Functional Structure and Approximation in Econometrics

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Preface to
Functional Structure and Approximation in Econometrics

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This book deals with five of the most fundamental problems in applied economics:

- How to estimate consumer preferences;
- How to estimate producer's production functions or any of the dual representations of technology;
- How to aggregate over economic agents;
- How to determine whether an economic aggregate *exists* that has the usual microeconomic properties and
- How to aggregate over commodities.

The first two problems are indeed fundamental. Preferences determine consumer demand and labor supply functions while production functions determine output supply and input demand functions. Virtually all important problems in applied economics depend on the accurate determination of these functions and the elasticities of demand and supply that they generate. Parts 1 and 2 of this volume deal extensively with these first two problems.

The third fundamental problem is an aggregation problem; namely, the aggregation over agents problem. In the household context, we ask how can we best aggregate over consumer demand functions and what properties will the resulting market demand functions possess. Chapter 1 can be viewed as a contribution to this literature. In the producer context, we similarly ask how can we best aggregate over individual firm supply and demand functions in order to obtain the corresponding market supply and demand functions. As Barnett and Zhou observe in chapter 16, a result in Debreu (1959; 45) can be used to prove the existence of an aggregate technology that satisfies all the usual firm microeconomic properties, provided that there is price taking behavior over all producers in the industrial aggregate and provided that there are no fixed inputs.[1]

The fourth problem listed above is also of some importance to applied economists. There are millions of goods and services that are used in a modern economy. Even with powerful computers, it is impossible to estimate flexible preferences or production functions once the number of goods in the model exceeds 100 or so.[2] However, if utility or production functions satisfy *separability conditions*,[3] then the number of parameters required to represent accurately these functions can shrink quite dramatically. Moreover, a large number of economic models simply assume various separability conditions. For example, many consumer models assume that labor supply or leisure demand is separable from the demands for goods and services. Virtually all intertemporal consumer models assume some type of separability between consumption in the present period versus consumption in future periods. Hence, it is very useful to have at our disposal some econometric techniques that enable us to test for the existence of separable aggregates. However, Blackorby, Primont and Russell (1977) showed that testing for separable aggregates in a flexible functional form context is not a trivial problem. The innovative work of Professor Barnett and his co-authors on this topic may be found in chapters 11, 12, 15 and 16 of this volume.[4]

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[1] Bliss (1975; 146) also noted that if all producers are competitive profit maximizers and face the same prices, then the group of producers can be treated as if they were a single producer subject to the sum of the individual production sets. This result can be traced back to May (1946) and Pu (1946). Thus aggregation over producers will be easier to justify if annual data are used instead of monthly or quarterly data, since many inputs are likely to be fixed in the short run.
[2] I have had some success estimating the Diewert and Wales (1988a) semiflexible normalized quadratic functional form for 50 or so goods in the production context but I have not gone beyond this limit.
[3] See Blackorby, Primont and Russell (1978) for the best systematic exposition of separability concepts and their role in economics.
[4] I have also worked in this area of research: see Diewert and Parkan (1985) for a nonparametric approach and Diewert and Wales (1995) for an econometric approach. This econometric approach is used as a building block in Chapter 16 of the present volume.

The fifth fundamental problem in applied economics is also an aggregation problem; given that we plan to aggregate over a number of commodities, what is the "best" way of forming this aggregate? This is the *index number problem*. Professor Barnett and his co-authors have done a considerable amount of work in this area as well; see the monograph edited by Barnett and Serletis (2000). Several of the chapters in the present volume draw on this earlier research on the index number problem and extend this research. In particular, chapter 16 looks at the problem of constructing commodity aggregates when there is uncertainty or risk in the model.

Let us return to the first two fundamental problems in applied economics listed above. The chapters in sections 1.3, 1.4 and 2.2 of this volume deal with virtually all of the problems that arise when it is attempted to estimate utility or production functions (or their dual representations) in a flexible way. The theory of flexible functional forms started with the contributions of Diewert (1971) and Christensen, Jorgenson and Lau (1971) (1973) (1975), who introduced the Generalized Leontief and Translog functional forms into the economics literature. These functional forms are capable of providing second order approximations to any twice continuously differentiable function (satisfying the appropriate regularity conditions) around any specified point. Thus these functional forms can provide first order approximations to arbitrary supply or demand functions without imposing unwarranted a priori restrictions on elasticities of supply or demand. However, a problem with these functional forms soon became apparent: as the number of commodities in the model grew, it proved to be impossible to impose the correct curvature conditions on these functional forms without destroying the flexibility of the functional form.[5] This negative result led to search for a *parsimonious flexible functional form* where the correct curvature conditions would hold globally without destroying the flexibility of the functional form. There was also a search for functional forms that could do better than just achieving a second order approximation. Several of the chapters in this volume contributed strongly to this literature; in particular, see chapters 4-10 and 14-16. However, in my opinion, this literature has not been completely successful: Diewert and Wales (1993; 89-92) reviewed this literature and found problems with *all* of the models that had been suggested up to that point in time, with one exception. Diewert and Wales suggested that the normalized quadratic functional form[6] was the "best" parsimonious flexible functional form, since it was the only known parsimonious form where curvature conditions could be imposed globally without destroying the flexibility of the functional form. However, subsequent research has revealed a flaw with this form: the elasticities of derived demand or supply that this functional form generates will tend to have systematic *trends* in them in the time series context, trends that are an artefact of the functional form rather than reflecting the "truth".[7] This defect of the normalized quadratic functional form can readily be fixed in the producer context by making the functional form flexible at two points: one near the beginning of the sample period, and one near the end; see Diewert and Lawrence (2002; 150-151). However, there is a significant cost in making this modification, since the resulting functional form is no longer parsimonious: it has double the number of parameters of the initial form.[8] Hence, it seems that all of the difficult issues involved in finding the "best" functional form for applied work in economics have still not been settled in a definitive manner.

In addition to the above general points about this volume, some specific comments on some of the chapters are made below.

In chapter 14, Barnett, Geweke and Wolfe use Bayesian methods in order to aid in the estimation of a flexible functional form model.[9] In general, I do not favor Bayesian procedures. There is nothing logically wrong about a Bayesian estimation method, but these methods introduce an additional layer of complexity to the overall modeling exercise. In a non Bayesian model, it is necessary to:

- Decide which variables are important and should be included in the model;
- Decide which variables are exogenous and which are endogenous;
- Decide on functional forms that relate the exogenous variables to the endogenous variables and

[5] See Diewert and Wales (1987) on this point.
[6] For the properties and applications of this functional form, see Diewert and Wales (1987) (1988a) (1988b) (1992) (1993).
[7] See Diewert and Lawrence (2002; 149-150).
[8] Of course, this is to be expected since we achieve flexibility at two points instead of one point using the modified functional form.
[9] Bayesian techniques did not play a large role in this chapter since classical estimation techniques were used as well.

