A Multivariate GARCH Model with Time-Varying Correlations

Y.K. Tse and Albert K.C. Tsui

Department of Economics
National University of Singapore
Kent Ridge, Singapore 119260

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Abstract: In this paper we propose a new multivariate GARCH model with time-varying correlations. We adopt the vech representation based on the conditional variances and the conditional correlations. While each conditional-variance term is assumed to follow a univariate GARCH formulation, the conditional-correlation matrix is postulated to follow an autoregressive moving average type of analogue. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive definite during the optimisation. Thus, our new model retains the intuition and interpretation of the univariate GARCH model and yet satisfies the positive-definite condition as found in the constant-correlation and BEKK models. We report some Monte Carlo results on the finite-sample distributions of the MLE of the varying-correlation MGARCH model. The new model is applied to some real data sets. It is found that extending the constant-correlation model to allow for time-varying correlations provides some interesting time histories that are not available in a constant-correlation model.

Key Words: BEKK model, constant correlation, Monte Carlo method, multivariate GARCH model, maximum likelihood estimate, varying correlation

JEL Classification: C12

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1 Introduction

Following the success of the autoregressive conditional heteroscedasticity (ARCH) model and the generalized ARCH (GARCH) model in describing the time-varying variances of economic data in the univariate case many researchers have extended these models to multivariate dimension. Applications of the multivariate GARCH (MGARCH) models to financial data have been numerous. For example, Bollerslev (1990) studied the changing variance structure of the exchange rate regime in the European Monetary System assuming the correlations to be time invariant. Kroner and Claessens (1991) applied the models to calculate the optimal debt portfolio in multiple currencies. Lien and Luo (1994) evaluated the multiperiod hedge ratios of currency futures in a MGARCH framework. Karolyi (1995) examined the international transmission of stock returns and volatility using different versions of MGARCH models. Baillie and Myers (1991) estimated the optimal hedge ratios of commodity futures and argued that these ratios are nonstationary. Gourieroux (1997, Chapter 6) presented a survey of several versions of MGARCH models. See also Bollerslev, Chou and Kroner (1992) and Bera and Higgins (1993) for surveys on the methodology and applications of GARCH and MGARCH models.

Bollerslev, Engle and Wooldridge (1988) provided the basic framework for a MGARCH model. They extended the GARCH representation in the univariate case to the vectorized conditional-variance matrix. Their specification follows the traditional autoregressive moving average time series analogue. While this vech representation is very general, it involves a large number of parameters. Empirical applications require further restrictions and simplifications. A useful member of the vech-representation family is the diagonal form. Under the diagonal form, each variance-covariance term is postulated to follow a GARCH-type equation with the lagged variance-covariance term and the product of the corresponding lagged residuals as the right-hand-side variables in the conditional-(co)variance equation.

It is often difficult to verify the condition that the conditional-variance matrix of an estimated MGARCH model is positive definite.[1] Furthermore, such conditions are often very difficult to impose during the optimisation of the log-likelihood function. Bollerslev (1990) suggested the constant-correlation MGARCH (CC-MGARCH) model that can overcome these difficulties. He pointed out that under the assumption of constant correlations, the maximum likelihood estimate (MLE) of the correlation matrix is equal to the sample correlation matrix. As the sample correlation matrix is always positive definite, the optimisation will not fail as long as the conditional variances are positive. In addition, when the correlation matrix is concentrated out of the log-likelihood function further simplification is achieved in the optimisation.

Due to its computational simplicity, the CC-MGARCH model is widely used in empirical research. However, while the constant-correlation assumption provides a convenient MGARCH model for estimation, some studies find that this assumption is not supported by some financial data.[2] Thus, there is a need to extend the MGARCH models to incorporate time-varying correlations and yet retain the appealing feature of satisfying the positive-definite condition during the optimisation.

Engle and Kroner (1995) proposed a class of MGARCH model called the BEKK (named after Baba, Engle, Kraft and Kroner) model. The motivation is to ensure the condition of a positive definite conditional-variance matrix in the process of optimisation. Engle and Kroner provided some theoretical analysis of the BEKK model and related it to the vech-representation form. Another approach examines the conditional variance as a factor model. The works by Diebold and Nerlove (1989), Engel and Rodrigues (1989) and Engle, Ng and Rothschild (1990) are along this line. One disadvantage of

[1] Engle, Granger and Kraft (1984) presented the necessary conditions for the conditional-variance matrix to be positive definite in a bivariate ARCH model. Extensions of these results to more general models are, however, intractable.

[2] For example, Tse (2000) found that the stock returns across different national markets exhibit time-varying correlations.

the BEKK and factor models is that the parameters cannot be easily interpreted, and their net effects on the future variances and covariances are not readily seen. Bera, Garcia and Roh (1997) reported that the BEKK model does not perform well in the estimation of the optimal hedge ratios. Lien, Tse and Tsui (1998) reported difficulties in getting convergence when using the BEKK model to estimate the conditional-variance structure of spot and futures prices.

In this paper we propose a new MGARCH model with time-varying correlations. Basically we adopt the vech representation. The variables of interest are, however, the conditional variances and conditional correlations. We assume a vech-diagonal structure in which each conditional-variance term follows a univariate GARCH formulation. The remaining task is to specify the conditional-correlation structure. We apply an autoregressive moving average type of analogue to the conditional-correlation matrix. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive definite during the optimisation. Thus, our new model retains the intuition and interpretation of the univariate GARCH model and yet satisfies the positive-definite condition as found in the constant-correlation and BEKK models.

The plan of the rest of the paper is as follows. In Section 2 we describe the construction of the varying-correlation MGARCH model. As in other MGARCH models, the new model can be estimated using the maximum likelihood estimation (MLE) method. Some Monte Carlo results on the finite-sample distributions of the MLE of the varying-correlation MGARCH model are reported in Section 3. Section 4 describes some illustrative examples of the new model using some real data sets. These are the exchange rate data, national stock market price data and sectoral stock price data. The new model is compared against the CC-MGARCH model. It is found that extending the constant-correlation model to allow for time-varying correlations provides some interesting empirical results. The estimated conditional-correlation path provides a time history that would be lost in a constant-correlation model. Finally, we give some concluding remarks in Section 5.

2 A Varying-Correlation MGARCH Model

Consider a multivariate time series of observations $\{y_t\}$, $t = 1, ..., T$, with K elements each, so that $y_t = (y_{1t}, ..., y_{Kt})'$. We assume that the observations are of zero (or known) mean. This assumption simplifies the discussions without straining the notations.[3]

The conditional variance of y_t is assumed to follow the time-varying structure given by

$$\mathrm{Var}(y_t|\, \Phi_{t-1}) = \Omega_t, \tag{1}$$

where Φ_t is the information set at time t. We denote the variance elements of Ω_t by σ^2_{it}, for $i = 1, ..., K$, and the covariance elements by σ_{ijt}, where $1 \leq i < j \leq K$. Denoting D_t as the $K \times K$ diagonal matrix with the ith diagonal element being σ_{it}, we let $\epsilon_t = D_t^{-1}\, y_t$. Thus, ϵ_t is the standardised residual and is assumed to be serially independently distributed with mean zero and variance matrix $\Gamma_t = \{\rho_{ijt}\}$. Of course, Γ_t is also the correlation matrix of y_t. Furthermore, $\Omega_t = D_t\, \Gamma_t\, D_t$.

