Evaluating Alternative Representations of the Choice Sets in Models of Labour Supply[◆]

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Abstract

During the last two decades, the discrete-choice modelling of labour supply decisions has become increasingly popular, starting with Aaberge et al. (1995) and van Soest (1995). Within the literature adopting this approach there are however two potentially important issues that are worthwhile analyzing in their implications and that up to know have not been given the attention that they might deserve. A first issue concerns the procedure by which the discrete alternative are chosen. For example Van Soest (1995) chooses (non probabilistically) a set of fixed points identical for every individual. This is by far the most widely adopted method. By contrast, Aaberge et al. (1995) adopt a sampling procedure and also assume that the choice set may differ across the households. A second issue concerns the availability of the alternatives. Most authors assume all the values of hours-of-work within some range [0, H] are equally available. At the other extreme, some authors assume only two or three alternatives (e.g. non-participation, part-time and full-time) are available for everyone. Aaberge et al. (1995) assume instead that not all the hour opportunities are equally available to everyone; they specify a probability density function of opportunities for each individual and the discrete choice set used in the estimation is built by sampling from that individual-specific density function. In this paper we explore by simulation the implications of

- the procedure used to build the choice set (fixed alternatives vs sampled alternatives)
- accounting vs not accounting for a different availability of alternatives.

The way the choice set is represented seems to have little impact on the fitting of observed values, but a more significant and important impact on the prediction of policy effects.

1. Introduction

The idea of modeling labour supply decisions as discrete choices has become more and more popular during the last two decades. In this paper we examine through a simulation exercise an issue that has received much less attention than it deserves: the implications of alternative methods of representing the choice set within the discrete choice approach.

The discrete choice approach has gained a prominent position as an outcome of the process aimed at solving or circumventing some theoretical and computational problems to be faced in microeconometric research when analyzing choices subject to complicated opportunity constraints. Let us consider the standard labour supply framework:

$$\max U(h,x) \quad s.t. \quad x \le wh + I \;\; and \;\; h \ge 0 \tag{1.1}$$

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where h represents hours of work, w is the (constant) hourly wage rate and I is the exogenous income. Using the Kuhn-Tucker conditions associated to (1.1) – and assuming for simplicity an interior solution – under appropriate conditions one can obtain the optimal labour supply $h*$ as a function of w and I :

(1.2) $$h* = h(w, I)$$

Then some empirical specification of $h(w, I)$ can be estimated and used for example to simulate the effects of policies implying changes in w and/or in I.

The linear budget constraint in problem (1.1), however, very rarely corresponds to reality. Considering a well-known example, taxes and transfers on income in general imply a non-linear constraint. The budget constraint would then be:

(1.4) $$x \leq wh + I - t\,(wh, I)$$

where t represents the tax-benefit rule that computes the net taxes to be paid given gross incomes (wh, I). Taking (1.4) into account, we might still be able to characterize the optimal solution as a function of w and I ,

(1.5) $$h* = h^t\,(w, I)$$

and estimate $h^t\,(w, I)$. However, $h^t\,(w, I)$ depends on the current tax-benefit rule t and therefore it cannot be used to simulate policies that introduce a different tax rule, say $t\,'$. The problem is that the behavioural function h^t in general mixes up preferences and constraints. More generally, the opportunity set might be defined by complicated budget and quantity constraints that do not even allow recovering a closed form solution for $h*$. What we really need is the estimate of the utility function $U(h, x)$ itself. Once preferences are estimated, in principle we are able to simulate the effect of any policy by solving $\max U(h, x)$ subject to the appropriate constraints.

A paper by J. Heckman (1974) probably for the first time takes full account of the non-linearity of the budget constraint in the estimation and simulation of microeconometric models. The problem addressed is the evaluation of a child related welfare policy that introduces significant complications in the budget set. Heckman proposes there a particular method of recovering preferences by using the conditions to be fulfilled by the marginal rate of substitution for h* to be located on a particular point of the budget set. Shortly after, a series of papers by J. Hausman and various co-authors proposed a method specifically addressed to piece-wise linear budget constraints (e.g. Hausman 1979). Both Heckman (1974) and Hausman (1979) work through the implications of the Kuhn-Tucker conditions. The solution can be located in different ranges of values along the budget constraint. Corresponding to each possible range of values there is a condition involving the preference parameters. Choosing a convenient stochastic specification, we can express the probability that those various conditions alternatively hold, write down the sample likelihood and estimate the preference parameters. Useful presentations of this class of methods are provided by Moffit (1986) and Blundell and MaCurdy (2000).

Soon it emerged that the approach described above presents three main problems. First, it works well with convex budget sets (e.g. those generated by progressive taxation) and a two-good application (e.g. h and x in the individual labour supply application) but it tends to become computationally cumbersome when the agents face non-convex budget sets and when more than two goods are object to choice (e.g. when the agent is a many-person household). Second, in view of the computational problems, the above approach essentially forces the researcher to choose relatively simple specifications for the utility function or the labour supply functions (e.g. MaCurdy, Green and Paarsch (1990)). Third, computational and statistical consistency of ML estimation of the model requires to impose a priori quasi-convexity of preferences.

