Classical Estimation of Multivariate Markov-Switching Models using MSVARlib

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Abstract

This paper introduces an upgraded version of MSVARlib, a Gauss and Ox-Gauss compliant library, focusing on Multivariate Markov Switching Regressions in their most general specification. This new set of procedures allows to estimate, through classical optimization methods, models belonging to the MSI(M)(AH)-VARX "intercept regime dependent" family. This research enhances the first package MSVARlib 1.1, which has been deeply inspired by the works of Hamilton and Krolzig. Not to mention the extension to a generalized multivariate regression framework, it notably augments the range of models with a possibly unlimited finite number of Markov states, offers automatic or manual intialization procedures and adds new statistical tests. The first part of this article provides the basic theoretical grounds of the related Markov-switching models. Following sections give some illustrations of the programs through univariate and multivariate examples. One is based on a non-linear reading of the american unemployment rate. A second study is focused on coincident stochastic models of US recessions and slowdowns. The paper concludes on possible extensions and new applications. Detailed guidelines in appendices and tutorial programs are provided to help the reader handling the Gauss package and the joined replication files.

Key Word: Multivariate Markov-Switching Regressions, Hidden markov Models, Open source Gauss library, Business cycle, EM algorithm, Kittagawa-Hamilton Filtering
JEL Classification: C32, E32, E44

The views expressed in this working paper are those of the author and do not represent those of the OECD. This article describes personal research in progress by the author and is published to elicit comments, to further debate and call for material collaboration in future developments of this library.

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1 Introduction.

MSVARlib 2.0, *Markov-Switching Vector Autoregression Library*, is an upgraded open-source basic package designed to model univariate or multivariate regime dependent time series. Among the available public resources in matrix languages, lots of programs have spread thanks to the work of Kim and Nelson (1999) and Hamilton (1989), however most of their routines were not designed in a general multivariate framework, but rather as specific programs not so easy to be extended.

Econometricians or statisticians may be eager to understand their estimate results and to master all the assumptions and implied choices related to the numerical procedures inside their programs. MSVARlib has been designed on this purpose: it offers an open-source set of procedures dedicated to estimate basic mixing law models inspired by Hamilton (1994) or generic Multivariate Markov-Switching regressions. Providing a wider array of specifications regarding autoregressive and exogenous systems, it is fully related to the MSVAR library developed by Krolzig (1998)[2] focusing on Markov-Switching Vector Autoregressions estimates through maximum likelihood methods. To my knowledge, there are few libraries covering the extent of the MSVARlib package. The Krolzig's MSVAR library offers a comprehensive and integrated toolbox, with lots of diagnostic tools[3]. However it is not available on a free access and on an open-source basis[4]. Khaled (2004), has also developped a promising Matlab package following a Bayesian approach and covering the same kind of specifications, but is not public for the time being.

Contrary to Ox, the library is not designed in an oriented-object environment, but still fully uses the power of a matrix language. It has been first developed in the Gauss environment, to propose a flexible way for applied economists to estimate hidden markov models in a familiar and widespread language matrix. However the use of specific gauss subroutines and matrix operators has been minimized so as the library to be easily translated in other languages. Thus, this package is being developed in the Scilab environment and might eventually be included in the GROCER Toolbox, a complete econometric package designed by Eric Dubois[5].

The paper is organized as follows. The first two sections introduce and deliver the usual copyright and disclaimers. The third section presents the general framework of Multivariate Markov-Switching Regression Models covered by the library and discusses its derivative models. The fourth sections hints at log-likelihood and related algorithms. The fifth section gives some illustrations through univariate and multivariate estimations. It introduces a stochastic model of the American unemployment rate and hints at two US-coincident business cycle models, already outlined in Ferrara (2003), Bellone, Gautier and Lecoent (2004) or Bellone (2004a). The fifth section concludes on possible extensions to co-integrated models and enhanced diagnosis tools. Detailed guidelines in

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[2]Because they have inspired this research a lot, a careful reading of Krolzig's and Doornik's works should be all the more useful. See http://www.economics.ox.ac.uk/research/hendry/krolzig/ and http://www.doornik.com/index.html.

[3]In this respect, MSVAlib 2.0 has been quite enhanced compared to the primal version 1.1 introduced in Bellone (2004b), but remains more modest.

[4]MVAR is a compiled, proprietary package.

[5]Grocer is availabe at http://dubois.ensae.net/ and is referenced as a contribution on the Scilab web site http://scilabsoft.inria.fr/

appendices are provided to help the reader handling the MSVARlib Gauss package and its related replication files.

2 Copyright, Conditions of Use and Disclaimer.

The GAUSS programs available on the website http://bellone.ensae.net[6] are written and copyrighted © 2004 by Benoit BELLONE, all rights reserved. They can be run on Gauss 3.2 or upper versions and should be OX-Gauss compliant, thanks to the routine M@ximize developped by Laurent and Urbain (2005)[7]. The code is licensed gratis to all third parties under the terms of this paragraph. Copying and distribution of the files in this archive is unrestricted if and only if the files are not modified. Modification of the files is encouraged, but the distribution of modifications of the code in this archive is unrestricted only if you meet the following conditions: modified files must carry a prominent notice stating

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If you use the code extensively in your research, you are requested to provide appropriate attribution and thanks to the author of the code. No representation is made or implied as to the accuracy or completeness of the programs which may indeed contain bugs or errors unknown to the author. Benoit Bellone takes no responsibility for results produced by MSVARlib programs which are used entirely at the reader's risk. This package is by no means finished yet, a enhancement "to do list" remains open. If you plan to extend the library, find any problems or have suggestions for improvement, contact the author[8].

3 The general multivariate Markov-switching regression model and its derivatives.

For a comprehensive presentation of Markov-Switching Vector Auto-Regression models, the reader should first report to Krolzig (1997) and to Krolzig (1998) or Krolzig (2003) for a complete introduction to the estimations of regime switching models with Ox. The library owes also a lot to the previous works of Roncalli (1995) who developed many econometric routines in Gauss that were heavily used or transformed in this package. For a basic presentation of Hidden Markov Models and simple gaussian distribution mixtures, one should report to Hamilton (1994). Note that contrary to MSVARlib 1.1, the 2.0 version of the library covers an extended range of models. Following the Krolzig taxonomy, this version includes the MSI(M)(AH)-VARX(q) "intercept regime dependent"

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[6]Also on the mirror site http://perso.magic.fr/benbellone

[7]This Ox package is an Optmum substitute for Ox users and is available at http://www.core.ucl.ac.be:16080/ laurent, Sebastien Laurent's web site.

[8]Email: benoit.bellone (at) ensae.org

family, but excludes the MSM(M)-VARX(q)"mean regime dependent" models. This version introduces a generic open source code and functions, that should be easy to extend to more complex specifications such as co-integrated Markov-Switching Models or regime dependent volatility models.