To specify the conditional variance of y_t, we adopt the vech-diagonal formulation initiated by Bollerslev, Engle and Wooldridge (1988). Thus, each conditional-variance term follows a univariate GARCH(p, q) model given by the following equation

$$\sigma^2_{it} = \omega_i + \sum_{h=1}^{p} \alpha_{ih}\, \sigma^2_{i,t-h} + \sum_{h=1}^{q} \beta_{ih}\, y^2_{i,t-h}, \qquad i = 1, \cdots, K, \tag{2}$$

where ω_i, α_{ih} and β_{ih} are nonnegative, and $\sum_{h=1}^{p} \alpha_{ih} + \sum_{h=1}^{q} \beta_{ih} < 1$, for $i = 1, ..., K$. Note that we may allow (p, q) to vary with i so that (p, q) should be regarded as the generic

[3]Additional parameters would be required to represent the conditional-mean equation in the complete model if the mean is unknown. Under certain conditions, the MLE of the parameters in the conditional-mean equation is asymptotically uncorrelated with the MLE of the parameters of the conditional-variance equation. Under such circumstances, we may treat y_t as pre-filtered observations (see Bera and Higgins (1993) for further discussions). Otherwise, the parameter vector has to be augmented to take account of the parameters in the unknown mean.

order of the univariate GARCH process. Researchers adopting the vech-diagonal form typically assume that the above equation also applies to the conditional-covariance terms in which σ_{it}^2 is replaced by σ_{ijt} and y_{it}^2 is replaced by $y_{it}\, y_{jt}$ for $1 \leq i < j \leq K$. We shall, however, deviate from this approach. Specifically, we shall focus on the conditional-correlation matrix and adopt an autoregressive moving average analogue on this matrix. Thus, we assume that the time-varying conditional-correlation matrix Γ_t is generated from the following recursion

$$\Gamma_t = (1 - \theta_1 - \theta_2)\, \Gamma + \theta_1\, \Gamma_{t-1} + \theta_2\, \Psi_{t-1}, \tag{3}$$

where $\Gamma = \{\rho_{ij}\}$ is a (time-invariant) $K \times K$ positive definite parameter matrix with unit diagonal elements and Ψ_{t-1} is a $K \times K$ matrix whose elements are functions of the lagged observations of y_t.[4] The functional form of Ψ_{t-1} will be specified below. The parameters θ_1 and θ_2 are assumed to be nonnegative with the additional constraint that $\theta_1 + \theta_2 \leq 1$. Thus, Γ_t is a weighted average of Γ, Γ_{t-1} and Ψ_{t-1}. Hence, if Ψ_{t-1} is a well-defined correlation matrix (i.e., positive definite with unit diagonal elements), Γ_t will also be a well-defined correlation matrix.[5]

It can be observed that Ψ_{t-1} is analogous to $y_{i,t-1}^2$ in the univariate GARCH(1, 1) model. However, as Γ_t is a standardised measure, we also require Ψ_{t-1} to depend on the (lagged) standardised residuals ϵ_t. Denoting $\Psi_t = \{\psi_{ijt}\}$, we propose to consider the following specification for Ψ_{t-1}

$$\psi_{ij,t-1} = \frac{\sum_{h=1}^{M} \epsilon_{i,t-h}\, \epsilon_{j,t-h}}{\sqrt{\left(\sum_{h=1}^{M} \epsilon_{i,t-h}^2\right)\left(\sum_{h=1}^{M} \epsilon_{j,t-h}^2\right)}}, \qquad 1 \leq i < j \leq K. \tag{4}$$

Thus, Ψ_{t-1} is the sample correlation matrix of $\{\epsilon_{t-1}, ..., \epsilon_{t-M}\}$. We define E_{t-1} as the $K \times M$ matrix given by $E_{t-1} = (\epsilon_{t-1}, ..., \epsilon_{t-M})$. If B_{t-1} is the $K \times K$ diagonal matrix

[4] For the sake of simplicity and at the risk of being not thorough, we shall describe a correlation matrix as being positive definite. It is not difficult to see that for some statements made in this section, the term "positive definite" should, strictly speaking, be replaced by the term "positive semi-definite".

[5] This statement is subject to the condition that the recursion starts with a well-defined correlation matrix Γ_0. Under such conditions, the diagonal elements of Γ_t are unity and Γ_t remains positive definite.

with the ith diagonal element being $(\sum_{h=1}^{M} \epsilon_{i,t-h}^2)^{1/2}$ for $i = 1, ..., K$, then we have

$$\Psi_{t-1} = B_{t-1}^{-1} E_{t-1} E_{t-1}' B_{t-1}^{-1}. \tag{5}$$

Note that when $M = 1$, Ψ_{t-1} is identically equal to the matrix of unity. Updating the conditional-correlation matrix with respect to the matrix of unity is of course not meaningful. Thus, taking first-order lag for the formulation of Ψ_{t-1} is not sufficient. Indeed, $M \geq K$ is a necessary condition for Ψ_{t-1} to be positive definite. When positive-definiteness is satisfied, Ψ_{t-1} is a well-defined correlation matrix. Thus, the condition $M \geq K$ will be imposed subsequently.

Equation (3) is analogous to the univariate GARCH equation, with the additional restriction that the sum of the coefficients is equal to 1. Indeed, Γ_t involves updating the conditional-correlation matrix with respect to the latest conditional-correlation matrix Γ_{t-1} and a sample estimate of the conditional-correlation matrix based on the recent M standardised residuals. We shall call the model specified by (2), (3) and (5) the varying-correlation MGARCH (VC-MGARCH) model.

Assuming normality, $y_t|\Phi_{t-1} \sim N(0, D_t \Gamma_t D_t)$, so that (ignoring the constant term) the conditional log-likelihood ℓ_t of the observation y_t is given by

$$\begin{aligned} \ell_t &= -\frac{1}{2} \ln |D_t \Gamma_t D_t| - \frac{1}{2} y_t' D_t^{-1} \Gamma_t^{-1} D_t^{-1} y_t & (6) \\ &= -\frac{1}{2} \ln |\Gamma_t| - \frac{1}{2} \sum_{i=1}^{K} \ln \sigma_{it}^2 - \frac{1}{2} y_t' D_t^{-1} \Gamma_t^{-1} D_t^{-1} y_t, & (7) \end{aligned}$$

from which we can obtain the log-likelihood function of the sample as $\ell = \sum_{t=1}^{T} \ell_t$. Here the log-likelihood function is conditional on Γ_0, Ψ_0 and y_0 being fixed. These assumptions have no effects on the asymptotic distribution of the MLE. Denoting $\theta = (\omega_1, \alpha_{11}, .., \alpha_{1p}, \beta_{11}, .., \beta_{1q}, \omega_2, .., \beta_{Kq}, \rho_{12}, .., \rho_{K-1,K}, \theta_1, \theta_2)$ as the parameter vector of the model, the MLE of θ is obtained by maximising ℓ with respect to θ. We shall denote this value by $\hat{\theta}$.

For parameter parsimony, (p, q) is usually taken to be of low order. For $p = q = 1$, the total number of parameters in the VC-MGARCH model is $3K + K(K+1)/2 + 2$. In comparison, an unrestricted BEKK model with order 1 for both the lagged conditional-covariance matrix term and the outer product of the lagged residuals term has $K(K+1)/2 + 2K^2$ parameters. For example, for $K = 2$, 3 and 4, the number of parameters in the VC-MGARCH model is 9, 14 and 20, respectively, while that for the BEKK model is 11, 24 and 42, respectively. The number of parameters in the VC-MGARCH model always exceeds that of the constant-correlation model by 2, due to the parameters θ_1 and θ_2. Indeed the CC-MGARCH model is nested within the VC-MGARCH model under the restrictions $\theta_1 = \theta_2 = 0$.

