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A Nonparametric Way of Distribution Testing

Ekrem Kilic
Istanbul Bilgi University

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Abstract

Testing the distribution of a random sample can be considered ,indeed, as a goodness-of-fit problem. If we use the nonparametric density estimation of the sample as a consistent estimate of exact distribution, the problem reduces , more specifically, to the distance of two functions. This paper examines the distribution testing from this point of view and suggests a nonparametric procedure. Although the procedure is applicable for all distributions, paper emphasizes on normality test.The critical values for this normality test generated by using Monte Carlo techniques.

1 Introduction

The distribution of a random variable is one of the most important question to be answered by many econometric studies. Econometricians needs to assume or know the distribution of a random variable to be able to make inference and sometimes efficient estimation as in the classical linear regression model where the unobserved disturbance vector , ε , is assumed to be normally distributed. Of course , then, this assumption of distribution must be tested. Therefore the literature on distribution testing is quite deep, there exists many studies on this topic. Since normality is the most common assumption in applying statistical procedures, most of the studies dealt with the normality. The normality tests can be simply divided into two classes; parametric and nonparametric test.

Firstly let us consider some parametric tests, one of the most common parametric normality test is the Jarque-Bera test. This test is based on the moment properties of the normal distribution. Jarque-Bera statistic is, simply, a function of skewness and kurtosis (see Jarque-Bera [5]) and is asymptotically chi-squared distributed. Therefore Jarque-Bera requires no special table for the critical values, that is why Jarque-Bera is so popular. Another parametric test is Shapiro-Wilk test. This test is also related with the moment of the distribution, however it uses a weighted sum of squared random variables. Shapiro and Wilk [9] provides the critical values of the test. Another family of distribution tests those exploits a feature of the normal distribution is proposed by Vasicek [12]. As described by Prescott [7];

> Among all distributions that posses a density function f and have a given variance σ^2, the entropy $H(f)$, defined as,
>
> $$H(f) = \int_{-\infty}^{\infty} f(x) \log[f(x)]dx \qquad (1)$$
>
> is maximized by the normal distribution.

Using this feature Vasicek defined a sample entropy test statistic. Vasicek's sample entropy test is a distribution free test.

There exists also some nonparametric tests. One important test is proposed by Kolmogorov and Smirnov. Kolmogorov and Smirnov's test is based on the empirical distribution function of the sample. The test statistic is maximum of the absolute difference between empirical distribution function and cumulative normal distribution. Kolmogorov and Smirnov test is strong in the sense that distribution of test statistic itself does not depend on the underlying distribution that is tested. Another nonparametric test that uses the empirical distribution function is Cramer-von Mises test. The notion of maximum difference which is used by Kolmogorov - Smirnov test, is replaced with the integrated squared differences. This test is more powerful than the Kolmogorov-Smirnov's test because it considers the whole distribution by integrating the squared differences whereas the Kolmogorov-Smirnov test uses just the maximum of the distance at data points. Approximate critical values can be found in [1] by Andersen and Darling.

This paper will introduce a nonparametric distribution test that is based on the kernel density estimation and simple euclidian measure of distance between functions. The organization of the paper will be as follows. In the next section the general framework will be presented. In the following section, the power of the test will be examined with the help of Monte Carlo simulations. In the other section, the empirical application of the test with USD/TRL exchange rate data will be presented. Finally, in the conclusion section, the results will be discussed.

2 Test Procedure

Parametric statistics defines the form of a distribution, $f(x, \theta)$, say normal density;

$$f(x; \theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \tag{2}$$

where θ is the parameter vector that includes μ and σ.

If the parameters, μ and σ , are set to 0 and 1 respectively, the distribution can be written as;

$$f(x; 0, 1) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{3}$$

this form is called as standard normal density.

Nonparametric methods, on the other hand, suggest histograms or smoothed histograms for density estimation problem. One important feature of the histograms is it integrates to unity. Histograms , however , produces rough and discontinues density estimates. Therefore Kernel estimators are more useful because of their continuity and smoothness. Kernel density estimators are just smoothed histograms. One can formally write a histogram function as follows;

$$f(x) = \frac{1}{nh}\sum_{i=1}^{n} I\left(-1/2 \leq \frac{x_i - x}{h} \leq 1/2\right) \tag{4}$$

where $I(.)$ is the indicator function and h is called as bandwidth or smoothing parameter. Rosenblatt [8] describes the kernel estimator as;

$$f(x) = \frac{1}{nh}\sum_{i=1}^{n} K(\psi_i) \tag{5}$$

where $\int_{-\infty}^{\infty} K(\psi)\, d\psi = 1$ and $\psi_i = \frac{x_i - x}{h}$. Obviously if one choose the indicator function as the kernel, 5 will be exactly same with 4 . Hence by replacing the indicator function with smoothing functions that satisfies the condition of unit integral, we can define different kernel estimators those are , indeed, just smoothed histograms.One of the most common kernel function is standard normal density function (as 3), generally called as gaussian kernel. For this kernel bandwidth can be chosen $h = n^{-\frac{1}{5}}\sigma$(see Pagan and Ullah [6]).