Due to these emerging problems, applied researchers have started to make use of another innovative research effort maturated also in the first half of the 70's, i.e. the discrete choice modelling approach developed by D. McFadden (1974). As far as the labour supply application is concerned, the approach essentially consists in representing the budget set with a set of discrete 'points'. Let $[0, H]$ be the (continuous) range of possible values for hour of work h is. Let us pick K points $h_1, h_2, ..., h_s$ to "represent" $[0, H]$. The utility level attained at point k is $U(x_k, h_k)$, where x_k is obtained through some budget rule such as the (1.4). Now let us assume that $U(x_k, h_k)$ can be decomposed into a systematic part containing the observables $V(x_k, h_k)$ and a random component e_k that accounts for the unobservables. The assumption that the random term e_k is Type I Extreme Value i.i.d. leads to the well known multinomial logit expression for the probability that point j (i.e. the job with hours h_j) is chosen:

(1.1) $$P(j) = \Pr\left(U\left(x_j, h_j\right) = \max\left(U\left(x_1, h_1\right), ..., U(x_s, h_s)\right)\right) = \frac{\exp\left(V(x_j, h_j)\right)}{\sum_i \exp\left(V(x_i, h_i)\right)}$$

The corresponding likelihood function can then easily be computed and maximized in order to estimate the parameters of the utility function. This approach is computationally very convenient when compared to the previous one, since it does not require going through complicated Kuhn-Tucker conditions involving derivatives of the utility function and of the budget constraints. As a consequence it is not affected by how complex it is the rule that defines the budget set or by how many goods are contained in the utility function. Equally important, the deterministic part of the utility function can be specified as very flexible without worrying for the computational implications.

During the last two decades, this approach has become increasingly popular, starting with Aaberge, Dagsvik and Strøm (1995) and van Soest (1995). Within the literature adopting this approach there are however two potentially important issues that are worthwhile analyzing in their implications and that up to know have not been given the attention that they might deserve.

A first issue concerns the procedure by which the discrete alternatives are included in the choice set. Most authors (e.g., among others, van Soest (1995), Duncan and Weeks (1997), Blundell, Duncan, McCrae and Meghir (2000)) choose (non probabilistically) a set of fixed points identical for every individual. By contrast, Aaberge, Dagsvik and Strøm (1995) and Aaberge, Colombino and Strøm (1999) adopt a sampling procedure and also assume that the choice set may differ across the households.

A second issue concerns the availability of the alternatives. Most authors assume all the values in $[0, H]$ are equally available. At the other extreme, some authors assume only two or three alternatives (e.g. non-participation, part-time and full-time) are available for everyone. More generally, Aaberge et al. (1995) assume that not all the hour opportunities in $[0, H]$ are equally available to everyone. More specifically, they assume that there is a probability density function of opportunities for each individual. The discrete choice set used in the estimation (and subsequently) in the simulations is built by sampling from that individual-specific density function.

In what follows we explore by simulation the implications of

- the procedure used to build the choice set (fixed alternatives vs sampled alternatives)
- accounting vs not accounting for a different availability of alternatives

upon the precision of the estimates and of policy simulation results (uniform availability vs heterogeneous availability).

As to the last issue, uniform availability (as for example in van Soest (1995) and Duncan and Weeks (1997) can be interpreted as a special case of heterogeneous availability (as in Aaberge et al. (1995, 1999) when the probability density functions of opportunities are assumed to be uniform and equal for everyone. Since the approach taken by Aaberge et al. (1995, 1999) is more general, we will use their model as the “true” one in order to generate a sample, which will then be used in the simulation experiments.

2. The “true” model

The "true" model is defined along the lines adopted in Aaberge et al. (1995) as well as in several successive papers[1] . The individuals maximise their utility by choosing among opportunities defined by hours of work, hourly wage and non-pecuniary attributes of the job. The utility functions for single males is assumed to be of the following form

(2.1) $$U\left(f(wh,I),h\right)=v(f(wh,I),h)+e$$

where w is the wage rate, h is hours of work, I is exogenous income including the husband's labour income, f is a function that transforms gross income into income after tax, i.e. $f\left(wh,I\right)$ is disposable income (income after tax), and e is a random variable that is supposed to account for unobservables affecting tastes for a given job across individuals as well as across job opportunities for a given individual. The individual is supposed to choose a "job" from a choice set B that may differ across individuals. Each job alternative in B contains a wage rate w, hours of work h and unobserved (to the analyst) job characteristics such as environmental characteristics and skill content of the job. Moreover, B contains also non-market activities, i.e. jobs with $w=0$ *and* $h=0$.

By assuming that e is type I extreme value distributed and that the specification (2.1) is valid, it turns out that the probability density that opportunities with hours h and wage rate w are chosen attains the following expression[2]

(2.2) $$j\,(h,w)\equiv \Pr\left[U(f(wh,I),h)=\max_{(x,y)\in B}U(f(xy,I),y)\right]=\frac{v(h,w)p(h,w)}{\iint v(x,y)p(x,y)dxdy}$$

where $p(h,w,s)$ is the density of choice opportunities which can be interpreted as the relative frequency (in the choice set) of opportunities with hours h and wage rate w. Opportunities with $h=0$ (and $w=0$) are non-market opportunities (i.e. alternative allocations of "leisure"). Thus, the density (2.2) will form the basis of estimating the parameters of the utility function and the choice sets. In practice, the estimation adopts a discretized version of (2.2). Let $q(h,w)$ be some known joint density function (e.g. empirically fitted to the observations on h and w). Let us represent the latent choice set B with a sample S containing M points, where one is the chosen (observed) point and the other M-1 are sampled from $q(h,w)$. It can be shown (McFadden 1978; Ben Akiva and Lerman 1985) that consistent estimates of $v(f(wh,I),h)$ and $p(h,w)$ can still be obtained when (2.2) is replaced by

(2.3) $$\Pr\left[U(f(wh,I),h)=\max_{(x,y)\in S}U(f(xy,I),y)\right]=\frac{\exp\left(v(f(wh,I),h)\right)p(h,w)/q(h,w)}{\sum_{(x,y)\in S}\exp\left(v(f(xy,I),y)\right)p(x,y)/q(x,y)dxdy}$$

We select a sample of married/cohabitating females. The systematic part of their utility function (2.1) is specified as follows

(2.4) $$\ln(v(h,w)=a_2\left(\frac{f(wh,I)^{a_1}-1}{a_1}\right)+(a_4+ \\ +a_5\log A+a_6\left(\log A\right)^2+a_7Ch_1+a_8Ch_8+a_9Ch_9)\left(\frac{L^{a_3}-1}{a_3}\right)$$

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[1] e.g. Aaberge, Colombino and Strøm, (1999 and 2000) and Aaberge, Colombino, Strøm and Wennemo (2000).