3.1 A general multivariate Markov-Switching framework.

Be $y_t = (y_{1,t}, \ldots, y_{p,t})$ a vector of size $(1, p)$, with p the number of variables of interest. We define $S_t = \{1 \ldots M\}$ as a M-state unobserved variable, following a first order Markov Chain. $S_t = 1$ (resp. $S_t = M$), means that the time series are said to be in the "lowest" (resp. the "highest") regime. The matrix P of transition probabilities is defined very classically, following notations in Hamilton (1994):

$$
P = \begin{pmatrix} p_{11} & \cdots & p_{M1} \\ p_{12} & \cdots & p_{M2} \\ \vdots & \vdots & \vdots \\ p_{1M} & \cdots & p_{MM} \end{pmatrix}, \text{ with } \sum_{j=1}^{M} p_{kj} = 1, \text{ and, } p_{kj} \geqslant 0, \ \forall k, j \in \{1 \ldots M\}.
$$

Let $I_{t-1} = (y_{t-1}, \ldots, y_1)$, the information set available in t-1, then :

$$
\begin{aligned}
p_{kj} &= P(S_t = j | S_{t-1} = k, \ldots, S_1 = l, I_{t-1}) \\
&= P(S_t = j | S_{t-1} = k), \forall k, j \in \{1 \ldots M\}.
\end{aligned}
$$

The "shadow random variable"[9] ξ_t denotes a $(M, 1)$ vector whose jth element is defined as follows, $\forall j \in \{1 \ldots M\}$:

$$
\xi_t^j = I_{S_t = j}, \text{ and, } \xi_t = \begin{pmatrix} \xi_t^1 & \cdots & \xi_t^M \end{pmatrix}.
$$

When $S_t = j$, then $\xi_t = e'_j$ where e_j is the jth column vector of the identity matrix I_M, which leads to :

$$
\xi_t = \sum_{j=1}^{M} e'_j . \xi_t^j.
$$

Thanks to these notations, the conditional probability related to the state j is expressed as:

$$
P(S_t = j | I_t) = E(\xi_t^j | I_t)
$$

and we note the vector of filtered probabilities:

$$
\begin{aligned}
P(S_t | I_t) &= E(\xi_t | I_t) &= \begin{pmatrix} P(S_t = 1 | I_t) & \cdots & P(S_t = M | I_t) \end{pmatrix} \\
&= \widehat{\xi}_{t/t} &= \begin{pmatrix} E(\xi_t^1 | I_t) & \cdots & E(\xi_t^M | I_t) \end{pmatrix}.
\end{aligned}
$$

Let $x_t = (x_{1,t}, \ldots, x_{n,t})$ and $z_t = (z_{1,t}, \ldots, z_{q,t})$ be $(1, n)$ (resp. $(1, q)$) vectors of exogenous regressors subject (resp. not subject) to switching regimes. We note respectively $\beta_{S_t} = (\beta_{S_t}^1, \ldots, \beta_{S_t}^p)$ the (n, p) matrix of regression-coefficients which are regime dependent, and $\delta = (\delta^1, \ldots, \delta^p)$ the (q, p) matrix of regression-coefficients which are regime independent. Let 1_p be a $(1, p)$ vector of 1's and define:

$$
\Gamma_{S_t} = (\beta_{S_t} | \delta)' \text{ and, } A_t = (x_t | z_t)
$$

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[9]See Hamilton (1994), pp 679.

The most general model estimated with MSVARlib is a Markov Switching Gaussian Model :

$$\left\{ \begin{array}{l} y_t = A_t.\Gamma_{S_t} + u_t \\ u_t|S_t \sim N(0, \Sigma_{S_t}) \end{array} \right.$$

with

$$\Sigma_{S_t} = \left(\begin{array}{ccc} \sigma_{11}(S_t) & \ldots & \sigma_{1M}(S_t) \\ \vdots & \ddots & \vdots \\ \sigma_{M1}(S_t) & \ldots & \sigma_{MM}(S_t) \end{array} \right)$$

such as [10],

$$\begin{array}{rcll} \sigma_{ii}(S_t) & = & \sigma_i^2(S_t), & \sigma_{ii}(S_t) \geqslant 0 \\ \sigma_{ij}(S_t) & = & \rho_{ij}(S_t).\sigma_i(S_t).\sigma_j(S_t), & \forall i \neq j \in \{1..M\}. \end{array}$$

Differencing independent and dependent regime regressors, we introduce the equivalent formula[11]:

$$\left\{ \begin{array}{l} y_t = x_t.\beta_{S_t} + z_t.\delta + u_t \\ u_t|S_t \sim N(0, \Sigma_{s_t}) \end{array} \right. \tag{1}$$

MSVARlib allows different specifications of regime dependent (resp. independent) covariance matrix such as :

- Generalized Covariance Matrix:

$$\Sigma_{S_t} = (\sigma_{ij}(S_t)) \text{ or, } \Sigma = (\sigma_{ij}), \forall i,j \in \{1..M\}$$

- Heteroskedastic Covariance Matrix:

$$\Sigma_{S_t} = \left(\begin{array}{ccc} \sigma_1^2(S_t) & \ldots & 0 \\ 0 & \ddots & 0 \\ 0 & \ldots & \sigma_M^2(S_t) \end{array} \right) \text{ or, } \Sigma_{S_t} = \Sigma = \left(\begin{array}{ccc} \sigma_1^2 & \ldots & 0 \\ 0 & \ddots & 0 \\ 0 & \ldots & \sigma_M^2 \end{array} \right)$$

- Homoskedastic Covariance Matrix:

$$\Sigma_{S_t} = \sigma^2(S_t).I_p \text{ or, } \Sigma = \sigma^2.I_p.$$

3.2   MSVARlib and its derivative models.

MSVARlib offers five different family models[12], that are a derived form of the equation (1):

(1) The Mean-Variance model:

$$y_t = \nu_{S_t} + u_t = 1_p.\beta_{S_t} + u_t, \tag{2}$$

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[10]The following representation is all the more important as it is used to constrain the parameters, see MSVARinit.prg.

[11]In fact, it corresponds to the Model 4, see further.

[12]Those models are related to the option _typmod, taking the value 1 to 5 in the program

(2) The MS-VAR regime dependent model:

$$y_t = \nu_{S_t} + y_{t-1}.\beta^1_{S_t} + \cdots + y_{t-q}.\beta^q_{S_t} + u_t = (1_p, y_{t-1}, \ldots, y_{t-q}).\beta_{S_t} + u_t, \qquad (3)$$

(3) The MS-VAR Intercept regime dependent model:

$$y_t = \nu_{S_t} + y_{t-1}.\delta_1 + \cdots + y_{t-q}.\delta_q + u_t = 1_p.\beta_{S_t} + (y_{t-1}, \ldots, y_{t-q}).\delta + u_t \qquad (4)$$

(4) The partially regime dependent MS-Regression model:

$$y_t = x_t.\beta_{S_t} + z_t.\delta + u_t, \qquad (5)$$

(5) The general MS-Regression model:

$$y_t = x_t.\beta_{S_t} + u_t, \qquad (6)$$

The first specification corresponds to the simpler version MSI(M)H-VAR(0) model which is used in Ferrara (2003) or Bellone (2004a) to extract a US-recession model. Following the Krolzig taxonomy, the Model 2 (resp. Model 3) can be identified as a MS(M)-VAR(q) (resp MSI(M)-VAR(q)) model. Eventually, both "Markov-switching" multivariate regressions are implemented via "mix" models with partial (Model 4) or full regime dependency (Model 5), which are close to MSI(M)-VARX Krolzig's modelling. Of course, each specification can be combined to different covariance options, as mentioned in the previous section.