- Specify something about the error terms that are added to the estimating equations or equivalently, specify something about the conditional distributions of the dependent variables, given the exogenous variables.

In a Bayesian model, it is necessary to make an additional layer of assumptions: namely, it is necessary to assume that the parameters in the above "classical" model satisfy some a priori distributional assumptions, which must be chosen by the econometrician. But on what informational base will these distributional assumptions be based? Many applied economics problems tend to be one of a kind, making use of unique data set and thus the typical applied economist will have no idea of how to parameterize a priori the unknown parameters in the model. Thus this problem of specifying prior distributions simply adds another layer of uncertainty to the outcome of the modeling exercise, leading to the *nonreproducibility* of the results. Results are *reproducible* if the average competent applied econometrician, when given the relevant data set, would come up with much the same answer to the applied economics problem that motivated the model. This concept of reproducibility is much broader than the usual narrow one where the econometrician hands in his programs and spreadsheets at the end of the modeling exercise and tells the client that the results can readily be replicated.[10] It can be seen that the Bayesian approach will tend to lead to a lack of reproducibility in the broad sense.[11]

There are a few more problem areas associated with the estimation of production and utility functions where the reproducibility issue again emerges. One of the major problem areas is concerned with the *endogeneity of the "exogenous" variables* or with exogenous variables that are measured with error, leading to correlation of the observed "exogenous" variables with the error terms in the regressions. I think that Professor Barnett and myself start out using similar methodologies; namely, represent preferences or technology using a dual function, differentiate this dual function in order to obtain derived demands or supplies (which are quantities) and then use these equations, which treat prices as exogenous, as the basis for estimating equations. Thus our regression equations tend to treat quantities as the endogenous variables and prices are regarded as exogenous. Thus we tend to estimate conditional mean functions for quantities, conditioning on prices. But of course, as Professor Barnett recognizes throughout the volume, the prices which appear in our regression equations are constructed using imperfect index number techniques and are certain to have measurement error in them. Under these conditions, classical OLS estimation will generally lead to biased estimates of the parameters that characterize tastes or technology, which is not a good thing.

One solution to this problem is to use *instrumental variable estimation* in place of OLS and its system variants.[12] But there is a lack of reproducibility associated with the use of instrumental variable estimation in finite samples. Davidson and Mackinnon (1993; 218-219) explain the problems associated with instrumental variable (IV) estimation methods as follows:

"The biggest problem with using IV procedures in practice is choosing the matrix of instruments **W**. Even though every valid set of instruments will yield consistent estimates, different choices will yield different estimates in any finite sample. … Thus, in practice, there are usually many reasonable ways to choose **W**. There are two conflicting objectives in the choice of **W**. On the one hand, we would like to obtain estimates that are as efficient as possible asymptotically. On the other hand, we would like to obtain estimates that have as small a finite sample bias as possible. Unfortunately, these objectives turn out to conflict with each other."

Thus as was the case with the use of Bayesian estimation, *the use of instrumental variable estimation will generally lead to a lack of reproducibility*. Even after choosing exactly the same economic model to explain the same data set, applied

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[10] I became aware of the broad sense reproducibility problem in working with statistical agencies who have to produce a consumer price index. These agencies try very hard to produce indexes which are reproducible, based on best practice and can be explained to the public. Thus they try to base their index procedures on principles that are broadly acceptable so that if two such agencies suddenly switched personnel and all their computer programs were lost in the changeover, then nevertheless, the switched agencies would still produce much the same index in each country.

[11] It could be argued that classical procedures are not immune to this type of criticism as well; after all, much of this volume is concerned with choosing a functional form for the underlying economic model and we have seen that this is a decision where there is not a lot of agreement. However, I am arguing that a Bayesian procedure will just make the reproducibility problem worse.

[12] Heckman (2000) provides a far ranging discussion and a very large and useful set of references dealing with this topic from different perspectives.

economists, using instrumental variable estimation, will generally choose different sets of instruments, leading to different parameter estimates and an overall lack of reproducibility in their economic advice.

Instead of using instrumental variable estimation techniques to solve the problem of errors in the exogenous variables, we might turn to the *cointegration literature*, which at first glance, seems relevant to our problem where both price and quantity variables contain some form of measurement or approximation error. We will explain the approach, following the exposition in Davidson and Mackinnon (1993; 716), in the context of a one price and one quantity model. Thus let x and y be N dimensional stochastic vectors[13] that are linked by a long run equilibrium relationship of the following form:

(1) $\eta_1 x + \eta_2 y = \eta_3 z + v$

where v is a stationary error vector and z is an N dimensional vector of nonstochastic explanatory variables and η_1, η_2 and η_3 are unknown parameters to be determined.[14] The two dimensional vector $[\eta_1, \eta_2]$ is called a *cointegrating vector*. A simple partial equilibrium supply or demand model would fit into this framework, where we let x and y be a vector of prices and quantities respectively. The applied econometrician is interested in determining the ratio of η_1 to η_2, since once this ratio is known, an elasticity of demand or supply can be calculated. Now equations (1) can be divided by η_2 and η_1 respectively in order to obtain the following two regression equations:

(2) $y = \alpha x + \beta z + v/\eta_2$;
(3) $x = \gamma y + \delta z + v/\eta_1$

where $\alpha \equiv -\eta_1/\eta_2$, $\gamma \equiv -\eta_2/\eta_1 = 1/\alpha$, $\beta \equiv \eta_3/\eta_2$ and $\delta \equiv \eta_3/\eta_1$. It can be verified that the regressions (2) and (3) do not satisfy the condition that the right hand side explanatory vectors are uncorrelated with the error terms. Nevertheless, as Davidson and Mackinnon (1993; 717) point out, the simplest way to estimate a cointegrating vector is to estimate the two models (2) and (3) using OLS. The OLS estimators for α and γ (the structural parameters of most interest to the applied economist) are defined as follows:

(4) $\alpha^* \equiv [z^Tz\, x^Ty - x^Tz\, z^Ty] / [x^Tx\, z^Tz - x^Tz\, z^Tx]$
$= x^T[I_N - M]y / x^T[I_N - M]x$;

(5) $\gamma^* \equiv [z^Tz\, y^Tx - y^Tz\, z^Tx] / [y^Ty\, z^Tz - y^Tz\, z^Ty]$
$= x^T[I_N - M]y / y^T[I_N - M]y$

where M is the following rank 1 projection matrix:

(6) $M \equiv z(z^Tz)^{-1}z^T$.

Davidson and Mackinnon (1993; 718) note that the above estimates for α and $\gamma \equiv 1/\alpha$ are consistent under their assumptions. But this is a large sample property. In small samples, there will be a systematic inequality between the two ways of estimating α, as we will now show.

