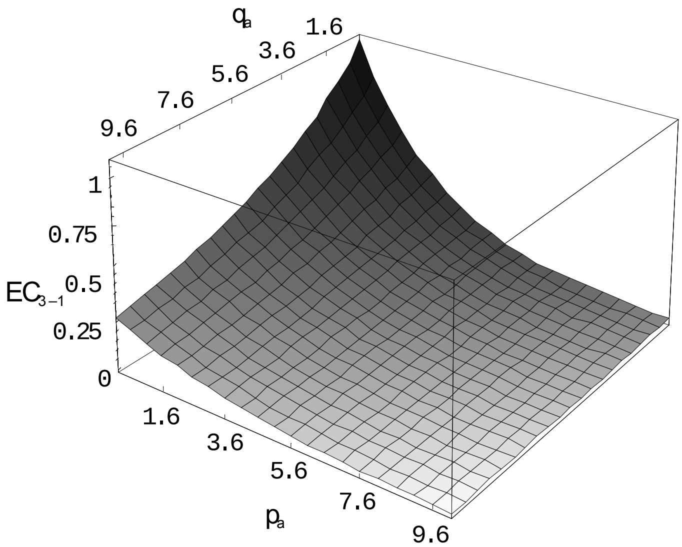

Fig. 8: EC3-1 $\lambda=\mu=1$, $\rho=0.5$, $q = 2.1, 0.001 \leq p_a \leq 10.1$, $0.001 \leq q_a \leq 10.1$

When q_a and p_a increase, EC3-1 decreases very quickly but remains above zero.
The conclusion is that the parameters have an influence only when set too “low”. When, for example, we have q set at the expected demand (in our case 2.1) and q_a set at half a standard deviation (1.1) for a price p_a of half a standard de variation above the expected spot price (3.1), we have an excess cost for the shipper in scenario 3 of: 0.289. This is almost equal to paying twice the spot price in scenario 3 versus what the shipper S would pay in scenario 1.

4.3. Scenario 4-1:

Let us call $EC_{4\text{-}1}$ the expression that the shipper has to minimize in this private information setting where both engage in opportunistic behaviour:

$$EC_{4-1} = \iint_{\Omega2\cup\Omega3\cup\Omega4} \left((Q-q)(P-p_a)\right) f(Q,P)\, dQdP + \iint_{\Omega5\cup\Omega6\cup\Omega7\cup\Omega8\cup\Omega9\cup\Omega10} \left(q_a(P-p_a)\right) f(Q,P)\, dQdP \tag{1.37}$$

Let us list the decision variables:
Contract parameters:

q : base capacity contracted contract parameter
q_a : additional capacity set in QF clause
p_a : price for additional capacity, set also in QF clause.
VC : variable cost limit of carrier C. Not a decision variable.

$\Omega 2 \cup \Omega 3 \cup \Omega 4 = [VC , \infty[\times [q , q + q_a]$: boundaries are a function of contract parameters

$\Omega 5 \cup \Omega 6 \cup \Omega 7 \cup \Omega 8 \cup \Omega 9 \cup \Omega 10 = [VC, \infty[\times [q + q_a , \infty[$: boundaries are a function of contract parameters.

So we write:

Optimization has to be done over the variables q, q_a, and p_a, neither penalties are present.

The first and second derivatives were not tractable so we again had recourse to the description of the EC4-1 expression for given values of the distribution parameters of demand and spot price.

We have fixed $\rho = 0.5$ $\lambda = 1$ and $\mu = 1$. This leads to marginal means for demand Q and Price P of 2. So we study the behaviour of EC_{4-1} for values of the variables ranging from near zero to 2 times the standard deviation above the mean of demand Q and spot price P. We find that for a fixed q of 6, which should cover almost two standard deviations above the mean demand distribution expected (which, as the reader will recall is supposed to be known by S), we still get deviations in cost. The shape of the surface is the same as the one represented below, even if in a more attenuated manner, meaning less differences between costs in scenario 4 and 1 (Fig. 9).

