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Aggregate investment dynamics when firms face fixed investment cost and capital market imperfections*

Abstract

In this paper a model of aggregate investment is derived which incorporates fixed investment costs and capital market imperfections on the micro-level. Aggregate investment reacts nonlinearly with respect to aggregate shocks to productivity and liquidity of firms. Employing non-parametric kernel estimation methods to analyze a sample of annual account data of UK companies, these nonlinearities also show up empirically.

Furthermore a difference in strength between the long- and the short-run effect of liquidity on investment is found, which is inconsistent with models that explain the empirical correlation of investment and liquidity solely as the result of some long-run relationship like liquidity-dependent costs-of-capital. Moreover, as a side-result when imposing a linear structure, we find, as reported in previous studies, that financially constrained firms appear to be less influenced by liquidity than unconstrained ones. However, this finding disappears when the investment function is estimated without any restrictions on the functional form and thus has to be attributed to a misspecification error, which is avoided by the estimation strategy of this paper.

JEL classification: E22; E44; G31; C32; C33

Keywords: Investment; aggregation; imperfect capital markets; debt constraints; adjustment costs; heterogeneity; nonlinear time series.

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1 Introduction

Economists knowledge of micro-level and aggregate investment is still far from being conclusive. The only thing seemingly well established is the empirical rejection of the standard neoclassical investment model.[1] Which assumption of the neoclassical model leads to its failure to what extend is yet to be found.

Beginning with Fazzari et al. (1988) the empirical literature has emphasized the role of financial factors in firm-level investment. More recently attention has been drawn to the role of non-convexities in the investment technology. This paper aims at merging both strands of the literature and shows that both, financial factors and non-convexities are not only each important, but also each important for the effect of the respective other and therefore play both a prominent role in determining micro-level investment. Yet, this interaction has not been analyzed up until very recently: Holt (2003) provides a theoretical real options model of irreversible investment that shows how financial frictions and irreversibility of investment work as complementaries, Whited (2003) provides evidence that firms which are identified as financially constrained exhibit investment spikes much less frequent, and Cagese (2003) develops a formal test for financial constraints based on the irreversibility of fixed investment.

Our approach differs in both methodology and focus from these studies: Firstly, we focus on the differential impact of liquidity on the optimal stock of capital and on investment to discriminate between various types of financial frictions. While we find only a very minor impact of liquidity on optimal capital, liquidity substantially speeds up investment. This effect apparently would be absent in a partial adjustment framework with liquidity-dependent costs of capital. Secondly, we use information on cross- and second-order derivatives to test for the different adjustment costs and financial frictions models. Moreover and as some side result, we show that omitting the interaction of liquidity and fixed adjustment cost can lead to a severe misspecification bias, by which the most constrained firms actually look least reactive to changes in liquidity.

Although the interaction we want to study is rather unanalyzed, non-convexities themselves such as irreversible investment or fixed costs of investment and other economies of scale have widely been discussed and have lately been theoretically analyzed in a very general framework by Abel and Eberly (1994). Non-convexities in the adjustment costs typically lead to an expected investment function that is convex in the underlying fundamentals.

Empirical evidence for non-convexities is mostly drawn from the Longitudinal Research Database (LRD): E.g. Doms and Dunne (1998) report that on the plant-level a small fraction of investment *activities* is associated with a large fraction of changes in the capital stock. Cooper et al. (1999) use the LRD to estimate a hazard model of investment and find a

[1] See Caballero (2000).

time-increasing investment hazard and thus evidence for non-convexities. A more direct approach has been taken by Caballero et al. (1995): Again using the LRD, they estimate a measure of "mandated investment" by imposing a long-run or cointegration relation between earnings, capital employed and the cost of capital. Explaining actual investment by mandated investment in a second stage, they empirically find a convex relationship between mandated and actual investment, which is inconsistent with the convex adjustment cost (and perfect capital market) framework - the neoclassical investment theory. Recently, there also has been additional evidence for non-convexities, drawn from other data than the LRD.[2]

Discussing the influence of micro-level non-convexities on aggregate investment, Caballero and Engel (1999) present a model of firm-level and aggregate investment that incorporates firm-specific stochastic adjustment costs and (to a certain degree) heterogeneous production functions. They estimate the resulting aggregate investment dynamics on the basis of 2-digit industry level panel data and obtain a better fit with their structural model than with any (simple) partial adjustment or other autoregressive models.

The empirical literature on financial factors in investment[3] has recently been tackled by a series of papers[4] emphasizing the problem of measurement errors and biased estimators based on q-theoretical regressions. Most disappointing for the financial factors in investment literature are the results of Gomes (2001): In a pecking order of finance framework he shows that the presence of financial frictions is neither sufficient nor necessary to obtain seemingly significant *positive coefficients* on *cash flow*s in a q-theoretic investment regression.[5] Therefore, in this paper we follow an approach that is not subject to Gomes' critique; we firstly do not rely on a q-measure and secondly do not use cash-flow to identify the extent of the financial constraint but the within variation of the equity ratio as the stock of liquidity. By using the within variation, we control for different baseline access to capital across firm but

[2] Abel and Eberly (2002) employ the Compustat data in a q-theoretic framework - their (1994) augmented adjustment cost model - and find evidence for non-convexities in the capital adjustment technology, especially for fixed costs. Goolsbee and Gross (2000) use micro-level airline industry data within the framework of Caballero et al. (1995) and find evidence for a convex investment-function. However, this evidence vanishes by aggregation - even on the airline (company) level there is nearly no evidence for non-convex adjustment costs.

[3] For a very exhaustive survey over the literature on capital market imperfections and investment see Hubbard (1998).

[4] See for instance Kaplan and Zingales (1997), Cummins et al. (1999), Erickson and Whited (2000) or Gomes (2001)

[5] The explanation for this can be summarized in the following way: On the one hand measurement errors can cause the regressor on cash flow to be positive without any financial frictions. On the other hand, true marginal q already (partly) measures the impact of financial frictions - if such indeed exist. A good applied example for the latter effect can be found in a paper of Cummins et al. (1999) in which a so called "real q" is constructed using analyst's EBIT forecasts. While "real q" is significant in explaining investment, cash flow is insignificant and the estimated parameter is negative. Interestingly the variable "real q" is (partly by construction) highly correlated with the current cash flow. Therefore, the regression results do not say much about whether it is fundamental effects or the value of internal funds that drive "real q". Moreover, "real q" will even have measurement error positively correlated with cash flow, if analysts' cash flow forecast errors were positively correlated with actual cash flow, e.g. due to omitted variables.

assume that all firms can in general be subject to a financing constraint.

Yet, there are other studies that find financial factors in investment and rely on more directly measuring capital productivity instead of focusing on q-theory. Examples for this strand of literature are Bond and Meghir (1994) or Gilchrist and Himmelberg (1998).[6]

Additionally to his scepticism on financial frictions Gomes (2001, p. 1279) also points out that the pecking order theory of finance "[...] is somewhat similar to those used in the investment with fixed cost literature."—depending on the specification. Therefore, by ignoring financial variables we might obtain a convex investment function in fundamentals only due to fixed costs of financing activities, even if cost of adjusting real capital are absent. Even worse, the similarity between the financial frictions and the fixed costs of investment framework also holds the other way round, i.e. the standard (S,s) investment-model already produces some correlation between investment and cash-flow.[7] Therefore, any empirical formulation of an investment function should not only use a more direct measure of fundamental investment incentives than Tobin's q to avoid measurement error bias, but should also allow for both financial frictions and non-convex adjustment costs simultaneously to avoid misspecification bias.

As briefly stated above there seems to be another empirical puzzle apparent in the literature which can be tackled by incorporating financial frictions in a model of fixed capital adjustment costs: Liquidity seems to affect investment much more in the short than in the long run. This evidence becomes even stronger when investment is estimated as an error-correction-process.[8]

As long as liquidity is measured as cash flow, this is easily explainable, but e.g. Guariglia (1999, pp.47) finds that firm size and stock based liquidity proxies are empirically independent, so there can be no long run relation. However, she does not comment on this point.

Moreover it is not quite clear whether cash-flow is a correct measure for liquidity or if a proxy for the line-of-credit would be preferable (Blinder, 1988, p. 199). If agency or other informational market imperfections are the driving force of the investment liquidity correlation, then the correct measure should be a stock- and not a flow-item and the dependency should appear mainly in the long run. If fixed transaction costs are the driving force of the liquidity dependence,[9] we would expect firms to have the same marginal cost-of-capital and thus the same long run capital stock regardless how indebted they are. Therefore, allowing

[6] Although Gilchrist and Himmelberg call their regressors "fundamental q" and "financial q", these are measures of capital productivity and the shadow costs of internal funds and are only loosely related to q-theory.

[7] To see this, suppose that the productivity-of-capital-process being a random walk. Because of the "region of inactivity" in the (S,s) model firms usually invest (disinvest) only after a series of positive (negative) changes in capital-productivity. However these changes in productivity correspond to a series of rises (falls) in cash-flow.

[8] See e.g. Hubbard (1998) or Mairesse et al. (1999) for an overview of the empirical literature on firm-level investment and the time structure of liquidity effects.

[9] See Gomes (2001) and Bond and Meghir (1994, p. 206).

for different impacts of liquidity in the short and in the long run, as will be done in this paper, can help to empirically distinguish between the transaction cost and the imperfect information view. Whether liquidity should be measured by a stock or a flow is ambiguous in a transaction cost framework. Therefore, the equity-ratio, a stock item, is used in this paper, because it proxies future profitability to a much lesser extent than cash flow does (Gilchrist and Himmelberg, 1998, p. 28) and thus causes less econometric problems once one controls for firm-specific baseline access to capital markets.

The starting point of this paper is a theoretical model in which the interaction of fixed investment costs and financing constraints is studied. The model is based upon the Caballero and Engel (1999) model of lumpy investment. This model is modified to incorporate an imperfect capital market as e.g. in Gilchrist and Himmelberg (1998). It yields possibility results which are strikingly different to existing investment models: In contrast to models with informational imperfections and continuous, convex adjustment costs expected investment rates may react more sensitively to the equity-ratio than the optimal stock of capital does. In contrast to pecking order of finance and convex adjustment cost models, the model predicts investment to be more sensitive to financial factors with increasing fundamental investment incentives. In contrast to pure non-convex adjustment cost models, there is an influence of liquidity on investment. Discussing the behavior of aggregate investment, it is shown that, although firm level investment behaves lumpily, aggregate investment is a smooth function in fundamentals.

When estimating investment non-parametrically from UK company accounts all the predicted effects show up empirically. Therefore, the alternative models mentioned before are rejected, while the model presented remains as an explanation. Especially the empirical larger sensitivity of investment than the sensitivity of capital on the equity-ratio supports the theoretical model.

The remainder of this paper is organized as follows: In section 2.1 a model that describes firm level investment under the assumption of capital market imperfections and fixed costs of investment is presented. In sections 2.2 and 2.3 the expressions for sectoral investment and its derivatives, and for aggregate investment and its dynamics are derived. Section 3 empirically assesses our model using firm level investment data. Section 4 concludes and an appendix follows.

2 A theoretical model

2.1 Firm-level investment

We will start off with presenting and discussing the representative *problem* of a firm which is at the same time subject to financial constraints and fixed adjustment costs .[10] For simplicity and as in Caballero and Engel (1999), the industry modelled in this chapter shall consist of a large number of monopolistically competitive single-plant and single-product firms which all are subject to a limited liability constraint. The investment decision is modelled in discrete time. Each firm faces an infinitely elastic supply of all factors. At the beginning of each period all uncertainty about that period is resolved and is common knowledge from then on. Thereafter, each firm decides upon investment.

2.2 Adjustment technology and financial constraint

If a firm wants to change its capital stock it has to pay some fixed costs; all other factors may be adjusted without costs. At the end of every period each firm has to pay back its last period's debt plus interest, has to pay for any new purchased capital goods and for all other factors. Moreover, it can issue new debt and pay out dividends.

Apart from the non-convexity in the investment technology, firms face a capital market imperfection: As in Gilchrist and Himmelberg (1998) and as an extremely simplified version of the pecking order of finance theory, there shall be a no-new-equity constraint, i.e. firms are unable to issue new shares or to have negative dividends once founded (which is in the absence of taxation equivalent on the margin):

Assumption 1: Once founded, firms are unable to issue new equity. Especially, dividend payments D_t must be non-negative at any point in time: $\forall t : D_t \geq 0$.

This simplified version of the pecking order theory is necessary to keep the model tractable. Nevertheless, the general results should not change if this assumption were replaced by a more complex version as in Gomes (2001).

Moreover and more important, this assumption is not strongly contradicted by empirical findings as e.g. Friedman (1982) shows empirically that firms hardly use any external equity finance at all. Moreover this assumption also has theoretical support: Fries et al. (1997)

[10] Note that typically the investment-functions in investment problems with fixed cost are not aggregable. Hence we can only discuss the typical investment problem - of course given a parameterization of the production technology, market structure etc. If these parameters vary across firms, this makes the aggregation problem more severe. This is quite analoguous to the aggregation problem in consumer-theory.

Moreover, when relating investment to consumer theory it should be noted, that under finacial frictions, the "law of supply" obviously does no longer necessarily hold as firms then face a budget constraint.

show how full collataralization- and "no new equity"-constraints may theoretically arise as an industry equilibrium.

Secondly, we assume that a firm must declare bankruptcy immediately if it has a negative book value of equity (at any point of time). Under assumption 1 this is equivalent to a full collataralization constraint, i.e., the amount of debt a firm may issue is limited by the actual stock of capital depreciated and discounted for one period.[11] To see the equivalence, suppose a firm would borrow above the constraint. Then the firm would be bankrupt next period. Hence, either all assets were transferred to the debtholders who continue operations or debt burden had to be renegotiated. Any combination of the two would contradict assumption 1. However, both procedures would imply a sure loss either for the stock- or the debtholders. Thus, at least one party will never allow for any increase in debt beyond that ceiling.