As described kernel density estimation methods provides a continues distribution function for every random sample. The problem of goodness-of-fit, then, becomes a problem of distance. We can simply measure the distance , in an Euclidian fashion, between kernel estimate of density and underlying parametric distribution as follows;

$$D = \sqrt{\int_{-\infty}^{\infty} [\hat{f}(x) - f(x)]^2 dx} \tag{6}$$

where $\hat{f}(x)$ is kernel estimation of the density, $f(x)$ is the underlying distribution's probability density function. To integrate this function, numerical integration methods can be used. Numerical integration approach in this paper is a piecewise method integrates the function using Newton-Cotes formulas (see Burden and Faires [2]). The method is called Composite Trapezoidal rule and can be written as follows;

$$\int_b^a f(x)\, dx = \frac{h}{2}[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)] - \frac{b-a}{12}h^2 f''(\mu)\,. \tag{7}$$

where $h = (b-a)/n$, and $x_j = a + jh$ for each $j = 0, 1, ..., n$. In this formula, $\frac{h}{2}[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)]$ is the numerical integrated result for the integral at the left hand side and $\frac{b-a}{12}h^2 f''(\mu)$ is the error of the numerical integration. When the error is equal to zero, the numerical and analytical integral will be the same. By this formula it is clear that as h goes to zero, the error part of the equation tends to zero. And also as n goes to infinity, h tends to zero. Then if n, number of integrated pieces, is large enough, the numerical integration will be approximately the same with analytical one. In this paper n is taken 200, a and b, upper and lower limits of the integral, are chosen 10 and -10 respectively.

The question, then, what is the critical values for the test? Critical values can be found using Monte Carlo techniques. The method is quite simple, one can generate a random sample coming from normal distribution with a certain number of elements, and can apply the test and can store the values that returned by the test. Repeating this process builds up a probability distribution for the test statistics of the given sample size. And repeating the experiment for different sample sizes completes the process. Since the test is a distance test, it has only positive values and one should check only right tail of the distribution. Table 1 obtained by repeating the process 100000 times for each sample size. Convergence graphs of the test statistics are shown at Figure 1.

3 Power and Size of the Test

After defining the test and obtaining critical values, we need to check power and size of the test. I used Monte Carlo simulations for this purpose and followed the methodology of Stengos and Wu [10].

The power can be defined as one minus probability of TypeII error or equivalently the probability that test rejects the null hypothesis when it should reject. I examined power of the distance to normality test against t distribution , mixture of two normals , lognormal distribution, chi distribution, exponential distribution and 4 different version of generalized lambda distribution.The normal distribution is also considered for the size of the test. For each of the distributions, 100000 random samples are generated at size $n = 20, 50, 100, 200, 500$ and 1000. Simulated distributions are listed below;

Norm	$N(0,1)$
t5	t_5
M2Norm	$z_1 I(p \leq 0.5) + z_2 I(p > 0.5)$ $where\ z_1 \sim N(-1,1), z_2 \sim N(1,1)\ and\ p \sim N(-1,1)$
LNorm	$\exp(z)\ where\ z \sim N(0,1)$
Chi3	χ_3^2
Exp	$-\ln(u)\ where\ u \sim U(0,1)$
Lam1	$\lambda_1 + \left(u^{\lambda_3} - \left(1 - u^{\lambda_4}\right)\right)/\lambda_2$ $where\ parameters\ \Lambda = [0, 0.19754, 0.134915, 0.134915]$
Lam2	$\lambda_1 + \left(u^{\lambda_3} - \left(1 - u^{\lambda_4}\right)\right)/\lambda_2$ $where\ parameters\ \Lambda = [0, -1, -0.8, -0.8]$
Lam3	$\lambda_1 + \left(u^{\lambda_3} - \left(1 - u^{\lambda_4}\right)\right)/\lambda_2$ $where\ parameters\ \Lambda = [0, 1, 1.4, 0.25]$
Lam4	$\lambda_1 + \left(u^{\lambda_3} - \left(1 - u^{\lambda_4}\right)\right)/\lambda_2$ $where\ parameters\ \Lambda = [0, -1, -0.0075, -0.03]$

Generalized Lambda distribution is an extended version of Tukey's Lambda Distribution. Inverse of the cumulative density function of generalized lambda distribution can be written as follows;

$$F^{-1}(u) = \lambda_1 + \frac{\left(u^{\lambda_3} - \left(1 - u^{\lambda_4}\right)\right)}{\lambda_2} \tag{8}$$

Generalized lambda distribution can provide a wide range of symmetric and asymmetric distributions. Lam1 is approximately the same with normal distribution(see Figure 2,Figure 3,Figure 4,Figure 5).