We assume that x, y and z are linearly independent so that the denominators in (4) and (5) will be positive. We also assume that x and y are correlated[15] in the subspace orthogonal to M; i.e., assume that

(7) $x^T[I_N - M]y \neq 0$.

The generalized Cauchy-Schwarz inequality[16] implies that

[13] These two vectors are assumed to be integrated of order one; i.e., their first differences are stationary.
[14] We assume η_1 and η_2 are both nonzero.
[15] Strictly speaking, this correlation terminology is only accurate if z is a vector of ones.
[16] See Rao (1965; 43). Our assumption that x, y and z are linearly independent implies that the strict inequality holds in (8) rather than the corresponding weak inequality.

(8) $(x^T[I_N - M]y)^2 < (x^T[I_N - M]x)^2 \, (y^T[I_N - M]y)^2$.

Using (4), (5) (7) and (8), it can be seen that the following inequalities hold between the two ways of estimating α:[17]

(9) $x^T[I_N - M]y > 0$ implies $\alpha^* < 1/\gamma^*$;
(10) $x^T[I_N - M]y < 0$ implies $\alpha^* > 1/\gamma^*$.

The results (9) and (10) are independent of the particular stochastic specification for the model defined by (1); *these results depend only on the algebra pertaining to the two least squares regression models*. The point is this: if we have a single linear regression with two jointly dependent variables and we decide to estimate the structural parameters in the model by running two conditional regressions, one where y is the dependent variable and one where x is the dependent variable,[18] and if we want a relatively large or small estimator for α (in order to please a client for example), then we can strategically choose to run either (2) or (3) to achieve this objective. This is a very unsatisfactory state of affairs.[19] Again, there is a lack of reproducibility due to the possibility that different applied economists will choose to run different conditional regressions.

What is the solution to the problems that arise in regression analysis when prices and quantities are measured with error? The problem is not that solutions have not been suggested; rather it is that the applied economist has a huge array of possible solutions and a consensus has not emerged on what is “the” best practice technique. We have already noted that instrumental variable techniques[20] could be used. But we could also turn to a regression model which treats x and y in a symmetric manner; the literature that follows this approach dates back to Adcock (1878).[21] The literature on models that have errors in the variables also has many additional suggestions on how to deal with the problem.[22] Heckman (2000) reviews many additional techniques that could be used to address the problem. It would be useful for Professor Barnett to reflect on this problem and try to distill his knowledge of econometric theory and his extensive practical experience in the area of preference and technology estimation into a readable “cookbook” of best practice methods for

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[17] The inequalities (9) and (10) generalize a related result due to Bartelsman (1995) where he set z equal to a vector of ones and considered only case (9). The above results can readily be generalized to the case where the vector z in (1) is replaced by an N by K matrix of exogenous variables Z, where K+1 is less than N, and x and y and the columns of Z are linearly independent. In this case, the inequalities (9) and (10) are still valid but the old rank one projection matrix M defined by (6) is now replaced by the rank K projection matrix, $M \equiv Z(Z^TZ)^{-1}Z^T$. The proof follows by using the Frisch, Waugh and Lovell Theorem; see Davidson and Mackinnon (1993; 19-21).

[18] Let y be a quantity vector and let x be the corresponding vector of prices. Then the regression (2) is a direct regression of price on quantity and generates a direct estimate, α^*, of the effects on quantity supplied or demanded of a change in price. The regression (3) generates an indirect estimate of the same effect, $1/\gamma^*$. In the case where we are estimating an input demand function or a consumer demand function, α^* and $1/\gamma^*$ will be negative and we will be in case (10). In the case where we are estimating an output supply function, α^* and $1/\gamma^*$ will be positive and we will be in case (9). In both cases, it can be seen that the absolute value of the direct estimate will generally be less than the absolute value of the indirect estimate; i.e., in both cases, we have $|\alpha^*| < 1/|\gamma^*|$. Thus own price elasticities estimated *directly* by regressing quantities on prices will generally be *less in magnitude* than when estimated *indirectly* by regressing prices on quantities. This is a very troublesome result since the direct and indirect estimates can be very different.

[19] Davidson and Mackinnon (1993; 721) also recognized that the fact that the different regressions corresponding to different normalizations of (1) will give rise to different results in finite samples is somewhat troublesome: “The OLS estimates of η depend on which one of the y_i’s is treated as the regressand. Changing the regressand will, in finite samples, change the residual vector v and hence change the calculated values of any cointegration test statistics based on that vector. This is rather unfortunate, because there are already a great many possible test statistics. Thus for cointegration tests even more than for unit root tests, there are likely to be plenty of opportunities for different tests to yield conflicting inferences.”

[20] For the history of the method and additional references to the literature, see Heckman (2000; 63).

[21] In the applied mathematics and science literature, this symmetric regression approach is known as “total least squares”; see Golub and. Van Loan (1980). See also Van Huffel and Vandewalle (1991) for an extensive review of this literature.

[22] See for example Madanski (1959).

doing the econometric estimation. Such a monograph or review article would be tremendously useful to the applied economics community, since forming a consensus on best practice would reduce the lack of reproducibility that current competing strands of econometric knowledge have generated for applied economists working in industry and government.

In my own work in this area of preference and technology estimation, I have tried to use prices as the exogenous variables as much as possible. From the viewpoint of economic theory, this seems appropriate in most cases, since our basic consumer and producer models assume that economic agents treat prices as exogenous signals and use these signals in their optimization problems in order to determine quantities demanded or supplied. But price and quantity data used in applied economics inevitably contain errors and this means that the usual conditions required to justify least squares estimation are not satisfied. However, it turns out that quantities are usually much more variable than prices.[23] Hence, it seems likely that the errors in the quantity aggregates are bigger than the errors in the corresponding price aggregates. This in turn suggests that biases due to the incorrect application of least squares methods will be lower using prices as the conditioning variables rather than using quantities. Hence, keeping in mind reproducibility considerations, I will probably continue to use least squares type estimation methods with prices as exogenous variables when estimating flexible functional forms. When Professor Barnett proposes the best practice alternative method of estimation, I will enthusiastically embrace it.