Assumption 2: The maximum debt B_{t+1} (repayable in period t+1) a firm may issue in period t is restricted by the book value of the stock of capital K_t, depreciated (with rate δ) and discounted (at the interest rate on debt r) for one period: $\frac{B_{t+1}}{K_t} \leq \frac{(1-\delta)}{1+r\left(\frac{B_{t+1}}{K_t}\right)}$.

The third, last, and weakest assumption regarding capital market imperfections is that the interest rate on debt r only depends on the leverage. As Gilchrist and Himmelberg (1998), we assume r to be homogenous of degree zero in B and K, and to be weakly increasing in B. This does not rule out r to be independent of B and K.

Assumption 3: $r = r\left(\frac{B_{t+1}}{K_t}\right)$ and $r'\left(\frac{B_{t+1}}{K_t}\right) \geq 0$.

2.2.1 Periodic sales

Now let K^* denote the frictionless stock of capital of a firm, i.e., the stock of capital that would be chosen in the absence of fixed costs for investment and capital market imperfections. Let K be the actual capital employed.

The semi-reduced function of earnings per-period (EBIT), Π, shall be linear homogenous in the frictionless stock of capital K^* and can be written as:[12]

$$\Pi(z, K^*) = \pi(z)K^*, \tag{1}$$

with z denoting the capital-imbalance at the beginning of each period before investment takes place, i.e. the ratio of actual capital employed to frictionless capital, $\frac{K}{K^*}$. π shall be

[11] This ensures that the firm always has a positive present value in the following period and will stop production before going bankrupt. Alternatively, this assumption can be viewed as a simplification of Hart and Moore's (1994) debt capacity model.

[12] This assumption and the assumptions below with respect to π are e.g. fulfilled if demand is iso-elastic and the production function is Cobb-Douglas. See Caballero and Engel (1999) for details.

strictly concave and fulfill the (Inada) conditions $\pi(0) = 0$ and $\lim_{z \to 0} \pi'(z) = +\infty$. Moreover, assume $\lim_{z \to +\infty} \pi'(z) < \psi\delta$ to make profits bounded. ψ denotes the discount factor.

When a firm invests, it faces a stop of production. The duration of this stop is determined by the random variable $w \in]0, 1[$ which represents the fraction of the period used for the installation of the new capital—similar to the adjustment costs assumption in Caballero and Engel (1999). So the costs, A, of adjusting the capital stock are given by:

$$A(k_t, K_t^*, w_t) := w_t \pi(k_t) K_t^*, \tag{2}$$

where k_t denotes the capital-imbalance after adjustment, $k_t := \frac{K_t + Investment_t}{K_t^*}$. Note that in the presence of depreciation the firm will typically invest up to a larger stock of capital than the frictionless optimal one, K_t^*.

2.2.2 Dynamics of the stochastic variables

So far there are no restrictions on the stochastic dynamics of the random variables K_t^* and w_t. Both variables together completely determine firm heterogeneity and investment dynamics, so any assumptions on these variables are crucial. A minimal assumption for keeping the model tractable is that both variables exhibit the Markov property.[13] Furthermore, we shall assume that w_t is i.i.d. and that K_t^* follows a geometric random walk (with drift d). The innovations ξ_t to K_t^* are normally distributed and serially uncorrelated (although possibly correlated across firms):

$$\frac{K_t^*}{K_{t-1}^*} = \exp(d + \xi_t). \tag{3}$$

2.2.3 Capital market and the firm's objective

Firms are assumed to be risk-neutral. Therefore, they seek to maximize the expected, discounted dividend stream. They do so by choosing some capital-imbalance k_t (respectively the amount of capital employed, $k_t K_t^*$) and the amount of debt used to finance production, B_{t+1}.

In order to finance investment, a firm can either cut back dividend payments D_t or raise debt B_{t+1}. As assumed, a firm is unable to sell any new shares or raise equity by negative dividends (assumption 1). Moreover, the amount of debt a firm can issue is limited by the actual stock of capital employed (assumption 2). Additionally, the interest rate is a function

[13] If shocks to productivity would be serially correlated, the analysis would just be complicated. Managers would have a true optimal stock of capital that is different from the one that maximizes current profits. This would make the financing problem more pronounced.

of $b_t := \frac{B_t}{K_{t-1}}$ and is weakly increasing in b_t (assumption 3).[14] Therefore, dividend payments, D_t, are given by

$$\begin{aligned} D_t &= D(k_t, B_{t+1}, K_t^*, w_t, z_t, B_t) \\ &:= \Pi(k_t, K_t^*) - A(k_t, K_t^*, w_t)\mathbb{I}_{\{k_t \neq z_t\}} - K_t^*(k_t - z_t) + B_{t+1} - (1 + r_t)B_t \end{aligned} \quad (4)$$

in which $\mathbb{I}$ is an indicator function. Moreover, let the time constant discount factor be denoted by ψ and the value of the firm be denoted by V. Then the following Bellman equation determines both V and the optimal investment policy:

$$\begin{aligned} &V(K_t^*, w_t, z_t, B_t) = \\ &\max_{(k_t, B_{t+1}) \in X} \left\{ D(k_t, B_{t+1}, K_t^*, w_t, z_t, B_t) + \psi \mathbb{E}_t[V(K_{t+1}^*, w_{t+1}, z_{t+1}, B_{t+1})] \right\}. \end{aligned} \quad (5)$$

In this expression $X := X(K_t^*, w_t, z_t, B_t)$ is the correspondence of financially feasible capital-imbalance and debt pairs. $\mathbb{E}_t$ denotes the expectations operator, conditional on information available at time t.

To reduce the number of state variables and to obtain a more convenient formulation of the problem at hand, we subtract the book value of equity from V and divide by K_t^*. This defines a new value function $v := \frac{V}{K_t^*} - e_t z_t$, which is the difference between "market-value" and book-value of equity relative to the optimal stock of capital. The ratio of equity to capital can be expressed as $e_t := e(b_t)$.[15] As both e_t and K_t^* are determined before the optimal policy decision is taken, maximizing v and maximizing V yield the same optimal policy.

Now, define firm value $\tilde{v}$ as v if the capital imbalance is not altered by investment in the current period. Rearranging terms,[16] we obtain

$$\begin{aligned} \tilde{v}(k_t, b_{t+1}) &:= c(k_t, b_{t+1}) + (1 - \delta)\psi k_t e(b_{t+1}) \\ &+ \psi \mathbb{E}_t \left\{ v \left[w_{t+1}, k_t \frac{1-\delta}{\exp(d + \xi_{t+1})}, e(b_{t+1}) \right] \exp(d + \xi_{t+1}) \right\}. \end{aligned} \quad (6)$$

For capital-imbalance and debt pairs ("plans") with strictly positive capital we define Y

[14] That the interst-rate for bonds increases does not follow from our model but is an assumption. And as we have explicitely ruled out bancruptcies, debt is even risk-free. Hence, the assumption in its strong form itself is somewhat inconsistent with the model. Yet, to rule out risky debt is only to simplify and concentrate the analysis. Introducing another risk-term that enters after the investment decisions are made and adding bancruptcy costs for the debt-holders would generate an upward sloping interest-function, but would only complicate the analysis a lot.

[15] See appendix for details.

[16] Again, see appendix.

to be the correspondence of financially feasible plans in terms of k and b :

$$
\begin{aligned}
&Y(w_t, z_t, e_t) \\
&\quad := \left\{(k_t, b_{t+1}) \in \mathbb{R}_{++} \times \mathbb{R}_+ | k_t - e_t z_t - \pi(k_t)[1 - w_t \mathbb{I}_{\{k_t \neq z_t\}}] \leq k_t b_{t+1} \leq k_t \widehat{b}\right\}. \quad (7)
\end{aligned}
$$

The first inequality represents the positive dividend constraint and the second inequality represents the debt-ceiling, in which $\widehat{b}$ denotes the maximum debt-to-capital ratio

$$\widehat{b} := \sup\left\{b \in \mathbb{R}_{++} |\ b \leq \frac{1-\delta}{1+r(b)}\right\}.$$

The following Lemma proves to be useful in order to write the Bellman equation (5) in terms of k and b in a short and accessible form. Basically the Lemma states that it is always possible and profitable to avoid bankruptcy in our model. That default is not profitable is due to the monopoly rents the firm would forgo by defaulting.

Lemma 1 *(a) Y is non-empty and*

(b) employing zero capital is suboptimal, i.e.

$$\max_{(k_t, b_t) \in Y} \tilde{v}(k_t, b_t) - \pi(k_t) w_t \mathbb{I}_{\{k_t \neq z_t\}} > \psi \mathbb{E}_t[v(w_{t+1}, 0, 0) > 0$$

Proof. *See appendix.* ■

Because of the above Lemma, firms never stop production completely respectively declare bankruptcy.[17] Therefore, the optimal policy is always an element of Y and thus the Bellman equation defining v is given by:

$$v(w_t, z_t, e_t) = \max_{(k_t, b_{t+1}) \in Y(w_t, z_t, e_t)} \tilde{v}(k_t, b_{t+1}) - \pi(k_t) w_t \mathbb{I}_{\{k_t \neq z_t\}}\ . \quad (8)$$

2.2.4 Adjustment process

In general, Y is only upper-hemicontinuous, thus v might not be continuous everywhere. The lack of continuity arises because of the fixed adjustment costs: If a firm employs a large stock of capital and is heavily indebted, it may find itself unable to repay the debt obligations if the capital-imbalance or the debt level rise marginally. Therefore, it will be necessary to distinguish two cases when describing firm level investment:

[17] This result seems a contradiction to empirical facts at a first glance, i.e. of course in reality firms do declare bankruptcy and are shut down. However, Lemma 1 should not be taken literally as "firms never disappear". Basically the Lemma states, that the monopoly power of a firm is always of some value, which would be lost upon bankruptcy. Hence, the Lemma may better be interpreted as "brands never disappear", which surely comes closer to reality than the former interpretation.

(a) The firm is in danger of becoming insolvent: this happens, if

$$1 - e_t - \frac{\pi(z_t)}{z_t} > \widehat{b} \ . \tag{9}$$

In this case the firm has a negative cash flow and cannot sustain the actual level of capital employed by issuing new debt. Therefore, it has to (heavily) cut back production to increase its average productivity. In consequence, a firm always disinvests if in financial distress.

(b) The firm is not in danger of becoming insolvent:

Then, denoting the (optimal) capital-imbalance after adjustment with z_{opt} and the ratio of debt to capital after adjustment with b_{opt}, a firm adjusts its stock of capital (i.e. it invests) in period t if the expected increase in discounted value outweighs the adjustment costs. That is if:

$$\begin{aligned} 0 \quad &\geq \quad \max_{b_{t+1} \in Z(z_t, e_t)} \tilde{v}(z_t, b_{t+1}) - \tilde{v}(z_{opt}, b_{opt}) + \pi(z_{opt}) w_t \qquad (10) \\ Z(z_t, e_t) \quad &: \quad = \left\{ b_{t+1} \in \mathbb{R}_{++} | 1 - e_t - \frac{\pi(z_t)}{z_t} \leq b_{t+1} \leq \widehat{b} \right\} \end{aligned}$$

or equivalently

$$w_t \leq \min_{b_{t+1} \in Z} \left\{ \frac{\tilde{v}(z_{opt}, b_{opt}) - \tilde{v}(z_t, b_{t+1})}{\pi(z_{opt})} \right\} \ . \tag{11}$$

As shown in the appendix, the value of a firm that adjusts is monotonically decreasing in w_t, so that for every (e_t, z_t) there exists an unique trigger $\bar{w}$ such that

$$\underbrace{\tilde{v}(z_{opt}(w_t, e_t), b_{opt}(w_t, e_t)) - w\pi(z_{opt}(w, e_t))}_{\text{Value of a firm adjusting the stock of capital}} \gtreqless \underbrace{\max_{b_{t+1} \in Z} \tilde{v}(z_t, b_{t+1})}_{\text{Value when only readjusting finance}} \quad \text{for } w \lesseqgtr \bar{w} \ . \tag{12}$$

2.3 Cross-sectional investment

2.3.1 Aggregation

Having reduced the firm's investment decision to a comparison of two values, it is now possible to define for every (e_t, z_t) a critical value Ω which is the largest value of the stoppage duration

w_t for which the firm chooses to invest.[18]

$$\Omega(e_t, z_t) := \begin{cases} \overline{w}(e_t, z_t) & \text{if } Z\left(z_t, e_t\right) \neq \emptyset \\ 1 & \text{if } Z\left(z_t, e_t\right) = \emptyset \end{cases} \tag{13}$$

As only contemporary state variables matter for the aggregation, the time indices of state variables are suppressed henceforth. Let $K_t^A, I_t^A, K_t(e,z), I_t(e,z)$ denote the aggregate stock of capital, aggregate gross investment, and the stock of capital and gross investment of firms with capital-imbalance z and equity-to-capital-ratio e (sectoral aggregates) in period t, respectively. Moreover, let $G(w)$ be the distribution of w. Then the investment hazard can be defined as $\Lambda(e,z) := G(\Omega(e,z))$ and we can define

$$\widetilde{z}_{opt}(e,z) := \Lambda(e,z)^{-1} \int^{\Omega(e,z)} z_{opt}(w,e,z) dG(w), \tag{14}$$

which is the average optimal capital-imbalance of firms that invest conditional on having an equity-ratio e and capital-imbalance z before investment.

