Using the correlation dimension to detect non-linear dynamics: Evidence from the Athens Stock Exchange

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Abstract : The standardised residuals from GARCH models fitted to three stock indices of the Athens Stock Exchange are examined for evidence of chaotic behaviour. In each case the correlation dimension is calculated for a range of embedding dimensions. The results do not support the hypothesis of chaotic behaviour; it appears that each set of residuals is *iid*.

Key words: Non-linear Dynamics, Stock Indices, Chaos, Correlation Dimension

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INTRODUCTION

A large body of literature has accumulated over the past three decades concerning the validity of the weak-form efficient markets hypothesis (EMH) in financial economics. The weak-form of the EMH postulates that successive one-period stock returns are independent and identically distributed (*iid*), i.e. the price levels resemble a random walk. At the same time it is well known that stock returns are characterised by volatility clustering. Additionally we usually observe large returns to be followed by large returns and small returns to be followed by small returns, leading to contiguous periods of volatility and stability. Although most of the empirical tests of the efficient markets hypothesis are based on linear models, interest in nonlinear processes has experienced a tremendous rate of development over the last few years (for an excellent review see Barnett and Serletis 2000). In this paper, we will examine how the introduction of the single European currency has affected earlier claims in the literature that the Athens Stock Exchange (ASE) is characterised by deterministic chaos as the ASE is in the process of becoming a fully developed capital market[1].

A limited number of studies have appeared in the literature providing empirical results for the ASE (for a review see Panagiotidis 2003). None has tested for the presence of nonlinear dynamics (other than GARCH) after the introduction of the common currency. Siriopoulos (1996) used monthly observations of the ASE General Index from 1974:1 to 1994:6. Using the BDS test statistic and the correlation dimension, it was concluded that a GARCH model could not explain the non-linearities of the series that might be generated by "semi-chaotic behaviour". Barkoulas and Travlos (1998) used daily observations of the ASE30, the 30 most

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marketable stocks, from January 1981 to December 1990. Models including an AR(p) and a GARCH (1,1) were employed and diagnostic tools such as BDS, correlation dimension and Kolmogorov entropy were estimated. They concluded that "the BDS test detects remaining unspecified hidden structure in the Greek stock returns" but " do not find evidence in support of a chaotic structure in the Athens Stock Exchange". Niarchos and Alexakis (1998) followed a different methodology to test the EMH in the Athens Stock exchange. They used error correction models and compared the speed of adjustment. Their evidence rejected the EMH. More recently, Apergis and Eleptheriou (2001) examined market volatility using daily observations of the ASE General Index for the period January 1990 to July 1999. They compared different GARCH models based on the log likelihood and concluded that "the presence of persistence in volatility clustering implies inefficiency of the ASE market". Lastly, Siourounis (2002) employs GARCH type models and tests for their validity using a data set of daily closings of the ASE General Index for the period of 4th January 1988 until 30th October 1998. The Ljung-Box test statistic is employed as a diagnostic tool and it was found that "the GARCH(1,1) and LGARCH(1,1) models can explain quite satisfactorily the dependencies of the first and second moments".

CORRELATION DIMENSION

Grassberger and Procaccia (1983) suggested the *correlation dimension* as a tool for distinguishing random from chaotic time series. To briefly discuss this, let us start with the 1-dimensional series, $\{x_t\}_{t=1}^{n}$, and from this form the sequence of $N = n - m + 1$ m-dimensional vectors $X_s = \{x_s, x_{s+1}, ..., x_{s+m-1}\}_{s=1}^{n-m+1}$. The selected

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[1] In July 2000 Morgan Stanley announced the change in the classification of the MSCI Greece Index from an emerging to a developed market index with effect from the 1st of June 2001 (see http://www.msci.com/pressreleases/archive/pr000731.html).

value of m is called the *embedding dimension* and each X_s is known as an m-history of the series $\{x_t\}_{t=1}^{n}$. This converts the original scalar series into a shorter series of N (m-dimensional) vectors with overlapping entries. Assuming that the true, but unknown, system which generated $\{x_t\}_{t=1}^{n}$ is θ-dimensional and provided that $m \geq 2\theta + 1$, then the set of m-histories recreates the dynamics of the data generation process and can be used to analyse the dynamics of the system - see Takens (1981).