Table 2 reports power and size tests. The simulations show that the distance to normality is less powerful with symmetric distributions like t5 and M2Norm. On the other hand, the test performs very good against asymmetric distributions so that it has quite high power even in small samples.For all asymmetric distributions having larger sample size than 200, the test's power is equal to one with 95% confidence level.

Size of the test is trivial for our case, because the critical values have already derived with Monte Carlo simulations. Table 2 shows that for all samples, size is equal to the significance level of the test. An interesting result is Lam1, an approximation to the normal distribution, have nearly same outputs with normal distribution. While sample size increases , the approximation starts to differ from exact normal distribution and test returns different results.

4 Empirical Results

In this paper, two types of normality problems will be considered. In the first model, a random sample will be tested for the normality. For this purpose three exchange rates will be used. The null hypothesis can be shown as follows;

$$H_0 : f(r_t) = \phi\left(\frac{r_t - \mu}{\sigma}\right) \tag{9}$$

where $r_t = \ln(x_t/x_{t-1})$, x_t is the exchange rate, ϕ is the probability density function of standard normal distribution (3) and f is probability density function of the data set.

First exchange rate is USD/TRL rate. The data includes 370 returns between, 05.01.2004 and 20.06.2005[1]. Its mean is -0.0001, so it has nearly zero mean. Its standard deviation is 0.0079. The maximum return is observed at 11.05.2004, in this day the exchange rate raised to 1.53 from 1.48. If the return is standardized, it will be 4.23. This value should be quite rare if the normal distribution assumption is hold actually. Its minimum is -0.02 that becomes -2.51 after standardizing. Second exchange rate is EUR/TRL rate. The data set covers the same interval. EUR/TRL exchange rate's mean and standard deviation are -0.0002 and 0.0074 and its maximum with in this interval is 0.03. This return was observed at 28.04.2004. The minimum return for the EUR/TRL is -0.02 at 01.06.2005. The standardized values for the maximum and minimum are 4.15 and -2.65 respectively. Finally the last exchange is GBP/TRL rate. The data comes from the same interval again. The GBP/TRL rate has approximately zero mean and its standard deviation is 0.0077, clearly its first two moment is very close to other rates. The maximum and minimum of the returns are 0.031 and -0.018 respectively. The maximum return was at 28.04.2004 and the minimum were at 13.05.2005. The standardized returns for maximum and minimum are 4.11 and -2.40 respectively. Kernel smoothed densities of three data are shown at Figure 6 and the descriptive statistics for three series are reported at Table 3.

Distance to normality test results for these data sets are given at Table 4. The Jarque-Bera tests are also available at this table. We can see that for all three data set Jarque-Bera test rejects the null hypothesis strongly, however distance test failed to reject the null hypothesis at 95% for GBP/TRL data set. As shown in the Figure 6 the GBP/TRL data is quite near to the normal distribution line with respect to other series. Therefore the distance test statistics for this data is also small so that the test could not reject the

[1]All data are taken from the web site of Turkish Central Bank. (http://tcmbf40.tcmb.gov.tr)

normality at 95% significance level. However at 90% confidence level for all three data sets the normality is rejected[2].

In second model, let us consider a regression. In this case the null hypothesis can be defined as;

$$H_0 : f(\varepsilon) = \phi\left(\frac{\varepsilon}{\sigma}\right) \tag{10}$$

where ε is the vector of residuals of a regression like $y = X\beta + \varepsilon$. In this study, the one-year interest rate yields of Turkish treasury bills are regressed on increases[3] on the number of firms in Turkey[4].The data set includes 107 monthly observations between 1995-01 and 2003-11.

Results of the regression are reported at Table 5. The results show that interest rates have a significant explanatory power on the increase in the number of the firms(see Figure 7). However the R^2 of the regression is only 22%. The distribution of the residulas is shown in Figure 8. The distance test statistic is 0.126 and the test strongly rejects the null hypothesis[5]. Then, common assumption pf classical regression does not hold for this model and the inference using normality assumption will not be efficient.

5 Conclusion

In this paper, a non-parametric density testing procedure is described. The test uses very simple euclidian definition of distance. Using this definition, a distance defined between nonparametric estimation of the density and underlying density for the test (In this paper normality is considered). Monte Carlo simulations pointed out the test is very powerful against asymmetric distributions and its power is very limited against t distribution. Against asymmetric distributions the test produces good results even in the small sample. Empirical evidences also supported the Monte Carlo results. Further studies might repeat same strategy to test, let's say, chi distribution at certain degrees of freedom.