At time t investment of firm j with capital-imbalance z_{jt} and equity e_{jt} that adjusts indeed is given by

$$I_{jt} = \left[z_{opt}(w_{jt}, e_{jt}, z_{jt}) - z_{jt}\right] K_{it}^{*} = \left[\frac{z_{opt}(w_{jt}, e_{jt}, z_{jt})}{z_{jt}} - 1\right] K_{jt}\ . \tag{15}$$

Therefore, with these quantities at hand, the expected (cross-sectional) investment conditional on (e,z) can be expressed as

$$\mathbb{E}[I_t(e,z)] = K_t(e,z) \left[\frac{\widetilde{z}_{opt}(e,z)}{z} - 1\right] \Lambda(e,z). \tag{16}$$

Since adjustment-cost shocks w are i.i.d., the cross-sectional average investment rate $i(e,z)$ follows directly from (16):

$$i(e,z) := \frac{I_t(e,z)}{K_t(e,z)} = \left[\frac{\widetilde{z}_{opt}(e,z)}{z} - 1\right] \Lambda(e,z) \tag{17}$$

Differentiating average investment i with respect to the equity-ratio e yields an interesting

[18] Since w is always smaller than one, $\Omega(z,e) = const \geq 1$ implies that a firm adjusts independent of its realized w. Furthermore, as a firm always disinvests if it is in danger of becoming insolvent ($:\Leftrightarrow Z = \emptyset$), $\Omega(z,e) = 1$ is a sensible value if $Z = \emptyset$.

decomposition of the effect of a change in the leverage (if $\frac{\partial \widetilde{z}_{opt}}{\partial e}$ and $\frac{\partial \Omega}{\partial e}$ exist):[19]

$$\frac{\partial i(e,z)}{\partial e} = \underbrace{\frac{\Lambda(e,z)}{z}\left(\frac{\partial \widetilde{z}_{opt}(e,z)}{\partial e}\right)}_{\text{level-effect}} + \underbrace{\frac{(\widetilde{z}_{opt}(e,z)-z)}{z}\frac{\partial \Lambda(e,z)}{\partial e}}_{\text{frequency-effect}} \tag{18}$$

While the first term represents a long-run or level effect of the equity-ratio, the second term represents an only short run or frequency effect. The latter effect is only short-run since an increase in Λ decreases the variance of the cross sectional capital-imbalance and therefore later on decreases the probability of investment. Due to this frequency effect investment can be more sensitive to the financial situation than the optimal stock of capital is:

Theorem 1 *(a) If* $[i(e,z)+\Lambda(e,z)]$ *is large enough*[20] *—but possibly smaller than one—then the investment rate is more sensitive to the equity-ratio than the optimal stock of capital, i.e. the elasticity of* $\widetilde{z}_{opt}$ *w.r.t. the equity-ratio is smaller then the semi-elasticity of the investment-rate w.r.t. the equity-ratio:*

$$\frac{\partial i(e,z)}{\partial \ln(e)} \geq \frac{\partial \widetilde{z}_{opt}(e,z)}{\partial e}\frac{e}{\widetilde{z}_{opt}}. \tag{19}$$

(b) In an environment around $(e, \widetilde{z}_{opt}(e,z))$ *we have*

$$\frac{\partial^2 i(e,\widetilde{z}_{opt}(e,z))}{\partial e \partial z} \bigg/ \frac{\partial i(e,z)}{\partial e} \leq 0. \tag{20}$$

Proof. *See appendix.* ■

Remark 1 *Differentiating* $\frac{\partial i(e,z)}{\partial \ln(e)}$ *with respect to* $\ln(z)$ *yields the following:*[21]

$$\begin{aligned} \frac{\partial^2 i(e,z)}{\partial \ln(e) \partial \ln(z)} &= \frac{\partial}{\partial \ln(z)}\left[[i(e,z)+\Lambda(e,z)]\,\eta_e^{z_{opt}} + \eta_e^{\Lambda}\cdot i\right] \\ &= \frac{\partial\,[i(e,z)+\Lambda(e,z)]}{\partial \ln(z)}\eta_e^{z_{opt}} + \frac{\partial^2 \widetilde{z}_{opt}}{\partial z \partial e}\frac{e}{\widetilde{z}_{opt}}z + i\frac{\partial \eta_e^{\Lambda}}{\partial \ln(z)} + \eta_e^{\Lambda}\frac{\partial i}{\partial \ln(z)} \end{aligned} \tag{21}$$

As z *and* e *enter* Y *only multiplicatively and as* Y *determines* $z_{opt}(w,e,z)$, $\frac{\partial^2 \widetilde{z}_{opt}}{\partial z \partial e}$ *can be*

[19] Proposition 4 in the appendix shows that $\frac{\partial \Omega}{\partial \bar{e}}$ exists almost everywhere (a.e.).

[20] *The qualification "*$[i(e,z)+\Lambda(e,z)]$ *large enough" pecludes situations in which firms invest only very rarely very small amounts. In such situations any increase in frequency would have virtually no effect on the investment rate. (Consider the extreme case of a firm that invests some* $\varepsilon \to 0$ *with probability 0.5+0.5e. Though there is a substantial frequency effect, the semi-elasticity is close to 0.5+0.5e times the change in the typical investment of an investing firm.*

[21] *This can be seen by differentiating expression* (46) *in the appendix.*

approximated by $\frac{\partial^2 \tilde{z}_{opt}}{\partial e^2}\frac{e}{z}$. *Therefore, we can state:*

$$\begin{aligned}\frac{\partial^2 i(e,z)}{\partial \ln(e)\partial \ln(z)} &\simeq \frac{\partial[i(e,z)+\Lambda(e,z)]}{\partial \ln(z)}\eta_e^{z_{opt}} + \frac{\partial^2 \tilde{z}_{opt}}{\partial e^2}\frac{e^2}{\tilde{z}_{opt}} + i\frac{\partial \eta_e^{\Lambda}}{\partial \ln(z)} + \eta_e^{\Lambda}\frac{\partial i}{\partial \ln(z)} \\ &= \frac{\partial \eta_e^{z_{opt}}}{\partial \ln(e)} + \frac{\partial i(e,z)}{\partial \ln(z)}\left[\eta_e^{z_{opt}} + \eta_e^{\Lambda}\right] + \left[\frac{i}{\Lambda(e,z)} + \eta_e^{z_{opt}}\right]\frac{\partial \Lambda(e,z)}{\partial \ln(z)}. \end{aligned} \tag{22}$$

Although the sign of these terms is not clear from analytic grounds, we would expect for negative $\ln(z)$*, the effect of equity on the optimal capital stock to be decreasing in* e*, the adjustment hazard to be decreasing in* z *and the expected investment to be a decreasing function of* z*, too. Hence, intuitively* $\frac{\partial^2 i(e,z)}{\partial \ln(e)\partial \ln(z)} < 0$ *if* i *is positive.*

2.4 Discriminating between our model and alternatives

The above theorem and remark are central in discriminating between the model of this paper and both, the Myers and Majluf (1984) pecking-order of finance and the liquidity-dependent cost of capital (but convex adjustment cost) models: In the liquidity-dependent cost of capital models with convex adjustment costs the long-run effect is clearly dominant. Any effect of liquidity on the speed of adjustment in these models is only a second-order effect. Slower adjustment marginally saves internal funds, so that the marginal gains of faster adjustment and the marginal-costs of internal funds have to be equalized. Therefore, liquidity can have an influence on the adjustment speed only via the second-order derivative of the costs of capital w.r.t. liquidity. In the fixed adjustment cost model however, investment is an extramarginal decision. Given any fixed cost of investment, a change in liquidity hence may render some projects unprofitable, so that there is a first-order effect of liquidity on the adjustment speed.

Whether the debt-ceiling or the liquidity dependence of the cost of capital is more important can be evaluated by comparing $\frac{\partial i}{\partial z} \times \frac{\partial \tilde{z}_{opt}}{\partial e}$ and $\frac{\partial i}{\partial e}$. If the liquidity dependence is important, $\frac{\partial i}{\partial z} \times \frac{\partial \tilde{z}_{opt}}{\partial e}$ and $\frac{\partial i}{\partial e}$ should be close to equal. In this case most of the effect of finance on investment frequency comes from rendering some investment projects unprofitable. If the debt-ceiling is the important financial friction, $\frac{\partial i}{\partial e}$ can be expected to exceed $\frac{\partial i}{\partial z} \times \frac{\partial \tilde{z}_{opt}}{\partial e}$ substantially. Here, liquididity corresponds to a number of investment options a firm can expect to have at most over some given period of time. The smaller the number of options is, the larger the value of each option will be. This option-value adds another factor to the fixed cost of adjusting the stock of capital.

In the pecking-order model there is an important short run effect of liquidity independent of the form of adjustment costs. In these models when adjustment costs are concave typically three regimes of firm-finance emerge.[22] These are stylized in figure 1.

The firms with a high z (low productivity of investment) are financially unconstrained,

[22] See Gomes (2001), Bond and Meghir (1994), or Whited (1992) for details.

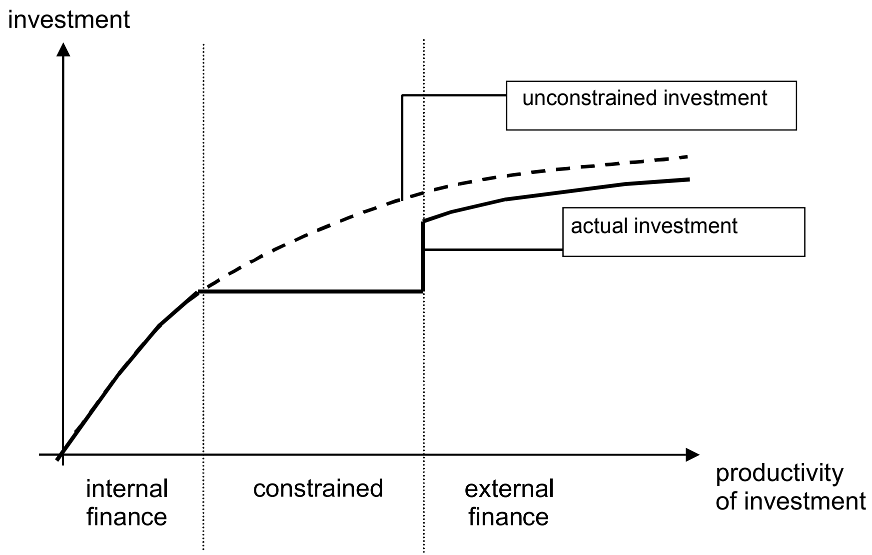


Figure 1: A stylized version of the pecking-order model

rely on internal finance, and their investment decision is independent of their liquidity constraint. Firms with low z rely on external finance and depending on the form of transaction costs, investment of these firms can be sensitive to liquidity. Firms with intermediate z are strictly constrained by liquidity and a change in liquidity changes investment. Firms with high productivity rely on external finance. For these firms increasing equity either has no effect or actually reduces the sensitivity of investment with respect to fundamentals, because these firms become liquidity constrained when equity rises. This can happen since the gains of obtaining external finance get smaller, the larger the internally financed amount of investment is. Therefore, the cross-derivative $\frac{\partial^2 i(e,z)}{\partial \ln(e) \partial \ln(z)}$ would be positive. Consequently, Theorem 1(b) and Remark 1 can be used to test the model of this paper against (simpler) pecking-order alternatives.[23] Another possibility to discriminate between fixed cost of investment and fixed cost of external finance models is the investment behavior for $z > 1$: Fixed disinvestment costs imply a range of inactivity, while transaction costs of finance yield immediate disinvestment, if firms can hold financial assets. In that sense, even if we later estimate the investment function non-parametrically and even if the estimated derivatives have no structural interpretation in the form of coefficients of an adjustment-cost function, we can identify the various investment models using first- and higher-order derivatives. Moreover, in its general formulation, the model nests the alternative models as it only partially differs

[23] This argument, however, only holds if transaction costs in a pecking-order model are deterministic. If transaction costs are stochastic, cross-sectional aggregation of the short run investment function may be governed by the *distribution* of transaction costs, which - if chosen from a set of *arbitrary* distributions - can have a dynamics similar to the one of our model as a result.

with respect to the "test objects" mentioned.

2.5 Aggregate investment and its dynamics

One obtains the time-series of aggregate investment by aggregating over the distribution of (e, z) pairs. Aggregate investment can therefore be expressed as:

$$I_t^A = \int \left[\int i(e,z) K_t(e,z) f_{z|e}(z,t|e) dz \right] f_e(e,t) de, \tag{23}$$

where $f_{z|e}$ denotes the conditional density of capital-imbalances and f_e denotes the cross-section density of the equity-to-capital ratio. f shall denote their common density.

To obtain a simplified expression for the aggregate investment equation, we make the following assumption:[24]

Assumption 4: Let $\frac{K_t(e,z)}{K_t^A}$ and $i(e,z)$ be independent in (e,z).

Then we get for the aggregate investment rate $\frac{I_t^A}{K_t^A}$:

$$\frac{I_t^A}{K_t^A} \simeq \int \int \left[\frac{\widetilde{z}_{opt}(e,z)}{z} - 1 \right] \Lambda(e,z) f(e,z,t) dedz. \tag{24}$$

Given equation (24), aggregate investment is fully determined by the distribution of equity and the conditional distribution of capital-imbalances. The dynamics of the aggregate investment series is then determined by the transition from $f(\cdot, t)$ to $f(\cdot, t+1)$. [25]

3 Empirical evidence

3.1 Measuring the capital imbalance

To test our model of interacting frictions against the alternatives mentioned, we will need an approach that *nests* the alternative models. Yet, this precludes any very structural approach, but forces us to rely on the reduced form representation of the two-step approach of Caballero et al. (1995). In this approach the capital imbalance z as a proxy of fundamental investment incentives is estimated in a first step. Thereafter, we regress investment on this proxy and the equity-ratio, to obtain the (short-run) expected investment function. However, without specifying the distributions involved one obtains no closed form representation of an investment

[24] See Caballero and Engel (1999, p. 816) for a detailed numerical justification of a closely related assumption.

