Limited Information Bayesian Analysis of a Simultaneous Equation with an Autocorrelated Error Term and its Application to the U.S. Gasoline Market

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revised July 2004

Abstract

Using Markov Chain Monte Carlo algorithms within the limited information Bayesian framework, we estimate the parameters of the structural equation of interest and test weak exogeneity in a simultaneous equation model with white noise as well as autocorrelated error terms. A numerical example and an estimation of the supply and demand equations of the U.S. gasoline market show that if we ignore autocorrelation we obtain unreasonable posterior distributions of the parameters of interest. Also we find that the hypothesis of the asymmetric effect of the changes in oil price on the changes in gasoline price is rejected. Oil inventory has a significant negative effect on the gasoline price.

Keywords: *limited information Bayesian estimation, exogeneity, identifying restrictions, MCMC algorithms, U.S. gasoline market*

JEL Classification Numbers: *C11, C30, C32*

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1 Introduction

There have been many studies on Bayesian limited information estimation of the parameters of the simultaneous equations model (SEM). Under diffuse prior, Dreze (1976) showed that posterior distribution is in poly-t family in some cases. Tsurumi (1985) also derived the exact posterior distribution for the structural parameters. Recently, Kleibergen and Van Dijk (1998) developed a framework for construction of prior probability density functions for the analysis of SEM. The use of the Jeffreys prior in SEM was examined by Chao and Phillips (1998, 2000). Zellner (1998) suggests a finite sample Bayesian method of moments.

Computational difficulties in obtaining the posterior distribution of the structural parameters have long hampered the use of the limited information Bayesian estimation procedures. Zellner et al. (1988) proposed to use Monte Carlo integration method. With the recent advent of Markov Chain Monte Carlo (MCMC) algorithms, the estimation and testing procedures in the limited information Bayesian (LIB) analysis has become practical. Gao and Lahiri (2001) focused on the weak instrument in the limited information analysis of the simultaneous equation and used simulation methods to examine the approaches of Chao and Phillips (1998), Kleibergen and Van Dijk (1998), Zellner (1998) and some non-Bayesian methods.

In this paper, we propose Markov Chain Monte Carlo (MCMC) algorithms to estimate not only the parameters of the structural equation of interest but also a parameter for testing a weak exogeneity of the right hand side endogenous variables. Also, we suggest a rank condition of identifiability. MCMC algorithms are developed first assuming that the error terms are white noise, and then assuming that they are autocorrelated. One of our primary objectives is to make Bayesian inference of the SEM with autocorrelated errors.

We estimate the demand elasticity and income elasticity of the gasoline price using the SEM with an autocorrelated error term. There has been a considerable interest in the analysis of the gasoline market in the last thirty years. Ramsey (1975), Dahl and Sterner (1991) among others estimate the gasoline demand and income elasticities using the aggregate data. Espey (1998) provides an extensive overview of this literature. Studies by Schmalensee and Stoker (1999), Yatchew and No (2001), and Nicol (2003) use the household data to estimate demand and income elasticities.

In the analysis of demand equation, we examine the effect of oil inventory on gasoline prices and allow the asymmetric effect of oil price changes. The relation between the gasoline prices and oil prices has been analyzed by many authors as well. Borenstein *et al.* (1997), Johnson (2000), Galeotti *et al.* (2003) present evidence of the asymmetric response of gasoline prices to changes in crude oil prices. Kaufmann and Laskowski (2004) do not find asymmetries in the gasoline price if inventories and capacity utilization rates are introduced in the model. Pindyck (2001) examines how oil inventories together with oil prices influence the gasoline price. The effect of anticipated and unanticipated changes in oil inventories and prices was analyzed by Radchenko (2004).

We find that gasoline is price inelastic but it is income elastic. We estimate the oil inventory elasticity for gasoline and show that oil inventory has a significant negative effect on gasoline price. In the analysis of asymmetric response of gasoline price to oil price increases and decreases, we do no find evidence of asymmetric response.

The structure of the paper is as follows. In Section 2 we explain the SEM estimation when the error terms are white noise and present an illustrative example. The model with an autocorrelated error term is discussed in Section 3. We examine the U.S. gasoline market using the procedures developed in Section 4. Concluding remarks are in Section 5.

2 Estimation of Parameters When the Error Terms are White Noise

2.1 Likelihood function of the structural equation of interest

The simultaneous equations system (SEM) is given by

$$Y\Gamma = XB + U \tag{1}$$

where $Y = [y_1, ..., y_n]$, $U = [u_1, ..., u_n]$; y_i, and u_i are $T \times 1$ vectors and Y is a $T \times n$ matrix of endogenous variables; X is a $T \times k$ matrix of exogenous variables; U is a $T \times n$ matrix of error terms which are assumed to follow a multivariate white noise process. Γ is an $n \times n$ nonsingular matrix of parameters and B is a $k \times n$ matrix of parameters. The structural equation of interest is given by

$$y_1 = Y_1\gamma_1 + X_1\beta_1 + u_1. \tag{2}$$

where y_1 and Y_1 are, respectively, $T \times 1$ and $T \times m_1$ endogenous variables and γ_1 and β_1 are, respectively, $m_1 \times 1$ and $k_1 \times 1$ structural coefficients.

As shown in Appendix A we derive the likelihood function

$$\begin{aligned} &L(\gamma_1, \beta_1, \Pi_2^*, \sigma_1^2, \rho^2, \theta, \Omega_{22} | data, \Pi_{21}^* = 0) \propto \\ &\quad \sigma_1^{-n}(1-\rho^2)^{-\frac{n}{2}} |\Omega_{22}|^{-\frac{n}{2}} etr\left(-\frac{1}{2}(Y_1 - X\Pi_2^*)'(Y_1 - X\Pi_2^*)\Omega_{22}^{-1}\right) \\ &\quad exp\left(-\frac{1}{2\tilde{\sigma}_1^2}(y_1 - Y_1\gamma_1 - X_1\beta_1 - V_1\theta)'(y_1 - Y_1\gamma_1 - X_1\beta_1 - V_1\theta)\right) \end{aligned} \tag{3}$$

where the notations used in (3) are defined in Appendix A.

Given the prior density function on $\tilde{\phi} = (\gamma_1', \beta_1', \Pi_2^{*\prime}, \sigma^2, \rho^2, \theta, \Omega_{22})'$ the posterior pdf is

$$p(\tilde{\phi}|data) \propto p(\tilde{\phi}) \times L(\tilde{\phi}|data).$$

where $p(\tilde{\phi})$ is the prior density.

2.2 Choice of the prior distribution

Two widely used priors are the diffuse prior (Zellner (1988), Tsurumi (1985), and Dreze(1976) among others) and the Jeffreys prior (Chao and Phillips (1998) and Kleibergen and Van Dijk (1998)). The diffuse prior has the form

$$p_d(\gamma_1, \beta_1, \Pi_2^*, \sigma_1^2, \rho^2, \theta, \Omega) \propto \sigma_1^{-\frac{1}{2}(m_1+2+\nu_0)}(1-\rho^2)^{-\frac{1}{2}(m_1+2+\nu_0)}|\Omega|^{-\frac{1}{2}(m_1+2+\nu_0)} \quad (4)$$

where $\nu_0 \geq 0$. The Jeffreys prior is given by

$$p_J(\gamma_1, \beta_1, \Pi_2^*, \Omega) \propto (\sigma_1^2)^{\frac{1}{2}(k_2-m_1)}|\Omega|^{-\frac{1}{2}(k+m_1+2)}|\Pi_{22}^{*\prime}X_2'Q_{X_1}X_2\Pi_{22}^*|^{\frac{1}{2}} \quad (5)$$

where $Q_{X_1} = I - X(X'X)^{-1}X'$.