[2]The critical values for the distance test are 0.056 and 0.062 at 90% and 95% significance levels respectively

[3]The definition is , in fact, the change in the number of firms but there is no decline in the number of firms with in the interval in which study examines. Always number of new firms were higher than number of firms closed.

[4]The interest rates are supplied by Riskturk MarketRisk module (www.riskturk.com) and increases in the number of firms are at the web site of DIE (www.die.gov.tr)

[5]The critical value at 95% significance level is 0.091

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Table 1: The critical values for the distance to normality test.

	α						
Sample Size	**0.5**	**0.75**	**0.85**	**0.9**	**0.95**	**0.975**	**0.99**
10	0.105	0.126	0.136	0.143	0.154	0.162	0.172
11	0.103	0.124	0.135	0.142	0.152	0.161	0.171
12	0.101	0.122	0.133	0.141	0.151	0.160	0.171
13	0.099	0.120	0.131	0.139	0.150	0.159	0.170
14	0.097	0.119	0.130	0.138	0.149	0.159	0.169
15	0.095	0.117	0.128	0.136	0.147	0.157	0.168
16	0.094	0.115	0.126	0.134	0.146	0.155	0.167
17	0.092	0.113	0.125	0.133	0.145	0.155	0.165
18	0.091	0.112	0.123	0.131	0.143	0.153	0.164
19	0.089	0.110	0.122	0.130	0.141	0.151	0.163
20	0.088	0.109	0.120	0.128	0.140	0.150	0.162
21	0.087	0.108	0.119	0.127	0.139	0.149	0.160
22	0.086	0.106	0.118	0.125	0.137	0.148	0.158
23	0.085	0.105	0.117	0.124	0.136	0.146	0.157
24	0.084	0.104	0.116	0.123	0.134	0.144	0.156
25	0.083	0.103	0.115	0.122	0.134	0.144	0.156
26	0.082	0.102	0.113	0.121	0.132	0.142	0.154
27	0.081	0.101	0.112	0.120	0.131	0.141	0.153
28	0.081	0.100	0.111	0.119	0.130	0.140	0.151
29	0.080	0.099	0.110	0.118	0.129	0.139	0.150
30	0.079	0.098	0.109	0.117	0.128	0.138	0.149
31	0.078	0.097	0.108	0.116	0.127	0.137	0.148
32	0.078	0.097	0.107	0.115	0.126	0.136	0.147
33	0.077	0.095	0.106	0.114	0.125	0.135	0.146
34	0.076	0.094	0.105	0.113	0.124	0.133	0.145
35	0.076	0.094	0.105	0.112	0.123	0.133	0.145
36	0.075	0.093	0.104	0.111	0.122	0.132	0.144
37	0.075	0.093	0.103	0.110	0.122	0.131	0.142
38	0.074	0.092	0.102	0.110	0.121	0.131	0.142
39	0.073	0.091	0.102	0.109	0.120	0.129	0.140
40	0.073	0.091	0.101	0.108	0.119	0.129	0.140
41	0.073	0.090	0.100	0.107	0.118	0.127	0.138
42	0.072	0.089	0.099	0.107	0.117	0.126	0.138
43	0.071	0.089	0.099	0.106	0.116	0.126	0.137

continues...	α						
Sample Size	**0.5**	**0.75**	**0.85**	**0.9**	**0.95**	**0.975**	**0.99**
44	0.071	0.088	0.099	0.106	0.116	0.125	0.137
45	0.071	0.088	0.098	0.104	0.115	0.124	0.135
46	0.070	0.087	0.097	0.104	0.115	0.124	0.135
47	0.070	0.086	0.097	0.104	0.114	0.124	0.135
48	0.069	0.086	0.096	0.103	0.113	0.122	0.133
49	0.069	0.085	0.095	0.102	0.113	0.122	0.132
50	0.069	0.085	0.095	0.102	0.112	0.121	0.132
51	0.068	0.085	0.094	0.101	0.111	0.120	0.131
52	0.068	0.084	0.094	0.100	0.111	0.120	0.130
53	0.067	0.084	0.093	0.100	0.110	0.119	0.130
54	0.067	0.083	0.093	0.099	0.109	0.119	0.129
55	0.067	0.083	0.092	0.099	0.109	0.118	0.129
56	0.066	0.082	0.092	0.098	0.108	0.117	0.128
57	0.066	0.082	0.091	0.098	0.108	0.117	0.127
58	0.066	0.081	0.091	0.097	0.107	0.116	0.126
59	0.065	0.081	0.090	0.097	0.107	0.116	0.126
60	0.065	0.080	0.090	0.096	0.106	0.115	0.125
61	0.065	0.080	0.089	0.096	0.106	0.114	0.125
62	0.064	0.080	0.089	0.095	0.105	0.114	0.124
63	0.064	0.079	0.089	0.095	0.104	0.113	0.123
64	0.064	0.079	0.088	0.094	0.104	0.113	0.122
65	0.064	0.079	0.088	0.094	0.103	0.112	0.122
66	0.063	0.078	0.087	0.094	0.103	0.112	0.122
67	0.063	0.078	0.087	0.093	0.103	0.111	0.122
68	0.063	0.078	0.087	0.093	0.102	0.111	0.120
69	0.063	0.077	0.086	0.092	0.102	0.110	0.120
70	0.062	0.077	0.086	0.092	0.101	0.110	0.119
71	0.062	0.077	0.085	0.092	0.101	0.109	0.119
72	0.062	0.077	0.085	0.091	0.101	0.109	0.119
73	0.061	0.076	0.085	0.091	0.101	0.109	0.119
74	0.061	0.076	0.085	0.091	0.100	0.108	0.118
75	0.061	0.075	0.084	0.090	0.099	0.107	0.117
76	0.061	0.075	0.084	0.090	0.099	0.108	0.117
77	0.061	0.075	0.083	0.089	0.098	0.106	0.116