[25] NOTE FOR THE REFEREE: The dynamics of aggregate investment can be generated by the system described in Appendix R1.

function, but only predictions on its first- and second-order derivatives to discriminate between the alternative models. To account for this, we will employ non-parametric estimation techniques and base our inference on the non-parametric average derivative estimates.

The first intermediate goal of this section is hence to construct an estimator for the capital-imbalance z. In contrast to Caballero et al. (1995), it cannot be assumed that the desired capital is proportional to the stock of capital K^* that a plant would hold in the absence of *adjustment costs*. We know that z and e enter only multiplicatively in z_{opt}. Taking logs of all variables except for $i(e,z)$ (without changing notation), we then can write the optimal capital imbalance, as defined in the previous chapter, as a function in two arguments. Abusing notation slightly, we replace $z_{opt}(w,e,z)$ by $z_{opt}(w,e+z)$. Under the assumption that z_{opt} is differentiable a Taylor-approximation of $z_{opt}(w,e+z)$ around $z_{opt}(0,0)$—neglecting cross- and higher-order-derivatives with respect to e—yields for desired capital $\widetilde{k}$ (by the definition of z) :

$$z_{opt}(w,e+z)=\tilde{k}_{it}-k^*_{it}=\alpha_{i0}+\sum_{j=1}^{\infty}\frac{1}{j!}\alpha_{ij}w^j+\beta_i(e+z)\ , \tag{25}$$

In this equation s_{it} denotes the logarithm of the stock of equity in the opening balance. Since w_t is i.i.d. and unobservable, it is useful to define $u_{it}:=\frac{1}{1-\beta_1}\sum\frac{1}{j!}\alpha_j w^j_{it}$. Furthermore, z can be approximated by:[26]

$$-z_{it}=k^*_{it}-k_{it}=\eta_i\left[\ln(b_i)+y_{it}-k_{it}-\theta_i\widetilde{co}_{it}\right], \tag{26}$$

where k denotes log-capital employed, y denotes log-sales, $\widetilde{co}$ denotes log cost-of-capital and b_i denotes the elasticity of sales to capital. Now combining (25) and (26) yields:

$$\begin{aligned}-\frac{x_{it}}{1-\beta_i}&=\frac{z^{opt}_{it}-z_{it}}{1-\beta_i}=\alpha_{i1}+\eta_i\left[y_{it}-k_{it}-\theta_i\widetilde{co}_{it}\right]+\tfrac{\beta_i}{1-\beta_i}e_{it}+u_{it}\\ \text{with }\alpha_{i1}&:=\tfrac{\alpha_{i0}}{1-\beta_i}+\eta_i\ln(b_i)\end{aligned} \tag{27}$$

Although a formal proof is not available so far, it is very likely that $z_{opt}-z$ (conditional on the predetermined e) is stationary, while y,k and $\widetilde{co}$ are most likely to be non-stationary, so that θ and β can be estimated from a panel-cointegration regression (Caballero et al., 1995, p. 15). Obviously better than just assuming (and even than proving within our model) would be to test for a cointegration relation being present in the data. However, as the panel data we have is large only along the cross-section- and not along the time series dimension, such a test would be meaningless. Nevertheless, estimating the cointegrating vector should

[26] See appendix 1 for details, especially for the definitions of b_i and η_i.

still be possible, as this is even possible in pure cross-sections (Madsen, 2001).[27]

Regarding (β_i, θ_i) we assume the cointegrating vector to be homogeneous (at least amongst industries Ind_j). Thus $(\beta_i, \theta_i) = (\beta, \theta)\, \forall i \in Ind_j$. Furthermore, since the number of parameters is larger than the number of predetermined variables, one needs to approximate η_i by $\frac{1}{1-\gamma_i}$ and b_i by γ_i, where γ_i is the cost share of equipment capital (Caballero et al., 1995, p. 15).

Given consistent estimators $\widehat{\theta}, \widehat{\beta}$ (and $\widehat{\alpha}_{i0}$) it is then possible to compute z and estimate $i\,(e,z)$, $f(e,z,t)$ non-parametrically.

3.2 Estimation procedure

For estimating the cointegration relation (28), Phillips' and Moon's (1999) "full-modified panel cointegration estimator" (henceforth PFM-OLS) is used. This estimator is $\sqrt{n}T$-consistent, asymptotic normal and corrects for possible endogeneity of the regressors.[28]

A drawback of the PFM-OLS estimator is that it is formulated for balanced panels with integrated regressors only. The data we have is an unbalanced panel and at least for e we would rather assume it to be an I(0) process. However, the PFM-OLS estimator is a generalization of the full-modified OLS estimator of Phillips and Hansen (1990) and Phillips (1995) and hence we expect the results of Phillips (1995) to carry over to the panel-case as well, i.e. the estimator is $\sqrt{n}T$-consistent and asymptotic normal for parameters of stationary regressors. The standard errors will thus be calculated in analogy to the time-series case.

To account for the unbalancedness, the unbalanced-panel equivalents of all items that appear in the formula of the estimator are calculated.[29] For inference the average number of observations per firm is used. The equation we use for estimation is:

$$\eta_i\,(k_{it} - y_{it}) = \alpha_{i1} + \gamma_t - \theta\eta_i\widetilde{co}_{it} + \kappa e_{it} + u_{it},\; \kappa := \frac{\beta}{1-\beta}. \tag{28}$$

The time-dummies γ_t have been added for two reasons: First, the data that will be used covers a period of large shocks to inflation, thus measuring the real interest rate correctly is something to be concerned about. The second reason is that we do not have data on taxational shocks—common to all firms. The usual within transformation will be used to remove time and individual effects.

With the estimator of θ at hand, we can use (26) to calculate end-of-period t capital

[27] Strictly speaking, of course we only estimate a long-run covariance. This covariance reflects the cointegration relation if there is such a relation, which is what we assume.

[28] See Appendix for the Referee (Note R3.3)

[29] For example when estimating the short-run covariance matrix, we calcualte the covariance for every firm in the sample using the firm-specific number of observations and then average over firms.

For inference in the I(1)-regressor case we calculate confidence bounds on the basis of T being the average number of observations per firm.

imbalances $\hat{z}_{t+1}$. From our model, we would need to know the beginning-of-period capital imbalance which is unobservable. Of course one could simply subtract investment from $\hat{z}_{t+1}$, which would yield beginning-of-period z consistently with our model. However, subtracting the regressand in generating the regressor is obviously problematic.[30] Instead, we will simply take $\widehat{z}_t$, as a proxy for z_t.

In a second step, the investment rate i_{jt} is regressed non-parametrically on $(e_{jt}, \widehat{z}_{jt})$. To compare the short- and long-run behavior, and to compare the results with other empirical studies, average derivatives of $i(e, z)$ are estimated, as well. These are the counterparts to the coefficients estimated in linear models. However, as we only observe average z on the company level, but not for every single plant, the second-order derivative of the investment function in z will be underestimated if the function is convex, i.e. the function will appear to be less curved than it is.

The nonparametric estimation procedure, that will be employed will basically be a generalized-nearest neighborhood estimator. However, accounting for firm-specific effects is not as straightforward for nonparametric estimation, as it is in the parametric one (Ullah and Roy, 1998). To account for fixed effects the investment function shall meet the following assumption:

Assumption 5: For each firm j at time t investment i is given by

$$i_{jt}(z_{jt}, e_{jt}) = f^i_{jt} + i(z_{jt} - f^z_{jt}, e_{jt} - f^e_{jt}) + v_{jt}\ . \tag{29}$$

Moreover, $\mathbb{E}(i(\cdot,\cdot)) = 0$ (to identify i). $f^x_{jt} := (\bar{x}_{j.} + \bar{x}_{.t} - \bar{x}_{..})$ to remove fixed effects.

Under this assumption the function i can be directly estimated using within-transformed quantities only. Alternatively, the nonparametric first-derivative estimator of Ullah and Roy (1998) could be applied. To obtain the investment function one has to integrate over the derivatives. For estimating average derivatives however, we will also apply the estimator of Ullah and Roy.

3.3 Data

The UK-data we employ is the BSO-dataset of the Cambridge/DTI Database.[31] This database contains annual accounting data from UK companies from 1976 to 1990. 50494 company-year observations are included in the data. About half of them come from manufacturing firms. For the subsequent analysis the dataset has been restricted to companies

[30] In an earlier version of this paper, this approach has been taken and the results were qualitatively the same.
[31] See Goudi et al., 1985. The database is freely available by HMSO after registration. It covers a representative sample of UK company accounts.

of the manufacturing sector with positive fixed capital and positive equity. Moreover, only firms with 5 or more consecutive observations available remain in the sample. After removing outliers (see below), the sample contains 7147 observations from 915 different firms.[32]

The BSO dataset contains capital and investment data for land and buildings as well as for tools and machinery. Since reported depreciation rates for machinery are about 40%, we restrict the analysis to land and buildings.[33] All data have been deflated to 1975 prices using the retail-price-index (RPI).

The user-cost of capital are computed as the average reported depreciation rate (on land and buildings) for each firm $\overline{\delta}_i$ plus the real interest-rate (reported in table 1). But then a fraction s_{it} of the investment spending is payed for by subsidies. In consequence, this fraction has to be subtracted from the cost:

$$\widetilde{co}_{it} = \ln((\overline{\delta}_i + r_t)(1 - s_{it})). \tag{30}$$

The real interest series is obtained by subtracting annual inflation rate (on the basis of RPI) from 3-month Euro-sterling deposit rates. Due to the logarithmic transformation the real-interest rate shocks are not completely removed by the time-dummies.

The data on subsidies is problematic as we obtain $s_{it} \geq 1$ for a few firm-years. Therefore, all these observation are removed by discarding all observations with cost-of capital below the 4% and above the 99.5% quantile.[34,35]

Table 1 reports descriptive statistics for the variables of the sample that we use. The within transformation is calculated before taking first differences, and thus before loosing the first observation for each firm. Hence, the regressors do not have mean zero exactly.

In the introduction we briefly summarized the recent literature on q-theoretic empirical investment models which has highlighted the role of measurement errors. Although the analysis does not rely on q, in two other systematic ways a measurement error might be present in the cost series. The first is a systematic risk effect, the second would be a difference between our constructed real interest series and the real interest rate on debt. If firms have different idiosyncratic risks, their costs of capital are different. If risk enters capital costs

[32] There are missing observations for few firms due to the way outliers are removed, i.e. we have an observation in $t-1$ and in $t+1$ but the observation in t is removed. The within-transformation we use takes care of this, but we simply neglect this fact for the following regressions. More strictly, we either would have to take the whole time series for the firm out of the sample or at least treat observations after the outlier and before differently. However, in any case we would lose quite many observations.

[33] Although our model might still hold true for fast depreciating capital goods, we would need data at a higher frequency to sensibly analyse the data. At a depreciation rate of 50% capital goods are replaced on average every second year on a regular basis, if the stock of capital stays constant. Hence, we can expect to hardly find any influence of fixed costs in yearly data.

[34] Additionally, we remove observations with the 0.05% highest or lowest costs after the within transformation.

[35] See Appendix for the Referee (Note R3.5)

Table 1: Descriptive statistics of the quantities used (within-transformed)

full sample				removing first obs. (as in PFM-OLS)				
	min	max	std		mean	min	max	std
ηy	-5.655	3.438	0.326	$\eta\,(y-k)$	0.001	-5.648	4.956	0.383
ηk	-3.389	6.302	0.486	$\eta\widetilde{co}$	-0.004	$-.4233$	0.627	0.062
$\eta\widetilde{co}$	$-.4233$	0.627	0.075	e	0.0005	-2.414	1.585	0.209
e	-2.414	1.585	0.210					
η^a	0.617	1.000	0.026	before individual demeaning				
z^b	-5.680	4.775	0.358	z	-4.08	-10.66	1.716	0.796
i	-10.37	0.778	0.514	i^c	0.339	-0.936	8.115	0.571

[a]Not demeaned

[b]After removing individual effects, the correlation of capital-imbalance and equity-ratio is about 0.032

[c]Gross-investment, calculated as annual differences of reported capital stocks.

multiplicatively, however, the fixed effects perfectly control for this (if risk is constant over time), otherwise they still should do most to remove the measurement-error bias.[36] The same argument also holds true for differences between the real-interest series used and the relevant real-interest series for firm debt.

3.4 Long-run optimal stock of capital

Table 2 reports estimates of (28) for the whole dataset as well as for the industries for which many observations are available. These are Food (21), Chemicals (26), Non-electrical engineering (33), Electrical Engineering (36), Paper, Printing & Publishing (48) and Transport & Communication (50). The standard errors are given in parentheses.

The estimated long-run elasticity of capital with respect to the user-costs is with 0.9752 statistically insignificantly different from 1. The elasticity with respect to the equity-ratio is with $\frac{\kappa}{\kappa+1}=0.077$ statistically significant under both the I(0) and I(1) assumption for e.[37] If the equity-ratio influences the long-run stock of capital through influencing the marginal cost of finance (interest rate on debt), these estimates imply an elasticity of the cost of finance with respect to the equity-ratio that is equal to $\frac{\kappa}{\theta}\simeq 0.081$, which is only moderate.

The OLS-estimates for the elasticity with respect to equity is slightly lower than the PFM-OLS-estimate. This is quite in line with what one would expect: High realizations of z induce losses and will therefore lower the equity-ratio. Other explanations, such as older firms being less productive and more equity financed are mostly controlled for by the fixed effect. Although there are differences among subsample estimates these are insignificant,

[36] Note that obviously, if financial frictions determine the cost of capital, our measure of costs measures with an error. However, the residual is just explained by the parameter of equity in our regression.

