Statistical properties of volatility in fractal dimension and probability distribution among six stock markets - USA, Japan, Taiwan, South Korea, Singapore, and Hong Kong

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Abstract

This study examines the statistical properties of volatility. Fractal dimension, probability distribution and two-point volatility correlation are used to measure and compare volatility among six different countries for the 12-year period from Jan. 1 1990 to Dec.31 2001. New York market is found to be the strongest among the six in terms of market efficiency. Moreover, the Tokyo and Singapore markets are found to be very similar in fractal dimension and probability distribution, but different in their resistance to volatility : Tokyo has a higher ability to dissipate volatility. This phenomenon implies that the Tokyo market is more efficient than the Singapore market. The Hong Kong market is similar to the Singapore market in its ability to dissipate volatility. Meanwhile, the Taiwanese and Korean markets are the two most volatile markets among the six. Notably, the Taiwanese market is weaker than the Korean market in dissipating volatility.

Keywords: Volatility, fractal dimension, probability distribution.

JEL Classification: G15

1 Introduction

Recently, developments in the study of nonlinear dynamics have strongly impacted thinking in many fields of science (Mantegna and Stanley, 1999). By employing the ideas and techniques used in the study of nonlinear dynamic systems, scientists can extract more information and capture more structures by analyzing time series generated by real systems. Such ideas and techniques include attractors, Lyapunov exponents, fractal dimensions, and so on, as well as statistical measures enabling the diagnosis of time series for which deterministic information is lacking. Recent developments in the study of nonlinear dynamics have also strongly impacted economics, particularly the study of stock market generated time series. For example, several empirical studies have examined the scale-invariant behavior of price change distribution (Gopikrishnan, Plerou, Amaral, Meyer and Stanley, 1999) and of long-range correlations in the absolute values of price changes (Liu, Gopikrishn, Cizeau, Peng, Meyer and Stanley, 1999).

This study builds on earlier research on this area by analyzing the stock markets of the USA, Japan, Taiwan, South Korea, Singapore, and Hong Kong. Japan, Taiwan, South Korea, Singapore, and Hong Kong are five important Asian markets, but they differ significantly from one another in many aspects, including degree of trading restrictions, industrial orientation of listed companies, and so on. Consequently, it is important to compare the performance of these markets. Moreover, since the US market is recognized traditionally as the most mature in the world, it is also included in the comparison to provide an enhanced understanding of the nature of these Asian markets.

Regarding the analysis of time series in financial markets, several observables can be used to describe market activity. Such variables include exchange rates, values of market indices, individual company prices, trading volume, and so on. Among these observables, the values of market indices are frequently used to characterize global market properties. Particularly, market index fluctuation, known as volatility, indicates the global efficiency and stability of a market.

This study analyzes and compares stock market volatility among the USA, Japan, Taiwan, South Korea, Singapore, and Hong Kong. Market volatility is estimated using average local price changes. To extract more information from the time series of volatility, this work uses the statistical measurements of fractal dimension, probability distribution function, and two-point autocorrelation function of the series.

Fractal dimension measurement is commonly applied for nonlinear dynamics. This study uses this measurement to compensate for the lack of measurement of standard deviation. Notably, fractal dimension considers fluctuation structure, which standard deviation fails to represent. Determining probability distribution function is also essential to understanding markets. The same fitting function used to construct the probability distribution data is also used to fit the data numerically, thus facilitating the identification of the similarities and differences among markets. Finally, the two-point correlation function is measured to determine the duration for which a certain effect impacts a market.

Many studies of individual market volatility already exist. Most of these studies concentrate on the effects of particular events, such as political risk event (Chan and Wei, 1996), price change spillover effects (Wei, Liu and Yang, 1995), price reversals and continuations (Chang, Rhee and Soedigno, 1995), U.S.stock crash event (Schwert, 1990), East Asian Crisis (Forbes and Rigobon, 1999), industry structure factor (Arshanapali, Doukas and Lang, 1997), new government restrictions, and so on. However, relatively few studies have attempted global comparisons of market volatility. This paper is organized as follows. Section II describes the databases used and discusses the quantification of volatility. Section III then measures the fractal dimension possessed by the time series of volatility. Next, Section IV determines the probability distribution function of volatility. Furthermore, Section V studies the volatility correlations. Finally, Section VI presents a summary and discusses the results.