The diffuse prior was criticized because it leads to improper posterior pdf's when the structural equation of interest is just identified. Although the Jeffrey's prior avoids this problem, the posterior pdf's using the Jeffrey's prior tend to have a wide variance such that the proper posterior pdf's are not much of practical use. As we see later, if the structural equation of interest is overidentified, both the diffuse and Jeffrey's priors give rise to proper posterior pdf's that are insensitive to the choice of the priors. Hence, we use the diffuse prior (4) in this paper and work with an over-identified equation.

Using Markov Chain Monte Carlo (MCMC) algorithms explained in Appendix B, we obtain the posterior pdf's of parameters of interest, in particular γ_1, β_1, ρ^2 and Π^*_{22}. The parameter ρ^2 is defined in (A-12). It is the canonical correlation between the endogenous variables on the right hand side of equation and the error term. The posterior density of ρ^2 is used to test weak exogeneity. If the highest posterior density interval of ρ^2 includes zero, we do not reject the hypothesis that Y_1 are weakly exogenous. Otherwise, we conclude that variables Y_1 are endogenous. We use the reduced form coefficients Π^*_{22} to extract the singular values to test the rank condition of identifiability. That is, after obtaining the draws of Π^*_{22}, we extract the singular values $\xi_1, ..., \xi_{m_1}$ from $\Pi^{*\prime}_{22}\Pi^*_{22}$ and obtain the posterior density of the minimum of ξ_i's, and test the rank condition.

2.3 A Numerical Illustration for a White Noise Error Terms

Let us illustrate the limited information Bayesian estimates for the white noise error terms. We generate the data using the following structural model

$$Y\Gamma = XB + U \tag{6}$$

where $Y = [y_1, y_2, y_3]$ is a $T \times 3$ matrix of endogenous variables, $X = [x_1, ..., x_7]$ is a $T \times 7$ matrix of exogenous variables, and x_1 is the vector of ones. The number of observations in the simulated example is $T = 300$. As for Γ, B, and the variance of the structural error term $u_t = (u_{t1}, u_{t2}, u_{t2})'$, we set

$$\Gamma = \begin{bmatrix} 1 & -0.267 & -0.87 \\ -0.222 & 1 & 0 \\ 0 & -0.046 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 0.12 & -0.05 & 0.08 \\ 0 & 0.74 & 0 \\ 0.7 & 0 & 0.53 \\ 0 & 0 & 0.11 \\ 0.96 & 0.13 & 0 \\ 0 & 0 & 0.56 \\ 0.27 & 0 & 0 \end{bmatrix},$$

$$Var(u_t) = 2 \begin{bmatrix} 1.0 & 0.25 & 0.0625 \\ 0.25 & 1.0 & 0 \\ 0.0625 & 0 & 1.0 \end{bmatrix} \tag{7}$$

The correlation matrix of the exogenous variables X is given in Table 1.

Table (1) Here.

The determinant of the correlation matrix of the exogenous variables is 0.0000393, indicating that a high degree of multicollinearity exists among the exogenous variables. Multicollinearity leads to poor posterior inference in that the posterior pdf's tend to have larger variances. Instead of equation (7), if we put $Var(u_t) = 2I_3$ then we obtain much tighter posterior pdf's. The structural equation of interest is

$$y_1 = \gamma_1 Y_1 + X_1 \beta_1 + u_1 \tag{8}$$

where $Y_1 = y_2$, $X_1 = [x_1, x_3, x_5, x_7]$, $\gamma_1 = 0.222$, $\beta_1 = [\beta_{11}, \beta_{13}, \beta_{15}, \beta_{17}]'$ with $\beta_{11} = 0.12$, $\beta_{13} = 0.7$, $\beta_{15} = 0.96$, $\beta_{17} = 0.27$. Note that the equation is over-identified because $(m_1 = 1) < (k_2 = 3)$ and the rank of Π^*_{22} is $m_1 = 1$ Because $m_1 = 1$, the singular value of $\Pi^{*\prime}_{22}\Pi_{22}$ is $(\pi^1_{22})^2 + ... + (\pi^{k_2}_{22})^2$ where π^i_{22} is the i-th element of Π^*_{22}.

Based on a diffuse prior, Tsurumi (1985) derived a marginal posterior probability density functions of the coefficients of the structural equation within the limited information framework. The posterior probability density function of the coefficient associated with the i-th endogenous variable on the right hand side of the equation, γ_{i1}, is given by

$$\begin{aligned} p(\gamma_{i1}|\text{data}) &\propto \left|Y_2' M Y_2\right|^{-\frac{1}{2}} \\ &\quad \cdot \frac{[1+(\gamma_{i1}-\widehat{\gamma}_{i1})^2 h_{11\cdot 2}]^{\frac{\nu_1}{2}}}{[1+(\gamma_{i1}-\tilde{\gamma}_{i1})^2 g_{11\cdot 2}]^{\frac{\nu_2-m_1+1}{2}}} \sum_{i=0}^{r} p_i \binom{r}{i}, \end{aligned} \tag{9}$$

while the posterior probability density function of β_{i1} is given by

$$p(\beta_{i1}|\text{data}) \propto |Y_1' M_1^* Y_1|^{-\frac{1}{2}} (\nu_2^* s_2^{*2})^{-(\nu_2^*-m_1)/2} \sum_{i=0}^{r} q_i^* \binom{r}{i} \tag{10}$$

notations are defined in Tsurumi (1985). The cumulant generating function is used to derive equations (9) and (10).

In Table 2 we present the posterior summary statistics of the MCMC draws. Figure 1 (a) exhibits the posterior pdf's of γ_1 that are derived in three different ways: (i) the exact posterior pdf using equation (9), (ii) the posterior pdf by MCMC draws using the diffuse prior (iii) the posterior pdf using Jeffrey's prior. All of these posterior pdf's are very close to each other. This shows two things: (i) the MCMC draws yield a posterior pdf that is indistinguishable from the exact posterior pdf, and (ii) the posterior pdf using the diffuse prior and that using Jeffrey's prior are practically the same, showing that for an overidentified model the posterior pdf is insensitive to the choice of prior. We present the posterior pdf of ρ^2, the test of weak exogeneity, in Figure 1 (b). The posterior pdf is centered around the true value, and clearly shows that the endogenous variable on the right hand side, Y_{t1}, is correlated with the error term u_{t1}.

Table 2 and Figures 1 (a) and 1 (b) here

3 Estimation of the Parameters When the Error Terms are Autocorrelated

3.1 SEM with ARMA errors

Let us extend our Bayesian analysis to the SEM with autocorrelated error terms. The structural error terms follow the following $ARMA(p, q)$ process:

$$u_t = \Phi(L)u_{t-1} + \nu_t + \Theta(L)\nu_{t-1} \tag{11}$$

where $\nu_t \sim N(0, \Xi)$ and

$$\Phi(L) = \Phi_1 L + \Phi_2 L^2 + ... + \Phi_p L^p, \quad \Theta(L) = \Theta_1 L + \Theta_2 L^2 + ... + \Theta_q L^q$$