The conditions $0 \leq \theta_1, \theta_2 \leq 1$ and $\theta_1 + \theta_2 \leq 1$ pose some problems in the optimisation. One way to get around this difficulty is through transformation. For example, we may define $\theta_i = \lambda_i^2 / (1 + \lambda_1^2 + \lambda_2^2)$ for $i = 1, 2$, where λ_1 and λ_2 are unrestricted parameters. The log-likelihood function may be initially optimised with respect to λ_1, λ_2 and other parameters of interest. The optimisation is then shifted to the original vector θ when convergence with respect to λ_1, λ_2 and other parameters has been achieved. This technique is used in the computations reported in this paper.

3 Some Monte Carlo Results

Research on the asymptotic theory of conditional heteroscedasticity models has been lagging behind their empirical applications. Weiss (1986), Pantula (1989), Bollerslev and Wooldridge (1992), Lee and Hansen (1994), Lumsdaine (1996) and Ling and Li (1997b) investigated the asymptotic distribution of the quasi MLE (QMLE) of the univariate ARCH/GARCH models. Sufficient conditions for consistency and asymptotic normality have been established. Recently, Ling and McAleer (2000) examined the asymptotic distribution of a class of vector ARMA-GARCH models. They established conditions for strict stationarity and ergodicity, and proved the consistency and asymptotic normality of the QMLE under some mild moment conditions. While the models considered by Ling and McAleer are quite general, the CC-GARCH framework is adopted and time-varying conditional correlation is not allowed. An extension of the results by Ling and McAleer to the VC-MGARCH model will be interesting. This, however, is beyond the scope of this paper.

An interesting issue for empirical applications concerns the properties of the MLE of the conditional heteroscedasticity models in small and moderate samples. In the univariate case, Engle, Hendry and Trumble (1985) and Lumsdaine (1995) examined the small-sample properties of the MLE of the ARCH and GARCH models. In this section we report some results on the small-sample properties of the MLE of the VC-MGARCH model based on a small-scale Monte Carlo experiment. It is not our intention to provide a comprehensive Monte Carlo study of the MLE. We shall focus our interest on the small-sample bias and mean squared error only. The reliability of the inference concerning the model parameters will not be examined. Our results, however, will provide some preliminary evidence with respect to the small-sample properties of the MLE of the VC-MGARCH model.

We consider bivariate VC-MGARCH models in which the conditional-variance equations are given by

$$\sigma_{it}^2 = \omega_i + \alpha_i\,\sigma_{i,t-1}^2 + \beta_i\,y_{i,t-1}^2, \qquad i = 1, 2, \tag{8}$$

with

$$\rho_t = (1 - \theta_1 - \theta_2)\,\rho + \theta_1\,\rho_{t-1} + \theta_2\,\psi_{t-1}, \tag{9}$$

where ψ_{t-1} is specified as

$$\psi_{t-1} = \frac{\sum_{h=1}^{2} \epsilon_{1,t-h}\,\epsilon_{2,t-h}}{\sqrt{(\sum_{h=1}^{2} \epsilon_{1,t-h}^2)(\sum_{h=1}^{2} \epsilon_{2,t-h}^2)}}. \tag{10}$$

with $\epsilon_{it} = y_{it}\,/\,\sigma_{it}$ for $i = 1,\, 2$.[6]

[6] All computations reported in this paper assume $M = K$ in the definition of Ψ_t.

We consider four experimental setups. The true parameter values of the data generating processes of these experiments, labelled E1 through E4, are given in Tables 1.1 and 1.2. Observations $\{y_t\}$ are generated from these models assuming the errors are normally distributed. We consider $T = 500$, 1000 and 1500. The MLE are calculated for each generated sample. Using Monte Carlo samples of 1000 runs, we estimate the bias and mean squared error (MSE) of the MLE.

E1 and E2 represent models with higher volatility persistence (as measured by $\alpha_i + \beta_i$), while E3 and E4 represent models with lower volatility persistence. The selected values of ρ in the experiments are 0.2 and 0.7. It can be seen from the Monte Carlo results that the biases of the MLE are generally quite small. The bias decreases with the sample size, although in some cases not steadily. Likewise, the same is true for the MSE. Overall, for the sample sizes and models considered, the bias and MSE appear to be small.

In the next section, we illustrate the application of the VC-MGARCH model with some real data sets.

4 Some Illustrative Examples

We examine three sets of financial data, denoted by DS1, DS2 and DS3. DS1 consists of two exchange rate (versus US dollar) series, namely, the Deutschmark (D) and the Japanese Yen (J). These series represent 2131 daily observations from January 1990 through June 1998. DS2 covers the stock market indices of the Hong Kong and the Singapore markets. We use the Hang Seng Index (H) for the Hong Kong market and the SES Index (S) for the Singapore market. There are 1942 daily (closing) prices for each series, covering the period from January 1990 through March 1998. DS3 consists of three sectoral price indices of the Hong Kong stock market. These are the Finance (F), Properties (P) and Utilities (U) sectors. Each series have 2389 daily observations covering the period from January 1991 through August 2000. DS1 was downloaded from the website of the Federal Reserve Bank of New York. DS2 was compiled from various issues of the Stock Exchange of Singapore Journal. Some adjustments were made to account for the differences in the holidays of the two exchanges. DS3 was downloaded from Datastream.

Figures 1 through 3 present the plots of the seven series in the three data sets. In Figure 1 the Japanese Yen (Y) series have been rescaled for easy presentation. This is similarly done for the Hang Seng Index (H) series in Figure 2. We can see that the exchange rates of the Deutschmark and the Japanese Yen generally moved in tandem against the US dollar during the sample period. As expected, the three sectoral indices in the Hong Kong stock market moved quite closely together. This is especially true for the Finance and Properties Indices. In contrast, the Utilities Index was quite sluggish in the mid 90s while the Finance and Properties Indices were undergoing a bull run during this period. It is quite clear from Figure 2 that the national stock markets of Hong Kong and Singapore experienced different phases of bulls and bears. The general impression is that Hong Kong has a more volatile market compared to Singapore.

Table 2 provides a summary of the descriptive statistics of the data. The summary statistics refer to those of the differences of the logarithmic series (expressed in percentage). It can be seen that all differenced logarithmic series exhibit excess kurtosis (compared to the normal distribution) in the unconditional distribution. While the exchange rate data (DS1) demonstrate no evidence of serial correlation, the stock return data (DS2 and DS3) have significant serial correlation as suggested by the Q_1 statistics. The Q_2 statistics show that there is serial correlation in the conditional variance for all data sets and GARCH type of modelling may be required.[7] In the subsequent analysis, we apply autoregressive filters to the differenced logarithmic series and model the fil-

[7] $Q_1(20)$ is the Box-Pierce portmanteau statistic of the differenced logarithmic series based on the autocorrelation coefficients up to order 20. Similarly, $Q_2(20)$ is the portmanteau statistic of the squared differenced logarithmic series.

tered residuals using MGARCH models. The autoregressive filters are estimated using ordinary least squares (OLS).

We fit the CC-MGARCH(1, 1) model to all data sets using Bollerslev's (1990) algorithm. The results are summarised in Panel A of Table 3.[8] It can be seen that the estimates of α, β and ρ are statistically significant at the 5 percent level for all data sets. In comparison, the exchange rate data have the highest intensity of persistence in volatility as measured by $\hat{\alpha}+\hat{\beta}$. With respect to the correlation coefficients, the returns of the national stock markets of Hong Kong and Singapore have the lowest correlation. In contrast, the correlations between the various sectoral indices of the Hong Kong stock market are the highest.