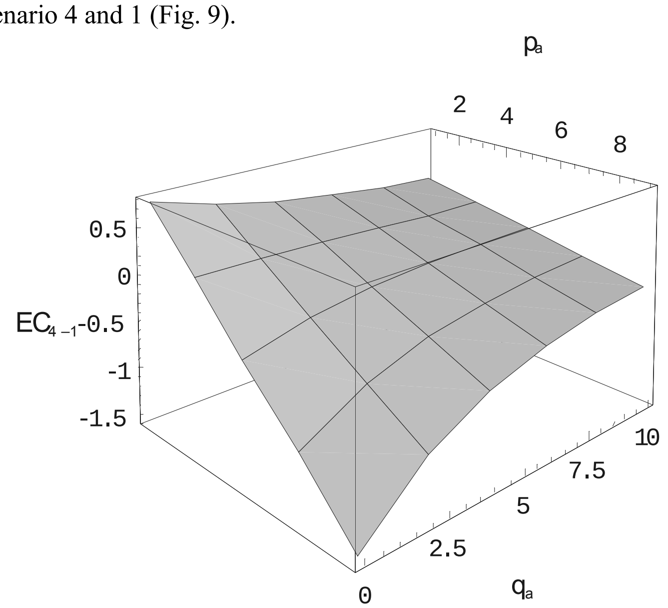

Fig. 9: EC_{4-1}, $\lambda = \mu = 1$, $\rho = 0.5$, $q = 2$, $0.0001 \le q_a \le 10$, $1 \le p_a \le 9$

When p_a is "high", for all values of q_a we have a negative EC_{4-1} meaning that the cost to S is higher in scenario 1 than in scenario 4. When q is increased to several standard deviations above the mean demand received, this effect is attenuated and the difference becomes insignificant.

The following figure gives an idea of the shape of the behaviour of the difference between costs when the correlation factor between demand and spot takes a value of 0.1 (Fig. 10). This leads to an expected demand of 1.1 and an expected spot price of 1.1.

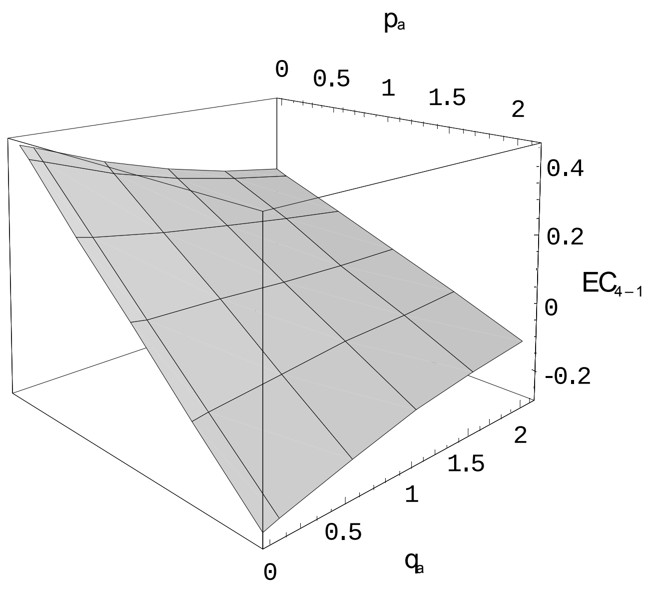

Fig. 10: $EC_{4\text{-}1}$, $\lambda = \mu = 1$, $\rho = 0.1$, $q = 1.1$, $0.01 \leq q_a \leq 2.1$, $0.01 \leq p_a \leq 2.1$

Both are similar and show a crossing from positive to negative signs when the contract parameters for the QF clause go from under the means of spot price and demand to higher. However, an interesting phenomenon is the fact that when correlation is at 0.5, p_a has to go over mean spot price plus 2 standard deviations in our graph, given ρ at 0.1, the mean of the spot price becomes 1.1 and the standard deviation at 1.1, p_a has to go to 3.3 for the difference to become null (for whatever level q_a is set at). On the other hand, when ρ is worth 0.1, the difference becomes null for values of p_a at barely mean spot price plus half a standard deviation, and this is true for whatever level of q.