Φ_i, Θ_i are $n \times n$ matrices. The reduced form of (1) is

$$Y = X\Pi + V \tag{12}$$

Given that u_t follows $ARMA(p, q)$ processes, it can be shown that v_t also follows $ARMA(\tilde{p}, \tilde{q})$ processes, where $\tilde{p}$ and $\tilde{q}$ are appropriately defined (Hamilton (1994), pp. 106-108). Accordingly, v_{ti}, $i = 1, ..., n$, can be represented as

$$(1 - \tilde{\Phi}_i(L))v_{ti} = \eta_{ti} + \tilde{\Theta}_i(L)\eta_{ti}, \tag{13}$$

where η_{ti} is a white noise process and

$$\tilde{\Phi}_i(L) = \tilde{\phi}_{1i} L + \tilde{\phi}_{2i} L^2 + ... + \tilde{\phi}_{\tilde{p}i} L^{\tilde{p}}, \quad \tilde{\Theta}_i(L) = \tilde{\theta}_{1i} L + \tilde{\theta}_{2i} L^2 + ... + \tilde{\theta}_{\tilde{q}i} L^{\tilde{q}}.$$

Let us postmultiply (12) by Λ defined in equation (A-3) in Appendix A:

$$
\begin{aligned}
Y\Lambda &= X\Pi\Lambda + V\Lambda \\
W &= X\Pi^* + V_* \qquad (14)
\end{aligned}
$$

where $W = Y\Lambda$, $\Pi^* = \Pi\Lambda$, $V_* = V\Lambda$, V_* follows Gaussian $ARMA$ processes and the variance-covariance matrix is a non-linear functions of $\tilde{\phi}_{ij}$, $\tilde{\theta}_{ij}$, γ_1 and of the elements of the variance-covariance matrix Ξ. Hence, we cannot proceed with the decomposition (A-11) in Appendix A that is for the white noise error terms.

The approach we take is to transform equation (14) into the model with white noise error terms and apply MCMC algorithms developed for the model in Section 3. Using (13), we transform (14) into

$$
\begin{aligned}
y_1 - \tilde{\Phi}_1(L)v_1 - \tilde{\Theta}_1(L)\eta_1 &= X_1\beta_1 + (Y_1 - \tilde{\Phi}_2(L)V_1 - \tilde{\Theta}_2(L)\eta_2)\gamma_1 + \eta_1 - \eta_2\gamma_1 \\
Y_1 - \tilde{\Phi}_2(L)V_1 - \tilde{\Theta}_2(L)\eta_2 &= X\Pi_2^* + \eta_2 \qquad (15)
\end{aligned}
$$

or

$$
\begin{aligned}
y_1^* &= Y_1^*\gamma_1 + X_1\beta_1 + v_1^* \\
Y_1^* &= X\Pi_2^* + V_1^* \qquad (16)
\end{aligned}
$$

where

$$
\begin{aligned}
y_1^* &= y_1 - \tilde{\Phi}_1(L)v_1 - \tilde{\Theta}_1(L)\eta_1 \\
Y_1^* &= Y_1 - \tilde{\Phi}_2(L)V_1 - \tilde{\Theta}_2(L)\eta_2 \qquad (17) \\
v_1^* &= \eta_1 - \eta_2\gamma_1 \\
V_1^* &= \eta_2
\end{aligned}
$$

We use the MCMC algorithms for VAR models to construct y_1^* and Y_1^*. We approximate the ARMA processes by AR processes. The main reason is that the estimation of a vector ARMA process requires stringent sets of assumptions about the ARMA parameters to guarantee the identification (Lutkepohl (1993), pp. 241-248), which makes estimation of a vector ARMA model unattractive. At the same time, an invertible vector MA process can be written as an infinite VAR process and infinite VAR process can be approximated by a finite VAR process. Having obtained y_1^* and Y_1^*, we apply the MCMC algorithms we explained in the previous section to equations (16).

3.2 A Numerical Illustration for the Case of an ARMA Error Process

We set that the structural error terms u_t follow the following $ARMA(1,1)$ process

$$u_t = \Phi u_{t-1} + \nu_t + \Theta \nu_{t-1} \tag{18}$$

where

$$\Phi = \begin{bmatrix} 0.97 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & -0.55 \end{bmatrix}, \quad \Theta = \begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}$$

In the reduced form error terms the ARMA process becomes an $ARMA(3,3)$ process. As we discussed above, we estimate VAR rather than VARMA process. To decide on the lag length of an AR process, either Akaike Information Criteria (AIC) or the Bayesian Schwartz Criteria (SC) can be used. Based on the AIC criteria in Table 3, we choose an $AR(4)$ process.

Table 3 here

In Table 4 we compare the estimates of the structural parameters of the first equation of the SEM in which y_1 and Y_1 are transformed into y_1^* and Y_1^* with the estimates we obtain if we erroneously assume that u_t is white noise. We see from Table 4 that the suggested approach works well. The estimates of the parameters with transformation are very close to the true values, while the estimates of the parameters ignoring transformation are very misleading. We present the posterior densities of the parameters γ_1, β_{11}, and ρ^2 in Figure 2 (a) - (f). One may notice that the posterior pdf's of the parameters with y_1^* and Y_1^* are much sharper and are centered around the true values of the parameters than the posterior pdf's with y_1 and Y_1. This illustrates that wrongly assumed error terms lead to misleading inference of the parameters.

Table 4 and Fig 2 Here

4 Analysis of the U.S. gasoline market

There has been a significant amount of interest in estimating price and income elasticities of gasoline and in analyzing the relation between gasoline prices and oil prices. The elasticity estimates of gasoline received a renewed attention recently because of environmental consequences of rapid growth in US gasoline consumption. Depending on the responsivness of gasoline price to changes in price or income, the government may consider policies to change the gasoline demand in order to decrease pollution. The estimates of price and income elasticities are mixed and seem to depend on the estimation approach used.

Espey (1998) presents an extensive overview of the literature on estimating the gasoline demand equation using the aggregate data. He reports that the range of short-run price elasticity is from 0 and -1.26 with the median estimate of -0.23; the range of short-run income elasticity is from 0 to 2.91 with the median estimate of 0.39.[3] Slightly different results were reported by Dahl and Sterner (1991) who present the price gasoline elasticity in the range between -0.26 and -0.86 while the long-run income elasticity seems to be elastic.

Another group of studies use expenditure survey data to estimate the demand elasticity. These studies generally find that gasoline demand is price and income inelastic. Schmalensee and Stoker (1999) and Yatchew and No (2001) provide evidence that high long-run income elasticities could be due to a failure to control for some household characteristics. Nicol (2003), Greening (1995) and Puller and Greening (1999) provide evidence that different household groups seem to respond to price and income changes differently.

We use time series data to estimate price and income elasticities of gasoline. Preliminary analyses of the data show that the data are autocorrelated, and thus we use the Bayesian analysis developed in Section 4.

We also address a question of the effect of oil prices and inventories on gasoline prices. Pindyck (2001) shows how dynamics of commodity prices, inventories and production are interrelated.

Several studies including Borenstein *et al.* (1997), Godby *et al.* (2000), Johnson (2002) and Borenstein and Shepard (2002) analyze the relation between oil and gasoline prices in US. They find that the effect of oil price changes may be asymmetric. Galeotti (2003) presents similar evidence for European markets. However, these studies generally do not take into account the effects of gasoline inventories or production and the error terms are assumed to be white noise. The only exception is the study by Kaufmann and Laskowski (2004) who analyze how refinery utilization rates and inventories influence the asymmetry in gasoline price.

We estimate two supply equations of gasoline. In one version of the supply equation we

[3] The long run income elasticity ranges from 0.05 to 2.73 with the median estimate of 0.81.

do not distinguish between oil price increases and decreases. In the second version we allow an increase and decrease in oil price to have different impact on gasoline price.