Panel B of Table 4 summarises the estimation results of the VC-MGARCH(1, 1) models for the three data sets.[9] Again, it can be seen that the estimates of α, β and ρ are statistically significant at the 5 percent level for all data sets. In addition, all estimates of θ_1 and θ_2 are statistically significant at the 5 percent level, indicating that the correlations are significantly time varying. We note that the intensity of the volatility persistence remains approximately unchanged compared to the CC-MGARCH models. Indeed, incorporating time-varying correlations does not have much effect on the estimates of α and β. The estimates of ρ in the varying-correlation models are all larger than the corresponding estimates of ρ in the constant-correlation models. This, however, does not imply that the correlations are on average higher in the varying-correlation model. It should be noted that the time-invariant component of the conditional correlation coefficient in the VC-MGARCH model is $(1-\theta_1-\theta_2)\,\rho$. A comparison of the correlation correlations in the two models will be provided below.

As the CC-MGARCH model is nested within the VC-MGARCH model, ignoring

[8]All parameter estimates reported in Table 3 are the MLE assuming normality. The standard errors are calculated using the robustified quasi-MLE (QMLE) of the covariance matrix of the parameters.

[9]The results are based on the assumption $M=K$. We have re-estimated the models with $M=K+1$. The results are qualitatively similar.

the extension would induce model misspecification. We now proceed to examine the model diagnostics of the constant-correlation and varying-correlation models. Table 4 summarises a battery of diagnostic tests for the fitted models. The constant-correlation assumption is tested using a Lagrange multiplier test (LMC) based on the estimates of the CC-MGARCH(1, 1) model and the likelihood ratio test (LR) based on the estimates of the VC-MGARCH(1, 1) model. LMC is the Lagrange multiplier test suggested by Tse (2000) for the assumption of constant correlation in a MGARCH model. It is asymptotically distributed as χ^2_R, where $R = K(K-1)/2$, under the null.[10] From Panel A of Table 4 we can see that the constant-correlation assumption is rejected for all data sets at the 5 percent level of significance. In Panel B of the table we present the likelihood ratio statistic LR, which tests for the restriction $H_0 : \theta_1 = \theta_2 = 0$. It can be seen that the constant-correlation assumption is rejected for all data sets at any conventional level of significance.

To further test for misspecification in the MGARCH models we adopt the regression-based diagnostics suggested by Wooldridge (1990, 1991). The methodology developed by Wooldridge applies to a wide class of possible misspecification. Here we focus on the problem of misspecification in the conditional heteroscedasticity. As shown by Wooldridge, the suggested tests are robust to departure from distributional assumptions that are not being tested. Since our main concern is misspecification in the conditional variance, we use the squared standardised residuals and the cross products of the squared standardised residuals as the indicators.

We first consider tests based on the squared standardised residuals. We denote $\hat{\epsilon}_{it}$ as the estimate of the standardised residual ϵ_{it} and $\hat{\sigma}^2_{it}$ as the estimated conditional variance of y_{it}. We define $\hat{\lambda}_{it} = (\hat{\epsilon}^2_{i,t-1}, \hat{\epsilon}^2_{i,t-2}, ..., \hat{\epsilon}^2_{i,t-Q})'$ as the vector of indicator variables, and $\bigtriangledown_\theta \hat{\sigma}^2_{it}$ as the gradient vector of σ^2_{it} with respect to θ evaluated at $\hat{\theta}$. Denoting $(\bigtriangledown_\theta \hat{\sigma}^2_{it})/\hat{\sigma}^2_{it}$

[10] Tse (2000) provided some Monte Carlo results for the finite-sample distributions of the LMC test. In particular, he showed that the test is robust against nonnormality in moderate samples.

as $\bigtriangledown_\theta \tilde{\sigma}^2_{it}$, we regress each element of $\hat{\lambda}_{it}$ on $\bigtriangledown_\theta \tilde{\sigma}^2_{it}$ to obtain the Q-element residuals $\hat{r}_{it}$. Finally, we regress unity on the vector of Q regressors $\hat{\phi}_{it}\hat{r}_{it}$, where $\hat{\phi}_{it} = \hat{\epsilon}^2_{it} - 1$. We calculate $W_{ii}(Q) = T - SSR$, where SSR is the sum of squares of the residuals of the last regression. If there is no model misspecification, $W_{ii}(Q)$ is asymptotically distributed as χ^2_Q.

The above diagnostic statistic can be calculated for the cross products of the standardised residuals from different equations. Specifically, we define $\hat{\lambda}_{ijt} = (\hat{\epsilon}_{i,t-1}\hat{\epsilon}_{j,t-1}, \hat{\epsilon}_{i,t-2}\hat{\epsilon}_{j,t-2}, ..., \hat{\epsilon}_{i,t-Q}\hat{\epsilon}_{j,t-Q})'$ and $\bigtriangledown_\theta \tilde{\phi}_{ijt}$ as the gradient vector of $\phi_{ijt} = \epsilon_{it}\epsilon_{jt} - \rho_{ijt}$ with respect to θ evaluated at $\hat{\theta}$. We regress each element of $\hat{\lambda}_{ijt}$ on $\bigtriangledown_\theta \tilde{\phi}_{ijt}$ to obtain the Q-element residuals $\hat{r}_{ijt}$, and then regress unity on the Q regressors $\hat{\phi}_{ijt}\hat{r}_{ijt}$, where $\hat{\phi}_{ijt} = \hat{\epsilon}_{it}\hat{\epsilon}_{jt} - \hat{\rho}_{ijt}$. We define the test statistic as $W_{ij}(Q) = T - SSR$ for $1 \leq i < j \leq K$, which is asymptotically distributed as χ^2_Q when there is no misspecification.[11]

We apply the W statistics to the MGARCH models with $Q = 4$. From the results in Table 4 we can see that both the CC-MGARCH and the VC-MGARCH models pass the diagnostic checks of the W statistics. Indeed, the W statistics of the two models are quite similar. As the constant-correlation assumption is not supported by the LMC and the LR statistics, one might expect the W statistics of the CC-MGARCH model to be significant. The fact that this is not the case may be an indication of loss in power when the test has no specific alternative.

Table 5 reports the summary statistics of the in-sample conditional variances, covariances and correlations of the VC-MGARCH(1, 1) models. It can be seen that the sample means of the conditional correlations are remarkably close to the MLE of the (constant) correlation coefficients of the CC-MGARCH(1, 1) models reported in Panel A of Table 3. Nonetheless, the range of the conditional correlations is quite large in some cases. For example, for the exchange rate data (DS1) the range of $\{\hat{\rho}_{DJt}\}$ is 0.4057,

[11] In this paper the gradient vectors required for the computation of the W statistics are calculated using numerical differentiation.

with a mean of 0.5226. For the cross-national stock market returns (DS2), the range of the conditional correlations is 0.6962. Indeed, the conditional correlation was once below zero. In contrast, the sectoral market indices (DS3) represent the case where the conditional correlations vary within the smallest range.

In Table 6 we present the summary statistics of the standardised residuals of the CC-MGARCH and VC-MGARCH models. It can be seen that the standardised kurtosis and the Q_2 statistics have dropped significantly compared to those of the raw data in Table 2.[12]

To obtain a clearer picture of the time history of the conditional correlations, we plot the time paths of the conditional correlations based on the VC-MGARCH(1, 1) models. The plots are presented in Figures 4 through 8, in which both the conditional correlations and the constant correlations (given by the dotted lines) are provided.

Figure 4 presents the correlations between of the Deutschmark and the Japanese Yen. Large, there were two subperiods when the conditional correlations of these two currencies were mostly above the average (constant) level, namely, October 1991 to June 1993 and March 1994 to October 1996. From October 1996 to June 1998, the conditional correlations were mostly below the average level.