We conclude that the centrally coordinated supply chain is dominated by the private information scenario for large prices of additional capacity and this is true for all studied values of q between 0.0001 and 30.

4.4. Scenario 5-1:

Let us call $EC_{5\text{-}1}$ the expression to be minimized here for the shipper:

$$EC_{5-1} = \iint_{\Omega 2^5} \left(\left(Q-q\right)\left(P-p_a\right)\right) f\left(Q,P\right) dQdP + \iint_{\Omega 5^5 \cup \Omega 8^5} \left(q_a\left(P-p_a\right)\right) f\left(Q,P\right) dQdP \tag{1.38}$$

$\Omega 2^5 = [VC\ ,\ p_a[\times[q\ ,\ q\ +\ q_a]$: boundaries are a function of contract parameters

$\Omega 5^5 \cup \Omega 8^5 = [VC\ ,\ p_a[\times[q+q_a\ ,\infty[$: boundaries are a function of contract.

Again, we solve the optimal *x*,*y* and *z*.

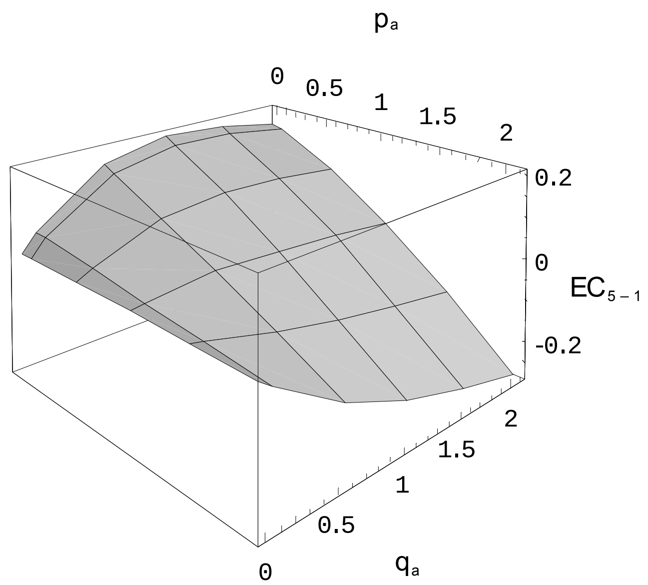

Fig. 11: $EC_{5\text{-}1}$, $\lambda = \mu = 1$, $\rho = 0.1$, $q = 1.1$, $0.01 \le q_a \le 2.1$, $0.01 \le p_a \le 2.1$

For all values of q the base capacity contracted the shape of the envelope is the same save for a scaling parameter. The higher q_a, the more important a correct value for p_a becomes. On the contrary, and as would be expected, for a low q_a, whatever the price p_a, the difference between both scenarios is negligible for all values of q the base capacity.

If q_a and p_a are chosen too high, the cost to S in scenario 5 is less than in benchmark scenario 1. So S has an inventive to hide rue demand from C and negotiate high values for both qa and pa beforehand. This will be resisted by C, of course, for the same reason.

The conclusion is therefore for the shipper and carrier to bargain for an additional capacity equal to the average demand expected and a price equivalent to the mean spot price observed. Only in this case can the supply chain expect to moderate the effects of hidden demand information. When the additional quantity qa is around one standard deviation of expected demand and the price for this additional capacity pa is also around one standard deviation of expected spot price, the difference between scenarios is almost null ($EC_{5\text{-}1} = 0.014$ for $p_a = q_a = 1.1$).