2 Time series of volatility

First, this study describe the nature of the data from the six stock markets. Subsequently, volatility is discussed and the method used here to quantify it is presented. This section also presents the time series of volatility obtained through the quantification.

2.1 Data analyzed

This article examines daily trading data from Jan. 1, 1990 to Dec. 31, 2001 for six stock markets: New York, Tokyo, Taiwan, Korea, Singapore and Hong Kong. Volatility is estimated over the entire sample period of 1990-2001, as well as over the two subperiods of 1990-1996 and 1997-2001. The later subperiod is the aftermath of the Asian Financial Crisis. The lower the sensitivity of the stock market to this economic catastrophe, the more stable that market can be considered. Conversely, markets that were highly sensitive to the Asian Financial Crisis can be considered to be highly volatile. In each case, an index heavily weighted with blue chip companies was chosen to represent a particular market. For example, the Dow Jones Industrial Index was chosen to represent the US market. The Dow Jones Industrial Average is a price-weighted average of 30 major stocks traded on the New York Stock Exchange, and is the oldest and most watched index in the world.

DJIA includes companies which are well-established, financially-sound and stable companies.

The Nikkei 225 Industry Index was chosen to represent the Japanese market. The Nikkei is the leading and most respected index of Japanese stocks. This index of blue-chip stocks includes 225 large Japanese firms and represents almost 50 percent of the total market capitalization of the Japanese stock market. The Nikkei 225 is a price-weighted index, similar to the Dow Jones Industrial Average, but it differs from the Dow in that dividends are not reinvested. However, cash dividends paid on most Japanese stocks are minor, meaning that the omission of these dividends is unimportant.

The Taiwanese stock market is represented using the Taiwan Weighted Index, which is a value-weighted arithmetic index including all stocks listed on the main board, except for those listed for less than a month and those requiring a 100% margin. This index represents more than 90% of the total market capitalization of the Taiwan Stock Exchange.

The Korean stock market is represented using the Seoul Composite Index, which is a also a value-weighted arithmetic index including 96 percent of the total market capitalization of the Korean stock exchange.

Hong Kong is represented by the Heng Seng Index, which includes the 33 largest firms in Hong-Kong and represents about 75 percent of the total equity capitalization of the Hong Kong stock exchange. The Heng Seng Index is a value-weighted arithmetic index.

Finally, the Singapore Straits Times Index, which includes 30 stocks, is used to represent the Singapore stock market. This index is an unweighted index, also known as an equally weighted index, and is the only index in Singapore.

Figure 1 displays the raw data from the six markets.

2.2 Quantify volatility

Many different ways of quantifying market index volatility exist (Chorafas, 1994, 1998).But there exist the assumptions limitations like normal distribution, homoschedasticity or heteroshedasticity. This study adopts the method used by the Boston group (Liu, Gopikrishnan, Gizeau, Meyer, Peng and Stanley, 1999), which exist no assumptions limitations. namely estimating volatility as the local average of absolute price changes over a proper time interval T. Generally, T is an adjustable parameter, but this work always takes $T = 5$ days for constructing the time series of volatility. The results of other choices of T and their implications are presented in the following sections.

To construct the time series of volatility using the time series of the market index $Z(t)$, first the price change $G(t)$ is defined as the change in the logarithm of the index,

$$G(t) = \ln Z(t + \delta t) - \ln Z(t), \tag{1}$$

where δt denotes the time interval of sampling with $\delta t = 1$ day in the data. Notably, for sufficiently small sampling time interval we have

$$G(t) \simeq \frac{Z(t + \delta t) - Z(t)}{Z(t)}. \tag{2}$$

Then, the volatility $V(t)$ is defined as as the average of the absolute value of $G(t)$ over a time window $T = 5\delta t$, i.e.

$$V(t) = \frac{1}{5} \sum_{n=0}^{4} |G(t + n\delta t)|. \tag{3}$$

Figure 2 shows the calculated time series of volatility for each of the six stock markets. Interestingly, the data indicates a clustering effect for the periods of high volatility (Gopikrishnan, Plerou, Gabaix and Stanley, 2000).