continues...	α						
Sample Size	**0.5**	**0.75**	**0.85**	**0.9**	**0.95**	**0.975**	**0.99**
78	0.060	0.075	0.083	0.089	0.098	0.106	0.116
79	0.060	0.074	0.083	0.089	0.098	0.106	0.116
80	0.060	0.074	0.083	0.089	0.098	0.106	0.115
81	0.060	0.074	0.082	0.088	0.097	0.105	0.115
82	0.060	0.074	0.082	0.088	0.097	0.105	0.115
83	0.059	0.073	0.081	0.087	0.096	0.104	0.113
84	0.059	0.073	0.081	0.087	0.096	0.104	0.113
85	0.059	0.072	0.081	0.087	0.096	0.104	0.113
86	0.059	0.072	0.081	0.087	0.095	0.103	0.112
87	0.058	0.072	0.080	0.086	0.095	0.103	0.113
88	0.058	0.072	0.080	0.086	0.095	0.103	0.112
89	0.058	0.072	0.080	0.086	0.094	0.102	0.112
90	0.058	0.071	0.080	0.085	0.094	0.102	0.111
91	0.058	0.071	0.080	0.085	0.094	0.102	0.111
92	0.058	0.071	0.079	0.085	0.093	0.101	0.111
93	0.057	0.071	0.079	0.085	0.093	0.101	0.110
94	0.057	0.070	0.078	0.084	0.093	0.100	0.109
95	0.057	0.070	0.078	0.084	0.092	0.100	0.109
96	0.057	0.070	0.078	0.083	0.092	0.100	0.109
97	0.056	0.070	0.078	0.083	0.092	0.100	0.109
98	0.056	0.070	0.078	0.083	0.092	0.099	0.108
99	0.056	0.069	0.077	0.083	0.091	0.099	0.108
100	0.056	0.069	0.077	0.083	0.091	0.099	0.107
200	0.045	0.056	0.062	0.066	0.073	0.079	0.086
500	0.034	0.041	0.045	0.048	0.053	0.057	0.062
1000	0.027	0.032	0.035	0.038	0.041	0.044	0.048

Table 2: Power and size of distance to normality test

	n=20		n=50		n=100		n=200		n=500		n=1000	
	0.95	0.99	0.95	0.99	0.95	0.99	0.95	0.99	0.95	0.99	0.95	0.99
Norm	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
t5	0.07	0.02	0.08	0.03	0.14	0.07	0.29	0.16	0.73	0.55	0.97	0.92
M2Norm	0.09	0.02	0.17	0.05	0.28	0.11	0.48	0.23	0.83	0.62	0.98	0.93
LNorm	0.83	0.66	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Chi3	0.47	0.24	0.89	0.72	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00
Exp	0.65	0.41	0.98	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Lam1	0.05	0.01	0.05	0.01	0.05	0.01	0.04	0.01	0.04	0.01	0.04	0.01
Lam2	0.65	0.55	0.97	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Lam3	0.41	0.17	0.86	0.63	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00
Lam4	0.28	0.13	0.30	0.14	0.90	0.77	1.00	0.99	1.00	1.00	1.00	1.00

Table 3: Descriptive statistics for three exchange rate returns

	USD/TRL	**EUR/TRL**	**GBP/TRL**
Mean	-0.00	-0.00	-0.00
StDev	0.01	0.01	0.01
Max	0.03	0.03	0.03
Min	-0.02	-0.02	-0.02
Skewness	0.60	0.50	0.57
Excess Kurtosis	1.28	1.10	1.34