We specify the following simultaneous equations:

$$\begin{aligned} ln\ P_t &= \gamma_{11} ln\ J_t + \beta_{11} + \beta_{12} ln\ P_{oil,t} + \beta_{13} D_s + \beta_{14} D_w + u_{t1} && (19) \\ ln\left(\frac{G_t}{W_t}\right) &= \gamma_{21} ln\ P_t + \beta_{21} + \beta_{22} ln\ Z_t + \beta_{23} D_s + \beta_{24} D_w + u_{t2} && (20) \\ ln\ J_t &= \beta_{31} + \beta_{32} ln\ J_{t-1} + \beta_{33} D_s + \beta_{34} D_w + u_{t3} && (21) \\ Q_t^s &= G_t + J_t - J_{t-1} && (22) \end{aligned}$$

where

Q_t^s = production of gasoline in month t, in millions of barrels,

P_t = real retail price of gasoline to end users in urban areas including taxes in month t, in dollars,

$P_{oil,t}$ = crude oil import FOB price from OPEC countries in month t, in dollars,

J_t = inventory of gasoline in month t, in millions of barrels,

W_t = stock of gasoline consuming vehicles at month t

G_t = gasoline consumption in month t, in millions of barrels,

Z_t = average real weekly earnings of production workers, not seasonally adjusted in month t, in dollars,

D_s = dummy variable for summer season; 1.0 for June – August and 0 otherwise

D_w = dummy variable for winter season; 1.0 for January – February and 0 otherwise

The parameters of interest in equations (19) and (20) are γ_{11}, β_{12}, γ_{21}, and β_{22} and we expect the following signs for these parameters:

$$\gamma_{11} < 0, \quad \beta_{12} > 0, \quad \gamma_{21} < 0, \quad \beta_{22} > 0 \tag{23}$$

Both of these equations are over-identified by the counting rule.

To analyze the asymmetric effect of oil price on gasoline price, we replace equation (19) with the following supply equation:

$$\begin{aligned} \triangle ln\ P_t &= \tilde{\gamma}_{11}\triangle ln\ J_t + \tilde{\beta}_{11} + \tilde{\beta}^{+}_{12}\triangle ln\ P^{+}_{oil,t} + \tilde{\beta}^{-}_{12}\triangle ln\ P^{-}_{oil,t} \\ &\quad + \tilde{\beta}_{13}\triangle D_s + \tilde{\beta}_{14}\triangle D_w + \tilde{u}_{t1} \end{aligned} \tag{24}$$

where

$\triangle ln\ P_{oil,t} = ln\ P_{oil,t} - ln\ P_{oil,t-1}, \triangle ln\ P^{+}_{oil,t} = max\{0, \triangle ln\ P_{oil,t}\}, \triangle ln\ P^{-}_{oil,t} = min\{0, \triangle ln\ P_{oil,t}\}.$

In equation (24) we see that if parameters $\tilde{\beta}^{+}_{12}$ and $\tilde{\beta}^{-}_{12}$ are found to be equal then the impact of oil price on gasoline price is symmetric.

We chose AR(4) error term in estimating the reduced form model based on AIC. In Table 5 and Table 6, we present the estimated results for the supply and demand equations. The tables present estimates assuming that the error terms are white noise and then assuming that the error terms are autocorrelated.

Table 5 and Table 6 Here

The estimates of supply equation (24) with the white noise and autocorrelated error term are presented in Table 7.

In the estimation of the supply equation (19) with the white noise error terms, the point estimate of γ_{11} is -1.6 and this shows that the level of inventories has an unreasonably large negative effect on gasoline price. With the assumption of autocorrelation in the error terms, the effect of inventory is much less: $\gamma_{11} = -0.24$. This is close to the estimate of $\tilde{\gamma}_{11} = -0.30$ in the autocorrelation version of (24). We may say that the estimation of SEM with autocorrelated error terms gives robust results for different specifications of supply function, while the results with the white noise error terms are not robust. An increase in oil inventory by 1% leads contemporaneously to $0.24 - 0.3\%$ decline in gasoline prices.

If we assume white noise error terms, we obtain a positive and significant estimate of price elasticity of demand with the point estimate of $\gamma_{21} = .184$. If we assume that the error terms are autocorrelated, however, we obtain a negative and significant price elasticity with the point estimate of $\gamma_{21} = -.543$. Since we expect that the price and quantity demanded are inversely related, the estimate obtained under autocorrelated errors is reasonable.

The Bayesian estimate of the price elasticity in the demand equation with autocorrelated error term is -0.54 and this is in the range of price elasticities reported by Espey (1998) or Dahl and Sterner (1991). The estimate of the income elasticity with autocorrelated error term is 1.685 which indicates that the gasoline demand is income elastic.

Table 7 Here

Table 7 presents the estimates of the effect of oil price decreases and increases, $\tilde{\beta}_{12}^{-}$ and $\tilde{\beta}_{12}^{+}$. Our estimates for these parameters are fairly close: 0.276 for an oil price increase and 0.233 for an oil price decline. Based on the estimates of the highest posterior density intervals for these parameters, we reject the hypothesis of an asymmetric response of gasoline price to changes in oil prices. This finding is not consistent with findings of Borenstein *et al.* (1997) or Johnson (2002), but Bettendorf *et al.* (2003) argue that conclusions on the asymmetry are dependent on the choice of the dataset in estimation. Our results support the findings of Kaufmann and Laskowski (2004). Using a different econometric model, the authors reported results similar to ours and argue that asymmetries in the price of gasoline are generated by refinery utilization rates and inventories.

For the white noise and autocorrelated error terms, the 95% highest posterior density interval's of ρ^2 in estimation of supply equation (19) show that the right hand side endogenous variable $ln\ J_t$ (log of gasoline inventory) is correlated with the structural error term u_{t1}. The test of weak exogeneity for supply equation (24) indicates endogeneity only for the autocorrelated error term case and does not reject the weak exogeneity for a white noise error term version of equation (24). We take it as another evidence that the proposed autocorrelated approach produces reasonable results, while proceeding with white noise assumption is misleading.

The 95% HPDI of ρ^2 in the demand equation shows that the right hand side endogenous variable $ln\ P_t$ (log of gasoline price) is correlated with u_{t2} if we assume that the error terms are white noise, but it is not correlated if we assume that the error terms are autocorrelated.

In conclusion, we compare the estimation results using the Bayesian procedures to those using the two stage least squares (TSLS) with white noise error term and with an autocorrelated error terms.[4] Table 8 and Table 9 present the estimates of gasoline demand and supply functions by TSLS procedures. We observe that the Bayesian estimates with white noise error terms and TSLS with white noise error terms are close to each other. Nevertheless, there is a big difference between Bayesian and TSLS estimates of SEM for autocorrelated case. Judged by the expected signs and sizes of the key regression parameters, the Bayesian estimates using the autocorrelated error terms are much better than the TSLS estimates with autocorrelated errors.

The SEM with autocorrelated error terms may be an alternative to the VAR analysis. In the VAR analysis an impulse response function is used to measure an impact of oil shocks on GDP (Jones *et. al.* (2004)) or on inflation (Hooker (2000)). If we use a SEM with

[4]TSLS with white noise error term and with an autocorrelated error term are canned procedures in EVIEWS. The estimation procedure with an autocorrelated error term is done by Fair (1972) method.

autocorrelated errors and construct a macro model, we may measure the impact of oil shocks through the structural coefficients rather than the reduced form coefficients in the case of a VAR model. Moreover, the LIB with autocorrelated errors allows us to test weak exogeneity leading to careful specification of structural equations. Rather than treating oil price as an endogenous variable, Hamilton (2003) uses a dummy variable for dates of oil shocks. A structural equation that includes oil price as a weak exogenous variable may explain an impact of oil shocks better than a VAR model that includes oil price as an endogenous variable.