3 Fractal dimension of volatility

Nature contains many irregularly shaped objects, such as clouds, coastlines, trees, and so on. One particular class of irregular objects appear self-semilar under a varying scale of magnification, and are referred to as fractals. It is impossible to characterize a fractal structure completely using Euclidean geometry. Unlike the topological dimension used in Euclidean geometry, the fractal dimension (Mandelbrot, 1983) becomes a basic notion for characterizing a fractal object. The fractal dimension is usually non-integral, and the most intuitive way of measuring it is the box-counting method. The dimension obtained by this method is called the box-counting dimension D_B , which is the same as the topological dimension for a regular object.

To illustrate the box-counting method, this study first considers a set of infinite numbers of points that form a curve with unit length. Suppose that the set of points is covered by at least N line segments with length ϵ. This supposition leads to the relation, $N\epsilon = 1$. Next consider a set of points defining a unit surface. Once again, suppose this set of points is covered by at least N squares with area ϵ^2. The covering area is then yielded as being $N\epsilon^2 = 1$. By deduction, define D_B is defined as

$$D_B = \lim_{\epsilon \to 0} \frac{\log N}{\log\left(\frac{1}{\epsilon}\right)}. \tag{4}$$

According to this definition of D_B, we have $D_B = 1$ for a straight line and $D_B = 2$ for a square. When a straight line is not very smooth but have a lot of up-down fluctuations, the D_B value would increase to $1 < D_B < 2$.

Moreover, if some subareas are digged out from a square, the corresponding D_B value would be less than two but larger than one.

We may use the D_B value as a statistic measure for the data points associated with a time series. The D_B value thus obtained is always in the range, $1 < D_B < 2$, for a time series. With more frequent appearence of up-down fluctuations in a time series, the corresponding D_B value would increase. Conversely, D_B decreases when the distribution of data points in the time series is smooth. Thence, the D_B measure is quite similar to the measure of standard deviation. However, these two quantities are not proportional to each other. There is no distinction between smooth and rapid changes in the measure of standard deviation. But, rapid changes contribute more to the D_B measure. Thus, it can be expected that the fractal dimension is more appropriate than alternatives in measuring the time series. Furthermore, the information revealed by the D_B value can be expected to be more reliable than that revealed by the standard deviations in studying market stability.

For the time series of volatility obtained in the last section, the following question arises: if the series has a fractal structure, does it increase naturally? However, because D_B is the same as the topological dimension for a regular object, the question can be disregarded and the box-counting method can be used to determine the D_B value of each time series. Since the region covered by the volatility distribution increases with D_B value, we can conclude that

Table 1: Fractal Dimension of Stock Market in Six Countries

Volatility	U.S.A.	Japan	Taiwan	Korea	Hong Kong	Singapore
1990-2001	1.484	1.501	1.566	1.509	1.457	1.442
1990-1996	1.502	1.500	1.569	1.460	1.496	1.482
1997-2001	1.420	1.440	1.520	1.503	1.385	1.390

stock market stability decreases with increasing D_B value.

Figure 3 shows $\log N$ as a function of $\log(1/\epsilon)$ for different markets, and the slope of the straight line gives the D_B value of each market. Table 1 lists the calculated D_B values.