CONCLUSION

In this paper, we present transport as an individualized supply chain member with proper characteristics. We have modelled the impact and influence that information sharing and coordination with a transport supplier have on the efficiency of the supply chain. We have established that:

- Better coordination is achieved by including a fixed capacity commitment and some additional flexibility in capacity (QF clause) in a contract in a mixed procurement strategy (contract + spot).
- Penalties should be included and proportionate to the standard deviation of the spot market price. We have shown that this ensures coordination.

- The information imbalances induced by keeping private information as to the real transport capacity by the carrier, and as to the real demand received by the shipper are detrimental to the overall efficiency of the supply chain, because it encourages deviant attitudes both from the carrier and the shipper and hence increases overall revenue or cost volatility.
- Carefully crafted ex-ante contractual arrangements can substantially correct this information asymmetry. These contractual arrangements depend only on demand and price distribution characteristics.

The aim of the supply chain manager should be to reduce standard deviation because increased cost standard deviation is an incentive, in a multi-period game, to increase margins at both levels of the supply chain, thus leading to the notorious double margining phenomenon. The shipper increases his budgeted costs because he cannot ensure regularity of his cost and hence must protect himself by padding his transport budget; the carrier increases price of services because he has to contend with fixed cost non-scalable capacity and so must also preserve his financial health by higher than warranted profit margins.

Annex I

Description of bivariate exponential distribution

We have chosen Downton's bivariate exponential distribution as described in Kotz et al.(2000) and discussed in Iliopoulos (2003) with the joint density function (pdf):

$$f(q,p,\lambda_1,\lambda_2,\rho)=\frac{\lambda_1\lambda_2}{1-\rho}\exp\left(-\frac{\lambda_1 q+\lambda_2 p}{1-\rho}\right)I_0\left(\frac{2(\rho\lambda_1\lambda_2 qp)^{1/2}}{1-\rho}\right) \quad (2.1)$$

where, to simplify, $q = Q$, $p = P$, λ_1 and $\lambda_2 > 0$ and

$$I_0(z)=\sum_{k=0}^{\infty}(z/2)^{2k}/k!^2 \quad (2.2)$$

is the modified Bessel function of the first kind of order zero.

Let it be clear that here we limit our consideration to the case where the **correlation coefficient is positive or null**: $0 < \rho < 1$. We will restrict our study to the cases where spot market prices for freight transport and demands addressed to the shipper are **positively correlated**.

The above density was initially derived by Moran (1967). The marginal distributions of both Q and P are exponential with means $1/\lambda_1$ and $1/\lambda_2$ respectively. Since $I_0(0)=1$, it is clear that Q and P are independent if and only if $\rho = 0$. Downton (1970) showed that ρ is the correlation coefficient of the two variates.

The marginal probability density functions can be written:

$$\begin{aligned} f_q(q)&=\lambda_1 e^{-\lambda_1 q}\\ f_p(p)&=\lambda_2 e^{-\lambda_2 p}\end{aligned} \quad (2.3)$$

We can write the marginal distribution functions as:

$$\begin{aligned} F_1(q)&=\int_0^q \lambda_1 e^{-\lambda_1 t}dt=1-e^{-\lambda_1 q}\\ F_2(p)&=\int_0^p \lambda_2 e^{-\lambda_2 t}dt=1-e^{-\lambda_2 p}\end{aligned} \quad (2.4)$$

But first, we summarize a few properties of double integrals of the modified Bessel function.