5 Concluding remarks

In this paper we developed the Bayesian MCMC algorithms to estimate the parameters of a simultaneous equations system. With the MCMC procedures, one can conduct exogeneity and rank identifiability tests. The MCMC algorithms are developed not only for the SEM with white noise errors but also for the SEM with autocorrelated errors. To deal with autocorrelation, we propose to transform the endogenous variables using the VAR model.

The MCMC algorithms are applied to analyze the U.S. gasoline market. The price elasticity of demand for gasoline is -0.543 and the income elasticity is 1.68. We reject the hypothesis of the asymmetric effect of oil price increases and decreases on gasoline price and document a significant negative effect of oil inventory on the gasoline price. The reported oil inventory elasticity estimate is approximately -0.3.

The MCMC procedures can provide the exogeniety and rank identifiability tests. Moreover, the analysis of the U.S. gasoline market reveals that the Bayesian estimation of SEM with autocorrelated error term produces results that satisfy the prior expectations, while the estimation using TSLS approach gives results that are hard to interpret.

Appendix A: Derivation of the Structural Equation of Interest from the Reduced Form Equation

The reduced form equation of SEM in (1) is given by

$$Y = X\Pi + V \tag{A-1}$$

where $\Pi = B\Gamma^{-1}$, and $V = U\Gamma^{-1} = [v_1, ..., v_n]$. Given that the t-th column of U, u_t, is distributed as $\mathrm{N}(0, \Sigma)$, we see that the column of V, v_t, is distributed as $\mathrm{N}(0, \tilde{\Sigma})$, where $\tilde{\Sigma} = \Gamma^{-1\prime}\Sigma\Gamma^{-1}$. We partition Π as

$$\Pi = \begin{bmatrix} \pi_{11} & \Pi_{12} & \Pi_{13} \\ \pi_{21} & \Pi_{22} & \Pi_{23} \end{bmatrix} \tag{A-2}$$

where π_{11} is $k_1 \times 1$, π_{21} is $k_2 \times 1$, Π_{12} is $k_1 \times m_1$, Π_{13} is $k_1 \times (n - m_1 - 1)$ matrices. We see that Π_{12} and Π_{22} are the reduced form coefficients associated with Y_1, and Π_{13} and Π_{23} are the reduced form coefficients associated with Y_2, the endogenous variables in the simultaneous equation system that are not included in the structural equation of interest.

Postmultiplying (A-1) by a $n \times m_1$ matrix Λ

$$\Lambda = \begin{bmatrix} 1 & 0 \\ -\gamma_1 & I_{m_1} \\ 0 & 0 \end{bmatrix} \tag{A-3}$$

we obtain

$$Y\Lambda = X\Pi\Lambda + V\Lambda, \quad or \quad W = X\Pi^* + V_* \tag{A-4}$$

where $W = (y_1 - Y_1\gamma_1, Y_1)$,

$$\Pi^* = \Pi\Lambda = \begin{bmatrix} \pi_{11} - \Pi_{12}\gamma_1 & \Pi_{12} \\ \pi_{21} - \Pi_{22}\gamma_1 & \Pi_{22} \end{bmatrix} = \begin{bmatrix} \Pi_{11}^* & \Pi_{12}^* \\ \Pi_{21}^* & \Pi_{22}^* \end{bmatrix}, \quad V_* = [u_1, V_1] \tag{A-5}$$

From equation (2) and (A-5), we see that

$$\beta_1 = \pi_{11} - \Pi_{12}\gamma_1, \quad and \quad \beta_2 = \Pi_{21}^* = \pi_{21} - \Pi_{22}\gamma_1 = 0$$

As a result, we re-write equation (A-4) as

$$y_1 = Y_1\gamma_1 + X_1\beta_1 + u_1 \tag{A-6}$$

$$Y_1 = X_1\Pi_{12}^* + X_2\Pi_{22}^* + V_1, \tag{A-7}$$

where $[u_{1t}, V_{1t}] \sim N(0, \Omega)$ and

$$\Omega = \Lambda'\tilde{\Sigma}\Lambda = \begin{bmatrix} \sigma_1^2 & \delta' \\ \delta & \Omega_{22} \end{bmatrix}, \tag{A-8}$$

where

$$\delta = Cov(u_{t1}, Y_{t1})$$

Notice that if $\delta = 0$ then the endogenous variables (i.e. stochastic regressors) in the right hand side of the structural equation of interest are independent of u_{t1}.

From equations (A-6) and (A-7) we obtain the likelihood function

$$L(\phi|data, \Pi_{21}^* = 0) \propto |\Omega|^{-\frac{n}{2}} exptr\left[-\frac{1}{2}(W - X\Pi_*)'(W - X\Pi_*)'\Omega^{-1}\right] \tag{A-9}$$

where $\phi = (\gamma_1, \beta_1, \Pi_2^*, \sigma_1^2, \Omega_{22}, \delta)$, $\Pi_2^{*'} = [\Pi_{12}^{*'}, \Pi_{22}^{*'}]$ and "etr" denotes "'exponential and trace"'. This likelihood may be transformed into

$$L(\tilde{\phi}|d, \Pi_{21}^* = 0) \propto (\tilde{\sigma}_1^2)^{-\frac{n}{2}} |\Omega_{22}|^{-\frac{n}{2}} exp\left(-\frac{1}{2} tr(Y_1 - X\Pi_2^*)'(Y_1 - X\Pi_2^*)\Omega_{22}^{-1}\right) \times exp\left(-\frac{1}{2\tilde{\sigma}_1^2}(y_1 - Y_1\gamma_1 - X_1\beta_1 - V_1\theta)'(y_1 - Y_1\gamma_1 - X_1\beta_1 - V_1\theta)\right) \quad \text{(A-10)}$$

Equation (A-10) is obtained by factoring Ω^{-1}:

$$\Omega^{-1} = \begin{bmatrix} 1 & 0 \\ -\Omega_{22}^{-1}\delta & I \end{bmatrix} \begin{bmatrix} \omega_{11\cdot 2}^{-1} & 0 \\ 0 & \Omega_{22}^{-1} \end{bmatrix} \begin{bmatrix} 1 & -\delta'\Omega_{22}^{-1} \\ 0 & I \end{bmatrix} \quad \text{(A-11)}$$

where

$$\omega_{11\cdot 2} = \sigma_1^2 - \delta'\Omega_{22}^{-1}\delta = \sigma_1^2(1 - \rho^2), \quad \rho^2 = \delta'\Omega_{22}^{-1}\delta/\sigma_1^2 \quad \text{(A-12)}$$

and

$$\tilde{\phi} = (\gamma_1, \beta_1, \Pi_2^*, \tilde{\sigma}_1^2, \Omega_{22}, \theta), \quad \theta = \Omega_{22}^{-1}\delta \quad \text{and} \quad \tilde{\sigma}_1^2 = \sigma_1^2(1 - \rho^2) \quad \text{(A-13)}$$

Appendix B: The Metropolis-Hasting Algorithm to Estimate the parameters $\tilde{\phi}$

Using the likelihood (A-10) in Appendix A and the diffuse prior (4) in the text we see that the posterior pdf is given by

$$p(\tilde{\phi}|data) \propto \text{equation (4)} \times \text{equation (A-10).} \quad \text{(B-1)}$$

We carry out MCMC algorithms in the following steps

- **Step 0:** Set initial values for the parameters ξ^0, Π_2^{*0}, $\tilde{\sigma}_1^{2(0)}$ and Ω_{22}^0.