[2] For the derivation of the choice density (2.2) see Aaberge, Colombino and Strøm (1999). Note that (2.2) can be considered as a special case of the more general multinomial type of framework introduced by Ben-Akiva and Watanatada (1981) and Dagsvik (1994).

where L is leisure, defined as $L = 1-(h/8736)$, A is age and Ch_1, Ch_2 and Ch_3 are number of children below 3, between 3 and 6 and between 7 and 14 years old. In the specification of the probability density of opportunities we will assume that offered hours and offered wages are independently distributed. The justification for this is that offered hours, in particular normal working hours, are typically set in rather infrequent negotiations between employers and employees associations, while wage negotiations are far more frequent in which the hourly wage tend to be set independent of working hours. Thus, we specify the density of opportunities requiring h hours of work and paying hourly wage w as follows

(2.5)
$$p(h,w) = \begin{cases} p_0 g_1(h) g_2(w) & \text{if} \quad h>0 \\ 1-p_0 & \text{if} \quad h=0 \end{cases}$$

where p_0 is the proportion of market opportunities in the opportunity set, and g_1 and g_2 are respectively the densities of hours and wages, conditional upon the opportunity being a market job.

In view of the empirical specification it is convenient to divide both numerator and denominator by $1-p_0$ and define $g_0 = \frac{p_0}{1-p_0}$. We can then rewrite the choice density (2.2) as follows:

(2.6)
$$j(h,w) = \frac{v(h,w) g_0 g_1(h)\, g_2(w)}{v(0,0,0) + \int\limits_{x>0}\int\limits_{y>0} v(x,y)\, g_0\, g_1(h) g_2(w) dxdy}$$

for $\{h,w\}>0$, and

(2.7)
$$j(0,0) = \frac{v(0,0)}{v(0,0) + \int\limits_{x>0}\int\limits_{y>0} v(x,y)\, g_0\, g_1(h) g_2(w) dxdy}$$

for $\{h,w\}=0$

Offered hours are assumed to be uniformly distributed except for possible peaks corresponding to part time (*pt*), 18-20 weekly hours, and to full time (*ft*), 37-40 weekly hours. Thus, g_1 is given by

(2.8)
$$g_1(h) = \begin{cases} g & \text{if} \quad h \in [1,17] \\ g \exp(p_1) & \text{if} \quad h \in [18,20] \\ g & \text{if} \quad h \in [21,36] \\ g \exp(p_2) & \text{if} \quad h \in [37,40] \\ g & \text{if} \quad h \in [41,H] \end{cases}$$

where H is the maximum observed value of h. Thus, this opportunity density for offered hours implies that it is far more likely to find jobs with hours that accord with full-time and standard part time positions than jobs with other working loads.

Since the density values must add up to 1, we can also compute g according to:

(2.9) $$g\left((17-1)+(20-18)\exp(p_1)+(36-21)+(40-37)\exp(p_2)+(H-41)\right)=1$$

Moreover, let $g_0=\exp(q_0)$. In Table 2.1 we refer to p and q as the parameters of the *job opportunity density*.

The density of offered wages is assumed to be lognormal with mean that depends on length of schooling (*Ed*) and on past potential working experience (E*xp*), where experience is defined to be equal to age minus length of schooling minus five, i.e.

(2.10) $$\log w = b_0 + b_1 Exp + b_2 Exp^2 + b_3 Ed + s\,h\ ,$$

where h is standard normally distributed.

Inserting for (2.7) and g_0 in (2.5) yields

(2.11) $$j\,(w,h)=\frac{\exp\left(V\left(f\,(wh,I),h\right)+\left(q_0+\ln g_2(w)\right)d_0(h)+p_1 d_1(h)+p_2 d_2(h)\right)}{\int_x\int_y \exp\left(V\left(f\,(xy,I),y\right)+\left(q_0+\ln g_2(x)\right)d_0(h)+p_1 d_1(y)+p_2 d_2(y)\right)dxdy}$$

where

$d_0(h)$ = 1 if $h>0$; 0 otherwise

$d_1(h)$ = 1 if $h\in[18,20]$; 0 otherwise,

$d_2(h)$ =1 if $h\in[37,40]$; 0 otherwise.

In what follows we will refer to $d_0(h)$ as the "job" dummy, since it capture the relative frequency of market opportunities to non market opportunities; we will refer to $d_1(h)$ and $d_2(h)$ as the "peaks" dummies, since they are meant to capture the "peaks" in the density of hours corresponding to part-time and full-time jobs.

The estimation of the model for single individuals is based on data for 1842 married/cohabitating females from the 1995 Survey of Level of Living. We have restricted the ages of the females to be between 20 and 62 years in order to minimize the inclusion in the sample of individuals who in principle are eligible for retirement, since analysis of retirement decisions is beyond the scope of this study.

The parameters appearing in expressions (2.3) – (2.10) are estimated by the method of maximum likelihood using the sampling procedure illustrated in Section 2.3.. Each of the choice sets are represented by a set S that includes the observed choice plus 999 independent draws (h, w) from densities q(w, h) previously fitted to the observed values of w and h. If (w_s,h_s) are the observed values for a particular individual, the corresponding contribution to the likelihood function is:

(2.12)
$$P(w_s,h_s\,|S)=\frac{\exp\left(V\left(f\,(w_s h_s,I),h_s\right)+\left(q_0+\ln g_2(w_s)\right)d_0(h_s)+p_1 d_1(h_s)+p_2 d_2(h_s)-\ln q\,(w_s,h_s)\right)}{\sum_{i\in S}\exp\left(V\left(f\,(w_i h_i,I),h_i\right)+\left(q_0+\ln g_2(w_i)\right)d_0(h_i)+p_1 d_1(h_i)+p_2 d_2(h_i)-\ln q(w_i,h_i)\right)}$$

The parameters appearing in expressions (2.3)-(2.10) are estimated by the method of maximum likelihood in line with the procedure suggested by McFadden (1978). The likelihood function is equal to the products of the individual-specific labour supply densities for single females. Each of the choice sets are represented by the observed choice and 999 independent draws (h,w) from previously estimated opportunity. The estimates of the opportunity density parameters and the parameters of the utility function are reported in Tables 2.1 and 2.2.