[37] If e exhibits lag-dependency, however, the parameter-estimate of β may be biased. In section 4.3 this issue is discussed in more detail using the German dataset. For the German data, we show the bias from lag-dependency to be negative, if present.

Table 2: Estimates from the cointegration regression
(PFM-OLS, Within)

full sample				By industries[c]				
	κ	θ	$\sum T_i n_i$	Ind.		κ	θ	$\sum T_i n_i$
PFM-OLS[a]	0.079	0.98	5944	21	PFM	0.191	1.73**	471
prelim. OLS	0.070	1.08			OLS	0.136	1.340	554
std. err. I(1)	0.026	0.07		26	PFM	0.108	1.119	760
std. err. I(0)[b]	0.035	0.14			OLS	0.097	0.738	897
OLS	0.082	0.71	7147	33	PFM	0.020	0.32**	836
					OLS	-0.019	0.075	1065
				36	PFM	0.164	0.957	534
					OLS	0.209	0.443	626
				48	PFM	-0.037	0.991	410
					OLS	-0.047	0.570	493
				50	PFM	0.296	0.627	378
					OLS	0.156	0.637	482

[a]Only observational units have been used for which (outliers removed) 5 or more observations are available.
[b]The standard errors are obtain as panel analogues to Phillips (1995, p. 1033ff).
[c]Full industry-sample OLS estimates are reported in brackets ( ).
** Significantly different from the remaining-sample-estimate at the 5%-level - based on PFM-OLS estimate under the assumption of an I(1)-regressor.

except for two estimates. Therefore, we conclude the model to be reasonably specified as a homogeneous panel.

Compared to the estimates of Caballero et al. (1995) the PFM-OLS estimates of the cost-elasticity are much closer to its neoclassical-theory value of 1. Moreover, the variation between the industry specific estimates is slightly smaller.

With the regression estimates it is now possible to construct a time series $\widehat{z}_{it}$ for each firm. The series is constructed using all 7147 observations.[38]

3.5 Investment behavior

3.5.1 Density and conditional expectations estimates

Since the structure of our investment model is highly nonlinear and the functional forms of the profit function and distributions involved are not quite clear, a nonparametric estimation is most fruitful.[39]

[38] However, 3% of the observations were discarded as outliers with respect to the within transformed investment-rate. We do so by removing the top 0.5 % observations and the bottom 2.5 %.

[39] The non-parametric estimates presented in this section were generated in autumn 2000. At that time, computational effort of the variable window width estimator still played some role . This forced us to rely on less computational intense estimation techniques, such as k-nn estimators. At the time, the estimates presented in section 4.3 were generated (Spring 2003), this issue was of much lesser importance, as the computer system available then was about 10 times faster.

Figure 2 shows the distribution of within transformed investment rates conditional on the capital-imbalance $\widehat{z}$. The distribution has been estimated with a normal-kernel estimator and a fixed window width of $0.12 \times n^{-\frac{1}{6}}$, both z and i have been standardized. The relationship is as expected: Firms with lower z invest more. More surprisingly and in line with the model the distribution of investment rates has two peaks for high mandated investments.

For firms with an unproductive capital stock, we find three peaks in the distribution, one where no disinvestment occurs, one with partial (unconstrained optimal) disinvestment and one peak for nearly full disinvestment, being again perfectly in line with the model. However, these findings are on the boundary of the support, where the estimator becomes less reliable.

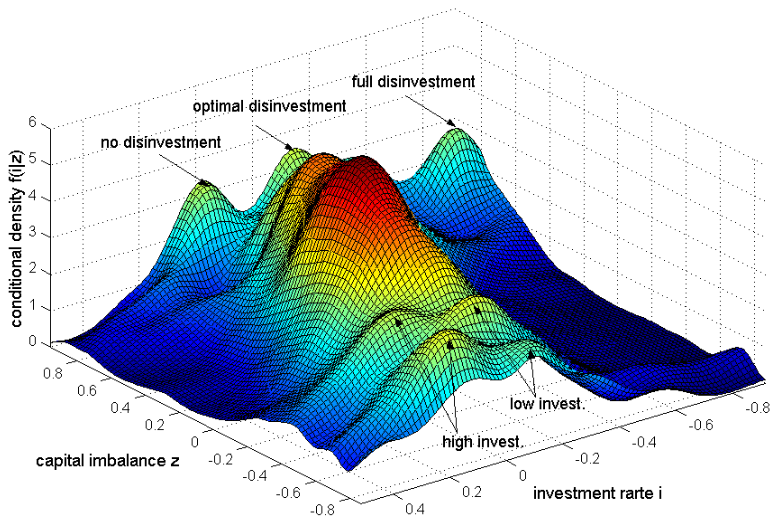


Figure 2: Density of the investment-rate distribution conditional on z

However the distributional findings are not completely supportive: Most firms do by far not remove their capital-imbalance completely in case they invest. One explanation for this could be time-to build constraints which cause investment to be spread over two accounting periods. Another explanation is that many of the firms in our sample are large firms. These are themselves aggregates of many establishments. Therefore, given z, the model presented above would yield an approximate investment rate of $\frac{n}{m} i(e, z)$ for a company with m independent establishments of which n establishments adjust. The distribution of n would then be given by a binomial distribution B with:[40]

$$P(n|z, e) = B(n|m, G(\Omega(e, z))$$

When the number m of establishments per firm is large enough we would expect to hardly ever see zero investment rates for larger z in our model.

Moreover, if heterogeneous capital goods indeed matters, aggregation over these goods will result in a downward bias of second order derivatives. (Goolsbee and Gross, 2000).

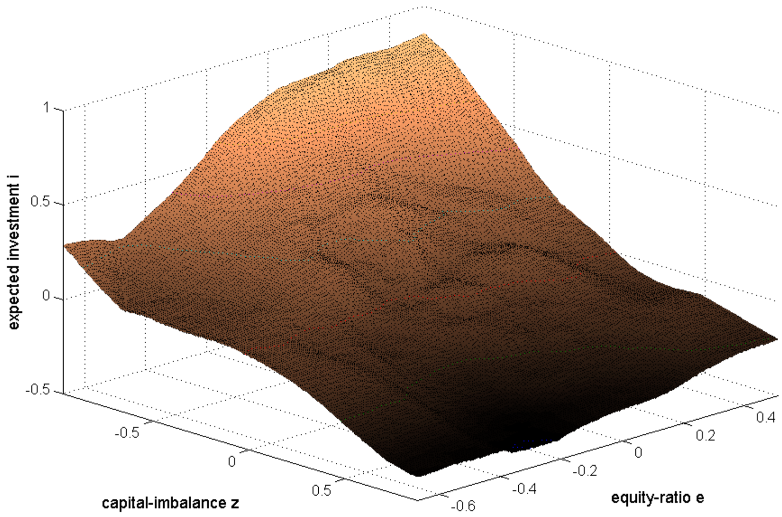


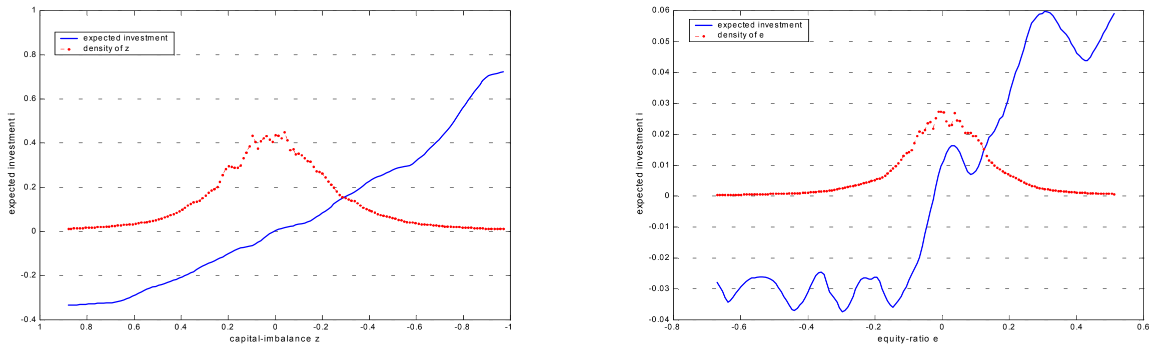
Figure 3: Expected investment-rate i conditional on e and z (demeaned)

Figure 3 presents the expected investment rates conditional on the log equity-to-capital ratio and mandated investment. For the estimation a 360-nearest neighborhood (k-NN), local linear, Epanechnikov-kernel estimator has been used. The choice of $k = 360$ is approximately equivalent to twice the fixed window-width used for density estimation.[41] The window-width has been chosen by "eye-balling", but tends to slightly undersmooth the conditional expectation estimator on the center of the support. Especially for second order-derivative estimation, this problem becomes much more apparent. Therefore, when estimating second-order derivatives (see below), a combination of 720-nearest-neighborhood and fixed window-width (of $h = 4n^{-1/10}$) kernel is used. This effectively approximates variable window-width

[40] Note that, investment decisions in heterogeneous capital goods as in Goolsbee and Gross (2000) or for different establishments or plants of one company would - strictly speaking - not be independent in our model, as investment decisions are linked through their effect on the balance sheet.

[41] The optimal k for k-NN estimators and the optimal window-width h are linked by $k = nh^{4/6}$ in the two dimensional case (Pagan and Ullah, 1999, p. 91).

Figure 4: Estimates for the generalized additive model



(a) $m_1(z)$ (b) $m_2(e)$

estimators, but is computationally much faster.

Figure 3 shows, that the investment function is nonlinear in a manyfold sense: For a large equity-ratio, investment is a convex function of the capital-imbalance, highly indebted firms investment mostly only disinvest and for these firms investment is concave in fundamentals. In our model this is a result of the concavity of the earnings-function and the fixed costs. A higher equity-ratio raises investment, but only when fundamentals allow so.[42] Therefore, this results supports our pecking-order, fixed-adjustment-cost model. Moreover, for very low productivity equity-ratio first has to reach some threshold to influence investment, i.e. to stop sharp disinvestment. Again this is very much in line with our pecking-order, fixed-cost model. And as a last (but not really surprising) deviation from the linear model, the second order derivative of equity seems to be negative, at least for low z.

To better visualize the effect of ignoring the important interaction of equity and capital-imbalance and thus the non-linear structure for investment, the generalized additive model

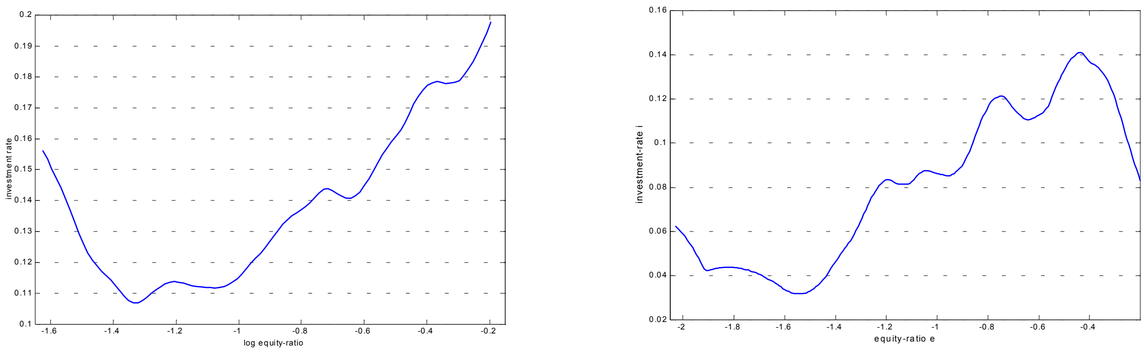
$$\mathbb{E}[i|(e,z)] = m_1(z) + m_2(e) \tag{31}$$

has also been estimated.

Figures 4(a) and (b) now present the estimates for the generalized additive model. As the identifying restriction in a generalized additive model is arbitrary, we employ different restrictions for figures 4(a) and 4(b): While in figure 4(a) $\mathbb{E}[m_2(e)] = 0$ was used, in figure 4(b) $\mathbb{E}[m_1(z)] = 0$ is chosen. This makes it easier to visualize the relative changes in the investment rate caused by changes in z and e. Still there is some sign of non-linearity of investment in z. However, this is very minor, as in general the function is close to linear. Clearly this shows, how misleading it can be if one neglects (non-linearity and) cross-effects.

[42] Note that we have not included any long-run effect of equity in generating the capital imbalance estimates.

Figure 5: Generalized additive model, untransformed data



(a) full sample: $m_2(e)$ (b) restricted sample: $m_2(e)$

This bias becomes even worse for the estimate of $m_2(e)$ in figure 4(b).

For low equity-ratios there is *no* effect, whereas for high equity-ratios the effect is clearly positive. However, we have seen a clearly positive effect for low equity as soon as productivity is high enough. Moreover from figure 3, this effect is *decreasing* if equity rises, while in figure 4(b) it is *increasing* in e. This no-effect result, when equity is low, has appeared in earlier studies in the form of *apriori* as financially constrained considered firms reacting less on increases in cash flow (Kaplan and Zingales, 1997). Therefore, figure 4(b) sheds some new light on these results, too: It may well be that a misspecification bias drives this evidence that has been brought forward as an argument against financial constraints playing a prominent role in investment decisions.

The effect of the within transformation can be seen, when comparing figures 4(a) and (b) to figure 5(a) and 5(b), where we report the estimates without transformation. This allows to use the full dataset and not only firms with at least five observations, therefore, the full dataset has been used for the estimates in figure 5(a). For figure 5(b) the restricted dataset was used.