Table 4: Test statistics for three exchange rates. (**Rejected at 0.95 and * rejected at 0.9)

	Distance Test	**Jarque-Bera**
USD/TRL	0.102**	45.35**
EUR/TRL	0.068**	33.12**
GBP/TRL	0.059*	46.19**

Table 5: Regression statistics

Variable	**Coefficient**	**Std.Error**	**t-Statistic**	**Prob.**
Constant	893.0792599	494.1753072	1.807211423	0.073592888
Interest	30.74945928	5.637318116	5.454625525	3.29E-07

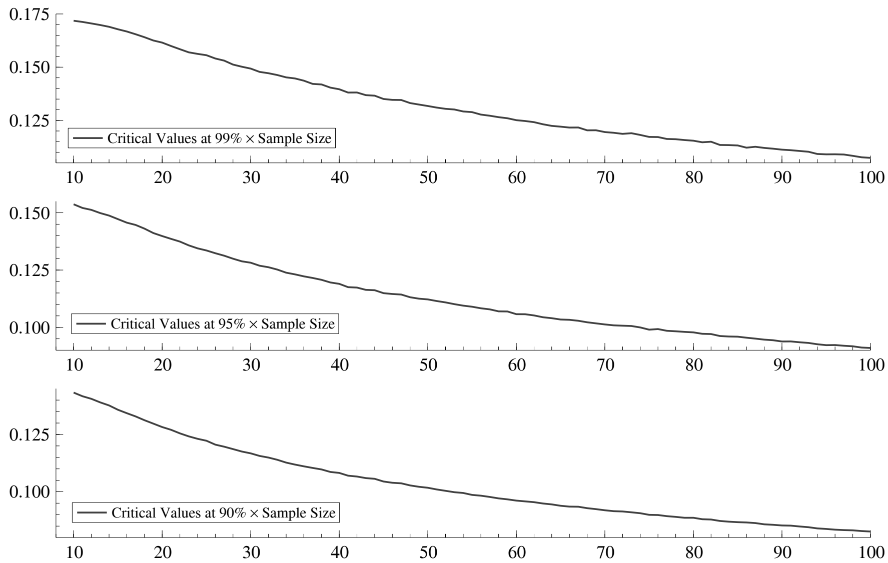

Figure 1: Convergence graphs of the test statistics at 0.99, 0.95, and 0.9 significance levels.