Chapter 16 is probably the most interesting and challenging chapter in the volume. This chapter by Barnett and Zhou presents an interesting intertemporal maximization problem for a financial firm using flexible functional forms but also taking the risk considerations that financial firms face into account. The authors also look at the implications of their model for the construction of index numbers in an uncertain context. The problem with this chapter is: there is a tremendous amount of interesting material presented all too briefly! There is a need for the authors to turn this chapter into a separate book, where all the various assumptions that go into the model could be discussed at greater length. In particular, given the intertemporal separability assumptions that the authors make, it would be useful to know how many of the implications of their complete model could be captured by a conditional one period model. I would also like to see a lengthier discussion of how expectations about future returns on the various asset classes were calculated. It would also be useful to have a more extended discussion on how best to choose the "correct" price deflator for financial variables, since this deflator plays a large role in the calculation of real quantities in the model. There are also some interesting econometric issues that arise when modeling economic behavior under risk since invariably, first order conditions involving a utility function crop up and usually the resulting estimating equations cannot be explicitly solved so that quantities are on the left hand side and prices are on the right hand side of the estimating equations. Usually, instrumental variable estimating techniques are used in this context but again, there is a bit of a worry about the reproducibility of the results in the small samples that the applied economist usually works with.[24] Finally, I note that the chapter uses the usual expected utility framework for modeling firm behavior. This is quite acceptable in such a pioneering work but it should be noted that the expected utility model has proven to be not quite flexible enough to *always* model economic behavior when there is risk.[25]

In chapter 17, Professor Barnett observes that in the literature on modeling the estimation of tastes and technology, it has become common to impose curvature globally, but not monotonicity. He further points out, that in some cases, these models generate isoquants that have positive slopes at one or both ends of the isoquant. Thus he concludes:

"For decades, econometricians have been searching for a model that would permit flexibility and global regularity to be attained simultaneously with a parsimonious model having a finite number of parameters. In my opinion, availability of that capability has still not been demonstrated."

It will be useful to look at the capabilities of the normalized quadratic functional form to attain global regularity in the light of the above observations. I agree that this functional form, with the correct curvature conditions imposed globally,

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[23] On this point, see for example, Allen and Diewert (1981).

[24] Estimation based on the method of moments is also subject to a lack of reproducibility in small samples. How many moments should be used in the estimation procedure? Are all the chosen moment equations equally important or should they be weighted somehow? And so on.

[25] See Diewert (1993; 405) for some references to the "paradoxes" literature. Diewert (1993; 415-432) (1995) shows how a relatively simple extension of the expected utility model can "explain" many of these paradoxes.

will frequently fail global monotonicity. But under what conditions will this matter to the applied economist? Consider for concreteness, a situation where we want to estimate a single output normalized quadratic cost function of the form C(y,p), where C is the cost function, y is output and p is a vector of input prices. Suppose that we use the following system of derived demand equations as the system of estimating equations:

(11) $x^t = \nabla_p C(y^t,p^t) + e^t$; $\qquad t = 1,2,\ldots,T$

where x^t is the positive input vector observed in period t, y^t is the corresponding observed output, p^t is the vector of observed period t input prices and e^t is a vector of period t error terms. If the fit in the system of estimating equations (11) is good, then monotonicity is very likely to hold over the convex hull of the set of sample prices. For most applications, this will be satisfactory. On the other hand, if the fit is poor, then monotonicity may well fail over part of the sample space. In this case, I agree with Professor Barnett that there is a problem. There is another way in which monotonicity could fail. Suppose that input data are not available but data, C^t, on observed costs in each period are available. Then in place of (11), we could run the following single equation regression model:

(12) $C^t = C(y^t,p^t) + u^t$; $\qquad t = 1,2,\ldots,T$

where u^t is a scalar error term. In this setup, monotonicity is very likely to fail, even over the sample region, due to multicollinearity problems. In this case, it will be necessary to impose monotonicity.

Thus Professor Barnett is right to caution us about the possible failure of monotonicity, but, keeping in mind the above limitations, I still lean towards the use of the normalized quadratic functional form with curvature imposed (but not monotonicity unless it becomes necessary to do this).

Section 4 of the volume is good fun. Professor Barnett and his co-authors take a look at the econometric problems involved in detecting whether economic variables exhibit chaotic behavior. I will not describe these chapters in detail except to say that I particularly enjoyed chapter 26.

I conclude with some words of praise for Professor Barnett and his co-authors.[26] The chapters in this volume demonstrate a tremendous grasp of economic theory, applied mathematics, statistics and econometrics. The list of Journals where many of these chapters first appeared is very impressive.[27] All in all, this volume is a substantial contribution to the five problem areas in applied economics that were listed at the beginning of this Preface.

Where do we go from here? I have the following research wish list that I hope that Professor Barnett will undertake in the future:

- Write a lengthy survey paper that would give applied economists his vision of what the best practice techniques are in estimating preferences and technology.
- Write a book or monograph that would present the material in chapter 16 in greater depth with more discussion of alternative approaches to modeling the financial firm's choice problems when there is uncertainty.
- Write a paper that would lay out the different options for a choice of a deflator for monetary variables.

I look forward to Professor Barnett's future contributions on the above topics.

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[27] These journals include the following ones: *Journal of Econometrics*, *Journal of Economic Behavior and Organization*, *Journal of the American Statistical Association*, *Journal of Political Economy*, *Federal Reserve Bank of St. Louis Review*, *Journal of Business and Economic Statistics*, *Econometrica*, *Advances in Econometrics*, *Economics Letters*, *Review of Economic Studies* .

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Editors' Introduction to Volume

William A. Barnett and Jane Binner

Economic theory defines and constrains admissible functional form and functional structure throughout the economy. Constraints on behavioral functions of individual economic agents and on the recursive nesting of those behavioral functions often are derived directly from economic theory. Theoretically implied constraints on the properties of equilibrium stochastic solution paths also are common, although are less directly derived. In both cases, the restrictions on relevant function spaces have implications for econometric modeling and for the choice of hypotheses to be tested and potentially imposed. This book contains state-of-the-art cumulative research and results on functional structure, approximation, and estimation: for (1) individual economic agents, (2) aggregation over those agents, and (3) equilibrium solution stochastic processes.

In the case of individual economic agents, the relevant function space often is the space of quasiconcave, monotonically increasing functions. We shall call that space the "neoclassical function space." That function space is too rich to be spanned by any approximating model containing a finite number of parameters. Hence economic theory does not produce parametric models, and any approximating model that contains only a finite number of parameters can span only a subset of the neoclassical function space. Similarly nonparametric and seminonparametric models can span the theoretical function spaces of economic theory only in the limit as the number of parameters goes to infinity. To be able to equate econometrics with economic theory, inference must be in function space and must relate to functional structure. Inferences in finite dimensional parameter space unavoidably restrict the span of econometric models to a strict subset of the function spaces produced by economic theory.