The figures in Table 1 indicate the fractal dimension. As shown in Table 1, the Taiwanese stock market has historically been the most volatile market in Asia. Taiwan clearly leads in terms of volatility over the 1990-2001 period, with a fractal dimension of 1.566. The high volatility of the Taiwanese market is considered to be the result of excessive speculation. Meanwhile, Korea shows the second highest fractal dimension of 1.509 over the same period. Conversely, the Singaporean stock market is the most stable over the 1990-2001 period, with a fractal dimension of 1.442. Moreover, the Hong Kong stock market ranks second in terms of stability, with a fractal dimension of 1.457. Both Singapore and Hong Kong are Asian financial centers and are viewed as very mature, open and well monitored markets. Finally, the New York and Tokyo stock markets also show a solid fractal dimension scores of 1.484 and 1.501, respectively. Both of these stock markets are viewed as having the strong characteristics of being liberalized and free of intervention.

During the 1997-2001 subperiod, the Taiwanese and Korean stock markets once again displayed the most volatile outlooks, with 1.520 and 1.503, respectively. The Korean market, which is widely considered the hardest hit by the 1997 crisis, showed higher volatility in 1997-2001 than in 1990-1996, of 1.503 compared to 1.460. This pattern demonstrates that the Korean market has historically not been especially volatile, but was hit hard by the 1997 crisis (Titman and Wei, 1999). In contrast, Taiwan displays the reverse pattern, with extremely high volatility of 1.569 before the financial crisis, reducing to 1.520 after the crisis. The lower sensitivity of the Taiwanese stock market to this financial crisis is somewhat ironic given its high volatility normally (Titman and Wei, 1999). Interestingly, the fractal dimensions of all of the other four markets, including the USA, Japan, Hong Kong and Singapore, are statistically lower during the aftermath of the 1997-2001 crisis. Especially, Hong Kong shows the lowest volatility over the 1997-2001 subperiod, of 1.385, despite the Hong Kong market containing a large number of firms in the very volatile property and financial sectors.

Taiwan and Korea are the most similar countries in the sample in terms of economies and capital markets, and consequently it is not surprising that both display a similar trend of high volatility. This tendency towards high volatility is probably one reason why both countries have strong government authorities that actively use policy tools to reduce volatility, with the Korean government being especially active in this regard.

Singapore, Hong Kong, the U.S. and Japan are all similar in that they display high stability, but the reasons are different in each case. The U.S. stock market is characterized by the accurate and timely reflection in prices of all available information. Also, investors in the U.S. market are diversified, and institutional investors, well known for approaching investment rationally, comprise two thirds of the market. Meanwhile, the Singaporean stock market is viewed as a modern and open market. Finally, the Hong Kong market basically satisfies the criteria of openness, breadth and flexibility, although government intervention does occasionally occur.

4 Probability distribution of volatility

To display the distribution of volatility more explicitly, this section constructs the probability distributions corresponding to the time series of volatility. The resultant distributions for the six markets are observed to have long tails and be asymmetrical across the peak of the curve. The log-normal distribution can take these characteristics into account, and thus the log-normal distribution function is used to fit the distribution data by adjusting the two parameters in the function.

To construct the probability distribution of volatility, this study first uses the histogram method to count the value of volatility $N(V_n)$ for the volatility ranging between $V_n = n(\Delta V)$ and $V_{n+1} = (n+1)(\Delta V)$ with $\Delta V = 0.0005$. Here, n is an integer ranging between 0 and ∞. The probability of the volatility in the interval between V_n and V_{n+1} is then given as

$$P(V_n) \cdot \Delta V = \frac{N(V_n)}{\sum_{m=0}^{\infty} N(V_m)}, \tag{5}$$

with the normalization,

$$\sum_{n=0}^{\infty} P(V_n) \cdot \Delta V = 1. \tag{6}$$

Figure 4 shows the resultant distribution of $P(V)$, and displays the plot of $P(V_n)$ versus V_n for each of the six markets.

From the distributions shown in Fig. 4, the average volatility value of a stock market and its standard deviation can be determined, and the calculation results are listed in Table 2. As mentioned previously, a D_B value increases with standard deviation, and the findings of this study are consistent with this expectation.