4 The Log-Likelihood, the Filters and the EM algortihm

4.1 A conditional innovation gaussian model.

From a practical point of view there are M "shadow models", which are piled up during the estimates with:

$$\Sigma_{S_t} = \Sigma_1 \xi^1_t + \cdots + \Sigma_M \xi^M_t \text{ and, } \beta_{S_t} = \beta_1 \xi^1_t + \cdots + \beta_M \xi^M_t.$$

So, M distinct models are simultaneously estimated such as:

$$(1_p \otimes y_t) = x_t.(\beta_1 | \ldots | \beta_M) + (1_p \otimes z_t).\delta + (u^1_t | \ldots | u^M_t).$$

Conditioning by S_t, M different $(p \times 1)$ gaussian vectors have to be taken into account. Thus, for a given regime $S_t = j$:

$$\begin{cases} y_t = x_t.\beta_j + z_t.\delta + u_t \\ u_t | S_t = j \sim N(0, \Sigma_j) \end{cases} \qquad (7)$$

Let's note $\mu_{t,j} = x_t.\beta_j + z_t.\delta$ and let Θ be the $(1, n_\Theta)$ vector of related parameters. Then, the conditional probability density function is:

$$f(y_t | S_t = j, I_{t-1}, \Theta) = (2\pi)^{-\frac{p}{2}}.det(\Sigma_j^{-\frac{1}{2}}).exp\Big(-\frac{(y_t - \mu_{t,j})'\Sigma_j^{-1}(y_t - \mu_{t,j})}{2}\Big). \qquad (8)$$

To a certain extent, the models can then be summed-up as a M "stacked" vector of conditional gaussian densities estimated by maximum likelihood with the following properties linked to the Kitagawa-Hamilton filter. To summarize the logic of the MSVAR main program:

- First, initial values of the vector of parameters are calculated. If the automatic intializer is choosen , the series are sorted, split in M parts on which initial conditional regressions are computed to launch the Maximum likelihood descent. If available, an initial a priori datation can be loaded and use to sort the data. Eventually, manual initialization remains possible, whether in practice, not often used.

- Second, the model is recursively estimated through the "EM" algorithm, starting from the unconditional density of y_t which is calculated by summing conditional densities over possible values for S_t:

$$f(y_t|I_{t-1}, \Theta) = \sum_{j=1}^{M} P(S_t = j, y_t|I_{t-1}, \Theta)$$

or,

$$f(y_t|I_{t-1}, \Theta) = \sum_{j=1}^{M} P(S_t = j|I_{t-1}, \Theta).f(y_t|S_t = j, I_{t-1}, \Theta). \tag{9}$$

The maximum likelihood estimate of Θ is obtained by maximizing the log-likelihood :

$$L(\Theta) = \sum_{t=1}^{T} ln(f(y_t|I_{t-1}, \Theta))$$

with $\Theta = (\theta_P, \theta_\beta, \theta_\delta), \theta_\Sigma)$, is the vectorized matrix of parameters[13].

4.2 A priori filtering and smoothing with the Kittagawa-Hamilton's algorithms.

The filtered probability is computed such as:

$$P(S_t = j|I_t, \Theta) = P(S_t = j|y_t, I_{t-1}, \Theta) = \frac{P(S_t = j, y_t|I_{t-1}, \Theta)}{f(y_t|I_{t-1}, \Theta)}$$

that is to say:

$$P(S_t = j|I_t, \Theta) = \frac{P(S_t = j|I_{t-1}, \Theta).f(y_t|S_t = j, I_{t-1}, \Theta)}{\sum_{k=1}^{M} P(S_t = k|I_{t-1}, \Theta).f(y_t|S_t = k, I_{t-1}, \Theta)} \tag{10}$$

Following the notations of Krolzig or Hamilton, the algorithm replicates the generic following formula to extract a $M \times T$ matrix of filtered probabilities, the components of which are expressed in the following matrix:

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[13]MSVAR_init.prg should be carefully examined to master all the details related to the constrained optimization and initialization of parameters.

$$P(S_t|I_t) = \widehat{\xi}_{t/t} = \frac{f(y_t|S_t) \odot \widehat{\xi}_{t/t-1}}{1'_M \left(f(y_t|S_t) \odot \widehat{\xi}_{t/t-1} \right)} \tag{11}$$

where $\odot$ (.* in the Gauss matrix language) designates the element by element product. A novel aspect of the MSVARlib package is also to define a more concise formulation of the smoothing algorithm in a multivariate specification, that of course applies in an univariate framework through the recursive formula:

$$P(S_t|I_T) = \left(P(S_t|I_t)' \odot \left\{ P' \left[P(S_{t+1}|I_T)' \oslash P(S_{t+1}|I_t)' \right] \right\} \right)', \ \forall t \in \{1 \dots T-1\} \tag{12}$$

where $\oslash$ (./ in the Gauss matrix language) designates the element by element division. For a more comprehensive cover of the Baum-Lindgren-Hamilton-Kim (BLHK) filters and smoothers in a multivariate framework, the reader should report to Krolzig(1997), chap 5, and Hamilton (1994), chap 22.

5 Three illustrations of MSVARlib estimations.

Many replication files are joined with the MSVARlib package. Here we propose to detail an univariate models MSI(2)-AR(q) and two multi state multivariate Mean-Variance model(MS(M)-VAR(0)) which refer to some illustrations of Ferrara (2003), Bellone, Gautier, Lecoent (2004) and Bellone (2004a).

5.1 A stochastic MSI(2)-AR(q) analysis of the US employment rate

In the program UNMod3.prg and UNMod1.prg, we assume that the dynamic of the US unemployment rate[14] is subject to two regimes: a low one identifying a steep rise in unemployment and a high regime, matching a stabilization or a decrease in the unemployment rate. Many papers, see for instance Rothman (1998) or Hamilton (2005) for a justification of such a model by asymmetric variations during NBER recessions calling for possible non-linear behaviors.

If we note y_t^* the reverse two-month growth rate of unemployment (ie $y_t^* = 100 \times (ln(un_{t-2}) - ln(un_t))$), with un_t the US unemployment rate, and y_t the related standardized growth rate.

The model MSI(2)H-AR(3) is summarized as follow:

$$\begin{cases} y_t = \nu_{S_t} + \delta_1.y_{t-1} + \delta_2.y_{t-2} + \delta_3.y_{t-3} + u_t \\ u_t|S_t \sim N(0, \sigma_{S_t}^2) \text{ with, } S_t \in \{1,2\}. \end{cases} \tag{13}$$

The model MSI(2)H-AR(0) is a simple Mean-Variance model:

$$\begin{cases} y_t = \nu_{S_t} + u_t \\ u_t|S_t \sim N(0, \sigma_{S_t}^2) \text{ with, } S_t \in \{1,2\}. \end{cases} \tag{14}$$

---

[14]Here we deal with $1/u_t$ to match a rise in the probability of being in a low regim with a rise in the unemployment rate

Here, intercepts and variance are regime dependent:

$$\nu_{S_t} = \nu_1 \xi_t^1 + \nu_2 \xi_t^2, \text{ with } \nu_1 < \nu_2 \text{ and, } \sigma_{S_t}^2 = \sigma_1^2 \xi_t^1 + \sigma_2^2 \xi_t^2.$$
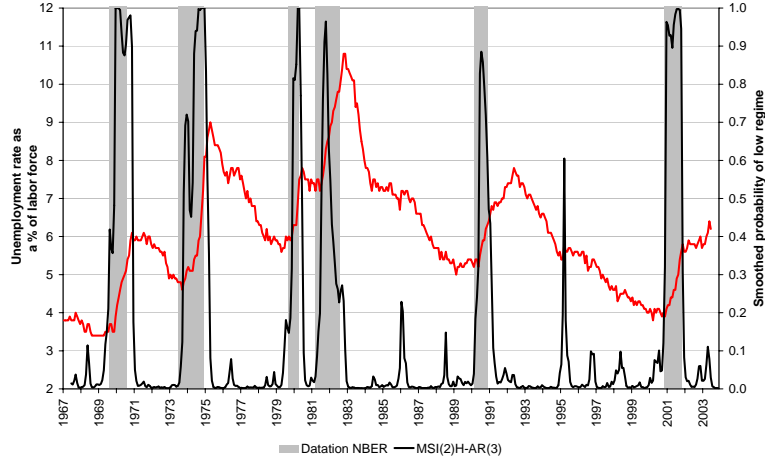