Figure 5 presents an interesting case in which we can see that the conditional correlations between the Hong Kong and the Singapore stock markets were experiencing an upward shift. From 1994 onwards, the conditional correlations were mostly above the average level, whereas the reverse was true before 1994. This finding has important implications for the international diversification of equity portfolios. While the increasing conditional correlations means that the two national markets were becoming more closely integrated, it also implies that there is reducing benefits for international diversi-

[12]We note that the Q_1 and Q_2 statistics are presented here for completeness. As pointed out by Li and Mak (1994) and Ling and Li (1997a) these statistics are not distributed as χ^2 under the null of no misspecification. While some of the Q_1 statistics appear to be large, we report that none of the lag autocorrelation coefficients is larger than 0.08 in absolute value.

fication. Using moving windows of unconditional correlations, Longin and Solnik (1995) showed that there was evidence of increasing correlations between international stock markets in 1960 – 1990.[13] Our similar finding for the Hong Kong and the Singapore markets is commensurate with the increasing importance of intra-Asian business in the 90s.[14]

Figures 6 through 8 show that the pairwise correlations between the three sectors in the Hong Kong stock market are quite similar. Broadly speaking, the conditional correlations were above average in the subperiods of 1993 to 1994 and mid 1997 to mid 1999. These two subperiods coincide with the time when the Hong Kong stock market was experiencing a downturn. In contrast, during the subperiods of the bull runs from 1995 to mid 1997 and post mid 1999, the conditional correlations were below average. At the risk of over-simplification, this casual observation agrees with the hypothesis that contagion is stronger for negative returns than for positive returns.[15]

We shall end this section by stating that it is not our intention to claim that the VC-MGARCH models as presented here represent the best MGARCH models for the data. Other MGARCH models could also provide the conditional-correlation structure. The VC-MGARCH model, however, does provide a viable alternative that is relatively easy to estimate. As the examples have illustrated, modelling correlations as a time-varying structure provides some interesting results that are not obtainable from constant-correlation models.

[13]For an update of the correlations of international stock markets in the recent crisis period, see Longin and Solnik (2000).

[14]In the second half of the 90s, many companies with business activities in Hong Kong were listed on the Singapore exchange. The most notable example is the listing of the five companies in the Jardine group.

[15]Bae, Karolyi and Stulz (2000) examined the financial contagion among Asian and Latin American economies using a multinomial logit model. They reported that the evidence of contagion being stronger for negative returns than for positive returns is mixed.

5 Conclusions

In this paper we propose a new MGARCH model with time-varying correlations. We assume a vech-diagonal structure in which each conditional-variance term follows a univariate GARCH formulation. The remaining task is to specify the conditional-correlation structure. We apply an autoregressive moving average type of analogue to the conditional-correlation matrix. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive definite during the optimisation.

We report some Monte Carlo results on the finite-sample distributions of the MLE of the varying-correlation MGARCH model. It is found that the bias and MSE of the MLE are small for sample sizes of 500 or above. The new model is applied to three data sets, namely, the exchange rate data, the national stock market data and the sectoral price data. The new model is found to pass the model diagnostics satisfactorily, while the constant-correlation MGARCH model is found to be inadequate. Extending the constant-correlation model to allow for time-varying correlations provides some interesting empirical results. In particular, the estimated conditional-correlation path provides an interesting time history that would not be available in a constant-correlation model.

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Table 1.1: Estimated Bias and MSE of the MLE of Bivariate VC-MGARCH(1, 1) Models

	Experiment: E1				Experiment: E2			
Parameters	True Value	Sample Size	Bias	MSE	True Value	Sample Size	Bias	MSE
ω_1	0.4	500	0.0907	0.0687	0.4	500	0.1166	0.0993
		1000	0.0363	0.0194		1000	0.0487	0.0273
		1500	0.0266	0.0116		1500	0.0328	0.0157
α_1	0.8	500	-0.0135	0.0033	0.8	500	-0.0183	0.0043
		1000	-0.0056	0.0012		1000	-0.0070	0.0016
		1500	-0.0046	0.0008		1500	-0.0050	0.0010
β_1	0.15	500	-0.0007	0.0013	0.15	500	0.0005	0.0017
		1000	-0.0005	0.0006		1000	-0.0010	0.0008
		1500	0.0005	0.0004		1500	-0.0004	0.0005
ω_2	0.2	500	0.0313	0.0095	0.2	500	0.0364	0.0118
		1000	0.0132	0.0031		1000	0.0123	0.0040
		1500	0.0076	0.0017		1500	0.0089	0.0024
α_2	0.7	500	-0.0170	0.0062	0.7	500	-0.0230	0.0079
		1000	-0.0094	0.0023		1000	-0.0075	0.0031
		1500	-0.0043	0.0015		1500	-0.0047	0.0018
β_2	0.2	500	-0.0018	0.0023	0.2	500	0.0011	0.0030
		1000	0.0013	0.0010		1000	-0.0003	0.0013
		1500	-0.0005	0.0008		1500	-0.0005	0.0009
ρ	0.7	500	-0.0011	0.0028	0.2	500	-0.0008	0.0077
		1000	-0.0027	0.0084		1000	-0.0012	0.0034
		1500	0.0010	0.0009		1500	0.0001	0.0022
θ_1	0.8	500	-0.0018	0.0014	0.8	500	-0.0358	0.0181
		1000	-0.0090	0.0023		1000	-0.0194	0.0065
		1500	0.0011	0.0004		1500	-0.0111	0.0029
θ_2	0.1	500	-0.0006	0.0008	0.1	500	0.0043	0.0016
		1000	-0.0064	0.0014		1000	0.0023	0.0008
		1500	0.0005	0.0003		1500	0.0011	0.0004

Notes: See equations (8), (9) and (10) for the data generating processes.

Table 1.2: Estimated Bias and MSE of the MLE of Bivariate VC-MGARCH(1, 1) Models

	Experiment: E3				Experiment: E4			
Parameters	True Value	Sample Size	Bias	MSE	True Value	Sample Size	Bias	MSE
ω_1	0.4	500	0.0293	0.0148	0.4	500	0.0315	0.0188
		1000	0.0137	0.0062		1000	0.0114	0.0088
		1500	0.0090	0.0037		1500	0.0051	0.0052
α_1	0.5	500	-0.0131	0.0092	0.5	500	-0.0181	0.0109
		1000	-0.0077	0.0036		1000	-0.0067	0.0051
		1500	-0.0053	0.0022		1500	-0.0025	0.0031
β_1	0.3	500	-0.0050	0.0037	0.3	500	-0.0032	0.0042
		1000	-0.0007	0.0017		1000	-0.0019	0.0021
		1500	0.0003	0.0011		1500	-0.0022	0.0015
ω_2	0.2	500	0.0197	0.0067	0.2	500	0.0219	0.0081
		1000	0.0089	0.0026		1000	0.0109	0.0032
		1500	0.0054	0.0016		1500	0.0089	0.0024
α_2	0.5	500	-0.0291	0.0216	0.5	500	-0.0352	0.0268
		1000	-0.0125	0.0090		1000	-0.0188	0.0110
		1500	-0.0083	0.0054		1500	-0.0089	0.0074
β_2	0.2	500	-0.0028	0.0030	0.2	500	-0.0008	0.0034
		1000	-0.0017	0.0014		1000	0.0016	0.0017
		1500	0.0001	0.0009		1500	0.0021	0.0013
ρ	0.7	500	0.0011	0.0064	0.2	500	0.0002	0.0139
		1000	0.0014	0.0025		1000	0.0007	0.0068
		1500	-0.0003	0.0015		1500	0.0001	0.0041
θ_1	0.6	500	-0.0026	0.0034	0.6	500	-0.0137	0.0055
		1000	-0.0015	0.0014		1000	-0.0035	0.0023
		1500	-0.0011	0.0010		1500	-0.0058	0.0016
θ_2	0.3	500	-0.0048	0.0019	0.3	500	0.0035	0.0023
		1000	-0.0019	0.0009		1000	-0.0009	0.0010
		1500	-0.0006	0.0006		1500	0.0019	0.0007

Notes: See equations (8), (9) and (10) for the data generating processes.