In mathematics, when parametric algebraic functions cannot be derived directly from theory, the relevant literature is approximation theory. Mathematical approximation theory often uses sequentially truncated series expansions, along with proofs of spanning theorems acquired from denseness of basis functions within the domain of approximation. That domain often is larger than the region of convergence of a series expansion about a fixed point. As a result, mathematicians use analytic continuation to piece together multiple expansions about different points. The intent is to acquire overlapping regions of convergence throughout the function space to be spanned. Evaluation of approximation methods usually is focused on remainder term properties within the domain of approximation. But economic theory constrains multivariate approximations to a subspace of the space of continuous functions, often to the neoclassical function space. Under those constraints, the relevant spanning theorems become substantially more complicated. Relative to formal statistical theory, a series expansion that spans a larger space of functions, such as the space of all continuous functions, can violate the maintained hypothesis in statistics and thereby become inadmissible. In addition, when approximations are estimated econometrically, evaluation of remainder term properties becomes stochastic and disconnected from any identifiable single point of approximation.

The related volume, Barnett and Serletis (2000), *The Theory of Monetary Aggregation*, comprises a focused and unified collection of the most important publications in monetary and financial aggregation by Barnett and his co-authors. The current volume comprises an analogously focused and unified collection of the most important publications of Barnett and his coauthors on functional structure, approximation, and econometric inference, with emphasis on nonlinear structural and time-series modeling and inference. The two coeditors of the current volume have organized the papers into logical sections, with unifying introductions and overviews. The result is a systematic development of the state of the art in consumer demand, factor demand, and output supply modeling and inference and in nonlinear time series inference, covering:

- production and consumer demand functional structure, aggregation, and estimation
- relevant econometric theory
- nonlinear structure in time series.

A. Functional Structure Modeling, Aggregation, and Estimation

Over the past 25 years, William Barnett, who is a coeditor of this volume, has advanced the state of the art of this subject in many directions. He has contributed many new modeling and inference approaches, such as the Laurent series flexible functional form approach, the Müntz-Szatz series seminonparametric approach, the generalized hypocycloidal utility tree approach, and an aggregated convergence approach within the space of stochastic differential equations. Many of Barnett's innovations contain the earlier Taylor series and CES approaches as nested special cases. He also has contributed extensively to the literature on aggregation over approximating specifications in econometrics, as well as to aggregation over economic agents and goods in economic theory. In addition, his work in those areas has motivated new approaches by others, such as the generalized symmetric Barnett approach originated by Diewert and Wales (1987).

Part 1 of this book contains Barnett's contributions to functional structure modeling and estimation for consumers, while Part 2 contains his contributions on those subjects for firms.

B. Statistical Theory

Barnett's contributions to statistical theory provide much of the asymptotic statistical theory needed to apply econometric inference procedures to the literature on economic functional structure and approximation. His contributions to the relevant statistical theory include discovery of the measure theoretic foundations for confidence regions in sampling theoretic statistics and the derivation of the asymptotic theory for joint maximum likelihood inference with closed-form systemwide models. He originated a multivariate extension of the Kolmogorov-Smirnov test to permit testing the disturbances of an equation system for multivariate normality.

Part 3 contains relevant results in statistical theory.

C. Nonlinear Time Series

Analogous approximation and function space problems arise in time series approaches. A Volterra expansion in the time domain with a finite number of terms cannot span the space of possible time-series solution processes from the state space structures of economic theory. Hence when sample size is finite, all structural and time-series approximating specifications, whether dynamic or static, drive an unavoidable wedge between econometrics and economic theory. No easy solution exists to this inherently deep problem in econometric modeling and testing.

In the time series literature, Barnett has designed and run a competition among tests for nonlinear and chaotic structure. The purpose was to investigate paradoxes that arose in that literature following his publication of findings of nonlinearity and chaos in some economic time series. The literature on modeling and filtering out linear structure from time series is now highly advanced. But many unsolved problems remain in the literature on modeling or filtering out various forms of nonlinear structure from time series. The results of Barnett's competition have cast much needed light on those problems and the relative properties of the various available competing approaches.

Contributions to time series modeling and inference in the time domain and the frequency domain are provided in Part 4.

D. Summary

In summary, this book contains a collection of Barnett's most important contributions to the subject of functional structure, approximation, and inference, organized and discussed in a systematic manner providing a unified perspective on the state of the art in this inherently difficult literature. The following table provides an overview of the structure of the book. Prior to each section, there is an introduction highlighting some of the more important contributions of that section and briefly summarizing each chapter. The table identifies the organization of the book, including the clustering of chapters into sections and subsections, and locates the pages on which the section introductions can be found.

Table 1: Section and Subsection Structure, and Page Location of Editors' Introduction to Each Section

Section and Subsection Structure	Section Introduction Location
PART 1: STRUCTURAL FUNCTION SYSTEM SPECIFICATION: CONSUMER DEMAND **Section 1.1:** Editors' Overview of Part 1…………………………………………………… **Section 1.2:** The Differential Approach Chapters 1, 2 **Section 1.3:** The Locally Flexible Functional Form Approach Chapters 3, 4, 5, 6, 7 **Section 1.4:** The Globally Flexible Functional Form Approach Chapters 8, 9, 10 **Section 1.5:** Recursively Nested, Homothetic, and Inverse Function Structures Chapters 11, 12, 13,	 p. ?
PART 2: STRUCTURAL FUNCTION SYSTEM SPECIFICATION: PRODUCTION **Section 2.1:** Editors' Overview of Part 2…………………………………………………….. **Section 2.2:** Production by Firms Chapters 14, 15, 16, 17 **Section 2.3:** Household Production Chapters 18	 p. ?
PART 3: RELEVANT ECONOMETRIC THEORY **Section 3.1:** Editors' Overview of Part 3…………………………………………………….. **Section 3.2:** Asymptotic Statistical Theory **Section 3.3:** Measure Theoretic Statistical Theory Chapter 19, 20, 21	 p. ?
PART 4: NONLINEAR STRUCTURE IN TIME SERIES **Section 4.1:** Editors' Overview of Part 4…………………………………………………...…… **Section 4.2:** Chaos Chapters 22, 23 **Section 4.3:** Frequency Domain Methods Chapters 24, 25 **Section 4.4:** A Competition Chapter 26	 p. ?

Part 1:
Structural Function System Specification:
Consumer Demand

Section 1.1: Editors' Overview of Part 1

William A. Barnett and Jane Binner

The following table contains a brief summary of the contents of each chapter in Part 1 of this book. This section of the book contains approaches to modeling consumer demand systems.