Figure 1: Smoothed probability of the MSI(2)-AR(3) low regime - Unemployment rate model

Appendix B (page 18) provides the detailed estimated parameters, standard deviations, transformed parameters, normality test on residuals, information criteria , graphs of filtered and smoothed probabilities and graphs of residuals . Figure 1 shows the orginal unemployement series, the NBER datation and the smoothed probabilities of contracting. The hypothesis of normality of residuals cannot be rejected in the specification of the first model, contrary to the simpler mean-variance model. However, if this specification is not statistically rejected, autoregressive components are often quite subject to critics. If they correct for non-gaussian disturbances, they might deteriorate the quality of the signal of the filtered probability. Thus, it should be noted that (see Figure 2) the more parcimonious model sends far clearer and even earlier signals of turning point, as already noticed by Albert and Chib (1993) and Lahiri and Wang (1994).

To conclude this section, as mentioned in Anas and Ferrara (2002) or in Bellone and Saint-Martin (2003), the unemployment rate remains one of the most reliable coincident indicator to detect recessions and confirm some non linear behavior. Appendix C estimates and graphs give some evidence of the following stylized facts : accelerations are steep in recession (the average standardized growth rate of rising unemployment is fourfold the average standardized growth rate of decrease in unemployment), decreases are slow in recovery phase, volatility is higher in recessionary phases (the variance of the low regime is twofold the high regimes' one).
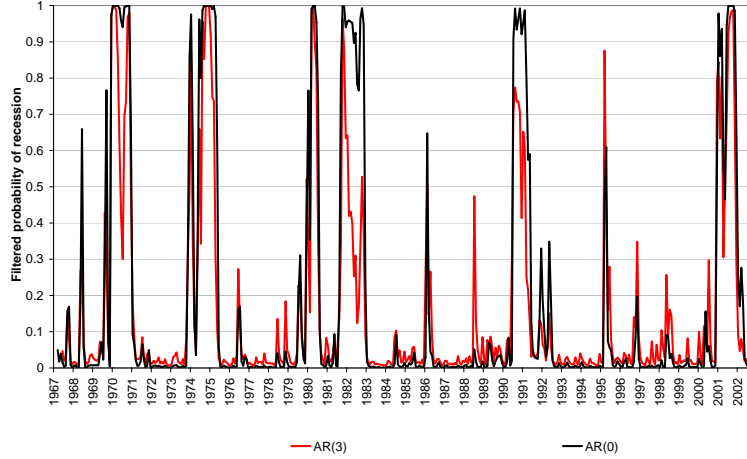
Figure 2: Filtered probabilities of the MSI(2)-AR(3) and MSI(2)-AR(0) low regime - Unemployment rate model

5.2 Detecting US recessions and slowdowns with Markov-Switching models.

Two stochastic models of the US business cycles are presented in this section. First a model estimated by Bellone (2004a) with the assumption of two regimes "recession vs expansion". A more refined model is estimated, extending the assumptions of Ferrara (2003) by Bellone, Gautier, Lecoent (2004), which introduce a third regime. The first model includes four series in the following order: the reversed unemployment rate, the help wanted advertising index, the Industrial Production index, 1000/Jobs hard to Get component (see the `MSVARRec4.txt` file and the `MSVAR.prg` program). Bellone (2004a) prefers a two-state model rather than a three-state model because of higher stability and robustness to changes in an extended sampling period of 1967-2000. The related model is a MS(2)H-VAR(0) :

$$
\begin{cases}
(y_{1,t}, \ldots, y_{4,t}) = (\nu_{1,S_t}, \ldots, \nu_{4,S_t}) + u_t \\
u_t | S_t = (u_{1,t}, \ldots, u_{4,t}) | S_t \sim N(0, \Sigma_{S_t})
\end{cases}
\tag{15}
$$

with,

$$
S_t \in \{1, 2\} \text{ and } \Sigma_{S_t} = \begin{pmatrix} \sigma_1^2(S_t) & \ldots & \sigma_{M1}(S_t) \\ \vdots & \ddots & \vdots \\ \sigma_{1M}(S_t) & \ldots & \sigma_M^2(S_t) \end{pmatrix}
$$

The figure 3 gives evidence that each period of a low regime of the filtered probability is coincident with a NBER recession. As expected, covariances in the lower regime are bigger than in the "growth" regimes. Note that the seven NBER recessions are clearly
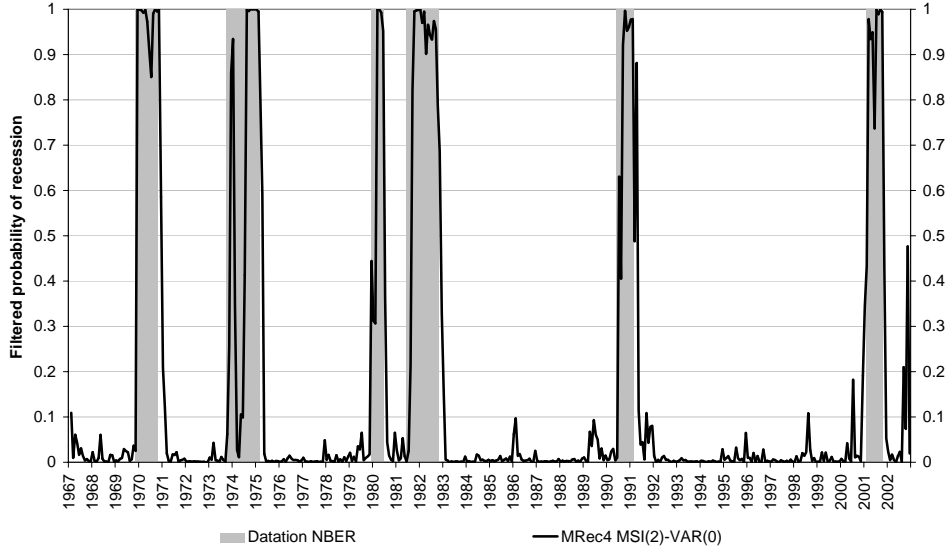
10

Figure 3: US coincident multivariate model - Filtered probability of a recession.

detected with the "low regime" filtered probability, with short lags: over the eighties peaks are detected on average with one month lags, and trough are coincident. Because three series out of four are un-revised on a short term basis, this indicator should deliver reliable and timely real-time signals of start and end of a recession. The reader can refer to Bellone (2004a) for the estimations or use the `MSVAR.prg` program to replicate those results[15].