Table 2: Summary Statistics of the Differenced Logarithmic Series of Various Data Sets

Variable (Code)	Mean	Std Dev	Minimum	Maximum	Std Skewness	Std Kurtosis	$Q_1(20)$	$Q_2(20)$	No of Obs
Panel A: Forex Market Data (DS1), 90/1 – 98/6									
Deutschmark (D)	0.0025	0.6746	–2.8963	3.1030	0.3715	16.6655	21.9957	464.2324	2131
Japanese Yen (J)	–0.0023	0.6750	–4.5228	3.2269	–9.5384	33.4012	27.6373	112.5759	2131
Panel B: National Stock Market Data (DS2), 90/1 – 98/3									
Hong Kong (H)	0.0721	1.7093	–14.7347	17.2471	–0.0533	120.2852	36.6618	759.7676	1942
Singapore (S)	–0.0010	1.0768	–7.7236	8.7867	–1.6643	95.7543	116.8250	846.2872	1942
Panel C: Hang Seng Sectoral Indices Data (DS3), 91/1 – 00/8									
Finance (F)	0.1061	1.8308	–17.6894	18.0011	–2.8257	108.1039	57.4268	962.9816	2389
Properties (P)	0.0562	2.1723	–14.2739	20.6846	5.4033	86.7263	91.6266	958.5099	2389
Utilities (U)	0.0626	1.7017	–14.4889	16.6176	6.5861	91.3024	44.7659	595.3206	2389

Notes: $Q_1(20)$ is the Box-Pierce portmanteau statistic of the differenced logarithmic series based on the autocorrelation coefficients up to order 20. Similarly, $Q_2(20)$ is the portmanteau statistic of the squared differenced logarithmic series.

Table 3: Estimation Results of Constant-Correlation and Varying-Correlation Models

Data	K	Variable	ω	α	β	θ_1	θ_2	Correlations
Panel A: CC-MGARCH(1, 1) Model								
DS1	2	D	0.0071	0.9349	0.0487	-	-	$\rho_{DJ} = 0.5234$
			(0.0035)	(0.0156)	(0.0099)			(0.0171)
		J	0.0102	0.9315	0.0479			
			(0.0072)	(0.0310)	(0.0188)			
DS2	2	H	0.2141	0.7776	0.1337	-	-	$\rho_{HS} = 0.3152$
			(0.0870)	(0.0595)	(0.0337)			(0.0253)
		S	0.0920	0.7229	0.1908			
			(0.0290)	(0.0633)	(0.0486)			
DS3	3	F	0.1660	0.8401	0.1024	-	-	$\rho_{FP} = 0.7597$
			(0.0679)	(0.0482)	(0.0302)			(0.0118)
		P	0.1418	0.8661	0.0956			$\rho_{FU} = 0.6811$
			(0.0389)	(0.0230)	(0.0160)			(0.0148)
		U	0.2126	0.7861	0.1287			$\rho_{PU} = 0.7103$
			(0.0471)	(0.0320)	(0.0212)			(0.0153)
Panel B: VC-MGARCH(1, 1) Model								
DS1	2	D	0.0056	0.9377	0.0501	0.9726	0.0146	$\rho_{DJ} = 0.6298$
			(0.0031)	(0.0145)	(0.0100)	(0.0071)	(0.0042)	(0.0462)
		J	0.0104	0.9316	0.0469			
			(0.0069)	(0.0295)	(0.0176)			
DS2	2	H	0.1761	0.8042	0.1222	0.9598	0.0285	$\rho_{HS} = 0.4839$
			(0.0728)	(0.0518)	(0.0302)	(0.0146)	(0.0101)	(0.0728)
		S	0.0888	0.7231	0.1899			
			(0.0258)	(0.0583)	(0.0462)			
DS3	3	F	0.1203	0.8630	0.0961	0.9744	0.0130	$\rho_{FP} = 0.8201$
			(0.0459)	(0.0350)	(0.0242)	(0.0064)	(0.0030)	(0.0223)
		P	0.1158	0.8703	0.0991			$\rho_{FU} = 0.7442$
			(0.0337)	(0.0213)	(0.0160)			(0.0284)
		U	0.1620	0.8143	0.1233			$\rho_{PU} = 0.7867$
			(0.0413)	(0.0291)	(0.0205)			(0.0254)

Notes: The parameter estimates are the MLE assuming normality. Figures in parentheses are standard errors. They are calculated using the robustified quasi-MLE (QMLE) of the covariance matrix of the parameters.

Table 4: Diagnostic Checks for Constant-Correlation and Varying-Correlation Models

Tests	Forex Market D-J	National Stock Markets H-S	Hang Seng Sectoral Indices F-P-U
Panel A: CC-MGARCH(1, 1) Model			
LMC	4.5663*	9.7552*	8.3815*
$W_{11}(4)$	5.905	3.7357	7.9761
$W_{22}(4)$	3.957	1.2229	4.2384
$W_{33}(4)$			3.7496
$W_{12}(4)$	7.886	4.9076	4.4561
$W_{13}(4)$			7.9565
$W_{23}(4)$			6.4020
Panel B: VC-MGARCH(1, 1) Model			
LR	43.3562*	38.2371*	79.7401*
$W_{11}(4)$	5.874	3.6733	8.6249
$W_{22}(4)$	3.955	1.2654	4.1959
$W_{33}(4)$			4.0204
$W_{12}(4)$	7.304	2.002	4.6163
$W_{13}(4)$			8.4929
$W_{23}(4)$			5.6295

Notes: $W_{ij}(4)$, for $1 \leq i \leq j \leq 3$ are Wooldridge's (1991) residual-based diagnostic statistics computed from the standardised residuals of variables i and j based on indicator variables up to 4 lags. The suffixes are according to the order of the coded variables. Thus, $W_{13}(4)$ in the system (F-P-U) is $W_{FU}(4)$. If there is no model misspecification, $W_{ij}(4)$ is asymptotically distributed as χ^2_4. LMC is the Lagrange multiplier test for constant correlation due to Tse (2000). It is approximately distributed as χ^2_1 for a bivariate system and χ^2_3 for a trivariate system when the correlations are time invariant. LR is the likelihood ratio statistic for $H_0 : \theta_1 = \theta_2 = 0$. Asterisks denote that the test is significant at the 5 percent level.

Table 5: Summary Statistics of the Conditional Variance, Covariance and Correlation of the Estimated VC-MGARCH(1, 1) Models

Data	System	Statistic	Mean	Std Dev	Minimum	Maximum
DS1	D-J	σ_D^2	0.4574	0.2455	0.1487	1.5607
		σ_J^2	0.4545	0.1864	0.2123	1.9258
		σ_{DJ}	0.2357	0.1109	0.0665	0.7847
		ρ_{DJ}	0.5226	0.0895	0.3020	0.7077
DS2	H-S	σ_H^2	2.7240	3.7569	1.0161	71.0641
		σ_S^2	1.0868	1.6044	0.3443	29.1311
		σ_{HS}	0.5586	1.0961	-0.0404	13.8341
		ρ_{HS}	0.2851	0.1596	-0.0606	0.6356
DS3	F-P-U	σ_F^2	3.2100	3.6028	1.0877	66.3576
		σ_P^2	4.4317	5.2476	1.1165	64.2797
		σ_U^2	2.7971	3.1568	0.9907	61.7953
		σ_{FP}	2.8350	3.1904	0.8517	48.0168
		σ_{FU}	2.0321	2.3842	0.6058	45.6830
		σ_{PU}	2.5052	2.9036	0.6342	47.2813
		ρ_{FP}	0.7547	0.0565	0.5318	0.8529
		ρ_{FU}	0.6738	0.0663	0.4877	0.8045
		ρ_{PU}	0.7135	0.0799	0.4776	0.8347

Notes: σ_i^2 and σ_{ij} are the conditional variance and covariance terms, respectively, and ρ_{ij} is the conditional correlation.