Integrals of Bessel functions

The following integral

$$L(x,y,p)=(1-p)\int_0^x\int_0^y e^{-u-v}I_0\left(2\sqrt{puv}\right)dudv,\ p\ge 0 \quad (2.5)$$

plays a crucial role in the analysis. For several properties we refer to Temme (1986), Luke (1962); see also Kotz et al (2000). This function can be written as

$$L(x,y,p)=\left(1-e^{py-y}\right)+e^{py-y}K\left(py,x\right)-e^{px-x}K\left(y,px\right) \tag{2.6}$$

where

$$K\left(x,y\right)=e^{-y}\int_0^x e^{-u}I_0\left(2\sqrt{uy}\right)du=1-e^{-y}\int_x^\infty e^{-u}I_0\left(2\sqrt{uy}\right)du\ . \tag{2.7}$$

This complementary property follows from (see 29.3.81 of Abramovitz – Stegun (1964))

$$e^y=\int_0^\infty e^{-u}I_0\left(2\sqrt{yu}\right)du \tag{2.8}$$

The starting point in Moran (1967) is the function

$$I\left(x,y\right)=\int_0^x\int_0^y e^{-u-v}I_0\left(2\sqrt{uv}\right)dudv \tag{2.9}$$

We have $I\left(x,y\right)=-\dfrac{\partial L\left(x,y,p\right)}{\partial p}$ at p=1 and

$$I\left(x,y\right)=x+\left(y-x\right)K\left(x,y\right)-e^{-x-y}\left[\frac{1}{2}\xi I_1\left(\xi\right)+xI_0\left(\xi\right)\right].\qquad \xi=2\sqrt{xy} \tag{2.10}$$

Observe that $I(x,y)=I(y,x)$, and from this symmetry relation it follows that

$$K\left(x,y\right)+K\left(y,x\right)=1-e^{-x-y}I_0\left(2\sqrt{xy}\right), \tag{2.11}$$

which formula is not given in Moran (1967). In (2.10), $I_1(\xi)$ is the modified Bessel function of the first kind of order one.

The analysis in Moran (1967) is focused on the asymptotic properties of $I(x,y)$ for large values of x and y. In fact the asymptotic properties of the function

$$F\left(x,y\right)=K\left(x,y\right)+\frac{1}{2}e^{-x-y}I_0\left(2\sqrt{xy}\right) \tag{2.12}$$

are studied. The symmetry rule for this function reads

$$F\left(x,y\right)+F\left(y,x\right)=1\,. \tag{2.13}$$

The first term approximation for $F(x,y)$ given in Moran (1967) reads

$$F\left(x,y\right)\sim\frac{\sqrt{x}+\sqrt{y}}{4\sqrt{\sqrt{xy}}}\operatorname{erfc}\left(\sqrt{y}-\sqrt{x}\right),\qquad y\geq x\,, \tag{2.14}$$

where erfc z denotes the complementary error function

$$\operatorname{erfc} z=\frac{2}{\sqrt{\pi}}\int_z^\infty e^{-t^2}dt \tag{2.15}$$

The approximation in (2.14) holds for large y, uniformly with respect to x, $0<\delta\leq x\leq y$, where δ is a fixed positive number. It gives the exact value ½ when x = y, and for y>> x it is exponentially small. This follows from the estimate

$$\operatorname{erfc} z\sim\frac{1}{z\sqrt{\pi}}e^{-z^2}\,, z\to\infty\,. \tag{2.16}$$

When x ≥ y we can use (2.13), and obtain

$$F(x,y) \sim 1 - \frac{\sqrt{x}+\sqrt{y}}{4\sqrt{\sqrt{xy}}}\,\mathrm{erfc}\,\left(\sqrt{x}-\sqrt{y}\right), \quad x \ge y \ge \delta > 0\,. \quad (2.17)$$

This non-uniform behaviour of *F(x,y)* for large *x* and *y* also occurs for *K(x,y)*. In fact, we have the following limiting values. First observe that from (2.11) it follows that

$$K(x,x) = \frac{1}{2}\left[1 - e^{-2x} I_0(2x)\right]. \quad (2.18)$$

For large values of *x* the right-hand side approaches ½, because we know that (see 9.7.1 of Abramovitz – Stegun (1964) or (9.54) of Temme (1986))

$$I_0(z) \sim \frac{e^z}{\sqrt{2\pi z}}, \quad z \to \infty \quad (2.19)$$

For fixed *x* and large *y* we can use (2.7) and (2.19) to show that *K(x,y)* tends to zero. From (2.7) and (2.8) we conclude that for fixed *y* and large *x*, *K(x,y)* tends to unity.