Based on the empirical distribution of the exogenous variable and on the estimates of Table 2.1 and 2.2 (to simulate the endogenous variables and choices) we generate a sample of 10000 observations, which is then used in the simulation exercise described in what follows.

Table 2.1. Hours and Wage densities, Norway 1994

	Parameter	Estimate	Std. Dev.
Job opportunity			
	q_0	-0.60	(0.10)
Hours			
Part-time	p_1	0.46	(0.10)
Full-time	p_2	1.57	(0.07)
Wage			
	b_0	0.24	(0.01)
	b_1	3.62	(0.05)
	b_2	2.41	(0.26)
	b_3	-3.67	(0.58)
	s	4.10	(0.35)

Table 2.2. Estimates of the parameters of the utility functions for married/cohabitating females. Norway 1994

Variable	Parameter	Estimate	Std. Dev.
Consumption			
	α_1	0.39	(0.11)
	α_2	4.42	(0.44)
Leisure			
	α_3	-4.57	(0.53)
	α_4	168.88	(27.47)
Log age	α_5	-94.29	(15.32)
Log age squared	α_6	13.35	(2.16)
Number of children below 3 years old	α_7	0.44	(0.23)
Number of children 3-6 years old	α_8	1.23	(0.24)
Number of children 7-14 years old	α_9	1.05	(0.19)

3. Alternative representations of the choice sets

As we have already mentioned in the Introduction, the first issue in choice set representation concerns the procedure used to select the alternatives. In many applications, including labour supply modelling, the choice set contains a very large (or even infinite) number of alternatives. For instance, if we model couples labour supply and the decision period is the year, considering 1 hour intervals, there are $2\times 24\times 365 =$ 17520. This implies a very heavy computational burden, since for each alternative we must compute the couple's budget possibly applying a complicated tax rule. Thus it is convenient to work with a smaller choice set somehow representative of the true one. Ben-Akiva and Lerman (1985) present a detailed treatment of the procedures that maybe used when the number of alternatives contained in the choice set is very large (or even infinite) so that a complete enumeration is computationally too costly:

- Aggregation of alternatives
- Sampling of alternatives

The procedure consisting in selecting a fixed number of hours' values can be interpreted as an aggregation procedure. Instead of using all the possible values between 0 and T, the (0,T) range is divided into sub-intervals and then the mid (or maybe the average) value of h in each interval is chosen to 'represent' all the values of that interval. The authors adopting this procedure realize that it introduces measurement errors, but tend to assume they are of minor importance. van Soest (1995) reports that some experiments with a different number of points did not show interesting differences in parameter estimates, however a systematic investigation of the implication of that procedure has never been done either theoretically or empirically. However, if one interprets the approximation as an aggregation procedure, the analysis provided by Ben-Akiva and Lerman (1985) can be applied to clarify the issue.

We will assume the average of h in each sub-interval is chosen as representative (instead of the more common procedure of choosing the mid point: of course the two are very close and in fact coincide if the values of h are continuous or if each interval contains a an uneven number of values). Let us define (we drop the subscript of the household to simplify the notation):

(3.1) $$V_j \equiv V(x_j, h_j) = V(f(wh_j, h_j)).$$

Furthermore, let $\bar{V}^L \equiv \frac{1}{N^L}\sum_{j\in L} V_j$ = average systematic utility in sub-interval L, where N^L = number of elements in L and $\bar{h}^L$ = average value of h in sub-interval L.

Ben-Akiva and Lerman show that the expected maximum utility attained on subinterval ℓ is

(3.2) $$\hat{V}^\ell = \bar{V}^\ell + \ln(N^\ell) + \ln\left(D^\ell\right)$$

where $D^\ell \equiv \sum_j \exp\left(V_j^\ell - \bar{V}^\ell\right)\frac{1}{N^\ell}$.

This last term is a measure of dispersion of V in sub-interval ℓ.

Accordingly, the probability that a value of h belonging to sub-interval L is chosen is

(3.3) $$P(\ell) = \frac{\exp\left(\bar{V}^L + \ln(N^L) + \ln\left(D^L\right)\right)}{\sum_\ell \exp\left(\bar{V}^\ell + \ln(N^\ell) + \ln\left(D^\ell\right)\right)}.$$

To compare this with the expressions used in the fixed-alternatives approach it is useful to Taylor-expand V_j up to 2-order terms to get

(3.4) $$P(L) \cong \frac{\exp\left(V\left(f(w\bar{h}^L, I), \bar{h}^L\right) + 0.5 s_{hh}^L V_{hh}^L + \ln(N^L) + \ln\left(D^L\right)\right)}{\sum_{\ell} \exp\left(V\left(f(w\bar{h}^\ell, I), \bar{h}^\ell\right) + 0.5 s_{hh}^\ell V_{hh}^\ell + \ln(N^\ell) + \ln\left(D^\ell\right)\right)}$$

where s_{hh}^{ℓ} is the variance of the values of h in sub-interval ℓ and V_{hh}^L is the second (total) derivative of $V\left(f(w\bar{h}^\ell, I), \bar{h}^\ell\right)$ with respect to $\bar{h}^\ell$.