Table 6: Summary Statistics of Standardised Residuals for Constant-Correlation and Varying Correlation Models

Variable (Code)	Mean	Std Dev	Minimum	Maximum	Std Skewness	Std Kurtosis	$Q_1(20)$	$Q_2(20)$	No of Obs
Panel A: CC-MGARCH(1, 1) Model									
Forex Market Data (DS1)									
Deutschmark (D)	0.0021	0.9994	–4.9721	4.1500	-1.3947	10.9156	21.4962	17.5600	2131
Japanese Yen (J)	0.0102	0.9984	–5.9169	4.1616	–11.3437	28.9282	23.9003	10.8972	2131
National Stock Market Data (DS2)									
Hong Kong (H)	0.0454	0.9990	–8.1404	4.8727	–9.6894	44.7941	32.7361	7.6177	1942
Singapore (S)	–0.0049	1.0010	–6.3663	5.7398	–0.7097	33.4642	29.5697	10.4003	1942
Hang Seng Sectoral Indices Data (DS3)									
Finance (F)	0.0647	0.9979	–6.2040	4.3923	–3.1430	21.6766	32.1809	17.9032	2389
Properties (P)	0.0270	0.9993	–6.9085	4.4455	–3.9588	22.7340	39.0274	20.1223	2389
Utilities (U)	0.0255	1.0001	–7.1906	4.5031	–4.3233	28.6097	28.9502	9.4468	2389
Panel B: VC-MGARCH(1, 1) Model									
Forex Market Data (DS1)									
Deutschmark (D)	0.0017	0.9975	–5.1912	4.1174	–1.6249	11.2586	21.6819	16.6230	2131
Japanese Yen (J)	0.0103	1.0016	–5.9272	4.1733	–11.3152	28.8989	23.9186	10.9223	2131
National Stock Market Data (DS2)									
Hong Kong (H)	0.0460	1.0024	–8.3075	4.6570	–10.1355	46.3682	32.3344	7.9557	1942
Singapore (S)	–0.0050	1.0100	–6.4123	5.7884	–0.6696	33.3612	29.6305	10.3820	1942
Hang Seng Sectoral Indices Data (DS3)									
Finance (F)	0.0661	1.0006	–6.3670	4.4115	–3.1965	22.2290	31.6209	20.8353	2389
Properties (P)	0.0279	1.0044	–7.1636	4.3127	–4.2219	23.5032	39.1166	16.0551	2389
Utilities (U)	0.0256	0.9986	–7.4710	4.4468	–4.7024	30.1182	28.5384	8.1040	2389

Notes: $Q_1(20)$ is the Box-Pierce portmanteau statistic of the standardised residuals based on the autocorrelation coefficients up to order 20. Similarly, $Q_2(20)$ is the portmanteau statistic of the standardised residuals.

Figure 1: Exchange Rates of Deutschmark and Japanese Yen

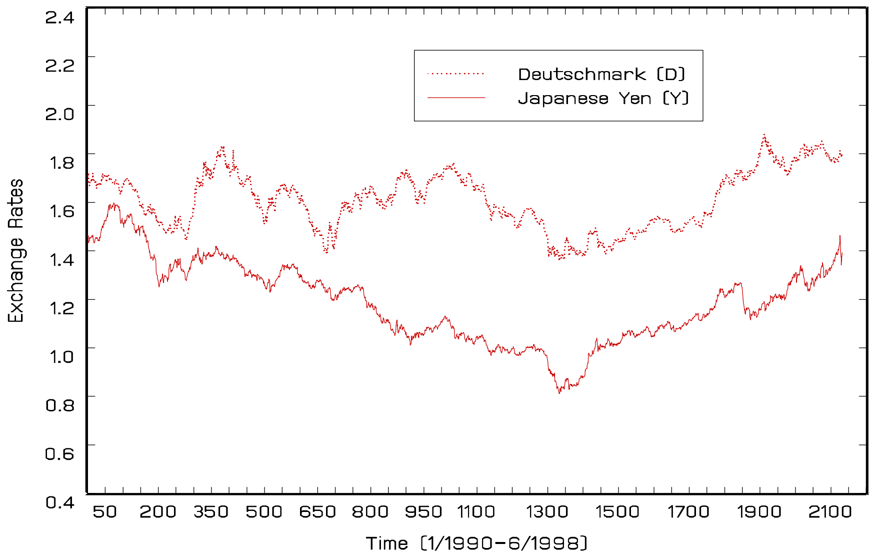

Figure 2: National Stock Market indices

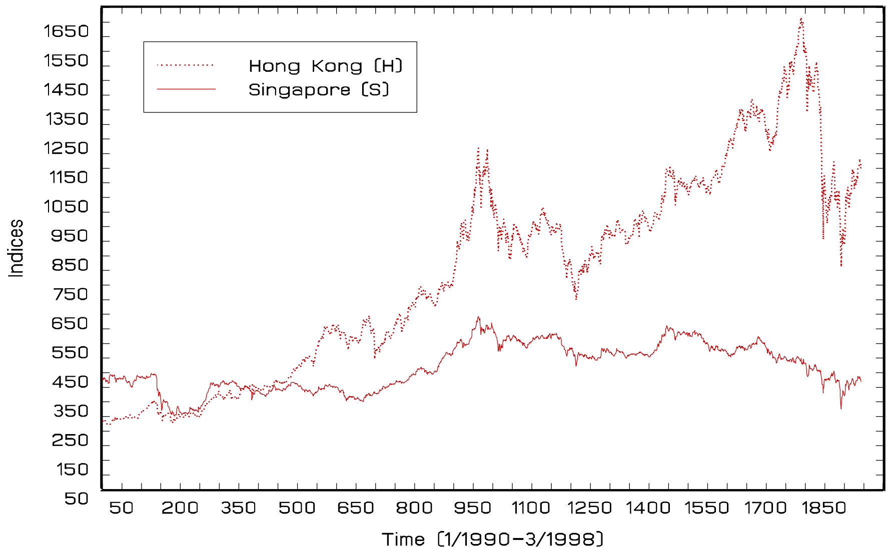

Figure 3: Hang Seng Sectoral Indices

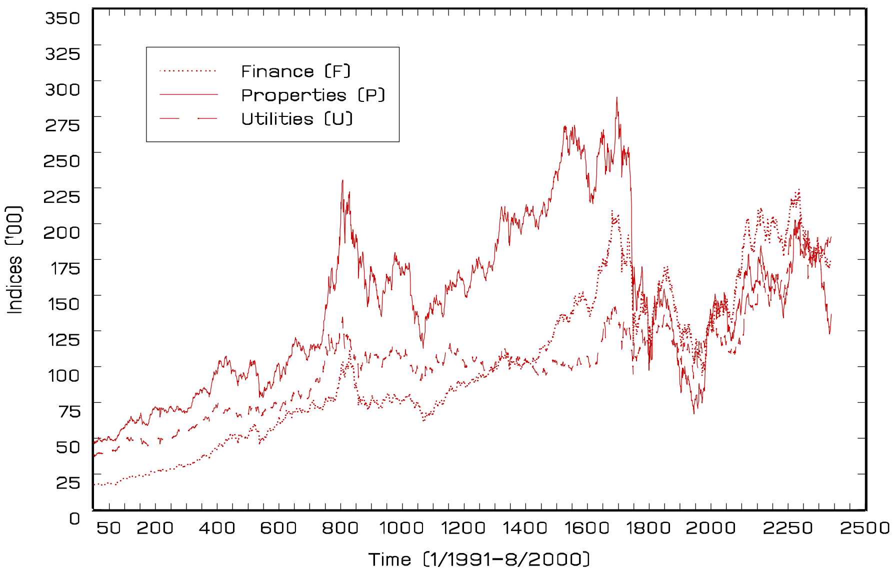

Figure 4: Conditional Correlation Coefficients of (D, J)