The correlation dimension is based on the *correlation function* (or *correlation integral*), $C(N, m, \varepsilon) = \dfrac{2}{N(N-1)} \#\{(s,t) \mid 1 \leq s, t \leq N\}$, where $\#$ denotes the number of elements in the set. The correlation dimension is defined as $D_C^m = \lim\limits_{\varepsilon \to 0} \dfrac{\log(C(N, m, \varepsilon))}{\log(\varepsilon)}$.

In practice, one estimates D_c^m for $m = 1, 2, 3, \dots, k$ for k no larger than around 10. If, as m increases, D_c^m continues to rise then this is symptomatic of a stochastic system. If, however, the data are generated by a deterministic process (consistent with chaotic behaviour), then D_c^m will reach a finite limit at some relatively small m. The correlation dimension can therefore be used to distinguish true stochastic processes from deterministic chaos (which may be low-dimensional or high-dimensional). Figure 1 illustrates the theoretical relationship between $\log(C(N, m, \varepsilon))$ and $\log(\varepsilon)$ (see Chappell & Eldridge, 1977). For $a \leq \log(\varepsilon) \leq b$, ε is 'too small' and very few m-histories lie with a distance ε of each other. For $\log(\varepsilon) > c$, ε is 'too large' and all m-histories will lie within a distance ε of each other. For $b < \log(\varepsilon) < c$, $C(N, m, \varepsilon)$ increases as m increases; $C(N, m, \varepsilon)$ is the *slope* of the line for $b < \log(\varepsilon) < c$. This slope will increase initially as m is increased

$\log\big(C(N,m,\varepsilon)\big)$
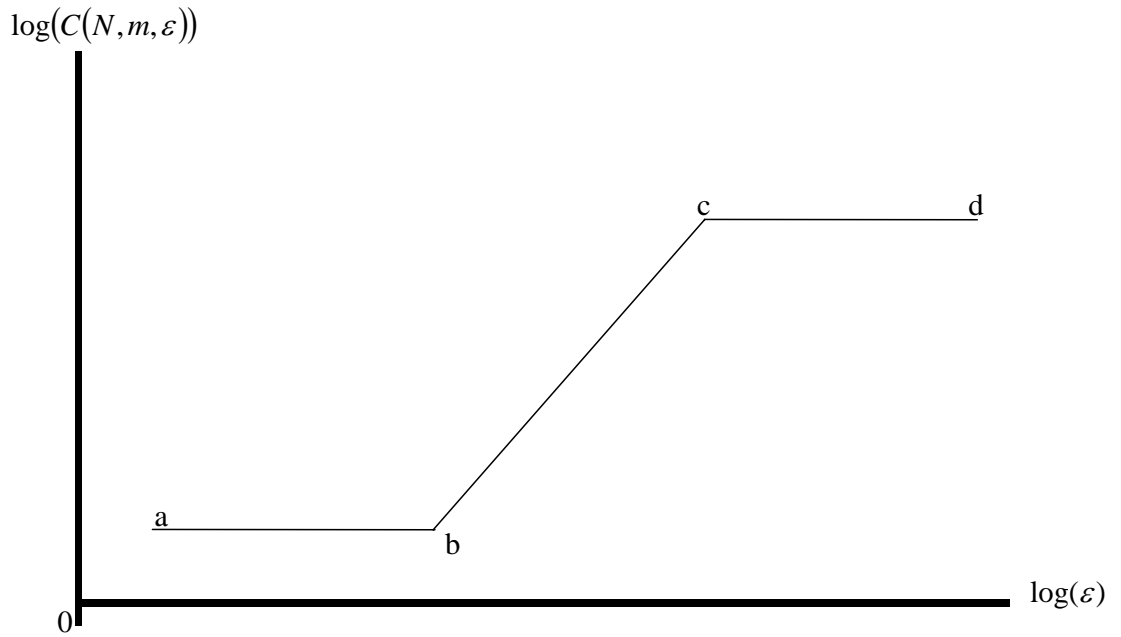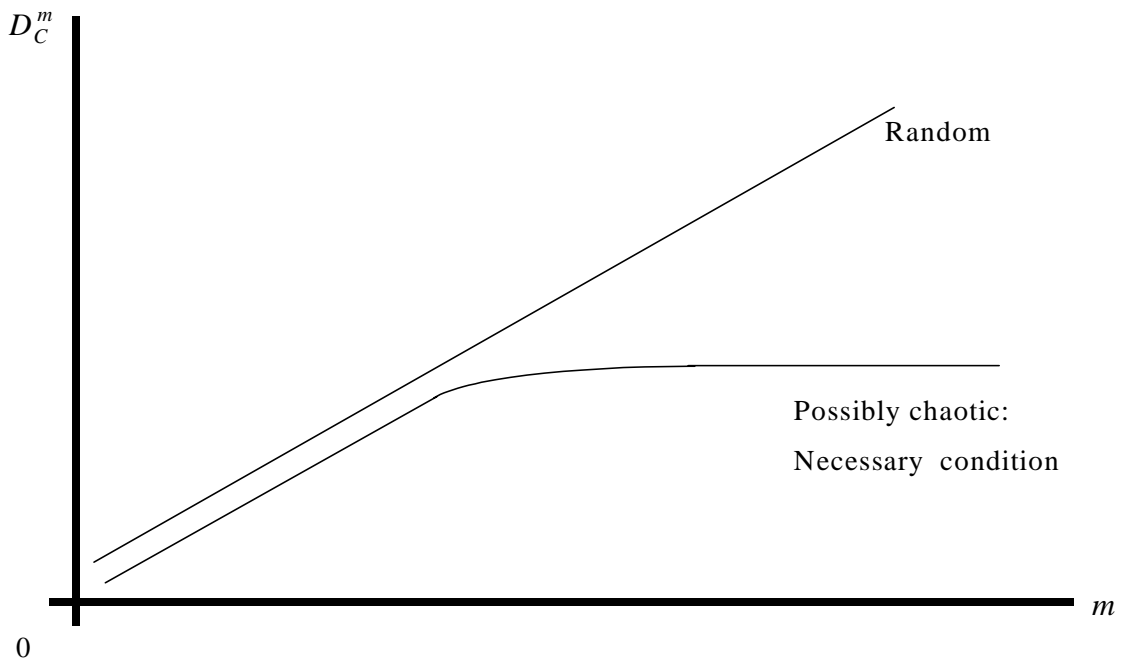
c   d