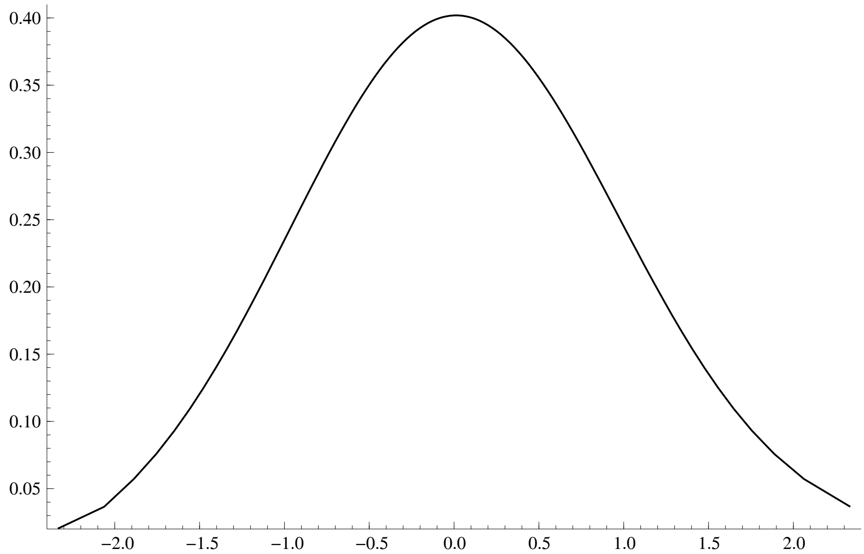

Figure 2: Graph of Lam1

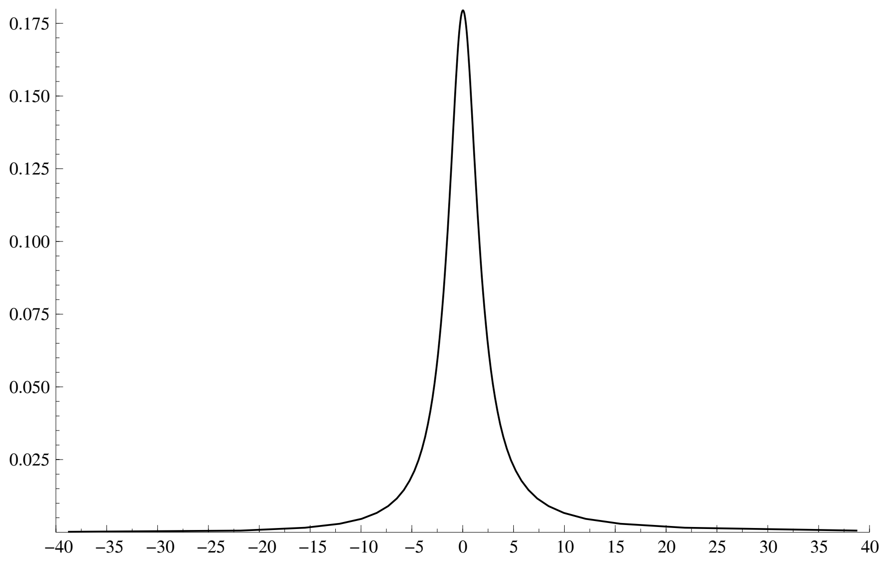

Figure 3: Graph of Lam2

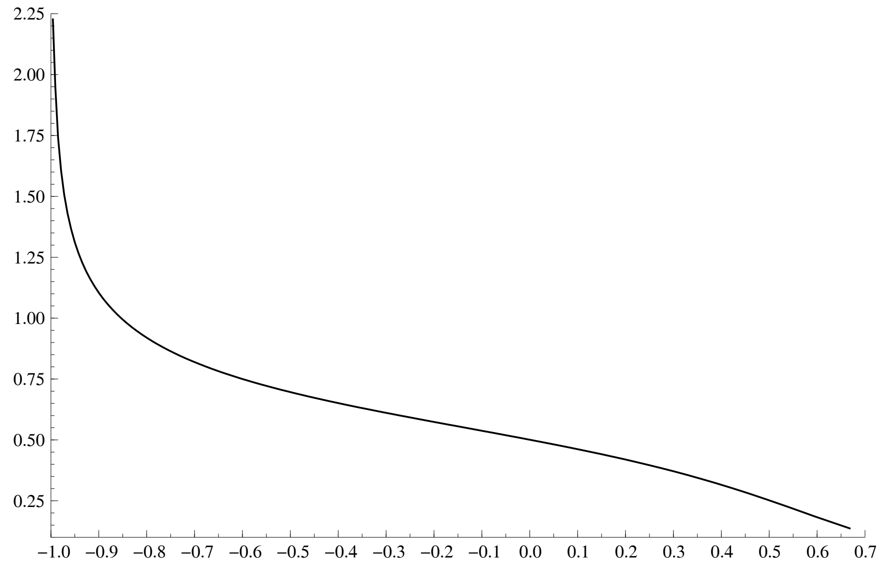

Figure 4: Graph of Lam3

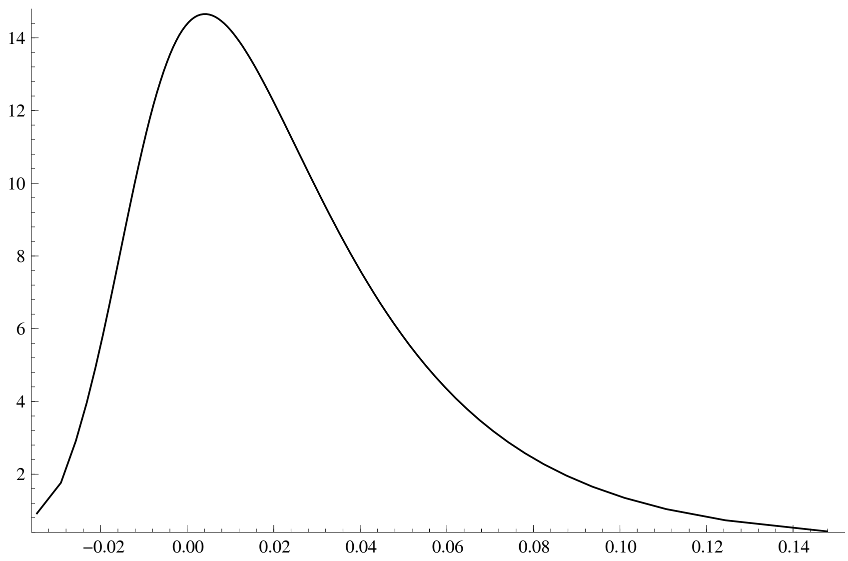

Figure 5: Graph of Lam4