It would be pointless to use (2.15) for estimation since it require the very same computations that one wishes to avoid by aggregating alternatives. However (2.15) is useful in order to understand the type and the extent of errors we incur by using various approximations. The expression typically used in the literature is:

(3.5) $$\hat{P}(L) = \frac{\exp\left(V\left(f(w\bar{h}^L, I), \bar{h}^L\right)\right)}{\sum_{\ell} \exp\left(V\left(f(w\bar{h}^\ell, I), \bar{h}^\ell\right)\right)}.$$

Clearly, in expression (3.5) all the terms $0.5 s_{hh}^\ell V_{hh}^\ell + \ln(N^\ell) + \ln\left(D^\ell\right)$ are dropped. If these terms were equal across all the sub-intervals they would cancel out from (3.4) and (3.5) would be exact. In general however they will not be equal, and dropping them will lead to biased estimates. Nonetheless there are ways by which we could improve upon (3.5) when adopting aggregation such an approximation strategy, which however have never been considered in the literature on labour supply modelling:

- The dimension of N^ℓ of the sub-intervals - when not equal for all of them - is typically known and can be explicitly accounted for;
- s_{hh}^ℓ can also be computed;
- Depending on the functional form used for the utility function, the term V_{hh}^ℓ might be explicitly evaluated and accounted for;
- The terms $\ln\left(D^\ell\right)$ in general will vary both across sub-intervals and across individuals; however we might capture at least some of their effect by introducing a set of dummies (as many as the number of sub-intervals - 1).

Summing up, the aggregation of alternatives implies biased estimates. The bias could be moderated by using various possible corrections suggested by expression (2.15) itself. Up to now, however, it must be said that the literature on labour supply has treated this issue in a rather superficial way (when compared, for instance to the literature on transportation or location choices).

Sampling of alternatives, on the other hand, offers the possibility of working with a relatively small choice set and at the same time preserving the consistency of the estimates. The basic results were established by McFadden (1978), Ben-Akiva and Lerman (1985) also provide a very useful and more practically oriented survey, together with some additional theoretical results.

Let us represent the true choice set B with a sample S containing M points, where one is the chosen (observed) point and the other M-1 are sampled from. Let q_i be the probability of sampling point h_i. It can be shown (McFadden 1978; Ben Akiva and Lerman 1985) that consistent estimates of $v(f(wh, I), h)$ and $p(h, w)$ can still be obtained when the true choice set B is replaced by S and the probability of observing choice j is evaluated as follows:

$$P(j\,|S) = \frac{\exp\left(V(f(wh_j, I), h_j)\right)/q_j}{\sum_{h_i \in S} \exp\left(V(f(wh_i, I), h_i)\right)/q_i} \tag{3.6}$$

If a simple random sampling is adopted, all the *q's* are equal and cancel out. Typically more sophisticated sampling procedures are used since they are expected to be more efficient. For instance, a common procedure consists in using as sampling probabilities the observed relative frequencies of choice possibly differentiated according to personal characteristics of the decision units. Besides Ben-Akiva and Lerman (1985), also Train et al. (1987) present a very detailed application of this procedure.

A second and possibly even more substantial issue is whether or not account is taken of the different availability of job-types on the market. Some authors (e.g. Zabalza et al. 1980) have made the extreme choice of assuming the choice set contains only a two or three alternatives (e.g. non-participation, part-time and full-time). This is taken to be a truthful representation of the "long-run" choice set, other arrangements being treated as temporary phenomena. More common, however, is the approach of choosing a few equally spaced points in the interval [0, H], without taking into account the possibility that some type of opportunities maybe more easily available than others. Other authors do account for this possibility as well as for the relative density of jobs as a function of personal characteristics (see section 2.2). In practice, their specification boils down to "augmenting" the term $V(.)$ with a set of appropriately defined dummy variables. Also van Soest (1995) introduced similar dummies, although he gives them a different interpretation.

In what follows we use the sample generated according to the true model to estimate various versions of models generated according the various possible representation of the choice set as discussed above.

The more general versions are

$$P(w_s, h_s\,|S) = \frac{\exp\left(V\left(f(w_s h_s, I), h_s\right) + q_0 d_0(h_s) + p_1 d_1(h_s) + p_2 d_2(h_s) - \ln q(w_s, h_s)\right)}{\sum_{i \in S} \exp\left(V\left(f(w_i h_i, I), h_i\right) + q_0 d_0(h_i) + p_1 d_1(h_i) + p_2 d_2(h_i) - \ln q(w_i, h_i)\right)} \tag{3.7}$$

when sampled alternatives are used, and

$$P(w_s, h_s\,|F) = \frac{\exp\left(V\left(f(w\,h_s, I), h_s\right) + q_0 d_0(h_s) + p_1 d_1(h_s) + p_2 d_2(h_s)\right)}{\sum_{i \in F} \exp\left(V\left(f(w\,h_i, I), h_i\right) + q_0 d_0(h_i) + p_1 d_1(h_i) + p_2 d_2(h_i)\right)} \tag{3.8}$$

when fixed alternatives are used. The more restricted versions of the models are generated by dropping the job dummy $d_0(h_i)$ and/or the peaks dummies $(d_1(h_i), d_2(h_i))$. The choice set S and F contains alternatively 6 or 24 points.

Altogether with have 16 models resulting from the combinations of the following possibilities:

1. *alternative generation*: fixed or sampled
2. *number of alternatives*: 6 or 24
3. *job dummy*: included or dropped
4. *peaks dummies*: included or dropped

In the following sections they are named as in Table 3.1. For each of the above models we have a version with fixed alternatives and a version with sampled alternatives. The parameter estimates of the 16 models are reported in the Appendix.