Figure 4 echoes a really close model as introduced in equation (15). IT differs only by the number of regimes: $S_t \in \{1, 2, 3\}$ and by an assumption of regime independency of covariance: $\Sigma(S_t) = \Sigma$, following Ferrara (2003) and some statistical tests. It is an enhanced estimation of the model proposed by Ferrara (2003): three filtered probabilities are calculated, associated to three states : "low growth ", "medium or below potential " and "high growth ". To sum this in two signals, the difference of high and low regime probabilities is calculated (bold line), varying between -1 and 1, and the intermediate regime is directly represented by the associated filtered probability. The slowdown periods are signaled by a rising medium regime probability , varying between 0 and -1, that is a drop of the dashed line in negative territory or by oscillations of the difference of filtered probabilities around 0 (see "High-Low" bold line). Recessions are signaled by a significant drop [16] in negative territory, and steady growth by a probability in a positive territory. This three-state model is implemented in the `Model1.prg` program,

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[15]The Mrec4 model is selected as the reference model in MSVAR.prg

[16]Usually with a probability above 50%.

full detailed results, of which transition matrix, parameters and ergodic probabilities, related standard deviations are printed out in the `Threestate\_output.txt` and a short summary is printed in Appendix C page 23. This model deals with the first three series previously mentioned and adds real construction spending [17].
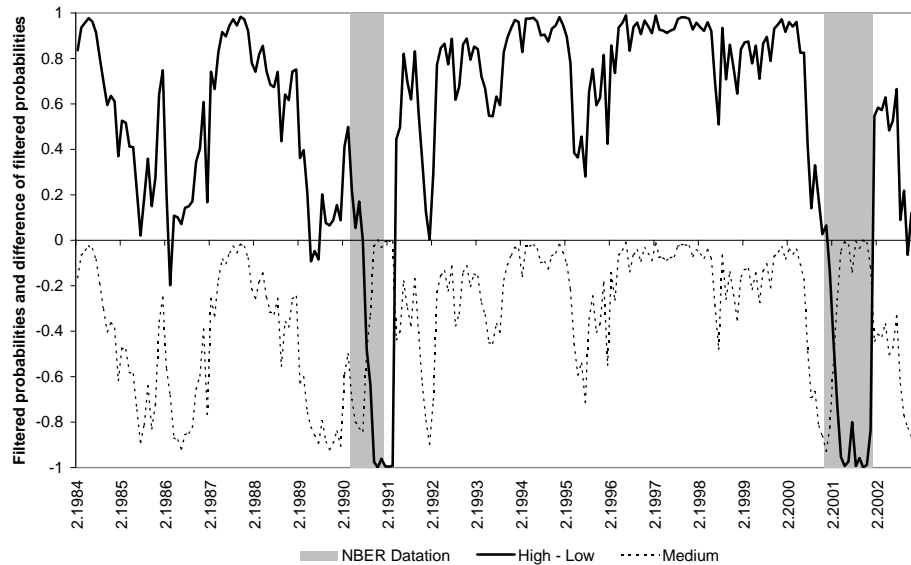

Figure 4: Filtered probabilities of the Three state model. Estimation period (1984-1 / 2003-2)

As introduced by Lahiri and Wang (1994) or Sichel (1994) three state-models may offer a more precise description of short term macroeconomic developments, exhibiting alternance of high growth, slow growth and recession regimes. The three-state models allows to detect "slowdowns", often early warning of a recessionary period, such as those observed in 1989 or in early 2000. It also hints at the qualitative intensity of a recovery or a slowdown in the business cycle as in early 1992 or in early 2003. However, the model seems extremely sensitive to the sampling period, which denotes a likely low robustness before 1984 and a lack of stability (or identification abilities) between the medium and the high regime[18]. The reader should experience varying estimation periods, different options of covariance matrix specification and a test of a MSI(M)H-VAR(q) extension to judge this effective sensitivity compared to a two-state specification.

---

[17]Since 2004, the BLS has stopped producing those series. New releases starts only at the beginning of the 90's. Besides, because of a less timely publication and those disrupted samples, Bellone (2004a) discarded the construction spending series and included a survey dealing with employment condition, the "Jobs hard to get" component.

[18]This is a well known feature of the US business cycle, studied in detail by Stock and Watson (2003) and discussed also in Bellone, Gautier, Lecoent (2004)

6   Conclusion.

MSVARlib should provide to the applied econometrician convenient and open source programs to estimate through EM algorithm basic and generic Multivariate Markov Switching Regression models. MSVARlib is still a modest package and a less "user friendly"' than the Krolzig's "MSVAR for Ox" compiled programs, but the package is transparent, quite flexible and could be easily adapted to new inclusive models such as Markov switching volatility models, Short-term forecasts automatic models... If a "to do list" had to be designed, here could be the next adds and improvement:

- a clearing and optimization of algorithms (there are probably some gains to make),

- an extension of the model to "Student" innovations to challenge the gaussian assumption, as proposed in Hamilton (2005),

- a development of news statistical tests and tools which would complete the new packages introduced in the 2.0 version (information criteria, normality tests...) such as correlograms, spectral densities, empirical densities, Davies Test, specification tests of Hamilton (1996) or Carrasco et al.(2004), and non-linearity tests such as the one proposed in Hansen (1992)...,

- an extension to forecasting and co-integration analysis,

- an adaptation to a Multivariate Bayesian approach with a Multimove Gibbs sampler as proposed in Khaled (2004).

Acknowledgements.

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Appendix A :Installation and Main structures

Before installing the package and execute the applications, you need to be able to run a GAUSS program. For the time being, MSVARlib 2.0 is written in the GAUSS® matrix language and requires the GAUSS® 3.2 or a later version of the software. It has been tested on the 3.2, 5.0 and 6.0 versions and has been designed in a WINDOWS 95/NT environment. It should be soon extended in the Scilab language.

To install and run these programs, you must absolutely create a directory `C:\GAUSS\MSVAR` where you should unzip the MSVARlib package. This package includes two directories, a readme file and this paper:

- the `MSLIB` directory includes the main program `MSVAR.prg`, program files and saved output results, - the `DATA` directory includes input data files and data sample spreadsheets with convenient templates. - Readme.txt exhibits those basic installation instructions - the `MSLIB\EXAMPLES` includes example program files related to this paper.

Once unzipped, you should place the first two directories in the following subdirectories:

```
C:\GAUSS\MSVAR\MSLIB
C:\GAUSS\MSVAR\DATA
```

To start theestimation, open the Gauss® program, select `C:\GAUSS\MSVAR\MSLIB` as your working library.

Data organization

In the `DATA` directory, some sample files are available. They are all ASCII files, saved in the ".txt" format with tab-spaces separators. These data are based upon American surveys and quantitative time series starting from January 1960, an depending on the vintage, to February 2004 or February 2003. The first two columns of an input file should start with a month series (m format: 1 to 12) and a year series (`yyyy` format), the following columns should refer to data. No label should be referenced A missing value is represented by a dot ".".

Here is a list of included sample files in the `DATA` directory:

- `MSVARrec4.txt` includes the four series used in Bellone (2004b) as a US recession two states hidden markov models: they belong to the "April 2004 vintage" (february 2004 is the minimal common date). Those four series are: the reversed unemployment rate, the help wanted advertising index, the Industrial Production index, 1000/Jobs hard to Get component.

- `MSVARrec3.txt` includes the three first indicators for the "March 2003 vintage",

- `MSVARrec4_2.txt` and `MSVARrec3_2.txt` include the same series but refer to an updated "April 2004 vintage", which allows to assess the possible impact of revisions between March 2003 and February 2004.

- `MSVARAnas.txt` deals with the four series used in Ferrara (2003) and in Bellone, Gautier, Lecoent(2004) as a US recession three states markov models: they are related to the "March 2003 vintage". Those four series are: the reversed unemployment rate, the help wanted advertising index, the Industrial Production index, the real construction spending.

- `MSVARUN.txt, MSVARHW.txt, MSVARIPI.txt`, include the first three series related to the "February 2004 vintage".

- `MSVARUNHWA.txt`, includes the reversed unemployment rate, the help wanted advertising index, the first three series related to the "February 2004 vintage".

On top of these files two spreadsheets include the whole data set and a utility to print output.