- **Step 1:** Draw parameters ξ^i from the proposal density

$$\xi^i \sim N(\hat{\xi}, \tilde{\sigma}^{2(i-1)}(Z^{(i-1)\prime}Z^{(i-1)})^{-1})$$

where $\hat{\xi} = (Z^{(i-1)\prime}Z^{(i-1)})^{-1}Z^{(i-1)\prime}y_1$ and $Z^{(i-1)} = [Y_1, X_1, V_1^{(i-1)}]$. We accept ξ^i with probability λ:

$$\lambda = min\left\{\frac{p(\xi^i, \Pi_2^{*(i-1)}, \tilde{\sigma}^{2(i-1)}, \Omega_{22}^{(i-1)}|data)}{p(\xi^{(i-1)}, \Pi_2^{*(i-1)}, \tilde{\sigma}^{2(i-1)}, \Omega_{22}^{(i-1)}|data)} \cdot \frac{g(\xi^{(i-1)}|Z^{(i-1)}, \tilde{\sigma}^{2(i-1)})}{g(\xi^{(i)}|Z^{(i-1)}, \tilde{\sigma}^{2(i-1)})}, 1\right\}$$

where $g(\cdot)$ is the proposal density.

- **Step 2:** Draw the parameter $\tilde{\sigma}^{2(i)}$ using the inverted gamma as the proposal density

$$\tilde{\sigma}^{2(i)} \sim IG(S, df) \tag{B-2}$$

where $S = (y_1 - Z^{(i-1)}\xi^i)'(y_1 - Z^{(i-1)}\xi^i)$ and *df* is the degree of freedom. We accept $\tilde{\sigma}^{2(i)}$ with probability λ:

$$\lambda = min\left\{\frac{p(\xi^{(i)}, \Pi_2^{*(i-1)}, \tilde{\sigma}^{2(i)}, \Omega_{22}^{(i-1)}|data)}{p(\xi^{(i)}, \Pi_2^{*(i-1)}, \tilde{\sigma}^{2(i-1)}, \Omega_{22}^{(i-1)}|data)} \cdot \frac{g(\tilde{\sigma}^{2(i-1)})}{g(\tilde{\sigma}^{2(i)})}, 1\right\}$$

- **Step 3:** Draw $\Pi_2^{*(i)}$ using the multivariate normal proposal density:

$$Vec(\Pi_2^{*(i)}) \sim N(Vec(\hat{\Pi}_2), \Omega_{22}^{(i-1)} \otimes (X'X)^{-1}) \tag{B-3}$$

whre $\hat{\Pi}_2 = (X'X)^{-1}X'Y_1$, and accept $\Pi_2^{*(i)}$ with probability λ:

$$\lambda = min\left\{\frac{p(\xi^{(i)}, \Pi_2^{*(i)}, \tilde{\sigma}^{2(i)}, \Omega_{22}^{(i-1)}|data)}{p(\xi^{(i)}, \Pi_2^{*(i-1)}, \tilde{\sigma}^{2(i)}, \Omega_{22}^{(i-1)}|data)} \cdot \frac{g(Vec(\Pi_2^{*(i-1)}))}{g(Vec(\Pi_2^{*(i)}))}, 1\right\}$$

- **Step 4:** Draw $\Omega_{22}^{(i)}$ using the inverted Wishart distribution

$$\Omega_{22}^{(i)} \sim IW(V_1^{'(i)}V_1^{(i)}, df)$$

and accept $\Omega_{22}^{(i)}$ with probability λ:

$$\lambda = min\left\{\frac{p(\xi^{(i)}, \Pi_2^{*(i)}, \tilde{\sigma}^{2(i)}, \Omega_{22}^{(i)}|data)}{p(\xi^{(i)}, \Pi_2^{*(i)}, \tilde{\sigma}^{2(i)}, \Omega_{22}^{(i-1)}|data)} \cdot \frac{g(\Omega_{22}^{(i-1)})}{g(\Omega_{22}^{(i)})} 1\right\}$$

- **Step 5:** Draws of $\rho^{2(i)}$ and the singular values $\xi_1, ..., \xi_{m_1}$ are made as follows. Draw $\Omega^{(i)}$ using the inverted Wishart as the proposal density

$$\Omega^{(i)} \sim IW[(u_1^{(i)}, V_1^{(i)})'(u_1^{(i)}, V_1^{(i)}), df]$$

where $u_1^{(i)} = y_1 - Y_1\gamma_1^i - X_1\beta_1^{(i)}$. The draw of $\sigma_1^{2(i)}$ is given by

$$\sigma_1^{2(i)} = \nu_{11\cdot 2}^{(i)} + \delta^{(i)'}\Omega_{22}^{-1}\delta^{(i)}$$

where $\nu_{11\cdot 2}^{(i)}$ is the 1-1 element of $\Omega^{-1(i)}$ and $\delta^{(i)} = \Omega_{22}^{(i)}\theta^{(i)}$. The draw of $\rho^{2(i)}$ is given by

$$\rho^{2(i)} = \theta^{(i)'}\delta^{(i)}/\sigma_1^{2(i-1)}$$

Appendix C: Sources of Data

Q_t^s = production of gasoline, millions of barrels per month, *Monthly Energy Review*

P_t = retail price of gasoline to end users in urban areas excluding taxes for month t, *Monthly Energy Review.* The price variable in the demand equation is deflated by CPI. CPI is taken from the *Federal Reserve bank of St. Louis*

J_t = inventory of gasoline, millions of barrels per month, *Monthly Energy Review*

W_t = stock of gasoline consuming vehicles for month t.We used the methodology described in Tsurumi(1980)

G_t = gasoline consumption, millions of barrels per month, *Monthly Energy Review*

Z_t = average weekly earnings of production workers, not seasonally adjusted at month t, divided by CPI, *Survey of Current Business*

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Table 1: The Correlation matrix of exogenous variables

	x_3	x_4	x_5	x_6	x_7
x_2	0.99	0.93	0.87	0.69	0.6
x_3		0.93	0.88	0.73	0.61
x_4			0.87	0.67	0.64
x_5				0.75	0.69
x_6					0.85

Table 2: Summary Statistics of Bayesian Estimate of (8) by using MCMC algorithms, White Noise Error Terms

	True values	Estimates
γ_1	0.222	0.205
		(0.09, 0.31)
β_{11}	0.120	-0.078
		(-0.75, 0.60)
β_{13}	0.700	0.704
		(0.59, 0.82)
β_{15}	0.960	0.988
		(0.93, 1.04)
β_{17}	0.270	0.267
		(0.25, 0.29)
σ^2	2	2.056
		(1.82, 2.31)
θ	0.637	0.651
		(0.64, 0.53)
ρ^2	0.685	0.683
		(0.59, 0.77)

(1) Figures are posterior means
(2) Figures in parenthesis are 95 % highest posterior density interval (HPDI)

Table 3: AIC and SC to select lag length in the transformation

p	SC	AIC
2	2.236	1.902
3	2.048	1.778
4	1.964*	1.777*
5	1.984	1.899

* denotes the minimum value

Table 4: Summary Statistics of Bayesian Estimates of (8)

		Error terms are assumed to be	
	True values	white noise*	autocorrelated
γ_1	0.222	0.532	0.241
		(0.02, 0.97)	(0.13, 0.34)
β_{11}	0.120	5.228	1.96
		(2.20, 8.11)	(1.16, 2.74)
β_{13}	0.700	0.419	0.674
		(-0.10, 1.02)	(0.56, 0.78)
β_{15}	0.960	0.813	0.967
		(0.65, 1.01)	(0.92, 1.02)
β_{17}	0.270	0.195	0.263
		(0.12, 0.27)	(0.25, 0.28)
σ^2	2	27.515	1.89
		(24.50, 31.19)	(1.66, 2.12)
θ	0.637	0.413	0.598
		(0.01, 0.47)	(0.48, 0.72)
ρ^2	0.685	0.584	0.650
		(0.13, 0.97)	(0.55, 0.75)