Table 3.1 Types of models

Model	Model Ia	Model Ib	Model Ic	Model Id	Model IIa	Model IIb	Model IIc	Model IId
Job dummy	*No*	*Yes*	*No*	*Yes*	*No*	*Yes*	*No*	*Yes*
Peaks dummies	*No*	*No*	*Yes*	*Yes*	*No*	*No*	*Yes*	*Yes*
Number of alternatives	*6*	*6*	*6*	*6*	*24*	*24*	*24*	*24*

4. Evaluation of the different modelling approaches

In order to evaluate the impact of alternative representations of the choice set on the performance of the models we proceed in the following way. First, for each of the 16 models we predict participation rates, hours of work and disposable income. The predictions, obtained individual by individual, are the aggregated into the 10 means of the 10 income deciles. Next, we introduce the following summary measure of prediction performance z_i for model i,

(4.1) $$z_k = \sqrt{\sum_{j=1}^{10}\left(\frac{(\tilde{y}_{kj} - y_j)}{y_j}\right)^2}$$, k=1, 2 …,16,

where y_j and $\tilde{y}_{kj}$ denote the outcomes in decile j of the true model and alternative model k, respectively. The outcomes are alternatively defined to be the job participation rate, hours of work and disposable income after tax.

Next, we carry out a regression analysis where z is treated as a response variable and the following variables are treated as co-variables,

x_1 = 1 if the choice alternatives are sampled (= 0 if the choice alternatives are fixed),

x_2 = 1 if the number of choice alternatives is equal to 24 (= 0 if the number of alternatives is equal to 6),

x_3 = 1 when it is accounted for job entry (= 0 when it is *not* accounted for job entry),

x_4 = 1 when it is accounted for part-time and full-time peaks (= 0 when it is *not* accounted for part-time and full-time peaks).

The following equation forms the basis of the evaluation of alternative modelling approaches,

(4.2) $$z = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + a_{34}(x_3 * x_4)$$

Since the most important application of labour supply models is the evaluation of tax and welfare policy reforms, we focus on the prediction performance under alternative tax regime. Namely, the steps above are repeated twice:

- Prediction of the outcomes under the current tax regime
- Prediction of the outcomes after the introduction of a Flat Tax (keeping total tax revenue constant).

4.1. Outcomes under the current tax regime

Tables 4.1 – 4.3 illustrate the results of the exercise under the current tax regime. Tab. 4.1 and 4.2 refer to the eight models with fixed alternatives. In order to simplify the illustration we limit ourselves to the models without job and peaks dummies and to the models with both types of dummies. For each of the models and each of the 10 income deciles, we report the predictions of participation rates and hours of work in Tab. 4.1 and of after tax disposable income in Tab. 4.2. We do not report here the analogous results fro the models with sampled alternatives, since they are very close to those with fixed alternatives. Even a causal inspection of the tables suggests that the prediction performance is pretty good whatever the model considered. Possibly the only entries where there seems to be some substantial error depending on the model used are the predictions of outcomes for the first decile. In any case, in order to systematically asses the impact of the characteristics of all the 16 models we run the regression (4.2), and report the results in Tab. 4.3.

Table 4.1 Prediction of participation rates and hours of work under the 1994 tax system. Fixed-alternatives models

Deciles	True model		Model Ia		Model Id		Model IIa		Model IId	
	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work
1	**58**	**568**	55	627	*43*	*514*	87	733	*55*	*568*
2	**65**	**715**	73	818	*61*	*730*	93	837	*67*	*708*
3	**79**	**937**	81	1000	*71*	*890*	95	989	*79*	*941*
4	**86**	**1157**	87	1179	*80*	*1130*	97	1125	*85*	*1153*
5	**91**	**1389**	92	1375	*87*	*1397*	96	1276	*90*	*1352*
6	**93**	**1527**	94	1494	*91*	*1541*	98	1429	*93*	*1528*
7	**93**	**1606**	95	1638	*91*	*1650*	99	1598	*94*	*1631*
8	**94**	**1695**	94	1701	*92*	*1735*	98	1667	*93*	*1672*
9	**94**	**1757**	95	1812	*93*	*1838*	99	1746	*96*	*1771*
10	**88**	**1523**	89	1631	*83*	*1566*	97	1676	*87*	*1567*
Mean	**84**	**1287**	86	1327	*79*	*1299*	96	1308	*84*	*1289*

Table 4.2 Prediction of disposable for couples under the 1994 tax system. Fixed-alternatives models

Deciles	True model	Model Ia	Model Id	Model IIa	Model IId
1	**168915**	170648	*169098*	171945	*168690*
2	**216080**	217801	*215357*	219415	*216333*
3	**244914**	245504	*243740*	245176	*243672*
4	**268880**	268308	*267340*	267880	*267659*
5	**290441**	290083	*290556*	288798	*289893*
6	**312088**	312113	*313719*	310410	*312446*
7	**336247**	335829	*337305*	334374	*336148*
8	**363833**	364607	*365453*	362513	*363739*
9	**403513**	405063	*405654*	403401	*404046*
10	**600841**	605283	*602163*	608705	*604516*
Mean	**320575**	321524	*321038*	321262	*320714*

Table 4.3. Estimates of equation (4.2): outcomes under the current tax regime

Outcome variable	a_0	a_1	a_2	a_3	a_4	a_{34}	R^2
Probability of participation	***.406*** ***(.123)***	-.075 (.100)	.126 (.100)	-.266 (.142)	.086 (.142)	-.076 (.201)	.54
Hours of work	***.266*** ***(.057)***	-.023 (.046)	.070 (.046)	***-.178*** ***(.007)***	-.004 (.007)	.031 (.092)	.60
Income after tax	***.021*** ***(.008)***	-.002 (.004)	***.007*** ***(.004)***	***-.009*** ***(.006)***	-.002 (.006)	-.003 (.008)	.50

*Standard deviation in parentheses

The results of Tab. 4.3 confirm the message conveyed by Tab. 4.1 – 4.2. Although most of the coefficients have the expected sign, almost none of them are significant at standard levels (the significant ones are in bold italics). Overall one can conclude there is little significant evidence for an important impact of alternative modes of representing the choice set as long as the replication of current values is concerned.