The distributions in Fig. 4 display two characteristics: first, the distribution is asymmetric with the peak, and second, the distributions have longer tails than the Gaussion distribution. A log-normal function can take both of these characteristics into account in calculating the volatility distribution. Moreover, previous studies on the volatility of the S&P 500 index in the U.S.A. have indicated that the distribution may be fitted by a log-normal function (Liu, Gopikrishnan, Gizeau, Meyer, Peng and Stanley, 1999). Thus, the distribution function is assigned the form

$$P(V) = \frac{1}{\sqrt{2\pi}wV} \exp\left[-\frac{1}{2w^2}\left(\ln\frac{V}{V_c}\right)^2\right] \tag{7}$$

to fit the distribution data. Notably, this distribution function is normalized as

$$\int_0^\infty P(V)\, dV = 1, \tag{8}$$

and the function contains two adjustable parameters, V_c and w, indicating the the peak probability location and distribution width, respectively.

The fitting curves of the log-normal distribution functions are represented by the solid lines in Fig. 4, and the corresponding values of the two parameters V_c and w are listed in Table 2.

Notably, besides listing the highest volatility probability and the width of the peak, Table 2, also shows the average volatility, indicating global volatility, and the standard deviation, indicating fluctuations in volatility.

Table 2: The parameters of log normal function among six countries' stock markets

Nations/ Parameters	V_c	w	average	standard deviation
USA	0.00630	0.50031	0.00711	0.00417
Japan	0.00999	0.50779	0.01128	0.00607
Taiwan	0.01106	0.55789	0.01374	0.00910
Korea	0.01162	0.60070	0.01414	0.00898
Singapore	0.00732	0.54949	0.00913	0.00648
Hong Kong	0.01024	0.55756	0.01207	0.00809

Notably, the standard deviations are consistent with the fractal dimension figures given in the previous section, and the two can be combined to clarify fluctuations in volatility.

From Table 2, volatility and probability differ significantly among the six countries studied. First, comparison with Fig. 4 reveals that the USA and Singapore seem to have the least volatile stock markets, as the V_c of USA and Singapore's markets are the smallest, at 0.0063 and 0.00732, respectively. Consequently, instances of high volatility are least likely in these two markets. Combining the $V_c$with the W, which represents the width of the peak, the USA once again displays the smallest value of 0.50031, while Singapore has a value of 0.54949. The Japanese market also shows a very small $V_c$of 0.00999 and a narrow peak width W 0.50779, second only to that of the USA. Thus, although the Japanese market displays a slightly higher probability of volatility than the Singaporean market, the width of the peak is narrower than for the Singaporean market. Given that the range of $V_c \pm W$ represents typical volatility conditions in a given stock market, then the 'volatility area' covered by the Singaporean and Japanese markets is similar, making it difficult to rank them. As for the Korean market, it possesses simultaneously both the largest volatility probability and width in the sample, of 0.01162 and 0.60070, respectively. The Korean stock market is thus the most unstable and volatile market in the sample. As for the Taiwanese market, it has the second largest volatility probability and width in the sample, of 0.01106 and 0.55789, respectively, implying that it too is highly volatile.

Comparing the figures in Table 1 and 2 reveals some differences in the volatility ranking assigned to the subject stock markets. Although the differences are trivial and do not influence our major findings, examining these differences in more detail is still interesting. Consequently, the following section investigates the two point volatility autocorrelation range from 1 to 60 days among the six markets studied.

5 Volatility correlations and market efficiency

To obtain more information on the evolution of markets over time, this study constructs the two-dimensional phase diagrams of volatility and calculates the corresponding two-point autocorrelation. Figure 5 shows an example of the phase diagram, and gives the plots of $V(t + \delta t)$ versus $V(t)$. Generally, market stability increases with reduced coverage area of volatility in the phase space. Moreover, market predictability increases with scarcity of points in the phase space. As the results indicated in Fig. 5, the distribution of volatility in all six markets is characterized by time-reversal symmetry.