Part 1 Section Contents

Structural Function System Specification: Consumer Demand

Chapter Number	Chapter Title	Contents
1	Theoretical Foundations for the Rotterdam Mod\el	A stochastic convergence approach to aggregation over economic agents under weak assumptions, and an application to the special case of the Rotterdam model's parameterization.
2	The Joint Allocation of Leisure and Goods Expenditure	A test of weak separability of consumer goods from leisure and thereby of the common dichotomy separating labor economics from consumption economics.
3	Definition of "Second Order Approximation" and of "Flexible Functional Form"	Proofs of the connections between the various definitions of local flexibility of functional forms.
4	The Global Properties of the Minflex Laurent, Generalized Leontief, and Translog Flexible Functional Forms	Comparison of the global regularity properties of those three flexible functional forms.
5	The Minflex Laurent Translog Flexible Functional Form	Derivation of a second-order Laurent-series flexible functional form using logarithmic transformations of the data.
6	The Three Dimensional Global Properties of the Minflex Laurent, Generalized Leontief, and Translog Flexible Functional Forms	Three dimensional graphical comparisons of the regular region of those three flexible functional forms in the three goods case.
7	The Global Properties of the Two Minflex Laurent Flexible Functional Forms	Comparison of the regularity properties of the second-order Laurent-series flexible functional forms with logarithmic data transformations versus square root data transformations.
8	The Müntz-Szatz Demand System: An Application of a Globally Well Behav Series Expansion	Derivation of the AIM seminonparametric demand model generated from the Müntz-Szatz series expansion in infinite dimensional parameter space.
9	Semiparametric Estimation of the Asymptotically Ideal Model: The AIM Demand System	Sampling theoretic seminonparametric estimation of the AIM demand system.
10	Seminonparametric Bayesian Estimation of the Asymptotically Ideal Model: the AIM Demand System	Bayesian seminonparametric estimation of the AIM demand system.
11	Recursive Subaggregation and a Generalized Hypocycloidal Demand Model	Derivation and recursive estimation of a globally regular, blockwise weakly separable consumer demand system.
12	A Monte Carlo Study of Tests of Blockwise Weak Separability	A Monte Carlo comparison of available tests for blockwise weak separability.
13	The Recent Reappearance of the Homotheticity Restriction on Preferences	Theoretical investigation and criticism of empirical models assuming linear homogeneity of utility functions.

Chapter 1: Chapter 1 contains a fundamental extension of Theil's stochastic convergence approach to aggregation over consumers. As is well known, the nonstochastic approach can recover microeconomic theory at the aggregate level only under very strong assumptions. Similarly Theil's stochastic approach has been shown to imply similarly strong assumptions on tastes, under the stochastic assumptions made by Theil. In this chapter, the strong assumptions in Theil's approach are eliminated. The average over consumers of their stochastically drawn Slutsky equations is produced, and the probability limit is acquired as population approaches infinity. Much microeconomic theory is found to be recovered at the aggregate level under very weak assumptions. But not all economic theory is recovered. For example, the aggregated equations are not integrable to a utility function. Hence no representative consumer is produced after aggregation. Barnett's aggregation theory is illustrated by use of one especially simplifying choice of parameterization. The result is Theil's Rotterdam model, but with a troubling remainder term having only partially understood properties. In addition, without Theil's very strong assumptions, the aggregated macroparameters are shown to be subject to aggregation bias. The fundamental aggregation theory produced in this chapter is in no way dependent upon the Rotterdam model parameterization used as an illustration at the end of the chapter.

Chapter 2: Chapter 2 consists of an empirical application of the theory developed in chapter 1. The conventional dichotomy between labor economics and consumer-demand-systems modeling is dependent upon the implicit assumption of blockwise weak separability of consumer goods from leisure demand. This chapter tests and rejects that assumption and thereby rejects the convention separation of labor economics from consumption allocation modeling.

Chapter 3: Chapter 3 explores the mathematical relationship between the various definitions of locally "flexible functional form" proposed in the literature. Most of the best known definitions are proved to be mathematically equivalent. But one less well know definition is proved to be sufficient but not necessary for the others.

Chapter 4: Chapter 4 compares the neoclassical functional regularity properties (monotonicity and quasiconcavity) of three locally flexible functional forms. The models are the generalized Leontief and translog flexible functional forms, derived from second-order Taylor-series expansions, and Barnett's minflex Laurent model, produced from a constrained second-order Laurent expansion. The degree of flexibility of the three models is identically, since each of the three models' parameterizatons is parsimonious within the class of flexible functional forms. But the minflex Laurent model is shown to have larger regular regions than the other two models within a broad range of settings of elasticities.

Chapters 5: Chapter 5 derives a variation on the minflex Laurent model explored in chapter 4. The version used in chapter 4 is based upon a constrained second-order Laurent-series expansion in the square roots of observed data. Chapter 5 modifies that functional structure by using the logarithmic transformation of the observed data, instead of the square root transformation.

Chapters 6: Chapter 6 further displays the results of chapter 4 by producing 3-dimensional plots of the regular regions of the three models. In the three goods case, these plots graphically display the comparative properties of the three models' regular regions for various settings of elasticities.

Chapter 7: Chapter 7 compares the global regularity properties of the two minflex Laurent models: the one produced in chapter 5 from the logarithmic transformation of the data and the earlier version using the square root transformation.

Chapter 8: Chapter 8 originates a new seminonparametric model for consumers and firms. In general, seminonparametric approaches link the order of the approximation to the sample size in a manner that permits spanning the entire relevant function space asymptotically as the sample size, and thereby the order of the approximation, approaches infinity. The approach was originated by Gallant (1981), using a Fourier series, which facilitates the relevant convergence proofs in function space. But the basis functions of the Fourier series are periodic functions that are outside the neoclassical function space of increasing quasiconcave functions. Hence, when used in economics, the Fourier series seeks to span the neoclassical function space, treated as a measure zero subset of the space of all continuous functions, from outside the neoclassical function space.

In contrast, chapter 8 shows that an infinite number of the elements of Müntz-Szatz series class can span the neoclassical function space from within the space. The basis functions in those cases are dense in the relevant function space, as is the objective of spanning theorems in mathematics. By spanning from within, the Müntz-Szatz series can maintain neoclassical properties at finite orders of approximation as well as asymptotically. The demand system derived from a Müntz-Szatz generating function (such as an indirect utility function or cost function) is called the Asymptotically Ideal Model (AIM). That name reflects the model's capability asymptotically to equate economic theory with econometrics. Other models produce a wedge between economic theory and econometrics by failing to equate theory with the models' spans.

Chapter 9: Chapter 9 applies the AIM consumer demand model of chapter 8 in a sampling theoretic manner by producing maximum likelihood estimates of the parameters. To keep the span within the neoclassical function space, a simple approach is used to impose regularity on the model. That approach, imposes nonnegativity on all parameters, and thereby forces the span, even with an infinite number of terms, to be a strict subset of the neoclassical function space. More recent research with the AIM model has used more complicated approaches to imposition of regularity, such as the Gallant and Golub (1984) approach, which can equate the span to exactly the theoretical function space.

Chapter 10: Chapter 10 applies the AIM consumer demand model of chapter 8 in a Bayesian manner. As in chapter 9, the imposition of regularity is accomplished by imposition of nonnegativity of parameters. While that simplified method provides a useful illustration of the Bayesian approach, more recent Bayesian applications of the AIM model have used newer regularity imposition methods that successfully equate the span to the complete neoclassical function space.