Main program files

A MSLIB\EXAMPLES subdirectory includes replication files that should be extracted and paste in the MSLIB directory to be launched. UNMod3.prg and UNMod1.prg are the main program used to estimate univariate MSI(2)H-AR(3) and MSI(2)H-AR(0) models associated to the American unemployment rate. The Model1.prg file provides the multivariate model estimated in Bellone(2004). Other models (3 and 5) can be assessed thanks to Model3.prg or Model5.prg.

The reader will find in the MSLIB directorty the set of programs of which:

- MSVAR.prg :runs the main program[19].
- MSVAR_call.prg : calls to all required subroutines[20].

Then, here comes the files[21] including subroutines dedicated to Input/Output relations, basic statistics tools. :

- MSVAR_load.prg: generates input files,
- MSVAR_Setsample.prg: performs transformation of input series (differences, dlog(), level, standardization),datation references, and selects the model among five generic versions,
- MSVAR_Setdatation.prg: transforms a dataset following lags options,
- MSVAR_moment.prg: computes descriptive statistics, sorts series and prepares automatic initializations,
- MSVAR_output.prg: delivers saving subroutines and drawing graphs.

Specific routines concentrating on Markov switching regressions and "EM algorithm" estimation programs follow:

- MSVAR_Initsort.prg: sorts sub-block to compute initialization estimates,
- MSVAR_Init.prg**: includes automatic or manual initialization subroutines of parameters, constraining functions for the optimization,
- MSVAR_vecmat.prg**: generates bijective functions to transform input and output parameters during the process of out-of sample filtering or during the EM-algorithm,
- MSVAR_FiltHmm.prg**: generates the Kittagawa-Hamilton filtered probabilities,
- MSVAR_SmoHmm.prg: computes smoothed probabilities,
- MSVAR_MaxHmm.prg: defines loglikelihood and maximum likelihood routines,
- MSVAR_stderr.prg: computes non linear Wald test and standard deviations, with associated statistics (t-stats, p-value...).

---

[19]All the program files stored in the subdirectory EXAMPLES are mild modifications of the MSVAR.prg

[20]This file si equivalent to a file.h recollection for C programmers or a succession of getf() for Scilab programmers.

[21]The most important files are noted **

Appendix B :Estimating the MSI(2)-AR(3) model of the US unemployment rate with MSVARlib

Guidelines to run a program

Here are some instructions to understand the `UNmod1.prg` program. One should pay attention to the number of series (here $2 + 1 + 2$[22]), to the input file name: `"MSVARUN"` and options linked to the model: `_ncol=5;` and `Data_init_file="MSVARUN";`.

First,the number of variable (`_K=1;`), the number of states (`_M=2;`), and the regime dependency option on the covariance matrix (here, it is regime independent : `_M_V=_M;`) have to be selected. Then, the specification of the covariance matrix (heteroskedastic (1), homoskedastic (2), generalized covariance (3) is implemented : `_Var_opt=1;` (here, the heteroskedastic case). The specifications have to be selected (family model and lags if VAR models) : `_typmod=1;` and `_p=1;` Transforming options of the input series are then selected (`F_option=1; k_lag=2;`). Note that, heterogenous series in need of different transformations should be first included in the input data file and those options should be neutralized in the main program (code 0). The model is estimated on the (1967-2/2004-2) period, that is `deb=86; fin= 530;`. If available, an apriori datation can be loaded and used as a benchmark to initialize the estimation. Here, we assume no apriori datation:`_apriori=0;`. At last, the optimization option (`estim =1`) has to be chosen. Output graphics are presented below and detailed output in UNmodel1_output.txt. The reader can change the periods of estimation, run new estimates and then change the option estim from 1 to 0. By running a second time the program, a set of parameters (saved in `C:\GAUSS\MSVAR\DATA\parameter.txt`) is imported and perform out of sample filtering.

Estimates of the MSI(2)H-AR(3) and related graphs

This model is estimated on the 1967-1/2004-2 period and gives the following results :

$$\begin{cases} y_t = \nu_{S_t} + 0.641.y_{t-1} - 0.458.y_{t-2} + 0.349.y_{t-3} + u_t \\ \nu_{S_t} = -0.80.\xi_t^1 + 0.148.\xi_t^2 \\ u_t|S_t \sim N(0, \sigma_{S_t}^2) \text{ with, } \sigma_{S_t}^2 = 0.706.\xi_t^1 + 0.3495.\xi_t^2 \end{cases} \tag{16}$$

The matrix of transition probabilities :

$$P = \begin{pmatrix} 0.851 & 0.027 \\ 0.149 & 0.973 \end{pmatrix}$$

The first column is associated to the transition from the low regime (that is associated to the State related to an increase in unemployment). Follows the original output results produced by MSVARlib with standard deviation and a full set of statistical tests and information criteria (log-likelihood, convergence criteria, AIC, BIC, matrix of residuals):

```
===MS OLS and MSVAR Analysis  with MSVARlib - Copyright (C) 2004 by Benoit BELLONE ===
Data File MSVARUN

Date:  2/23/05   Initial time:  0:22:52


Start and ending period
      86.000000        530.00000  (1967-2/2004-2)
```

---

[22]an apriori datation is provided in the input file

```
Options
_K,         1.0000000 ,  _M,         2.0000000 ,  _M_V,          2.0000000 ,
 _Var_opt, 3.0000000 ,  _typmod, 3.0000000 ,  _p,            3.0000000


....

=================Final Results of the Maximum likelihood optimization =================

Log likelihood   -449.27843949
Convergence code :       0.00000000
The EM algorithm has converged.


Degree of freedom    434.00000000

Number of observations :           443
Number of parameters :             9


        Estimates      Gradient      Standard-errors      T-student        Pvalue.
-------------------------------------------------------------------------------------
X01    0.850897       0.000399         0.079567         10.694052        0.000000
X02    0.973396      -0.000579         0.012862         75.678896        0.000000
X03   -0.800580      -0.000596         0.224159         -3.571487        0.000395
X04    0.148863      -0.004172         0.051813          2.873082        0.004264
X05    0.705779       0.000843         0.200345          3.522828        0.000472
X06    0.349502      -0.003230         0.031437         11.117596        0.000000
X07    0.641453       0.000645         0.057776         11.102430        0.000000
X08   -0.458772       0.004277         0.051199         -8.960605        0.000000
X09    0.349122       0.000826         0.050938          6.853869        0.000000

Duration of estimation, in secs 8.0

.....

=================Final Parameters   =================


============Final matrix of markovian transition probabilities P[i,j]: =============

      0.85089725        0.02660377
      0.14910275        0.97339623


=================Final ergodic probabilities :=================

      0.15141027
      0.84858973

====Final transposed conditional beta, covariances, var_res by regime ======

=============Regime  1 =============

Beta
```

```
        -0.80057959

Delta

         0.64145339
        -0.45877174
         0.34912150

Sigma
         0.70577942

==============Regime  2 =============

Beta
         0.14886273

Delta

         0.64145339
        -0.45877174
         0.34912150

Sigma
         0.34950180

==============Block matrix of beta, delta,  covariances =============

Beta:
        -0.80057959        0.14886273

Delta:

         0.64145339        0.64145339
        -0.45877174       -0.45877174
         0.34912150        0.34912150

Sigma:
         0.70577942        0.34950180


=== Residual analysis and diagnostics - Copyright (C) 2004 by Benoit BELLONE ===


===Covariance matrix of residuals===
         0.34816237

 === Information Criteria based upon Residual Covariance Matrix Analysis ===
-----------------------------------------------------------------------------
       BIC       AICa      AICc       SIC       FPE       AIC        HQ
-----------------------------------------------------------------------------
     -1.014    -1.035    -0.046     0.354     0.355    -1.041    -1.030

===========  Multivariate analysis of residuals - Jarque Berra =========
```

```
Jarque and Bera Test, Under Ho : 'Residuals are normal'
JB stat:  2.295
Pvalue :  0.317
 The Normality hypothesis cannot be rejected
```
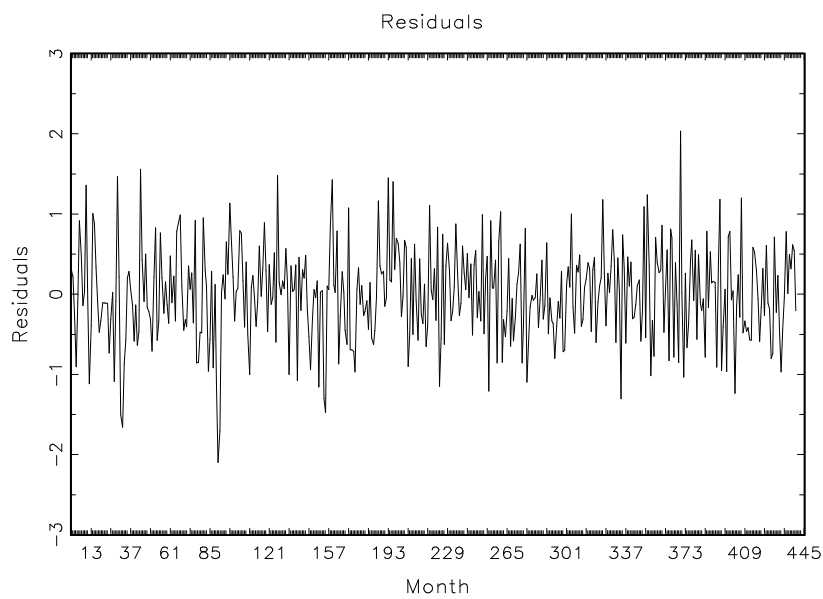
Residuals