Interestingly, the effect seems to be U-shaped. This could be due to some underlying economic structure or to the unreliability of nonparametric estimates near the boundary of the support. If there is an economic explanation for this, it has to rely on firm- or industry-characteristics of the very indebted firms. If there is an economic explanation, it has to rely on firm- or industry charcteristics of the very indebted firms. Yet, an exact analysis is beyond the scope of this paper.

3.5.2 Average derivative estimators

A major drawback of the nonparametric estimates are their wide confidence bounds. Thus, to draw more reliable conclusions, it is necessary to estimate average derivatives of $i(z, e) =$

$\mathbb{E}(i|z,e)$ directly. This yields much closer confidence bounds because nonparametric average-derivative estimators converge with parametric rates of convergence (Rilstone, 1991). The estimates are reported in tables 3 and 4.

Several nonparametric estimators for the average derivative are available in our panel data setting (Ullah and Roy, 1998). We concentrate on derivative estimates from a local linear estimation which is basically a locally weighted version of an OLS estimator and derived from a first order Taylor expansion around the point (z,e), at which the function is estimated, i.e.

$$i_{jt} = \mathbb{E}(i|z,e) + (z_{jt} - z)\, b_z\,(z,e) + (e_{jt} - e)\, b_e\,(z,e) + u_{jt} \quad . \tag{32}$$

From this regression, we can numerically generate $b^* := \partial_{(z,e)} \widehat{\mathbb{E}}(i|z,e)$ or alternatively take $b^{**} := \left(\widehat{b}_z, \widehat{b}_e\right)$ as their direct estimates. The weights are generated according to the kernel-function chosen. Both estimators are asymptotically normally distributed, but have a different variance. Moreover, in small samples the bias is different and they perform differently. In most cases the numerical estimator b^* should be preferable (Ullah and Roy, 1998). However, a drawback of this estimator is that the variance of its averaged form is not yet known (Pagan and Ullah, 1999). Both estimators use the within transformed data for the regressions and for evaluating the kernel.

In our panel data setting, additionally the fixed effects estimator of Ullah and Roy (1998) is available.[43] This estimator b^{FE} uses the within transformed data for the regression but calculates the weights with the kernel on the basis of the original data. The advantage of this estimator is that assumption 5 is no longer needed to let the estimates be interpretable in a sensible manner. All we need to assume is the investment function to exhibit fixed idiosyncratic and time effects and to be otherwise homogenous among the firms in the original (not within transformed) quantities. As we will see below, results do not qualitatively depend on the estimator chosen.

The average derivative estimators are generated as the mean of the pointwise estimates. Average-derivatives over a subset of observations are calculated by using the conditional mean (conditional on the observation falling into the subsample) and not by re-estimating for the subsample.

The cross- and higher-order-derivatives are computed as numerical estimates of these quantities using (32). As for the first-order derivative numerical estimator the asymptotic variance is not yet known.

For the second-order-derivatives the undersmoothing of the nearest neighborhood estimator in the center of the distribution becomes a more apparent problem. Therefore, we generate the weights (kernel) in the local linear regression as average of a 720 nearest-neighborhood

[43] Note that we also use within-transformed data in estimating (32).

Table 3: Average first-order derivatives of the investment rate $i(e,z)$

Number of observations: $N = 6950$					std. deviation
derivative	$\overline{b}^{*}$	$\overline{b}^{**}$	$\overline{b}^{FE}$	$\overline{b}^{*}_{720,fix}$ [a]	for $\overline{b}^{**}$
$-\frac{\partial \widehat{i}}{\partial z}$	0.5057	0.5146	0.5341	0.5137	0.0068
$\frac{\partial \widehat{i}}{\partial e}$	0.1555	0.1588	0.1782	0.1450	0.0097

[a] estimated with a mixture of fixed window-width and 720-NN-Kernel.

Epanechnikov kernel and a fixed window-width Epanechnikov kernel with $h = n^{-1/10}$. This practically gives a variable window-width, which is neither too small on the border nor on the center of the support, so that the estimator should not be too heavily under- or oversmoothed. Furthermore and within a certain range, window-width selection has been reported to be of a lesser issue for average derivative estimators, as they are generally not too sensitive to window-width selection,[44] as long as the averages are not dominated by a few outliers.

3.5.3 Average derivative estimates

The average derivative estimates for all four estimators are reported in table 3. Although the various estimators yield quantitatively slightly different estimates, they qualitatively do not differ: Both the equity-ratio and the capital-imbalance have a significant effect. Moreover and more surprisingly, the derivative with respect to the equity-ratio is about twice as large as the elasticity of the optimal stock of capital with respect to the equity-ratio. The difference is both economically and statistically significant. This also holds true if one requests higher levels of significance to account for the dependence of the short-run and long-run estimates.[45] Therefore, finance matters both in the short and in the long-run but influences the frequency of adjustment much stronger, than the optimal stock of capital.

Table 4 reports the higher-order derivative estimates and reveals some interesting results as well: We find a clear evidence for convexity in z for both the total sample average and the local averages over low z. Moreover and as predicted by the theoretical model, for low z the cross-derivative is clearly negative. Therefore, all empirical results are in line with the investment model that includes fixed adjustment cost and capital market imperfections.

Hence, we can conclude that all empirical results are in line with the investment model that includes fixed adjustment cost and capital market imperfections.

[44] See Pagan and Ullah (1999).

[45] Taking the standard deviations as reported above and the asymptotic distributions as approximation, the 5% one-sided upper confidence bound for β is 0.1368, while the lower 5% one-sided confidence bound for $\overline{b}_{e}^{**}$ is 0.1425. If both estimators were independent, this would of course equal an one-sided 0.25% test.

Table 4: Average second-order derivatives of the investment rate $i(e, z)$

derivative	full Sample[a]	low z[b]	high e[c]
$\frac{\partial^2 i}{\partial z^2}$	0.2931	0.3838	0.3565
$\frac{\partial^2 i}{\partial e^2}$	0.1079	-0.106	-0.150
$\frac{\partial^2 i}{\partial e \partial z}$	0.1078	-0.123	0.2257
N	6922	2284	2353

[a] e and z within the 0.5% and 99.5% quantiles .

[b] e within the 0.5% and 99.5%, z within the 0.5% and 33% quantiles.

[c] z within the 0.5% and 99.5%, e within the 66% and 99.5% quantiles.

4 Conclusion

In this paper a model of investment that incorporates an imperfect capital market and fixed investment costs was presented. Even though no closed form was deduced and some questions of stability in the sense of stationarity are still open to further research some new results were obtained by analyzing that model. One major result was to identify the difference between a short-run effect of liquidity on the frequency of investment and a long-run effect on the optimal stock of capital. While the standard agency or oligopoly models of investment and finance[46] predict only a strong influence of liquidity on the chosen stock of capital, the model in this paper rather suggests a strong short-run effect. This stronger short-run than long-run influence of the equity-to-capital ratio on investment, is exactly what is empirically found, so that empirical evidence supports our theoretical model.

Furthermore, the investment rate is empirically a highly nonlinear function of the capital-imbalance (investment opportunities) and equity (liquidity). Thus even only imposing additive separability leads to a severe error. This error could well be the cause of the puzzling finding reported in the literature that "*apriori* unconstrained" firms react stronger to changes in their financial variables than constrained ones. For further empirical research, this therefore suggest a need to estimate investment equations in a generalized error correction framework[47] as done in this paper.

The econometric approach taken in this paper to estimate the short-run dynamics (or error correction function) has been a completely nonparametric one. Even though the findings suggest a nonlinear investment function, a nonlinear, but (more) parametric approach might produce additional insights. Especially in discriminating between a transaction cost of finance model and a fixed adjustment cost model, the approach taken seems only a first step. More research is needed to clarify the quantitative importance of both.

[46] See e.g. Myers (1977) or Brander and Lewis (1986).

[47] See e.g. de Jong (2001).

Our model also has important policy implications: Suppose there are shocks to the balance sheet positions of firms (e.g. through exchange rates as in Céspedes et al. (2000), Aghion et al. (2001) or Devereux and Lane (2001)), then this paper's model predicts a much stronger short run real impact of these shocks than the usual financial accelerator model of Bernanke et al. (1998). Moreover, this impact will depend on the position of the economy along the business cycle. Thus—if fixed adjustment cost are present—policies that influence the balance sheet (shocks) will be differently rated along the business cycle and also differently compared to the convex adjustment cost framework. Therefore e.g. the central bank not only should try to observe the distribution of capital-imbalances, but also consider the financial situation of firms, in order to predict policy implications. More specifically both effects cannot be considered separately as the magnitude of each effect depends on the state of the other variable.

Another example for policy implications would be a (corporate) tax reform. There the implications on the costs of retaining earnings have to be taken into account,too, since a rise in the average equity-to-capital ratio, not only raises investment in the short run, but would also - at least in the partial model presented here - increase efficiency, as average (absolute) capital-imbalances would fall.

5 Appendices

In this appendix, we derive the Bellman equation which is central to our model. Thereafter, we show the existence and uniqueness of a solution to this equation. At the end of this appendix, some properties of the induced optimal-policy function are discussed.

5.1 Deriving the Bellman Equation

All variables, functions etc. are defined as in the main text, unless stated differently. The correspondence, X, of financial feasible capital-imbalance and debt pairs is given by:

$$\begin{aligned} X\left(K_t^*, w_t, z_t, B_t\right) &= \left\{ \begin{array}{c} k_t, B_{t+1} \in \mathbb{R}_+^2 | D(k_t, B_{t+1}, K_t^*, w_t, z_t, B_t) \geq 0 \\ \wedge B_{t+1} \leq \hat{b} k_t K_t^* \end{array} \right\} \\ &= \left\{ k_t, B_{t+1} \in \mathbb{R}_+^2 \left| \begin{array}{c} (1 + r(\frac{B_t}{K_{t-1}}))B_t - \Pi(k_t, K_t^*)[1 - w_t \mathbb{I}_{\{k_t \neq z_t\}}] \\ +K_t^*(k_t - z_t) \leq B_{t+1} \leq \hat{b} k_t K_t^* \end{array} \right. \right\} \end{aligned} \tag{33}$$

Dividing the expression by K_t^* and using $b_t := \frac{B_t}{K_{t-1}}$ and for the stock of capital before investment $K_{t-1} = \frac{K_t}{1-\delta}$ yields:

$$\begin{aligned} &\hat{X}\left(K_t^*, w_t, z_t, b_t\right) := \\ &= \left\{ k_t, b_{t+1} \in \mathbb{R}_{++} \times \mathbb{R}_+ \left| \begin{array}{c} \frac{(1+r(b_t))}{1-\delta} b_t z_t - \pi(k_t)[1 - w_t \mathbb{I}_{\{k_t \neq z_t\}}] \\ +(k_t - z_t) \leq b_{t+1} k_t \leq \hat{b} k_t \end{array} \right. \right\} \cup \{(0,0)\} \end{aligned} \tag{34}$$

To obtain a more accessible form define e_t to be the equity-ratio in the opening balance and thus

$$e_t := e(b_t) = 1 - \frac{(1 + r(b_t))}{1 - \delta} b_t.$$

Define furthermore $c(z, b)$ to be the cash flow per unit of capital (including cash flow from newly issued debt, and "costs" for "buying back" the capital stock), that is

$$c_t := c(k_t, b_{t+1}) = \frac{\pi\left(k_t\right)}{k_t} + \left(b_{t+1} - 1\right).$$

We then get for $\hat{X}$:

$$\begin{aligned} &\hat{X}\left(K_t^*, w_t, z_t, b_t\right) = \\ &\left\{ k_t, b_{t+1} \in \mathbb{R}_{++} \times \mathbb{R}_+ \left| \begin{array}{c} e(b_t) \frac{z_t}{k_t} + c(k_t, b_{t+1}) - w_t \mathbb{I}_{\{k_t \neq z_t\}} \frac{\pi(k_t)}{k_t} \geq 0 \\ \wedge \; 0 \geq \left(b_{t+1} - \hat{b}\right) \end{array} \right. \right\} \cup \{(0,0)\}. \end{aligned} \tag{35}$$

Next define

$$Y\left(w_t, z_t, e_t\right) := \left\{ k_t, b_{t+1} \in \mathbb{R}_{++} \times \mathbb{R}_+ \;\middle|\; \begin{array}{c} e_t \frac{z_t}{k_t} + c(k_t, b_{t+1}) - w_t \mathbb{I}_{\{k_t \neq z_t\}} \frac{\pi(k_t)}{k_t} \geq 0 \\ \wedge\ 0 \geq \left(b_{t+1} - \hat{b}\right) \end{array} \right\}. \quad (36)$$

For $e_t = e(b_t)$ we have $\hat{X} = Y \cup \{(0,0)\}$.

Lemma 2 (Lemma 1 main text) *(a) Y is non-empty and*

(b) employing zero capital is suboptimal, i.e.

$$\max_{(k_t, b_t) \in Y} \tilde{v}(k_t, b_t) - \pi(k_t) w_t \mathbb{I}_{\{k_t \neq z_t\}} > \psi \mathbb{E}_t[v(w_{t+1}, 0, 0) > 0$$

Proof. *(a) Obviously, $e_t z_t \geq 0$ holds, so it is sufficient to show, that*

$$\exists k^*(w_t) : k^* - \pi(k^*)[1 - w_t \mathbb{I}_{\{k \neq z_t\}}] \leq 0.$$

Because $\lim_{x \to 0} \pi'(x) = +\infty$ and $\frac{\pi(x)}{x} \geq \pi'(x)$ since π is concave , this "self-financing" k^* always exists.