Chapter 11: Chapter 11 produces a globally regular, blockwise weakly separable consumer demand model. While Brown and Heien's (1972) S-branch utility tree model was popular for many years as a means of producing a globally-regular, separable, consumer-demand model, that utility tree imposes blockwise strong separability rather than blockwise weak separability. The S-branch model is a nested special case of the model derived and estimated in chapter 11. To our knowledge, the model in chapter 11 is the only globally-regular, blockwise weakly separable model that has so far appeared in the literature. The most general form of the model is called WS-branch, and it contains a useful special case called the generalized hypocycloidal model, which is a generalization of the hypocycloid of 4 cusps in mathematics.

Chapter 12: Chapter 12 is a Monte Carlo study of available tests of blockwise weak separability. The implicit assumption of blockwise weak separability is logically prior to all empirical research in economics. When relative prices change over time, so that Hicksian aggregation is impossible, only weak separability can justify aggregation over goods or separation of sectors that can be modeled and estimated independently of the rest of the economy. Without weak separability, the entire disaggregated economy must be modeled and estimated simultaneously, with the consequence that sample size becomes far less than the number of equations and parameters to be estimated. But blockwise weak separability is a subtle assumption on functional structure: the existence of a composite function representation of the structure. Chapter 12 finds that no approach to testing blockwise weak separability has yet been shown to be successful; and one of the most popular of the tests is so severely biased against accepting separability as to reject separability, even when the data was generated by a completely strongly separable Cobb-Douglas model.

Chapter 13: Chapter 13 investigates the implications of the unfortunately common assumption of linear homogeneity of utility functions in empirical research. The damage of that assumption to the Slutsky equation biases own price elasticities in a manner that has created serious problems and controversies in previously published research.

Section 1.2: The Differential Approach

Section 1.3: The Locally Flexible Functional Form Approach

Section 1.4: The Globally Flexible Functional Form Approach

Section 1.5: Recursively Nested, Homothetic, and Inverse Function Structures

Part 2:
Structural Function System Specification:
Production

Section 2.1: Editors' Overview of Part 2

William A. Barnett and Jane Binner

The following table contains a brief summary of the contents of each chapter in Part 2 of this book. This section of the book contains approaches to modeling systems of factor demand and output supply functions for firms.

Part 2 Section Contents

Structural Function System Specification: Production

Chapter Number	Chapter Title	Contents
14	Seminonparametric Bayesian Estimation of the Asymptotically Ideal Production Model	Bayesian seminonparametric estimation of the AIM factor demand system.
15	Financial Firm Production of Monetary Services: A Generalized Symmetric Barnett Variable Profit Function Approach	Modeling and estimation of bank technology and financial services output aggregation with the Diewert and Wales (1987) generalized symmetric Barnett variable profit function specification.
16	Financial Firm's Production and Supply-Side Monetary Aggregation under Dynamic Uncertainty	Modeling and estimation of bank technology and financial services output aggregation with the Diewert and Wales (1987) generalized symmetric McFadden variable profit function specification.
17	Tastes and Technology: Curvature is not Sufficient for Regularity	Exploration of the regularity (curvature and monotonicity) properties of the generalized McFadden model, when elasticities are set at plausible estimated values.
18	Pollak and Wachter on the Household Production Function Approach	Theoretical exploration of the Pollak and Wachter critique of Becker's household production function approach, and determination of a structural approach that solves the problem.

Chapter 14: Chapter 14 has the same objectives, and uses the same approach, as chapter 10, but with production data rather than consumption data. The AIM factor demand system is less nonlinear and has fewer parameters than the AIM consumer demand function system of the same order. As a result, extension to more recent approaches to imposing regularity have most often been applied to production rather than consumption. As in the consumption case, attaining regularity by imposing nonnegativity of parameters, while easily accomplished, reduces the span to a strict subset of the neoclassical function space and thereby does not fully exploit the capabilities of the AIM modeling approach. Because of the increased computational demands of the newer methods of imposing regularity, the AIM production model in future applications is likely to prove more widely useful than the more complex AIM consumer demand model.

Chapter 15: Chapter 15 models and estimates bank technology with the Diewert and Wales (1987) generalized-symmetric-Barnett variable profit function specification. The model is used to test for blockwise weak separability of monetary service outputs of banks. Since monetary service outputs of banks contribute to the economy's supply of inside money, the ability to test for blockwise weak separability of bank outputs and to track the resulting output aggregator function is potentially important in monetary policy. The generalized symmetric Barnett model is a variation on Barnett's minflex Laurent specification. Neither is a nested special case of the other, and hence each has its own relative advantages and disadvantages.

Chapter 16: Chapter 16 has the same objectives as chapter 15, but extends the bank model to include stochastic dynamic growth. Since that extension complicates estimation, this chapter uses Diewert and Wales' (1987) generalized symmetric McFadden specification, which now more commonly is called the generalized quadratic. That model is less nonlinear and hence easier to estimate than the generalized symmetric Barnett model. But the generalized symmetric Barnett model appears to have regularity advantages, which were not exploited in this chapter and have never been fully characterized.

Chapter 17: Chapter 17 explores the regularity properties of the generalized quadratic (i.e., generalized McFadden) model with elasticities set at reasonable values. The model can be estimated with curvature imposed globally, but because of the parsimonious nature of the model's parameterization, monotonicity cannot also be imposed globally without severe loss of flexibility. This chapter finds that when monotonicity is imposed only at a point, while curvature is imposed globally, plausible settings of elasticities can result in frequent violations of monotonicity within the range of the data. Further forthcoming research is exploring the regularity properties of this model under a wide range of plausible settings of elasticities to determine conditions under which this problem is most and least troublesome.

It has sometimes been stated in this literature that positive values of all variables assures monotonicity within the range of the data. This statement is false. If it were true, it would imply that the model is inherently globally regular, since those measured variables are globally positive. But if monotonicity were implied globally, then the model's flexibility would be lost. The source of this fallacy is failure to recognize that fitted and actual values of variable are not the same, and hence signs of measured values do not impose restrictions on functional structure.

Chapter 18: Chapter 18 explores Pollak and Wachter's critique of Becker's household production function approach. That critique correctly identifies a simplified special case that has been used frequently and is defective. Chapter 18 proposes a structural approach that fully captures the potential of Becker's approach and is preferable to the reduced form approach advocated by Pollak and Wachter. Their reduced form approach, although easier to implement than the structural approach we advocate, fails to permit full use of the microeconomic theory that is at the heart of Becker's approach.