The two-point autocorrelation function is given as

$$C_v = \frac{1}{\sigma_V^2} \left[\langle V(t_m) V(t_m + v\delta t) \rangle - \langle V(t_m) \rangle^2 \right], \tag{9}$$

where σ_V denotes the standard deviation,

$$\langle V(t_m) V(t_m + n\delta t) \rangle = \frac{1}{N} \sum_{m=1}^{N} V(t_m) V(t_m + n\delta t), \tag{10}$$

and $\langle V(t_m) \rangle$ represents the average value of volatility,

$$\langle V(t_m) \rangle = \frac{1}{N} \sum_{m=1}^{N} V(t_m) = \Delta V \left(\sum_{n=0}^{\infty} V_n P(V_n) \right), \tag{11}$$

given in Table 2. The numerical results of C_v as a function of v is shown in Fig. 6.

6 Discussion and conclusions

This study uses fractal dimension, probability distribution and autocorrelation to measure and compare volatility among six stock markets. Volatility characteristics in the sample markets differed with all three measurement methods.

Fractal dimension found Singapore and Hong Kong to be the most stable stock markets, while Taiwan and Korea are the most volatile, and the U.S. and Japanese markets are ranked somewhere in between these extremes. However, combining the fractal dimension with the probability distribution reveals that the U.S. stock market has the smallest peak width, the lowest probability at the peak, the lowest global volatility and fluctuation degree, and a strong ability to dissipate volatility from outside (low autocorrelation). Thus the U.S. market can be considered the strongest of the six markets in terms of market efficiency.

The Japanese and Singapore stock markets are similar in that both rank behind the New York market in terms of smallestness of probability at the peak and narrowness of peak width. However, these two markets differ from each other in their ability to dissipate volatility. The Japanese market has a better ability than the Singaporean market to dissipate volatility, assessed in terms of ability of trading activity to cause residual volatility to converge to zero. While both the Singapore and Japanese markets dissipate volatility relatively quickly, within the first five days in both cases, the residuals do not converge in the case of the Singapore market, and a much higher ratio is left in autocorrelation. This phenomenon implies although both markets have similar volatility probability distributions, the Japanese market is more efficient than the Singapore market. Notably, the Hong Kong market is similar to the Singapore market in terms of its ability to dissipate volatility, with residual volatility being relatively long lasting, and ultimately unable to converge to zero.

Meanwhile, despite having a stable volatility fractal dimension, the Taiwanese and Korean markets are the most volatile among the six. The Taiwanese market has the second largest volatility peak width and probability at the peak, second only to the Korean market. Moreover, the Taiwanese market is the weakest of all the markets sampled in terms of its ability to dissipate volatility, and residual volatility remains persistently high following episodes of volatility. While the Korean market displays the most risky probability distribution of volatility in terms of both peak width and probability, its ability to dissipate volatility seems better than that of the Taiwanese market, as evidenced in the convergence of residual volatility and autocorrelation.

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7 Figure Captions

Fig. 1 Index movement for the six sample stock markets: (a) New York, (b) Tokyo, (c) Taiwan, (d) Korea, (e) Singapore, and (f) Hong Kong.

Fig. 2 The time series of volatility for the six sample stock markets: (a) New York, (b) Tokyo, (c) Taiwan, (d) Korea, (e) Singapore, and (f) Hong Kong.