(b) Using $k^*$from part (a) a firm can always pay out a larger dividend than e_t and can also set $b_t = 0$ as well. By paying a larger dividend in the current period and having the same debt as if it was to stop production but with a larger stock of capital, the expected value for $t+1$ must be larger than $E_t[v(w_{t+1}, 0, 0)]$, so that

$$\tilde{v}(k^*, 0, w_t, z_t, b_t) > \psi \mathbb{E}_t[v(w_{t+1}, 0, 0)]$$

follows. Because the plan $\forall t : k_t = k^*(w_t)$ is always feasible and leads to positive dividends, $v(\cdot)$ must be bounded from below by a positive real number, so that $\psi \mathbb{E}_t[v(w_{t+1}, 0, 0)] > 0$. ∎

Now denote the value-function by V. For notational convenience define $Y := Y\left(w_t, z_t, e(b_t)\right)$.

Then V is determined by the following Bellman equation:

$$
\begin{aligned}
& V(K_t^*, w_t, z_t, b_t) \\
& \quad := \max_{(k_t,b_{t+1})\in X(K_t^*,w_t,z_t,b_t)} \left\{ D(k_t, b_{t+1}, K_t^*, w_t, z_t, b_t) + \psi \mathbb{E}_t \left[V(K_{t+1}^*, w_{t+1}, z_{t+1}, b_{t+1}) \right] \right\} \\
& \quad = K_t^* \max_{\substack{(k_t,b_{t+1}) \\ \in Y}} \left\{ e(b_t)z_t + c(k_t, b_{t+1})k_t - w_t \mathbb{I}_{\{k_t \neq z_t\}} \pi(k_t) + \psi \mathbb{E}_t \left[\frac{V(K_{t+1}^*, w_{t+1}, z_{t+1}, b_{t+1})}{K_t^*} \right] \right\} \\
& = K_t^* \left[e(b_t)z_t + \max_{\substack{(k_t,b_{t+1}) \\ \in Y}} \left\{ \begin{array}{c} c(k_t, b_{t+1})k_t - w_t \mathbb{I}_{\{k_t \neq z_t\}} \pi(k_t) + \\ +\psi \mathbb{E}_t \left[\left(\frac{V(K_{t+1}^*, w_{t+1}, z_{t+1}, b_{t+1})}{K_{t+1}^*} - e(b_{t+1})z_{t+1} + e(b_{t+1})z_{t+1} \right) \frac{K_{t+1}^*}{K_t^*} \right] \end{array} \right\} \right] \\
& = K_t^* \left[e(b_t)z_t + \max_{\substack{(k_t,b_{t+1}) \\ \in Y}} \left\{ \begin{array}{c} c(k_t, b_{t+1})k_t - w_t \mathbb{I}_{\{k_t \neq z_t\}} \pi(k_t) + \psi e(b_{t+1})k_t(1-\delta) \\ +\psi \mathbb{E}_t \left[\left(\frac{V(K_{t+1}^*, w_{t+1}, z_{t+1}, b_{t+1})}{K_{t+1}^*} - e(b_{t+1})z_{t+1} \right) \frac{k_t(1-\delta)}{z_{t+1}}) \right] \end{array} \right\} \right]
\end{aligned} \tag{37}
$$

The first equality follows from the linear homogeneity in K^* of the function D and the linearity of the $\mathbb{E}_t-$operator, the second equality stems from the fact, that $e(b_t)z_t$ is no function of (k_t, b_{t+1}) and is thus not affected by the maximization. The third equality is obtained by some simple term replacements. (3) yields $\frac{K_{t+1}^*}{K_t^*} = \exp(d + \xi_{t+1})$ and $k_t(1-\delta) = z_{t+1} \cdot \exp(d + \xi_{t+1})$. Now define $v_t := \frac{V(K_t^*, w_t, z_t, b_t)}{K_t^*} - e(b_t)z_t$. Note that v does not depend on K_t^*, due to the homogeneity of V. Thus we obtain:

$$
v(w_t, z_t, e(b_t)) := \max_{\substack{(k_t,b_{t+1})\in \\ X(1,w_t,z_t,b_t)}} \left\{ \begin{array}{c} c(k_t, b_{t+1})k_t - w_t \mathbb{I}_{\{k_t \neq z_t\}} \pi(k_t) + \psi e(b_{t+1})k_t(1-\delta) \\ +\psi \mathbb{E}_t [v(w_{t+1}, k_t \frac{(1-\delta)}{\exp(d+\xi_{t+1})}, b_{t+1}) \exp(d + \xi_{t+1})] \end{array} \right\} \tag{38}
$$

Maximizing v is by definition equivalent to maximizing V.

5.2 Existence and uniqueness

From now on time-indices will be suppressed. Due to Lemma 1 we can drop "no production" from the set of alternatives X and express the value function v as

$$
v(w, z, e) = \max_{(k,b)\in Y(w,z,e)} \left[\begin{array}{c} c(k, b) - w\pi(k) I_{\{k \neq z_t\}} + (1-\delta)\psi e(b)\, k \\ +\psi \iint \left[v\left(\epsilon, k \frac{1-\delta}{\exp(d+\xi)}, e(b) \right) \exp(d+\xi) \right] dF(\xi) dG(\epsilon) \end{array} \right] \tag{39}
$$

or respectively as:

$$
v(w, z, e) = \left\{ \begin{array}{cl} \max \{ V_{no \text{ adj}}(z, e), V_{\text{adj}}(w, z, e) \} & \text{for } Z \neq \emptyset \\ V_{\text{adj}}(w, z, e) & \text{for } Z = \emptyset \end{array} \right. \tag{40}
$$

with

$$
\begin{aligned}
V_{no\text{ adj}}(z,e) \quad &: \quad = \max_{b\in Z}\left\{\begin{array}{c} c(z,b)+(1-\delta)\psi e(b)z \\ +\psi\iint\left[v\left(\epsilon, z\frac{1-\delta}{\exp(d+\xi)}, e(b)\right)\exp(d+\xi)\right]dF(\xi)dG(\epsilon)\end{array}\right\} \\
V_{\text{adj}}(w,z,e) \quad &: \quad = \max_{\substack{(k_t,b_{t+1})\\ \in Y(w,z,e)}}\left\{\begin{array}{c} c(k,b)-w\pi(k)+(1-\delta)\psi e(b)k \\ +\psi\iint\left[v\left(\epsilon, k\frac{1-\delta}{\exp(d+\xi)}, e(b)\right)\exp(d+\xi)\right]dF(\xi)dG(\epsilon)\end{array}\right\}.
\end{aligned}
$$

Assumption A.6: $\mu_\xi := \psi E[\exp(\xi+d)] < 1.$[48]

Lemma 3 *Consider the operator T defined by posing $(Tv)(w,z,e)$ equal to the right hand side of (39). This operator is defined on the set B of all real-valued, a.e. continuous and bounded functions with domain $D=[0,1]\times\mathbb{R}_+\times\mathbb{R}_+$*
Then the mapping T (a) preserves boundedness, (b) preserves continuity a.e., and (c) satisfies Blackwell´s conditions.

Proof. *(a) To show that T preserves boundedness, one has to show that for any bounded function u $(Tu)(\cdot)$ is bounded.*
Consider $u\in B$, that is bounded from above by $\overline{u}$ and bounded from below by $\underline{u}$, then$(Tu)(\cdot)$ is bounded from above because

$$
\begin{aligned}
(Tu)(w,z,e) &\le \mu_\xi\overline{u} + \sup_{(k,b)\in Y(w,z,e)}\left\{c(k,b)-w\pi(k)\mathbb{I}_{\{k\neq z_t\}}+(1-\delta)\psi e(b)k\right\} \\
&\le \mu_\xi\overline{u} + \sup_{0\le k, 0\le b\le\widehat{b}}\left\{c(k,b)+(1-\delta)\psi e(b)k\right\} \\
&= \mu_\xi\overline{u} + \sup_{0\le k, 0\le b\le\widehat{b}}\left\{[1-\psi(1+i(b))]bk+\pi(k)-(1-\psi(1-\delta))k\right\} \\
&\le \mu_\xi\overline{u} + \sup_{0\le k, 0\le b\le\widehat{b}}\left\{(1-\psi)k+\pi(k)-(1-\psi(1-\delta))k\right\} \\
&= \mu_\xi\overline{u} + \sup_{0\le k}\left\{\pi(k)-\psi\delta k\right\}
\end{aligned}
$$

where the supremum is bounded, because $\pi(k)-\psi\delta k$ obtains its maximum, as

$$
\lim_{k\to 0}\pi(k)-\psi\delta k > 0 > \lim_{k\to\infty}\pi(k)-\psi\delta k
$$

[48] This assumption is equivalent to assumption A.6 in Caballero and Engel (199, p. 811). Assume ξ is normally distributed with variance σ^2. Then this assumption is equivalent to $\exp\left(d+\frac{\sigma^2}{2}\right) < 1+r$. Approximately, this is $r-d > \frac{\sigma^2}{2}$. Economicly this means that productivity and hence value of a given stock of capital grows at a smaller rate than the market rate of return.

Suppose, this assumption would not hold and neglect adjustment costs for the moment. It is easy to see that a firm could obtain infinite expected value by choosing a stock of capital that is small enough to reproduce its depreciation plus the interest rate in the first period. In the next period it can be expected, that this stock of capital (depreciated capital replaced) generates a positive profit, which grows at a larger rate than the interest rate.

In this sense, assumption 1 is an equilibrium condition for the capital-market.

by assumption. The third inequality follows from $i(b) \geq 0, 0 \leq \widehat{b} < 1$.
That $(Tu)(\cdot)$ is bounded from below follows from

$$\begin{aligned}(Tu)(w,z,e) &\geq \mu_\xi \underline{u} + \sup_{(k,b)\in Y(w,e,z)} \left\{c(k,b) - w\pi(k)\mathbb{I}_{\{k\neq z_t\}} + (1-\delta)\psi e(b)\,k\right\} \\ &> \mu_\xi \underline{u}\end{aligned}$$

where the last inequality follows straightforward from the proof of Lemma 3.1.
(b) First note that for every u, that is continuous a.e. the parameter integrals in (40) are continuous. Then $(Tu)(w,z,e)$ is continuous because of Berge´s Theorem since Y is a continuous correspondence and Z is continuous except for $(e,z) \in A := \{(m,n)|m = (1-\widehat{b}) - \frac{\pi(n)}{n}\}$. But as A is a curve in R^2 it has measure of 0, so (Tu) is continuous a.e.
(c) To show that T satisfies Blackwell´s conditions, one first notes that if $f_1, f_2 \in B$ and if $\forall (w,z,e) \in D : f_1(w,z,e) \leq f_2(w,z,e)$, then (because $\exp(d+\xi) > 0$) the expected value in (40) preserves the inequality, and so does the max-function. Thus

$$(Tf_1)(w,z,e) \leq (Tf_2)(w,z,e)$$

A simple calculation shows, that

$$(Tf + a)(w,z,e) = (Tf)(w,z,e) + \mu_\xi a$$

Assumption A.6 now yields the second Blackwell condition. ■

Proposition 1 *Equation (39) has exactly one solution (which belongs to B).*

Proof. *Lemma A.1 yields that T defines a contraction mapping on the metric space B with a modulus strictly smaller than one. The existence and uniqueness now follows from the contraction mapping theorem (See Theorem 3.2 in Stockey, Lucas and Prescott, 1989)* ■

5.3 Optimal policy

Now define the following functions related to the solution of the Bellman equation of $v(w,z,e)$ in the main text.

$$J(z,b,w) := c(z,b) - w\pi(z) + (1-\delta)\psi e(b)\,z + I(z,b) \tag{41}$$

$$I(z,b) := \psi \iint \left[v\left(\epsilon, z\frac{1-\delta}{\exp(d+\xi)}, e(b)\right)\exp(d+\xi)\right] dF(\xi)dG(\epsilon) \tag{42}$$

Lemma 4 *The function $J(z,b,w)$ is bounded from above, so that*
$\sup_{(k,b)\in Y(w,e,z)} J(z,b,w)$ *is finite.*

Proof. *As v satisfies the Bellman-equation it must be bounded. However, since*

$$v(w,z,e) = \max_{(k,b)\in Y(w,e,z)} J(k,b,w) + w\pi(k)\mathbb{I}_{\{k=z\}} \geq \begin{cases} \max\limits_{(k,b)\in Y(w,z,e)} J(k,b,w) \\ \max\limits_{b\in Z(z,e)} J(z,b,w) + w\pi(z) \end{cases}$$

always hold, J must be bounded, too. ■

Corollary 1 *$\widehat{J}_{\max}(z,e) := \sup\limits_{b\in Z(z,e)} J(z,b,w)+w\pi(z)$ and $J_{\max}(w,z,e) := \sup\limits_{(k,b)\in Y(w,z,e)} J(k,b,w)$ are finite, too.*

Lemma 5 *(a) J and $J_{\max}$ are strictly monotonously decreasing in w.*
(b) $J(z,e,w)+w\pi(z)$ is independent of w,
(c) $J_{\max}$ and $\widehat{J}_{\max}(z,e)$ (and therefore v, too) are monotonously increasing in e.

Proof. *(a) For any $w_1, w_2 \in [0;1] : w_1 < w_2$ we have:*

$$\begin{aligned} J(z,b,w_2) &= c(z,b) - w_2\pi(z) + (1-\delta)\psi e(b)\, z + I(z,b) \\ &< c(z,b) - w_1\pi(z) + (1-\delta)\psi e(b)\, z + I(z,b) \\ &= J(z,b,w_1) \end{aligned}$$

And since $Y_2 := Y(w_2,z,e) \subset Y(w_1,z,e) =: Y_1$, we get:

$$\begin{aligned} J_{\max}(w_2,z,e) &= \max_{(k,b)\in Y_2} J(k,b,w_2) \leq \max_{(k,b)\in Y_1} J(k,b,w_2) \\ &< \max_{(k,b)\in Y_1} J(k,b,w_1) = J_{\max}(w_1,z,e) \end{aligned}$$

(b) follows directly from the definition of J,
(c) since both Z and Y strictly grow with e this follows straightforward. ■

Proposition 2 *Define for $Z \neq \emptyset$ as an implicit function $\overline{w}(z,e)$ by*

$$J_{\max}(\overline{w},z,e) - \widehat{J}_{\max}(z,e) = 0$$

Then firms adjust if their current adjustment cost factor w is smaller than $\overline{w}(z,e)$ or if $Z = \emptyset$.