4.2 Outcomes under a Flat Tax reform.

In this second part of the simulation exercise, the models are run as after a tax reform. Namely, a fixed proportional tax (Flat Tax) replaces the current tax system. The behavioural responses of the individual in the sample and the total net tax revenue are computed. The flat tax rate is updated and the model iterated until the total net tax revenue is the same as under the current regime. Tables 4.4 and 4.5 are analogous to Tab. 4.1 and 4.2. Tables 4.6 and 4.7 replicate 4.4 and 4.5 but with sampled-alternatives models. When it come to reforms simulation rather that current values replication the differences in outcomes are somewhat more marked, and this is confirmed by Tab. 4.8 where eq. (4.2) is estimated, analogous to Tab. 4.3 but with reference here to post-Flat-Tax outcomes. There is a clear pattern of the effects of different modelling strategies in particular on the prediction of disposable income. For example, using 24 alternatives instead of 6 reduces the average percentage error by 0.8%. Using sampled alternatives instead of fixed alternatives reduces it by 1.9%. Introducing job and peaks dummies reduces it by 3.2%

Table 4.4 Prediction of participation rates and hours of work under a flat tax reform. Fixed-alternatives models

Deciles	True model		Model Ia		Model Id		Model IIa		Model IId	
	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work
1	**69**	**987**	62	835	*55*	*826*	89	946	*63*	*890*
2	**75**	**1022**	77	943	*68*	*966*	95	1041	*74*	*943*
3	**84**	**1160**	83	1100	*76*	*1117*	96	1145	*83*	*1134*
4	**89**	**1315**	89	1260	*83*	*1279*	97	1271	*87*	*1291*
5	**93**	**1491**	93	1432	*89*	*1488*	97	1392	*91*	*1459*
6	**94**	**1609**	94	1542	*92*	*1626*	98	1543	*93*	*1609*
7	**94**	**1659**	95	1677	*92*	*1717*	99	1685	*94*	*1670*
8	**95**	**1742**	94	1735	*92*	*1786*	98	1727	*93*	*1720*
9	**95**	**1794**	96	1843	*94*	*1898*	99	1811	*96*	*1821*
10	**88**	**1549**	89	1647	*84*	*1619*	97	1721	*88*	*1606*
Mean	**88**	**1487**	87	1401	*82*	*1432*	96	1428	*86*	*1414*

Table 4.5 Prediction of disposable income under a flat tax reform. Fixed-alternatives models

Deciles	True model	Model Ia	Model Id	Model IIa	Model IId
1	**194076**	171081	*177612*	173092	*177934*
2	**234263**	214268	*220564*	222704	*220524*
3	**259189**	242704	*250457*	247374	*248492*
4	**279624**	266384	*272361*	271441	*271579*
5	**301124**	289038	*294062*	293453	*294681*
6	**323777**	314124	*320755*	319278	*319492*
7	**350809**	342509	*349310*	346358	*344397*
8	**383958**	375740	*379893*	378941	*377972*
9	**431297**	426513	*431747*	430622	*428668*
10	**651815**	649764	*651885*	657771	*652667*
Mean	**340993**	329213	*334865*	334103	*333641*

Table 4.6 Prediction of participation rates and hours of work under a flat tax reform. Sampled-alternatives models

Deciles	True model		Model Ia		Model Id		Model IIa		Model IId	
	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work
1	**69**	**987**	76	915	*65*	*883*	76	921	*65*	880
2	**75**	**1022**	83	982	*74*	*993*	84	985	*75*	992
3	**84**	**1160**	90	1159	*83*	*1131*	90	1151	*83*	1133
4	**89**	**1315**	92	1288	*88*	*1330*	93	1307	*89*	1338
5	**93**	**1491**	94	1449	*91*	*1493*	94	1460	*91*	1485
6	**94**	**1609**	95	1580	*94*	*1650*	95	1579	*94*	1646
7	**94**	**1659**	95	1671	*93*	*1691*	96	1675	*93*	1695
8	**95**	**1742**	97	1759	*96*	*1775*	97	1771	*96*	1774
9	**95**	**1794**	98	1806	*96*	*1811*	98	1807	*96*	1814
10	**88**	**1549**	92	1606	*88*	*1587*	92	1617	*88*	1586
Mean	**88**	**1487**	91	1422	*87*	*1434*	91	1427	*87*	*1434*

Table 4.7 Prediction of disposable income for couples under a flat tax reform. Sampled-alternatives models

Deciles	True model	Model Ia	Model Id	Model IIa	Model IId
1	**194076**	175360	*178959*	175829	*178558*
2	**234263**	221008	*223384*	220745	*222943*
3	**259189**	248332	*249373*	247584	*249304*
4	**279624**	272276	*275414*	273516	*275739*
5	**301124**	293241	*296123*	293368	*295567*
6	**323777**	318317	*321883*	318698	*321400*
7	**350809**	346147	*348328*	346124	*348868*
8	**383958**	377469	*379296*	378295	*378984*
9	**431297**	430380	*430587*	429954	*431015*
10	**651815**	651514	*650805*	652383	*650766*
Mean	**340993**	333404	*335415*	333650	*335314*

Table 4.8 Contributions to the prediction performance: outcomes under a flat tax reform

Outcome variable	a_0	a_1	a_2	a_3	a_4	a_{34}	R^2
Probability of participation	***.215*** ***(.008)***	-.090 (.064)	.060 (.062)	-.110 (.091)	.077 (.091)	-.073 (.129)	.47
Hours of work	***.093*** ***(.013)***	***-.061*** ***(.010)***	***-.023*** ***(.010)***	-.003 (.014)	-.001 (.014)	.012 (.020)	.82
Income after tax	***.128*** ***(.006)***	***-.019*** ***(.005)***	***-.008*** ***(.005)***	***-.029*** ***(.007)***	***-.039*** ***(.007)***	***.036*** ***(0.01)***	.84

*Standard deviation in parentheses

4.4 Computational costs

The different representations of the choice set imply different computational burdens, particularly with regards to the number of alternatives and to the procedure used to generate the alternatives. Depending on the availability of computing resources and time, the advantages of the various approaches to represent the choice set should be balanced against the computational costs. Table 4.9 reports the relative elapsed time (= 1 for the simplest model[3]) of a typical estimation run with four different type of models: fixed vs sampled alternatives and 6 vs 24 alternatives (accounting or not for job and peaks dummies does not make any significant difference in terms of computing time).