Section 2.2: Production by Firms

Section 2.3: Household Production

Part 3:
Relevant Econometric Theory

Section 3.1: Editors' Overview of Part 3

William A. Barnett and Jane Binner

The following table contains a brief summary of the contents of each chapter in Part 3 of this book. This section of the book contains econometric and statistical theory needed in structural inference with systems of nonlinear demand and supply functions.

Part 3 Section Contents
Relevant Econometric Theory

Chapter Number	Chapter Title	Contents
19	Maximum Likelihood and Iterated Aitken Estimation of Non-Linear Systems of Equations	The first proofs of the asymptotic properties of the maximum likelihood estimator of the parameters of nonlinear systems of equations in the class relevant to consumer and firm modeling.
20	A Test of Normality in Nonlinear Systems of Consumer Demand Equations	A multivariate extension of the Kolmogorov-Smirnov test to test for multivariate normality of the disturbances of a system of consumer demand functions.
21	Random Sets and Confidence Procedures	The first measure theoretic foundations for confidence regions as realizations of random sets. The relevant measure space is proved to be the one generated by the neighborhood system topology.

Chapter 19: At the time that Barnett began working on consumer and firm modeling, asymptotic theory for the maximum likelihood estimator of the resulting joint systems of equations was not available, except for linear models. Otherwise the proofs were for random sampling from a fixed distribution. Most of the models advocated in this book are nonlinear, and the endogenous variables are not sampled from a fixed distribution, since their means depend upon the exogenous variables. This chapter contains the first published proofs of consistency, asymptotic normality, and asymptotic efficiency for the maximum likelihood system estimators for the class of equations systems relevant to the research in this book. The chapter also produce the asymptotic theory for the likelihood ratio test statistic, and for the computationally simplified statistics produced from the concentrated likelihood function.

Chapter 20: Some inferences in econometrics depend heavily upon the assumption of normality of disturbances. The Kolmogorov-Smirnov test of normality resulted from one of the most brilliant proofs that ever appeared in the field of statistics. But that proof by Kolmogorov and Smirnov is univariate, while most of the models in this book are systems of equations. Chapter 20, using a pooled residuals transformation first proposed by Theil, extends the Kolmogorov-Smirnov test to a test for multivariate normality of the disturbances in a system of equations. The new test is applied to testing for multivariate normality of the disturbances in an estimated consumer demand system.

Chapter 21: In measure theory, it has long been known that random variables are measurable point valued mappings (functions), with the measurability being relative to a Borel measure space. Confidence regions similarly are realizations of mappings: set valued mappings, called confidence procedures. But the relevant measure theory had not been produced in the statistics literature prior to the appearance of Barnett's article that produced this chapter. This chapter proves that set valued mappings are confidence procedures, if and only if those mappings are measurable relative to a particular measure space: the measure space generated by the neighborhood system topology.

Section 3.2: Asymptotic Statistical Theory

Section 3.3: Measure Theoretic Statistical Theory

Part 4:
Nonlinear Structure in Time Series

Section 4.1: Editors' Overview of Part 4

William A. Barnett and Jane Binner

The following table contains a brief summary of the contents of each chapter in Part 4 of this book. The first three sections of the book concentrate on structural modeling and inferences. Part 4 contains results on nonlinear time series modeling and inferences.
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Part 4 Section Contents
Nonlinear Structure in Time Series

Chapter Number	Chapter Title	Contents
22	The Aggregation-Theoretic Monetary Aggregates are Chaotic and Have Strange Attractors: An Econometric Application of Mathematical Chaos	Tests of chaos using test procedures from the physics literature with Divisia monetary aggregate data
23	Robustness of Nonlinearity and Chaos Tests to Measurement Error, Inference Method, and Sample Size	Using monetary aggregate data, this chapter compares robustness of inferences regarding chaos and nonlinearity to variations in test methods, sample size, and aggregation method.
24	Time Series Cointegration Tests and Nonlinearity	Exploration of the implications of non-Gaussian disturbances for cointegration tests producing linear models.
25	Has Chaos Been Discovered with Economic Data?	An overview of the state of the controversies regarding claims of discovery of chaos with economic data.
26	A Single-Blind Controlled Competition among Tests for Nonlinearity and Chaos	A competition among tests of nonlinearity and chaos with simulated data.

Chapter 22: Chapter 22 finds that time series observations on certain Divisia monetary aggregates pass the tests for chaos used by physicists. But the tests have no way to determine whether the source of the chaos is nonlinear dynamics within the domestic economy, or within the international economy of the world, or within the planet's weather, producing chaotic shocks to agriculture and thereby to the economy. Hence the inference of chaos in this context is not surprising, since climatologists have found extensive evidence of chaos in the weather, which otherwise would have stabilized long ago.

Chapter 23: As a result of controversies produced by the results in chapter 22, chapter 23 explores robustness of test results to competing tests, measurement methods, and sample size. The comparisons are made with monetary aggregate data, as in chapter 22. Surprising inconsistencies in inferences are found across competing tests, aggregation methods, and sample sizes.

Chapter 24: Use of linear models often is justified by cointegration tests finding cointegrating linear combinations of stochastic processes. But the inference of model linearity implicitly assumes that the resulting linear combination is itself a linear stochastic process. This chapter demonstrates that this inference is not robust to the underlying assumption of Gaussianity of the innovations in the underlying VAR. Loss of Gaussianity of those disturbances not only results in loss of Gaussianity of any discovered cointegrating linear combination, but more ominously in the loss of lineary of that stochastic process. Hence without Gaussianity of the VAR innovations, the cointegrating linear combination is a nonlinear stochastic process and hence cannot be viewed as producing a linear model. We empirically explore this theoretical result by testing for linearity of a cointegrating linear combination found by the Johansen and Juselius (1990) method. We reject linearity of the resulting process.

Chapter 25: Chapter 25 surveys the controversies produced by chapters 22 and 23 and explores the credibility of the published finding of chaos.

Chapter 26: Barnett designed and ran a controlled competition of tests for nonlinearity or chaos. The competition was motivated by the controversies in chapters 22, 23, and 25. The purpose was to determine whether the conflicting results were produced by nonrobustness of the inferences to the competing tests. In this competition, simulated data from five generating models and two sample sizes were produced. Those ten data sets were transmitted as e-mail attachments to advocates of the best known available tests of nonlinearity and chaos. In most cases, the person who ran the test was the person who originated the test. In each of the ten cases, each competitor was required to conclude: (1) linear, (2) chaotic, or (3) nonlinear nonchaotic. No information was provided to the competitors about how the data sets were generated. Substantial inconsistencies were found across test results, with surprising failures in some cases. In most cases, the source of the disagreement was traced to differences in the definitions of "linear stochastic process" in the original derivations of the tests.

Section 4.2: Chaos

Section 4.3: Frequency Domain Methods

Section 4.4: A Competition