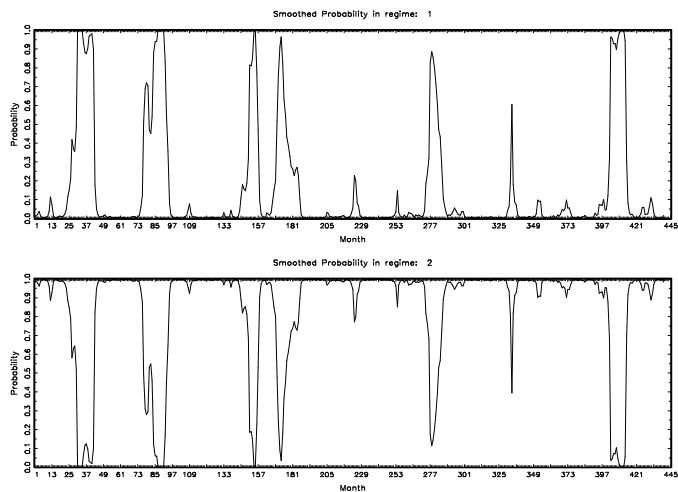Figure 5: Unemployment rate MSI(2)-AR(3): residuals

Figure 6: Unemployment rate MSI(2)-AR(3) - Smoothed probabilities
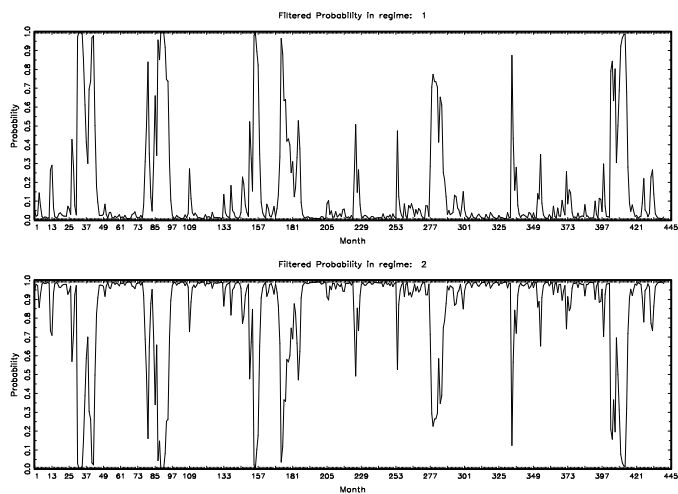

Figure 7: Unemployment rate MSI(2)-AR(3)- Filtered probabilities

Appendix C: excerpt of estimates of the MSI(3)H-VAR(0) US coincident stochastic indicator

This section gives an overview of the output files delivered by the program Model1.prg. To shorten the size of the appendix, some statistics (such as standard deviations) or printed results have been cut. The complete output file is available in `Threestate_output.txt`.

```
===MS OLS and MSVAR Analysis  with MSVARlib - Copyright (C) 2004 by Benoit BELLONE ====
Data File MSVARANAS


Date:  2/21/05   Initial time: 22:55:25


Start and ending period 289.0 517.0


Options
_K,4.0,  _M,3.0 ,  _M_V,1.0 ,
 _Var_opt,3.00000000 ,  _typmod,1.0 ,  _p, 0.0


........

============Final Results of the Maximum likelihood optimization ===========


Log likelihood  -1220.34592691
Convergence code :      0.00000000
The EM algorithm has converged.



Degree of freedom    200.00000000


Number of observations :            228
Number of parameters :          28



Duration of estimation, in secs 51.0



....

=================Final Parameters  =================


=======Final matrix of markovian transition probabilities P[i,j]: ========

        0.88441592       0.02900211       0.00100000
        0.06490924       0.91654270       0.03913726
        0.05067485       0.05445519       0.95986274



=======Final ergodic probabilities :=================

        0.08978919
        0.33811697
        0.57209384
```

```
====Final transposed conditional beta, covariances,
var_res by regime, y1..yk series in column :=======

===================Regime       1.00000000 =================

Beta
     -1.20869689    -1.06686388    -1.41437143    -0.98841911

Delta
      0.00000000     0.00000000     0.00000000     0.00000000

Sigma

      0.83980560    -0.00746475     0.14233815     0.09914807
     -0.00746475     0.86927102     0.11304900     0.05967819
      0.14233815     0.11304900     0.70791669     0.02670463
      0.09914807     0.05967819     0.02670463     0.87896069

==============Regime       2.00000000 ======================

Beta
     -0.03170946    -0.05895193    -0.27084231    -0.09803882

Delta
      0.00000000     0.00000000     0.00000000     0.00000000

Sigma

      0.83980560    -0.00746475     0.14233815     0.09914807
     -0.00746475     0.86927102     0.11304900     0.05967819
      0.14233815     0.11304900     0.70791669     0.02670463
      0.09914807     0.05967819     0.02670463     0.87896069

==============Regime       3.00000000 ======================

Beta
      0.20849722     0.20242343     0.38336724     0.21349575

Delta
      0.00000000     0.00000000     0.00000000     0.00000000

Sigma

      0.83980560    -0.00746475     0.14233815     0.09914807
     -0.00746475     0.86927102     0.11304900     0.05967819
      0.14233815     0.11304900     0.70791669     0.02670463
      0.09914807     0.05967819     0.02670463     0.87896069

=====Residual analysis and diagnostics - Copyright (C) 2004 by Benoit BELLONE =====


===Covariance matrix of residuals===
```

```
 0.83171267     -0.01624330      0.10779953       0.08437932
-0.01624330      0.86119396      0.08333086       0.04730917
 0.10779953      0.08333086      0.61427796      -0.00952972
 0.08437932      0.04730917     -0.00952972       0.86345343
```