Table 4.9 Relative computation time (estimation) for different models

	6 alternatives	24 alternatives
Fixed alternatives	1	4.62
Sampled alternatives	6.70	8.46

5. Conclusions

We have performed a series of simulation exercises aimed at exploring the performance of different versions of a labour supply model, where different approaches to represent choice sets are used. The various models are estimated using a large sample generated by a “true” model, to which they can then be compared. In evaluating the models, we focus upon their ability replicate the “true” outcomes under different tax regimes. It turns out that as far as the replication of the current-tax-regime outcomes are concerned, there is little evidence for important effects of alternative choice-set-representation procedures. Not even the number of alternatives contained in the choice set seems to matter. All the models predict very well, although there are some indications favouring the sampled-alternatives procedure. However, when it comes to predicting outcomes under a flat-tax reforms, the indications are more clear-cut: using sampled alternatives and accounting for heterogeneity of opportunities seem to significantly reduce the prediction errors (at least for the prediction of incomes). Clearly the sampled-alternative procedure is more costly computationally, so the benefits should eventually be balanced against the increased computational costs. The fact that the prediction performance of current values is not able to discriminate between different models and instead the prediction performance of post-reform is, conveys the important message that the ability of a model to replicate observed outcomes is not very informative: ultimately, the models should be judge in their ability to do the job they are mainly built for, i.e. predicting the outcomes of policy changes.

[3] The absolute computing time for estimating the simplest model was 2.42 seconds on a Alpha ES45, 1 Gb Mhz, 8 Gb workspace memory

Appendix

Here we report the parameter estimates of the true model and of the 16 alternative models.

Table A.1. Fixed-alternatives models Ix and IIx (x = a, b, c)

Variable	Para-meter	True model	Model Ia	Model IIa	Model Ib	Model IIb	Model Ic	Model IIc	Model Id	Model IId
Consumption										
	α_1	**0.39**	0.35	0.54	0.43	0.46	0.43	0.50	0.43	0.44
	α_2	**4.42**	2.46	3.70	3.97	4.55	4.05	4.64	4.17	4.38
Leisure										
	α_3	**-4.57**	-7.53	-3.18	-7.31	-6.72	-2.07	-0.14	-3.99	-4.15
	α_4	**168.88**	54.20	184.85	64.76	92.39	232.99	351.30	156.91	171.12
Log age	α_5	**-94.29**	-30.46	-102.83	-36.27	-51.64	-128.78	-193.30	-87.38	-95.45
Log age squared	α_6	**13.35**	4.32	14.62	5.15	7.33	18.27	27.48	12.40	13.54
Number of children below 3 years old	α_7	**0.44**	0.13	0.51	0.13	0.19	0.61	0.95	0.38	0.40
Number of children 3-6 years old	α_8	**1.23**	0.48	1.68	0.53	0.76	1.86	2.99	1.25	1.40
Number of children 7-14 years old	α_9	**1.05**	0.40	1.37	0.44	0.62	1.53	2.47	1.04	1.14
Job dummy	q_0	**-0.60**	-	-	-1.08	-2.33	-	-	-0.78	-2.10
Part-time dummy	p_1	**0.46**	-	-	-	-	-0.23	0.14	0.15	0.28
Full-time dummy	p_2	**1.57**	-	-	-	-	0.99	1.53	0.78	1.19

Table A.1. Sampled-alternatives models Ix and IIx (x = a, b, c)

	Para-meter	True model	Model Ia	Model IIa	Model Ib	Model IIb	Model Ic	Model IIc	Model Id	Model IId
Consumption										
	α_1	**0.39**	0.54	0.55	0.53	0.54	0.55	0.55	0.52	0.53
	α_2	**4.42**	3.96	3.93	4.72	4.64	4.56	4.51	4.70	4.62
Leisure										
	α_3	**-4.57**	-5.15	-5.27	-5.94	-6.10	-2.40	-2.49	-3.52	-3.60
	α_4	**168.88**	125.90	121.50	112.19	106.31	234.88	231.26	195.26	190.72
Log age	α_5	**-94.29**	-70.17	-67.75	-62.54	-59.28	-129.94	-128.03	-108.43	-105.95
Log age squared	α_6	**13.35**	9.96	9.62	8.88	8.42	18.46	18.19	15.39	15.04
Number of children below 3 years old	α_7	**0.44**	0.33	0.30	0.25	0.23	0.66	0.58	0.50	0.44
Number of children 3-6 years old	α_8	**1.23**	1.07	1.05	0.91	0.87	1.94	1.95	1.56	1.57
Number of children 7-14 years old	α_9	**1.05**	0.88	0.88	0.75	0.73	1.61	1.65	1.29	1.33
Job dummy	q_0	**-0.60**	-	-	-0.88	-0.86	-	-	-0.63	-0.60
Part-time dummy	p_1	**0.46**	-	-	-	-	0.44	0.44	0.53	0.52
Full-time dummy	p_2	**1.57**	-	-	-	-	1.66	1.63	1.56	1.54

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