a

b

$\log(\varepsilon)$

0

Figure 2 shows the theoretical relationship between D_C^m and m for a purely random series and a possibly chaotic series (Chappell & Eldridge, 1997).

Figure 2: Theoretical relationship between D_C^m and m for a purely random series and a possibly chaotic series.

D_C^m

Random

Possibly chaotic:

Necessary condition

m

0

While the correlation dimension measure is therefore potentially very useful in testing for chaos, the sampling properties of the correlation dimension are, unfortunately, unknown. As Barnett et al (1997, pp. 306) put it "if the only source of stochasticity is observational noise in the data, and if that noise is slight, then it is possible to filter the noise out of the data and use the correlation dimension test deterministically. However, if the economic structure that generated the data contains a stochastic disturbance within equations, the correlation dimension is stochastic and its derived distribution is important in producing reliable inference".

Moreover, if the correlation dimension is very large as in the case of high-dimensional chaos, it will be very difficult to estimate it without an enormous amount of data. In this regard, Ruelle (1990) argues that a chaotic series can only be distinguished if it has a correlation dimension well below $2\log_{10} N$, where N is the size of the data set, suggesting that with economic time series the correlation dimension can only distinguish low-dimensional chaos from high-dimensional stochastic processes - see also Grassberger and Procaccia (1983) for more details.

This paper will investigate the following conflicting claims. Panagiotidis (2003) used a battery of tests which signalled that the standardised residuals of the preferred GARCH models are *iid* processes. However, this does not exclude the case of deterministic chaos (looks random, but isn't). On the other hand, there are claims in the literature (see Barkoulas and Travlos 1998, and Siriopoulos 1996)) that the ASE is being determined by chaotic dynamics. To proceed, the standardised residuals from the estimated GARCH models[2] in Panagiotidis (2003) are examined and the correlation dimension is calculated for a range of embedding dimensions.

As mentioned above, if the data under consideration contain a detectable non-linear deterministic component, the correlation dimension should increase with

increasing values of the embedding dimension. However, this should level off at some point and remain constant for all further values of the embedding dimension (see figure 2). On the other hand, if the true data generating process is purely random, then we would expect the correlation dimension always to increase with the embedding dimension.

The outcome is presented in Figures 3, 4 and 5. In each of these, the embedding dimension is on the horizontal axis and the correlation dimension is on the vertical axis and the calculations were carried out using a program by Sprott (1998). It is clear in each of these figures that the correlation dimension keeps on increasing as a function of the embedding dimension and there is no sign that this levels off at some point in any of the series. Consequently, we could argue that there is no evidence to suggest that any form of chaotic non-linear deterministic process is present in the standardised residuals of the preferred GARCH models for the three indices. This finding further reinforces our argument that the series under investigation are *iid*.