=== Information Criteria based upon Residual Covariance Matrix Analysis ===

     BIC   AICa   AICc   SIC   FPE   AIC   HQ

    -0.639   -0.809   0.120   0.629   0.430   -0.880   -0.782

====== Multivariate analysis of residuals - Jarque Berra ==================

Jarque and Bera Test, Under Ho : 'Residuals are normal'
JB stat: 13.112
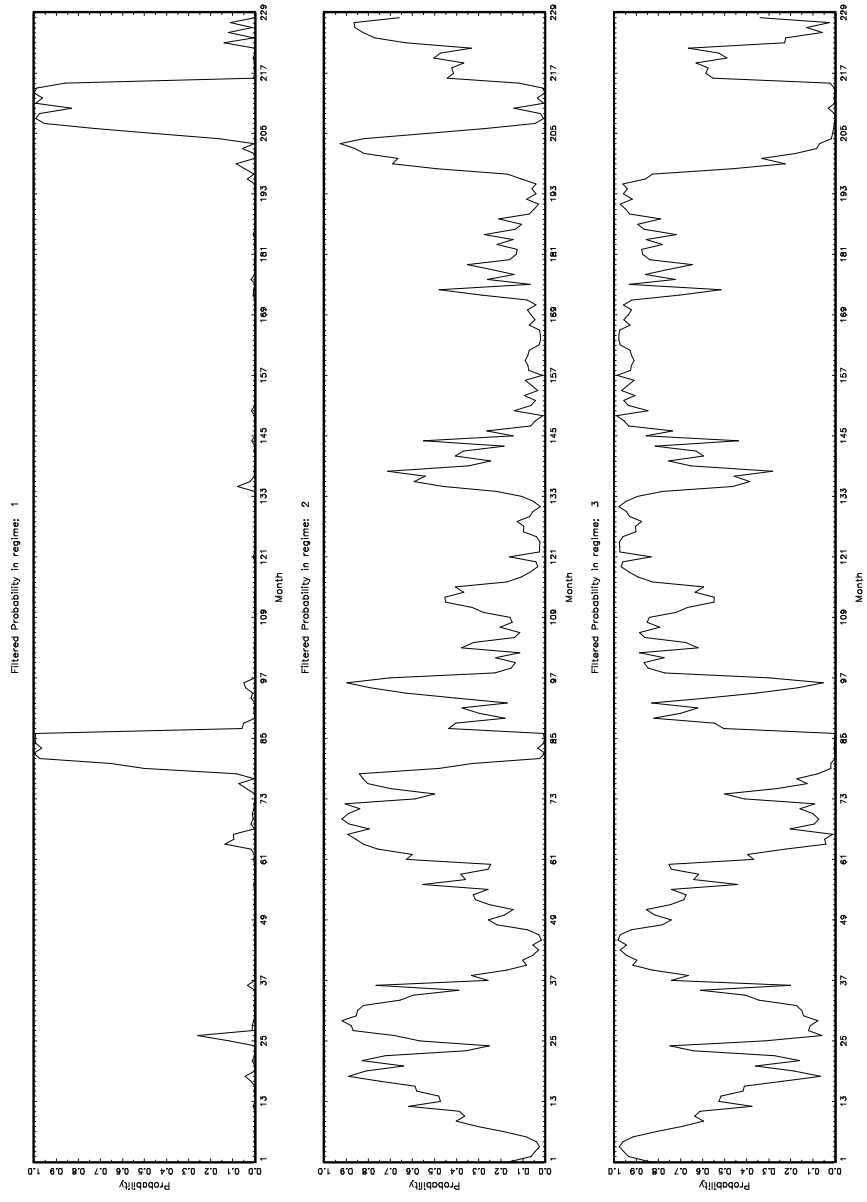Pvalue : 0.108
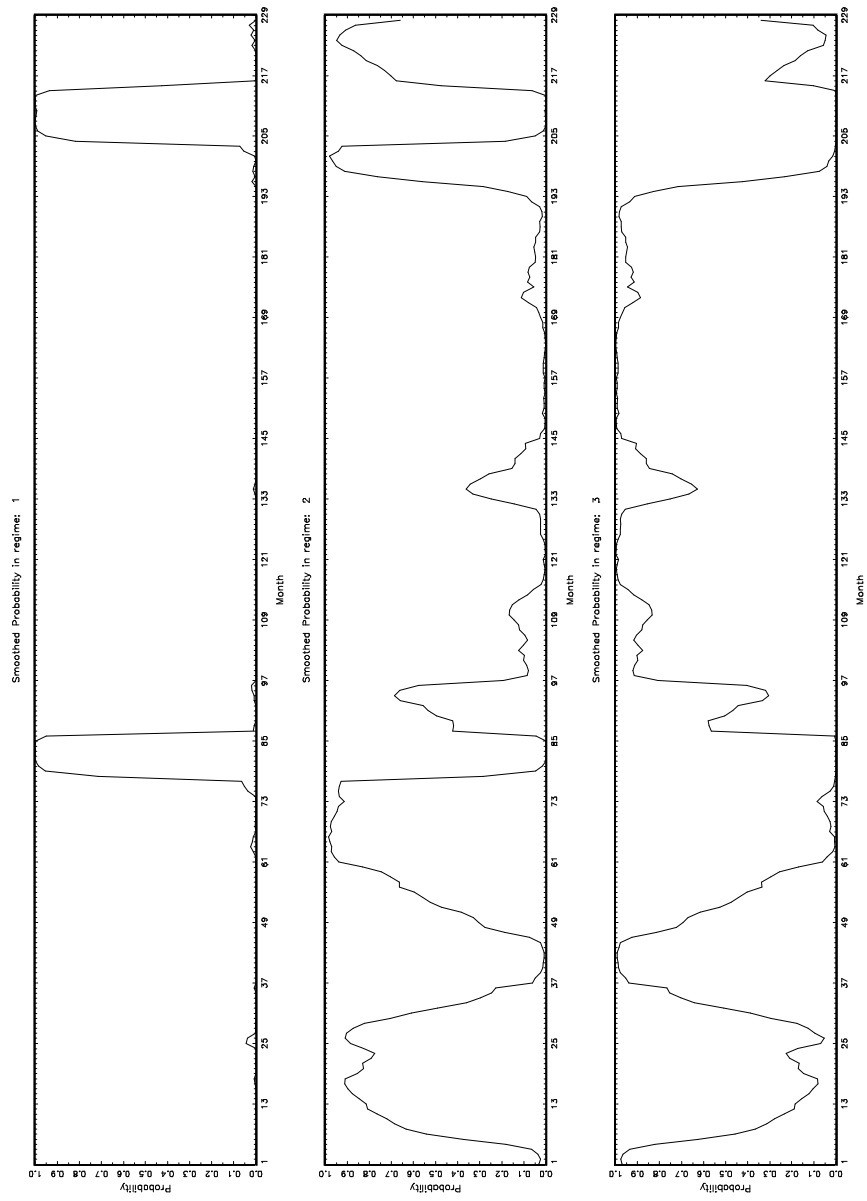 The Normality hypothesis cannot be rejected

Figure 8: MSI(3)H-VAR(0) - Filtered probabilities

Figure 9: MSI(3)H-VAR(0)- Smoothed probabilities