In other words,

$$\begin{aligned} &\lim_{y\to\infty} K(x,y) = 0, \ x \text{ fixed}, \\ &\lim_{x\to\infty} K(x,x) = \frac{1}{2}, \\ &\lim_{x\to\infty} K(x,y) = 1, \ \ y \text{ fixed}. \end{aligned} \quad (2.20)$$

Solving the double integral

For the intervals $\Omega_x = [x_l, x_h], \Omega_y = [y_l, y_h]$, we assume that $x_l = y_l = 0$ because we can always write $[x_l, x_h] = [0, x_h] \backslash [0, x_l]$, and the same for Ω_y. For infinite intervals $\Omega_x = [x, \infty[$ and/or $\Omega_y = [y_l, \infty[$, we also can concentrate on finite intervals, because of complementary rules based on (2.8).

For the evaluation of the objective function A defined in (1.32), we need the following functions

$$\begin{aligned} A_0 &= \iint_\Omega f(x,y,\lambda_1,\lambda_2,\rho)\,dxdy, \\ A_1 &= \iint_\Omega xf(x,y,\lambda_1,\lambda_2,\rho)\,dxdy \\ A_2 &= \iint_\Omega yf(x,y,\lambda_1,\lambda_2,\rho)\,dxdy \\ A_3 &= \iint_\Omega xyf(x,y,\lambda_1,\lambda_2,\rho)\,dxdy \\ A_4 &= \iint_\Omega x^2 f(x,y,\lambda_1,\lambda_2,\rho)\,dxdy \end{aligned} \quad (2.21)$$

We evaluate the function A_j of (2.21) and use the notation

$$x = x_h, \quad y = y_h, \quad \lambda = \frac{\lambda_1}{1-\rho}, \qquad \mu = \frac{\lambda_2}{1-\rho} \quad (2.22)$$

For A_0, we have

$$A_0 = L\left(\lambda x, \mu y, \rho\right) \tag{2.23}$$

and the relation with the K-function follows from (2.6).

For Aj, j = 1,2,3, it is useful to have a different representation of the L-function. By using the expansion (2.2) of the Bessel function in (2.5), we find

$$L\left(x, y, p\right) = \left(1-p\right)\sum_{k=0}^{\infty} p^k \frac{\gamma\left(k+1, x\right)}{k!}\frac{\gamma\left(k+1, y\right)}{k!}, \tag{2.24}$$

where $\gamma\left(a, z\right)$ is the incomplete gamma function (see [9, chapter 11])

$$\gamma\left(a, z\right) = \int_0^z t^{a-1}e^{-t}dt, \qquad \Re a > 0 . \tag{2.25}$$

For A_1, we obtain

$$A_1 = \left(1-\rho\right)\lambda\mu\sum_{k=0}^{\infty}\frac{\rho^k\lambda^k\mu^k}{k!k!}\int_0^x u^{k+1}e^{-\lambda u}du\int_0^y v^k e^{-\mu v}dv , \tag{2.26}$$

giving

$$A_1 = \frac{1-\rho}{\lambda}\sum_{k=0}^{\infty}\rho^k \frac{\gamma\left(k+2, \lambda x\right)}{k!}\frac{\gamma\left(k+1, \mu y\right)}{k!} . \tag{2.27}$$