(1) Figures are posterior means
(2) * Erroneously assumed error process
(3) Figures in parenthesis are 95 % highest posterior density interval (HPDI)

Table 5: Supply equation (19)

		Error terms are assumed to be	
		white noise	autocorrelated
$ln\ J_t$	γ_{11}	-1.631	-0.239
		(-1.85, -1.43)	(-0.32, -0.15)
Const	β_{11}	7.11	3.422
		(5.98, 8.25)	(2.97, 3.87)
$ln\ P_{oil,t}$	β_{12}	0.523	0.216
		(0.48, 0.57)	(0.20, 0.23)
D_s	β_{13}	-0.001	0.002
		(-0.04, 0.03)	(-0.010, 0.015)
D_w	β_{14}	0.070	0.004
		(0.03, 0.11)	(-0.009, 0.02)
	σ^2	0.018	0.0022
		(0.015, 0.020)	(0.002, 0.0025)
	θ	1.533	0.496
		(0.94, 2.12)	(0.32, 0.66)
	ρ^2	0.153	0.125
		(0.05, 0.25)	(0.05, 0.20)

(1) Figures are posterior means

(2) Figures in parenthesis are 95 % highest posterior density interval (HPDI)

Table 6: Demand equation (20)

		Error terms are assumed to be	
		white noise	autocorrelated
ln p_t	γ_{21}	0.184	-0.543
		(0.10, 0.26)	(-1.01, -0.07)
Const	β_{21}	-1.670	0.095
		(-2.65, -0.64)	(-2.84, 3.01)
ln Z_t	β_{22}	1.422	1.685
		(1.18, 1.64)	(1.51, 1.85)
D_s	β_{23}	0.038	0.039
		(0.02, 0.05)	(0.02, 0.05)
D_w	β_{24}	-0.051	-0.047
		(-0.07, -0.03)	(-0.06, -0.03)
	σ^2	0.0043	0.0025
		(0.0039, 0.0048)	(0.0022, 0.0028)
	θ	-0.147	0.349
		(-0.24, -0.06)	(-0.15, 0.84)
	ρ^2	0.167	0.063
		(0.02, 0.32)	(0.00, 0.19)

(1) Figures are posterior means
(2) Figures in parenthesis are 95 % HPDI

Table 7: Supply equation (24)

		Error terms are assumed to be	
		white noise	autocorrelated
$\triangle ln\ J_t$	$\tilde{\gamma}_{11}$	-0.106	-0.300
		(-0.63, -0.42)	(-0.47, -0.14)
Const	$\tilde{\beta}_{11}$	0.0021	0.001
		(-0.003, 0.007)	(-0.003, 0.006)
$\triangle ln\ P^{+}_{oil,t}$	$\tilde{\beta}_{12}$	0.239	0.276
		(0.14, 0.33)	(0.18, 0.38)
$\triangle ln\ P^{-}_{oil,t}$	$\tilde{\beta}_{15}$	0.225	0.233
		(0.13, 0.31)	(0.14, 0.32)
$\triangle D_s$	$\tilde{\beta}_{13}$	0.006	0.003
		(-0.006, 0.018)	(-0.05, 0.012)
$\triangle D_w$	$\tilde{\beta}_{14}$	0.002	0.014
		(-0.013, 0.017)	(0.004, 0.02)
	$\tilde{\sigma}^2$	0.0008	0.0006
		(0.0007, 0.009)	(0.0005, 0.0007)
	θ	0.298	0.528
		(-0.24, 0.84)	(0.33, 0.71)
	ρ^2	0.169	0.343
		(0.00, 0.49)	(0.18, 0.49)

(1) Figures are posterior means

(2) Figures in parenthesis are 95 % HPDI

Table 8: TSLS of the supply equation (19)

		Error terms are assumed to be	
		white noise	autocorrelated
$ln\ J_t$	γ_{11}	-1.631	0.161
		(0.10)	(0.06)
Const	β_{11}	4.918	-2.070
		(0.55)	(0.42)
$ln\ P_{oil,t}$	β_{12}	0.522	0.181
		(0.02)	(0.02)
D_s	β_{13}	-0.001	0.009
		(0.018)	(0.003)
D_w	β_{14}	0.071	0.003
		(0.019)	(0.003)
	σ^2	0.017	0.006

(1) Figures are TSLS estimates.

(2) Figures in parenthesis is estimated standard error.

Table 9: TSLS of the demand equation (20)

		Error terms are assumed to be	
		white noise	autocorrelated
$ln\ p_t$	γ_{21}	0.185	0.200
		(0.04)	(0.02)
Const	β_{21}	6.581	6.739
		(0.30)	(0.272)
$ln\ Z_t$	β_{22}	1.419	1.333
		(0.11)	(0.23)
D_s	β_{23}	0.039	0.033
		(0.009)	(0.005)
D_w	β_{24}	-0.051	-0.046
		(0.009)	(0.005)
	σ^2	0.0036	0.0025

(1) Figures are TSLS estimates.

(2) Figures in parenthesis is estimated standard error.

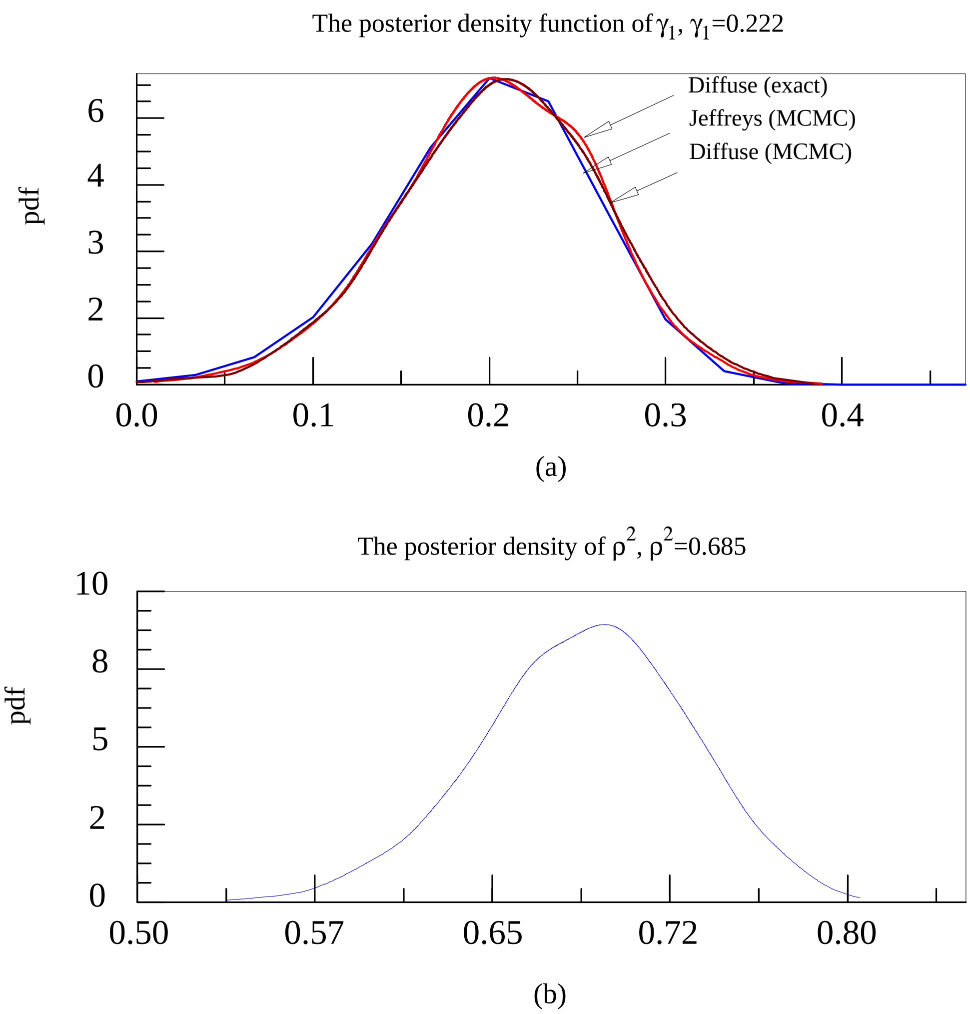

Figure 1: Estimation of parameters with white noise error term. (a) The posterior density of parameter γ_1, true value $\gamma_1 = 0.222$. (b) The posterior density of ρ^2, true value $\rho^2 = 0.685$.