CONCLUSIONS

In this paper, we have examined the claim that there is chaotic behaviour in the ASE . We feel that this was an interesting exercise since this stock market has recently joined the Euro zone.

To sum up, we argued against the chaos hypothesis in the case of the ASE. Firstly, there are theoretical reasons, which are explained in Lalley (1999). Lalley discusses the restricted number of cases where it is impossible to recover the original time series when there is an added noise component. Secondly, on an empirical level, the correlation dimension failed to provide any evidence in favour of chaotic dynamics.

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[2] The three estimated equations are reproduced in the appendix.

Figure 3: Correlation Dimension for the standardised residuals of the preferred GARCH model of the ASE ─ FTSE
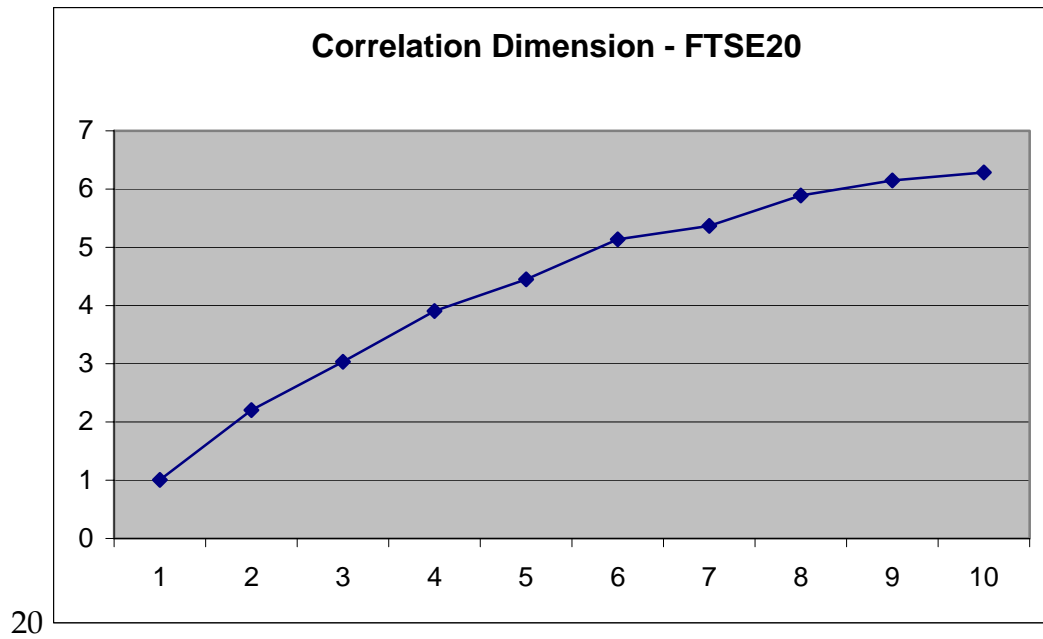