Using the recursion

$$\gamma\left(a+1, z\right) = a\gamma\left(a, z\right) - z^a e^{-z} \tag{2.28}$$

we obtain

$$A_1 = \frac{1-\rho}{\lambda}\sum_{k=0}^{\infty}\left(k+1\right)\rho^k \frac{\gamma\left(k+1, \lambda x\right)}{k!}\frac{\gamma\left(k+1, \mu y\right)}{k!} - \left(1-\rho\right)xe^{-\lambda x}\sum_{k=0}^{\infty}\frac{\rho^k\lambda^k x^k}{k!}\frac{\gamma\left(k+1, \mu y\right)}{k!} . \tag{2.29}$$

To evaluate the first series we use (2.24) and in the second one we interchange summation and integration. This gives

$$A_1 = \frac{1-\rho}{\lambda}\frac{\partial}{\partial\rho}\left[\frac{\rho L\left(\lambda x, \mu y, \rho\right)}{1-\rho}\right] - \left(1-\rho\right)xe^{-\lambda x(1-\rho)}K\left(\mu y, \rho\lambda x\right) . \tag{2.30}$$

In the same way,

$$A_2 = \frac{1-\rho}{\mu}\frac{\partial}{\partial\rho}\left[\frac{\rho L\left(\lambda x, \mu y, \rho\right)}{1-\rho}\right] - \left(1-\rho\right)ye^{-\mu y\left(1-\rho\right)}K\left(\lambda x, \rho\mu y\right) \tag{2.31}$$

For A_3, we obtain

$$A_3 = \frac{1-\rho}{\lambda\mu}\sum_{k=0}^{\infty}\rho^k \frac{\gamma\left(k+2, \lambda x\right)}{k!}\frac{\gamma\left(k+2, \mu y\right)}{k!} . \tag{2.32}$$

By using (2.28), we write

$$A_3 = B_1 + B_2 + B_3 + B_4 , \tag{2.33}$$

where

$$
\begin{aligned}
B_1 &= \frac{1-\rho}{\lambda\mu}\sum_{k=0}^{\infty}(k+1)^2\rho^k\,\frac{\gamma(k+1,\lambda x)}{k!}\,\frac{\gamma(k+1,\mu y)}{k!},\\
B_2 &= -\frac{(1-\rho)\,ye^{-\mu y}}{\lambda}\sum_{k=0}^{\infty}(k+1)\frac{\rho^k\mu^k y^k}{k!}\,\frac{\gamma(k+1,\lambda x)}{k!},\\
B_3 &= -\frac{(1-\rho)\,xe^{-\lambda x}}{\mu}\sum_{k=0}^{\infty}(k+1)\frac{\rho^k\lambda^k x^k}{k!}\,\frac{\gamma(k+1,\mu y)}{k!},\\
B_4 &= (1-\rho)\,xye^{-\lambda x-\mu y}\sum_{k=0}^{\infty}\frac{\rho^k\lambda^k\mu^k x^k y^k}{k!k!}.
\end{aligned}
\tag{2.34}
$$

For B_1 we obtain

$$
B_1 = \frac{1-\rho}{\lambda\mu}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial}{\partial\rho}\left\{\rho\frac{L(\lambda x,\mu y,\rho)}{1-\rho}\right\}\right]. \tag{2.35}
$$

Next,

$$
B_2 = -\frac{(1-\rho)\,ye^{-\mu y}}{\lambda}\sum_{k=0}^{\infty}(k+1)\frac{\rho^k\mu^k y^k}{k!}\,\frac{\gamma(k+1,\lambda x)}{k!}\,, \tag{2.36}
$$

$$
B_3 = -\frac{(1-\rho)\,xe^{-\lambda x}}{\mu}\sum_{k=0}^{\infty}(k+1)\frac{\rho^k\lambda^k x^k}{k!}\,\frac{\gamma(k+1,\mu y)}{k!}\,, \tag{2.37}
$$

Finally,

$$
B_4 = (1-\rho)\,xye^{-\lambda x-\mu y}I_0\left(2\sqrt{\rho\lambda\mu xy}\right). \tag{2.38}
$$

Armed with these results, we can now proceed to study the particular expressions resulting from differences between scenarios and come to conclusions.

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