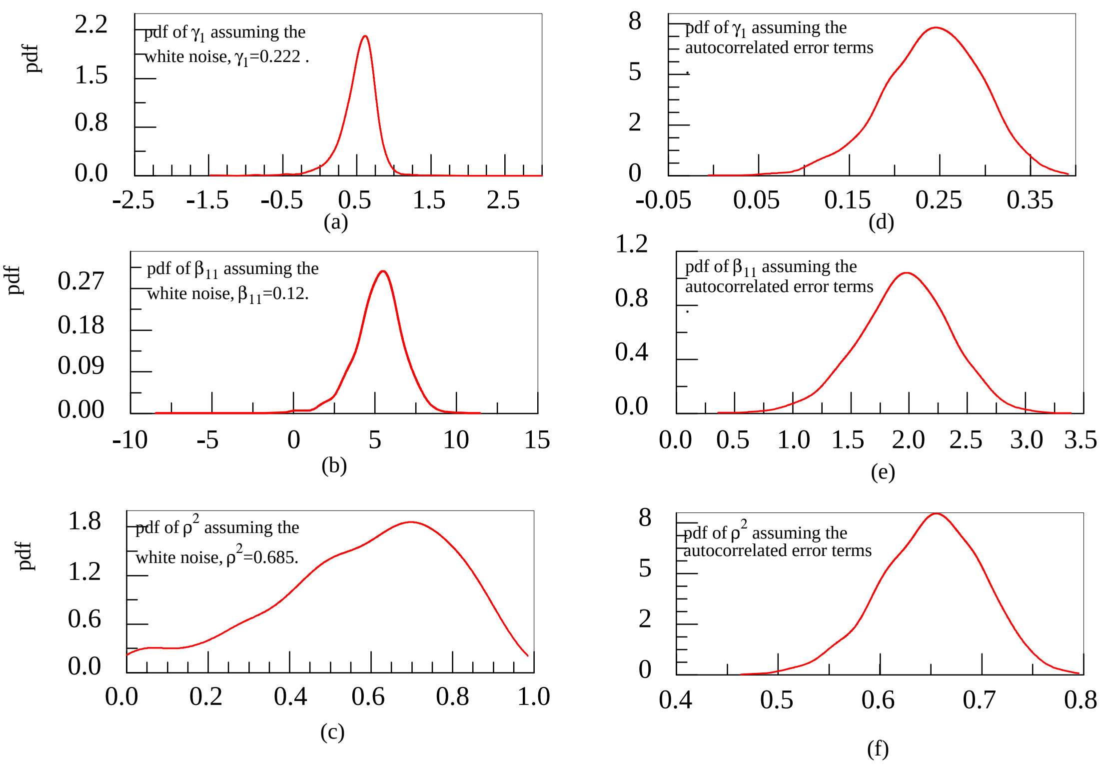

Figure 2: The error terms are autocorrelated in the data generating process. (a) The posterior density of parameter γ_1 assuming that error term is white noise. The true value of $\gamma_1 = 0.222$. (b) The posterior density of parameter β_{11} assuming that error term is white noise. The true value of $\beta_{11} = 0.12$. (c) The posterior density of ρ^2 assuming that error term is white noise. The true value of $\rho^2 = 0.685$. (d) The posterior density of parameter γ_{11} assuming $ARMA$ error term. (e) The posterior density of parameter β_{11} assuming $ARMA$ error term. (f) The posterior density of ρ^2 assuming $ARMA$ error term.

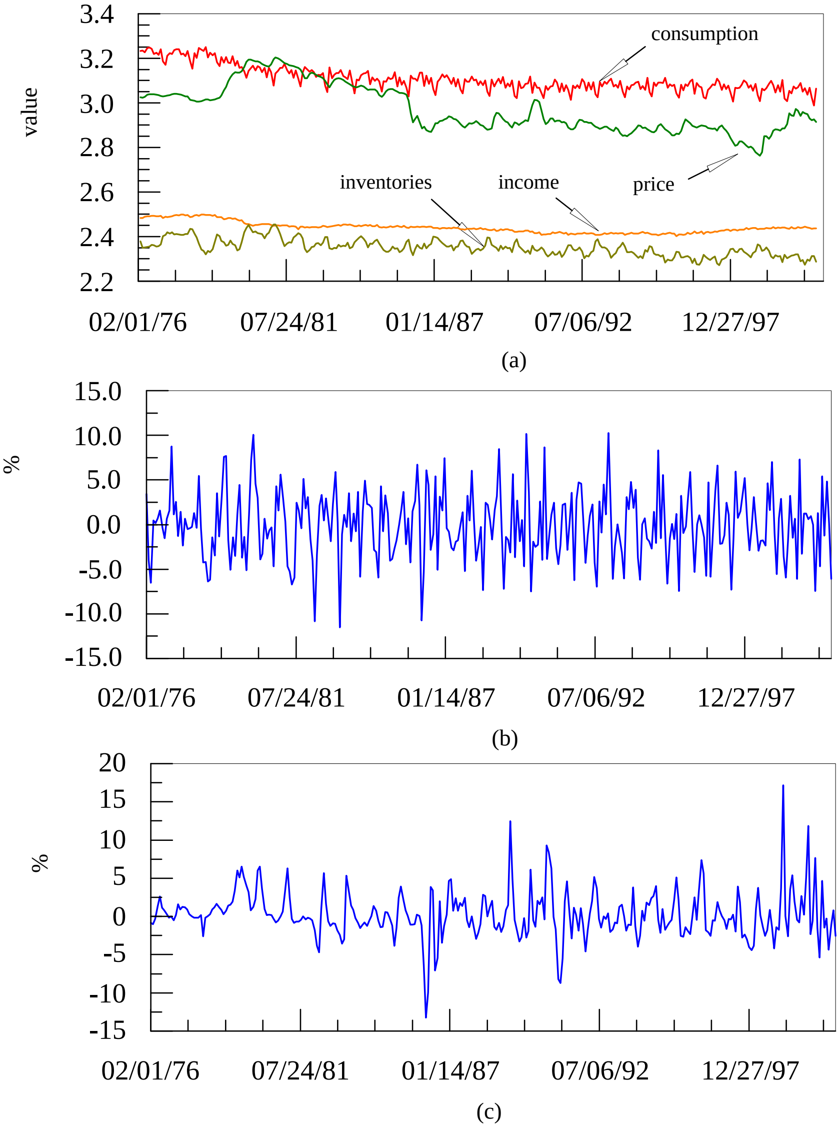

Figure 3: Data for the estimation of gasoline model. (a) Graphs of the main data series. (b) Graph of the monthly change in inventories, in %. (c) Graph of the monthly change in price of gasoline, in %.