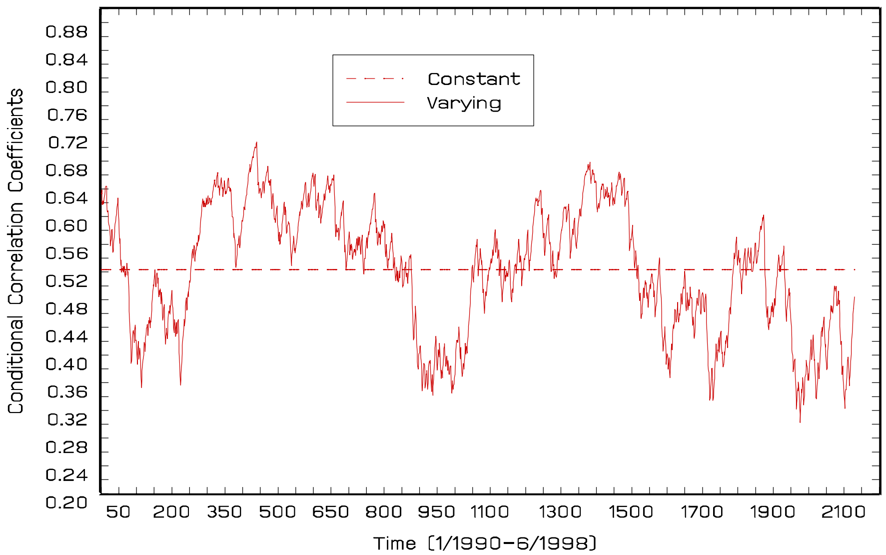

Figure 5: Conditional Correlation Coefficients of (H, S)

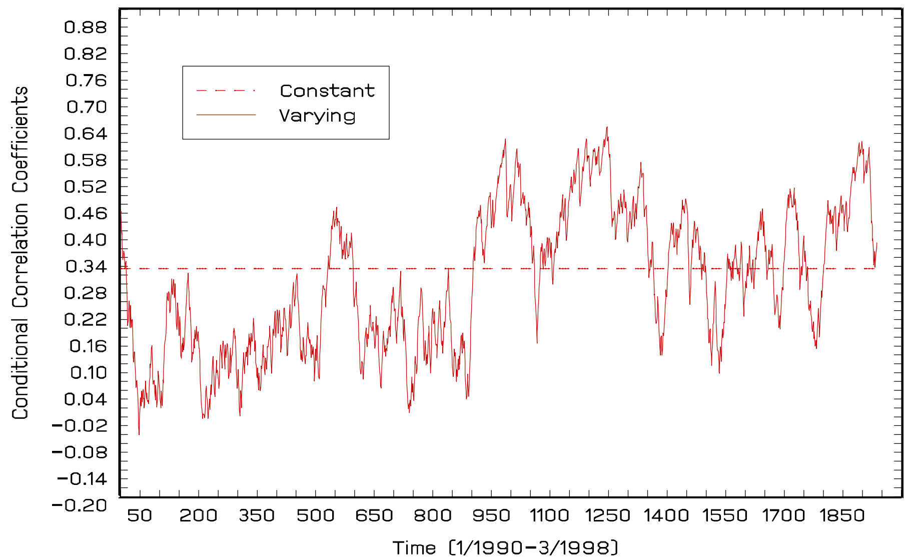

Figure 6: Conditional Correlation Coefficients of (F, P)

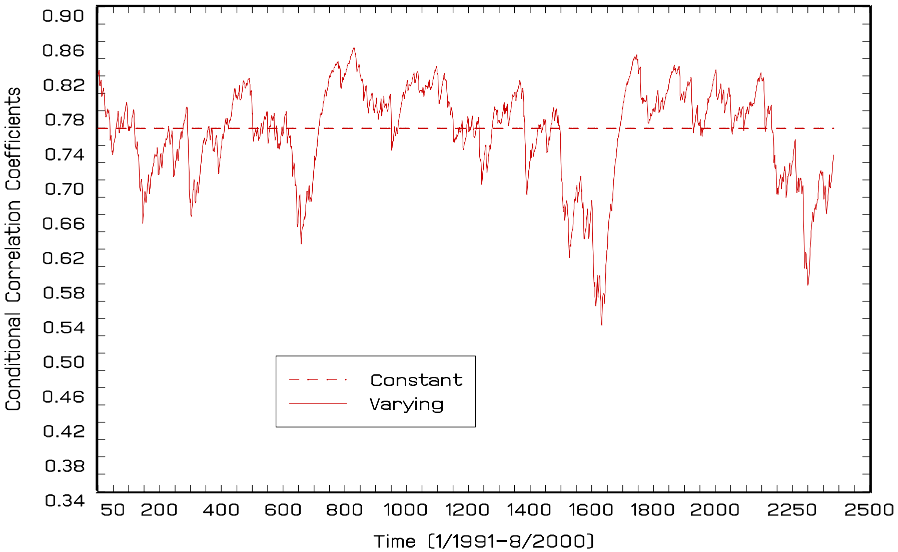

Figure 7: Conditional Correlation Coefficients of (F, U)

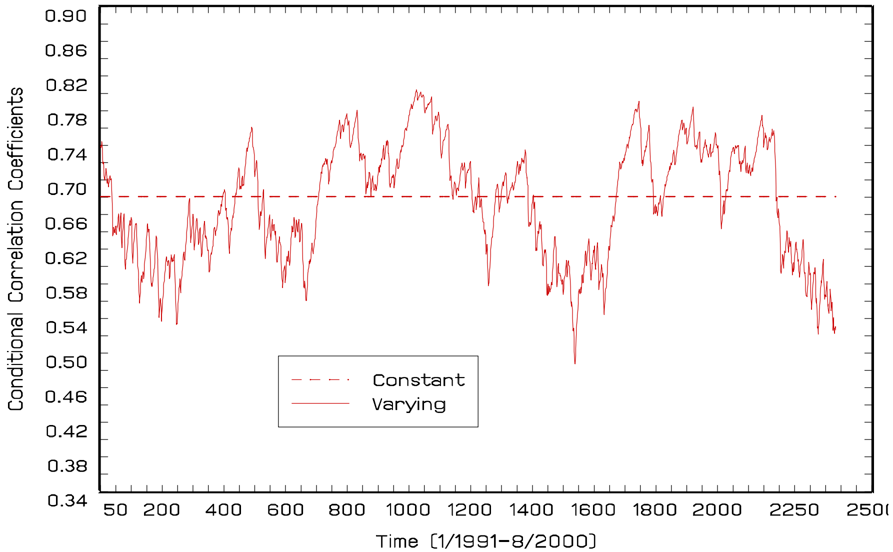

Figure 8: Conditional Correlation Coefficients of (P, U)