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Figure 4: Correlation Dimension for the standardised residuals of the preferred GARCH model of the ASE ─ FTSE Mid 40
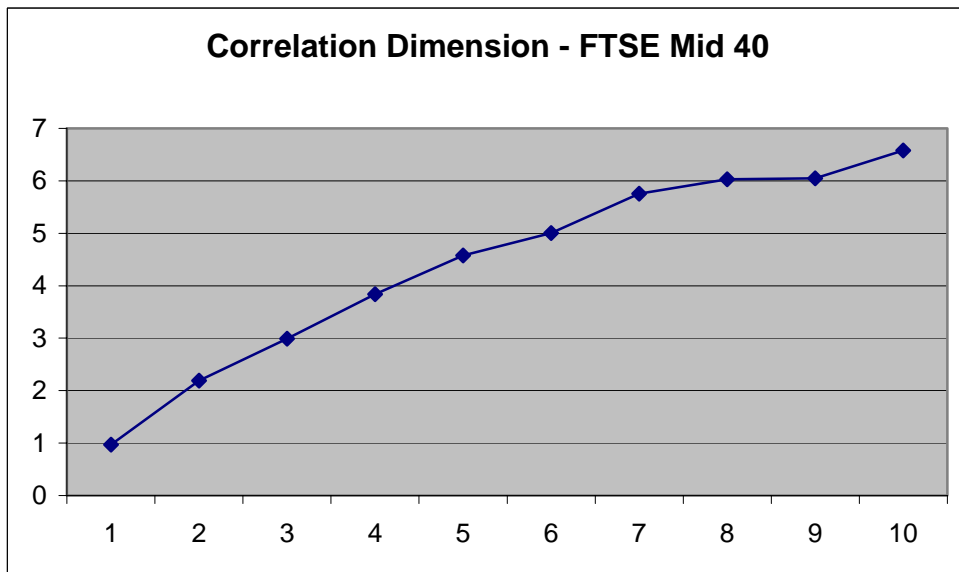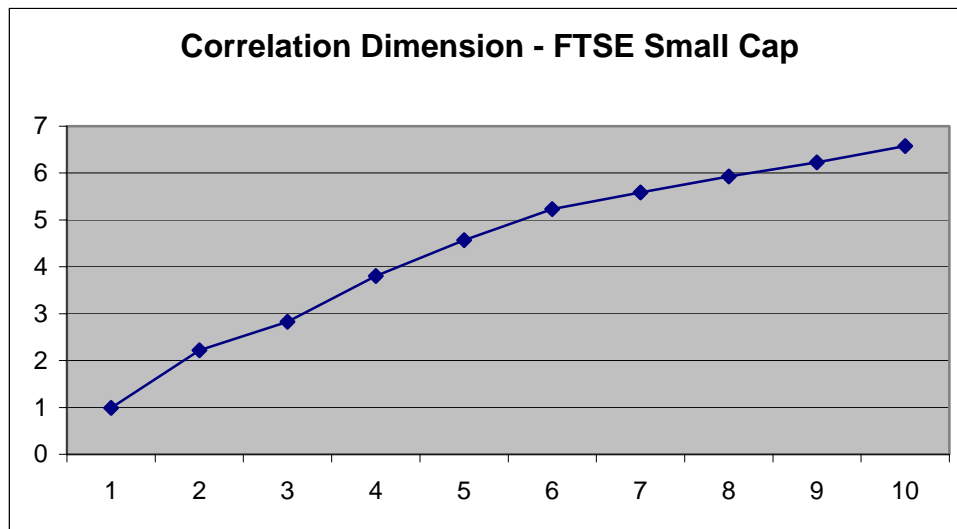
Figure 5: Correlation Dimension for the standardised residuals of the preferred GARCH model of the ASE ─ FTSE Small Cap Index

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APPENDIX

The three estimated equations from Panagiotidis (2003) are given below. Data are daily returns for the three stock indices, calculated from daily closing prices, and the sample period is from 1st June 2000 to 31st December 2002. R_t is the daily return, σ_t^2 the conditional variance and ε_{t-1}^2 the lagged squared residual. Numbers in parentheses are the corresponding t statistics. It is the standardised residuals from each of these equations for which correlation dimensions are calculated, and illustrated in Figures 3 – 5 in the main text[3].

1. The ASE FTSE 20 Index

$$R_t^{ASEFTSE20} = \underset{(2.32)}{-0.0006355} + \underset{(1.67)}{0.314054}\,\sigma$$

$$\sigma_t^2 = \underset{(3.54)}{0.000339} + \underset{(5.73)}{0.16374}\,\varepsilon_{t-1}^2 + \underset{(12.88)}{0.7033}\,\sigma_{t-1}^2$$

2. The ASE FTSE Mid 40 Index

$$R_t^{ASEFTSEMID40} = \underset{(2.54)}{-0.004866} + \underset{(2.92)}{0.1226}\,R_{t-1}^{ASEFTSEMID40} + \underset{(1.79)}{0.21473}\,\sigma$$

$$\sigma_t^2 = \underset{(2.33)}{0.0000714} + \underset{(6.84)}{0.122}\,\varepsilon_{t-1}^2 + \underset{(46.01)}{0.862}\,\sigma_{t-1}^2$$

3. The ASE FTSE Small Cap Index

$$R_t^{ASEFTSESMALLCAP} = \underset{(2.2)}{-0.0001725} + \underset{(2.34)}{0.1387}\,R_{t-1}^{ASEFTSESMALLCAP}$$

$$\sigma_t^2 = \underset{(2.11)}{0.0000183} + \underset{(5.82)}{0.1521}\,\varepsilon_{t-1}^2 + \underset{(18.78)}{0.7983}\,\sigma_{t-1}^2$$

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[3] For more information on the indices and their composition http://www.ase.gr and http://www.ftse.com . The data are available free from http://www.enet.gr/finance/finance.jsp .