Proof. *That a unique $\overline{w}(z,e)$ equating $J_{\max}$ and $\widehat{J}_{\max}$ exists, follows from $J_{\max}(0,z,e) \geq \widehat{J}_{\max}(z,e)\ \forall(z,e)$ together with the monotonicity of $J_{\max}$ in w.*
As argued in the main text firms adjust if $Z = \emptyset$ or $J_{\max}(w,z,e) > \widehat{J}_{\max}(z,e)$. Since $J_{\max}$ is monotonously decreasing in w this inequality holds if and only if $w < \overline{w}$. ■

Proposition 3 *$J(z,b,w)$ is analytic and thus the set Q of $(k,b) \in \widehat{Y}(w,e) \cap \{k \geq k^*\}$*

(Lemma 3.1) such that $J(k,b,w) = J_{\max}(w,z,e)$ is a non-empty set with a finite number of points.

Proof. *To show that J is analytic it suffices to show that I is analytic. Therefore note that I can be written as a convolution of an a.e. continuous function and a normal density:*

$$\begin{aligned} I(z,b) &:= \psi \int K(z,b,\xi)dF(\xi) \\ K(z,b,\xi) &:= \int \left[v\left(\epsilon, z\frac{1-\delta}{\exp(d+\xi)}, e(b)\right) \exp(d+\xi)\right] dG(\epsilon) \end{aligned}$$

However, as K is integrable, the convolution of K and a normal density is analytic (see e.g. Theorem 9 on p. 59 in Lehmann (1986)).
As J is analytic is must be continuous, too. Since $Y(w,z,e)$ is compact this ensures that J attains its maximum within a non-empty compact set Q. Since J is analytic the maxima are isolated, so that Q contains a finite number of elements. ■

Proposition 4 *The function $\overline{w}(z,e)$ is analytic on the open and convex set $C := \left\{(e,z) \in R_+^2 | e > (1-\widehat{b}) - \frac{\pi(z)}{z}\right\}$. Therefore $\Omega(z,e)$ is analytic a.e. and so has derivatives of all order on $R_+^2 \backslash A$*

Proof. *The proposition follows straightforward from proposition A.3, the differentiability of the functions constructing Y, the implicit-function- and the envelope-theorem.* ■

5.3.1 Proof of Theorem 1

Theorem 2 (Theorem 1 main text) *(a) If $[i(e,z) + \Lambda(e,z)]$ is large enough - but possibly smaller than one - then the investment rate is more sensitive to the equity-ratio than the optimal stock of capital, that is*

$$\frac{\partial i(e,z)}{\partial \ln(e)} \geq \frac{\partial \widetilde{z}_{opt}(e,z)}{\partial e} \frac{e}{\widetilde{z}_{opt}} \tag{43}$$

(b) In an environment around $(e, \widetilde{z}_{opt}(e,z))$ we have

$$\frac{\partial^2 i(e, \widetilde{z}_{opt}(e,z))}{\partial e \partial z} \bigg/ \frac{\partial i(e,z)}{\partial e} \leq 0. \tag{44}$$

Proof. *(a) From equation (17) we obtain*

$$\frac{i}{\Lambda(e,z)} = \frac{(\widetilde{z}_{opt}(e,z) - z)}{z}.$$

Using this to rewrite (18) *in terms of elasticities* η, *we obtain*

$$\begin{aligned}\eta_e^i &= \frac{e}{i(e,z)}\frac{\Lambda(e,z)}{z}\left(\frac{\partial \widetilde{z}_{opt}(e,z)}{\partial e}\right)+\frac{e}{i(e,z)}\frac{(\widetilde{z}_{opt}(e,z)-z)}{z}\frac{\partial\Lambda(e,z)}{\partial e}\\ &= \frac{\Lambda(e,z)}{i(e,z)}\frac{e}{z}\left(\frac{\partial \widetilde{z}_{opt}(e,z)}{\partial e}\right)+\eta_e^{\Lambda}=\frac{\left[\frac{\widetilde{z}_{opt}(e,z)}{z}-1+1\right]\Lambda(e,z)}{i(e,z)}\eta_e^{\widetilde{z}_{opt}}+\eta_e^{\Lambda}\\ &= \frac{i(e,z)+\Lambda(e,z)}{i(e,z)}\eta_e^{\widetilde{z}_{opt}}+\eta_e^{\Lambda}. \end{aligned} \tag{45}$$

If $[i(e,z)+\Lambda(e,z)]+i\,(e,z)\,\frac{\eta_e^{\Lambda}}{\eta_e^{\widetilde{z}_{opt}}}\geq 1$, *this yields*

$$\frac{\partial i(e,z)}{\partial \ln(e)}=\eta_e^i\cdot i(e,z)=[i(e,z)+\Lambda(e,z)]\,\eta_e^{\widetilde{z}_{opt}}+\eta_e^{\Lambda}\cdot i(e,z)\geq \eta_e^{\widetilde{z}_{opt}} \tag{46}$$

(b) Suppose $\frac{\partial i(e,z)}{\partial e}>0\ (<0)$, *now let e increase marginally then we have for some firms an increase (decrease) in realized z. Now assume contradictory* $\frac{\partial^2 i(e,\widetilde{z}_{opt}(e,z)))}{\partial e\partial z}>0\ (<0)$, *then investment rates would rise (fall) further, contradicting* $\widetilde{z}_{opt}$ *to be optimal. Thus the stated inequality must hold.* ■

5.4 Approximation of the optimal stock of capital

Suppose a firm produces with a production function is of the Cobb-Douglas-type and with two Input factors k and l, then - taking logarithms:

$$\begin{aligned} y &= a+bk+cl \quad ; \quad b+c<1\\ &\Longrightarrow \frac{\partial y}{\partial k}=b \ ; \ \frac{\partial y}{\partial l}=c \ ; \ \frac{\partial^2 y}{\partial k^2}=\frac{\partial^2 y}{\partial l^2}=\frac{\partial^2 y}{\partial k\partial l}=0\end{aligned}$$

Therefore a Taylor-extension of the first order condition yields:

$$\begin{aligned} & y(k^*,l^*(k^*))+\ln(b)\overset{!}{=}k^*+\ln(r)\\ \Longleftrightarrow\ & y(k,l(k))+(b+c\frac{\partial l^*}{\partial k})(k^*-k)+\ln(b)=k^*+\ln(r)\end{aligned}$$

After some calculations one gets

$$k^*-k=\left(\frac{1-c}{1-b-c}\right)(y-k+\ln(b)-\ln(r))$$

Now b is approximated by γ (the cost share of capital), then $\left(\frac{1-c}{1-b-c}\right)\approx\frac{1}{1-\gamma}$.[49]

[49] See Caballero et al. (1995, p. 37) for details.

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Aggregate investment dynamics
Appendix for the Referee

Abstract

This is an appendix to the paper "Aggregate Investment dynamics when firms face fixed investment cost and capital market imperfections". In this appendix some lengthy footnotes that are omitted in the main text are reproduced, the data for the aggregate variables used is displayed and the way investment evolves in the time-series domain is derived in detail.

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1 Appendix R1 - Time series behavior of aggregate investment

To describe the time-series behavior of aggregate investment let the sequence of shocks/adjustment be:

1. adjustment as described
2. idiosyncratic shocks ξ
3. aggregate shock v_t and depreciation δ.

Denoting the density of idiosyncratic shocks by $g(\xi)$, one can describe the transition from $f(\cdot, t-1)$ to $f(\cdot, t)$ as in Caballero et al. (1995, p. 35) by

$$\begin{aligned} f(e,z,t) &= \widetilde{f}_1\left((e+\delta)(1-\delta), z\frac{\exp(v_{t-1})}{1-\delta}, t-1\right), &(1)\\ \widetilde{f}_1(e,z,t\text{-}1) &= \int \widetilde{f}_2(e, z\exp(d+\xi), t\text{-}1)g(\xi)d\xi, &(2)\\ \widetilde{f}_2(e,z,t\text{-}1) &= \int dH(o,p|ep+\tfrac{o}{p},\tfrac{z}{p},t\text{-}1)f(ep+\tfrac{o}{p},\tfrac{z}{p},t\text{-}1)dodp, &(3)\end{aligned}$$

where H denotes the distribution of the stochastic variables O and P, which are defined as

$$P_{jt} := 1 + i_{jt} \quad , \quad O_{jt} := 1 + (1 + r(b_{jt}))\, b_{jt} - \frac{(1 + r(b_{jt}))}{1-\delta} b_{jt-1} - P_{jt} \ ,$$

so that P is the growth rate of capital, and O the change in leverage. Since w_t are i.i.d., H is stationary, so that combining (1) - (3) yields

$$\begin{aligned} f(e,z,t) = \int & dH(o,p|e\cdot p+\tfrac{o}{p},\tfrac{z}{p}|e\cdot o, z\cdot p) \\ & \times f\left(\left(e\cdot p+\tfrac{o}{p}+\delta\right)(1-\delta), \tfrac{z}{p}\tfrac{\exp(d+\xi+v_{t-1})}{(1-\delta)}, t-1\right) g(\xi)d\xi dodp \ . \quad (4)\end{aligned}$$

Therefore the aggregate investment series can be characterized as a generalized Markov-chain. Note that because of the presence of aggregate shocks f is non-stationary.[1] Furthermore, as H derives from the investment decision, we obtain from the aggregate investment equation (maintext: 23)for the first two moments of P:

$$\begin{aligned} E(P|e,z) &= i(e,z)+1, &(5)\\ Var(P|e,z) &= Var\left[i(e,z)\right]. &(6)\end{aligned}$$

[1] If f were stationary in the absence of aggregate shocks, the density of disequilibria and equity to capital ratios through which a firm goes over its lifetime would be ergodic (Caballero and Engel (1992)). That is, if the operator T defined below converges to a non-degenerated density.

$$\begin{aligned} (Tf)(e,z) \ : &= \int dH(o,p|e\cdot p+\frac{o}{p},\frac{z}{p}|e\cdot o, z\cdot p) \\ & \times f\left(\left(e\cdot p+\frac{o}{p}+\delta\right)(1-\delta), \frac{z}{p}\frac{\exp(d+\xi)}{(1-\delta)}\right) g(\xi)d\xi dodp\end{aligned}$$

2 Appendix R2 - Prices, interest rates and inflation

Table 1: Nominal interest-rate i, real interest-rate r, change in the RPI, deflator P

year	1975	76	77	78	79	80	81	82	
i [%]	11.4	13.5	8.9	10.4	13.9	16.7	13.9	12.3	
$\frac{RPI_{t+1}}{RPI_t}$ [%]	17	16	9	13	17	12	9	5	
r [%]	-4.75	-2.18	-0.09	-2.28	-2.65	4.22	4.53	6.93	
P	1	1.17	1.357	1.479	1.672	1.956	2.191	2.388	
year	83	84	85	86	87	88	89	90	91
i [%]	10.1	10.0	12.2	11.0	9.7	10.3	13.9	14.8	11.5
$\frac{RPI_{t+1}}{RPI_t}$ [%]	5	5	4	4	5	6	8	7	5
r [%]	4.89	4.72	7.91	6.70	4.48	4.07	5.46	7.26	6.22
P	2.507	2.632	2.764	2.875	2.990	3.139	3.327	3.594	3.845

3 Appendix R3 - Additional explanatory Footnotes

1. This value is given by

$$\tilde{v}(z_{opt}(w_t, e_t), b_{opt}(w_t, e_t)) - w_t \pi(z_{opt}(w_t, e_t)) \ .$$

2.

$$\begin{aligned}\frac{\partial^2 i(e,z)}{\partial \ln(e) \partial \ln(z)} &\simeq \frac{\partial [i(e,z)+\Lambda(e,z)]}{\partial \ln(z)} \eta_e^{z_{opt}} + \frac{\partial^2 \tilde{z}_{opt}}{\partial e^2} \frac{e^2}{\tilde{z}_{opt}} + i \frac{\partial \eta_e^{\Lambda}}{\partial \ln(z)} + \eta_e^{\Lambda} \frac{\partial i}{\partial \ln(z)} \\ &= \frac{\partial \eta_e^{z_{opt}}}{\partial \ln(e)} + \frac{\partial i(e,z)}{\partial \ln(z)} \left[\eta_e^{z_{opt}} + \eta_e^{\Lambda} \right] + \left[\frac{i}{\Lambda(e,z)} + \eta_e^{z_{opt}} \right] \frac{\partial \Lambda(e,z)}{\partial \ln(z)} .\end{aligned}$$

3. Yet, the consistency result is obtained by letting T (the time series dimension) and n (the number of observations per individual) tend to infinity sequentially, i.e. it is a consistent estimator for samples which are larger along the time-series than along the cross-sectional dimension. Nevertheless, the estimator should still be superior to OLS. Moreover and mentioned before, Madsen (2001) shows, how inference on the cointegrating vector can even be obtained from a cross-section in a similar framework.

4. Apriori might sound doubtful that the *yearly average* subsidy fraction s_{it} is equal to the *expected marginal* fraction of subsidies. In this sense, there could be a measurement error problem present in this specification. However, there are no obvious instruments available and the actually estimated coefficient looks in no way biased downwards, as it is insignificantly different from its theoretical value of 1.