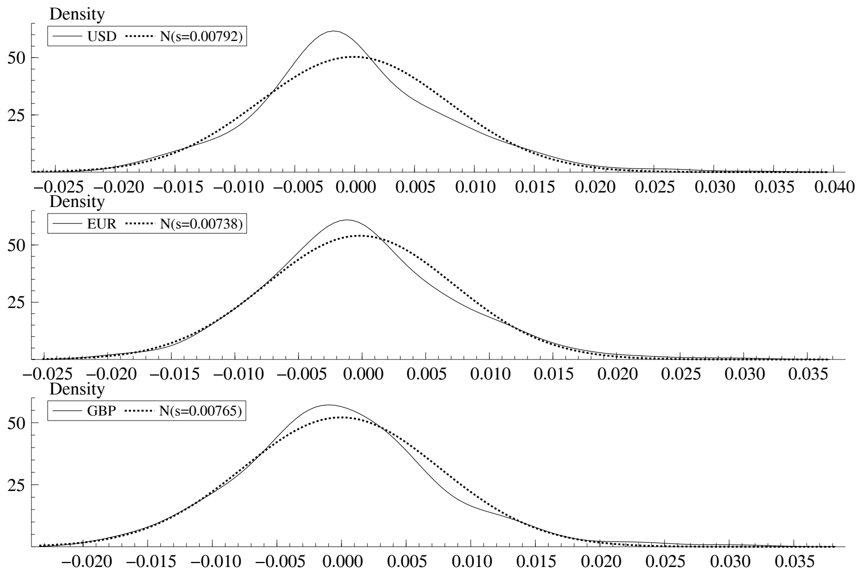

Figure 6: Kernel vs. Normal distribution for three exchange rate returns

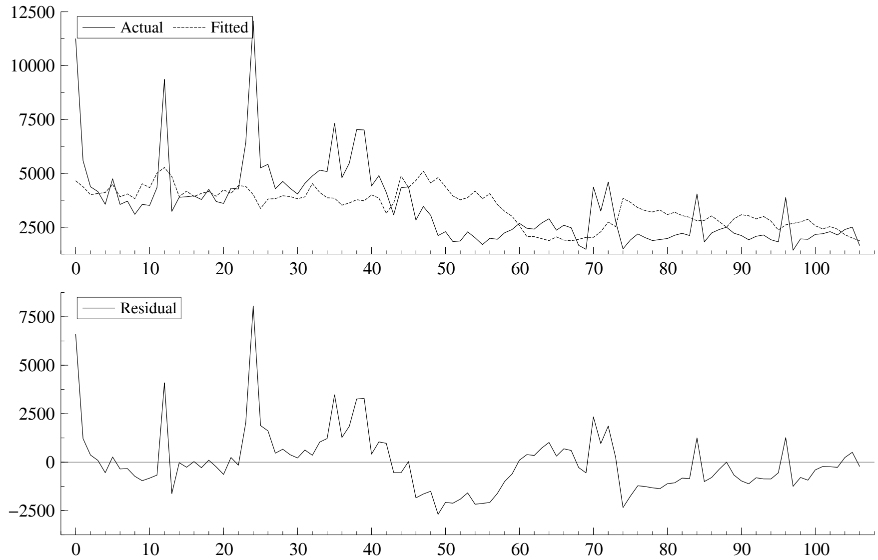

Figure 7: Graph of the regression

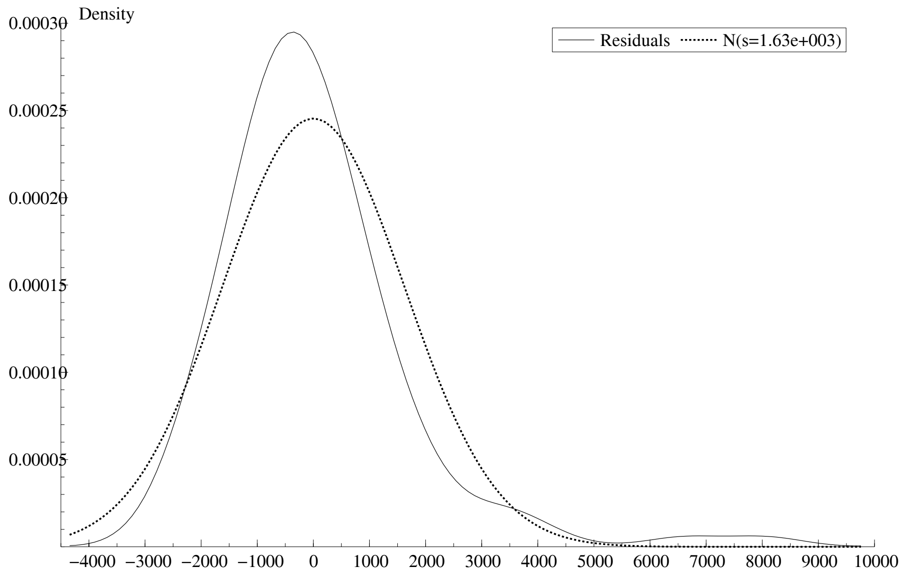

Figure 8: Density graph of the residuals