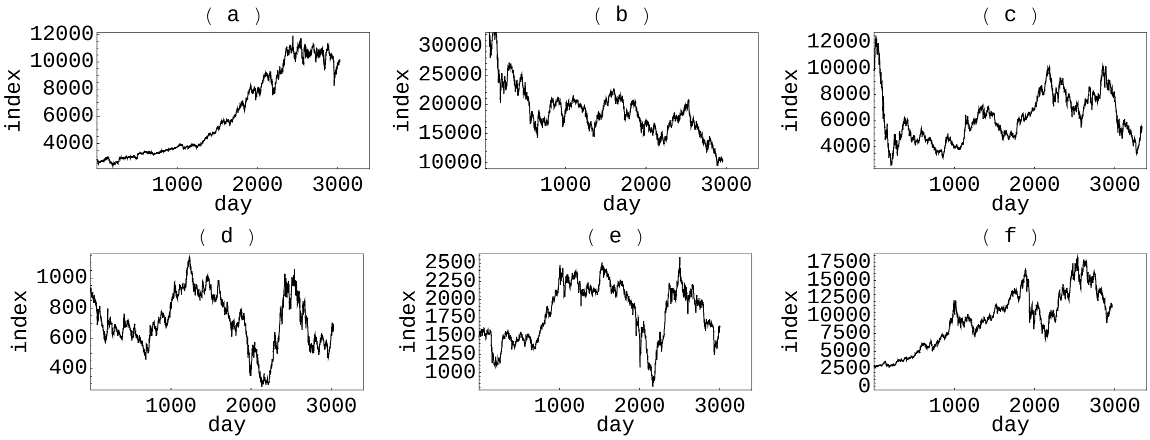
Fig. 3 The numerical values of $\log N$ as a function of $\log(1/\epsilon)$ for the six sample stock markets: (a) New York, (b) Tokyo, (c) Taiwan, (d) Korea, (e) Singapore, and (f) Hong Kong with ($\triangle$) 90˜96, ($\bigtriangledown$) 97˜01, and ($\diamond$) 90˜01. The slope of the straight line gives the value of fractal dimension D_B.

Fig. 4 The probability distribution function $P(V)$ as a function of volatility V for the six sample stock markets: (a) New York, (b) Tokyo, (c) Taiwan,

(d) Korea, (e) Singapore, and (f) Hong Kong. The solid line represents the fitting curve of the log-normal distribution function.

Fig. 5 The phase diagram of $V(t+\delta t)$ versus $V(t)$ with $\delta t = 1$ for the six sample stock markets: (a) New York, (b) Tokyo, (c) Taiwan, (d) Korea, (e) Singapore, and (f) Hong Kong.

Fig. 6 The two-point autocorrelation function C_v between two points, t and $t + v\delta t$, with $\delta t = 1$ for the six sample stock markets: New York, Tokyo, Taiwan, Korea, Singapore, and Hong Kong.


(a)
(b)
(c)
(d)
(e)
(f)
index
day
12000
10000
8000
6000
4000
30000
25000
20000
15000
10000
1000
2000
3000
1000
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10000
7500
5000
2500
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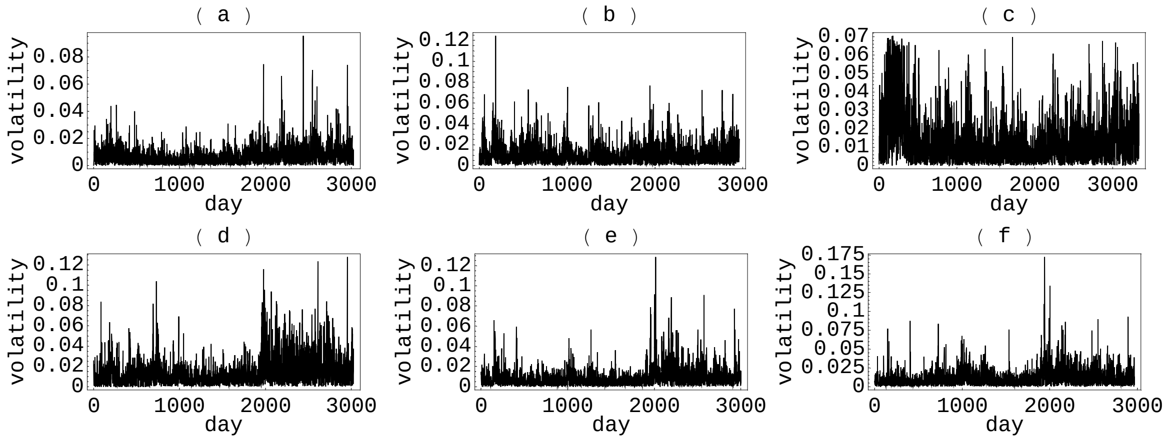
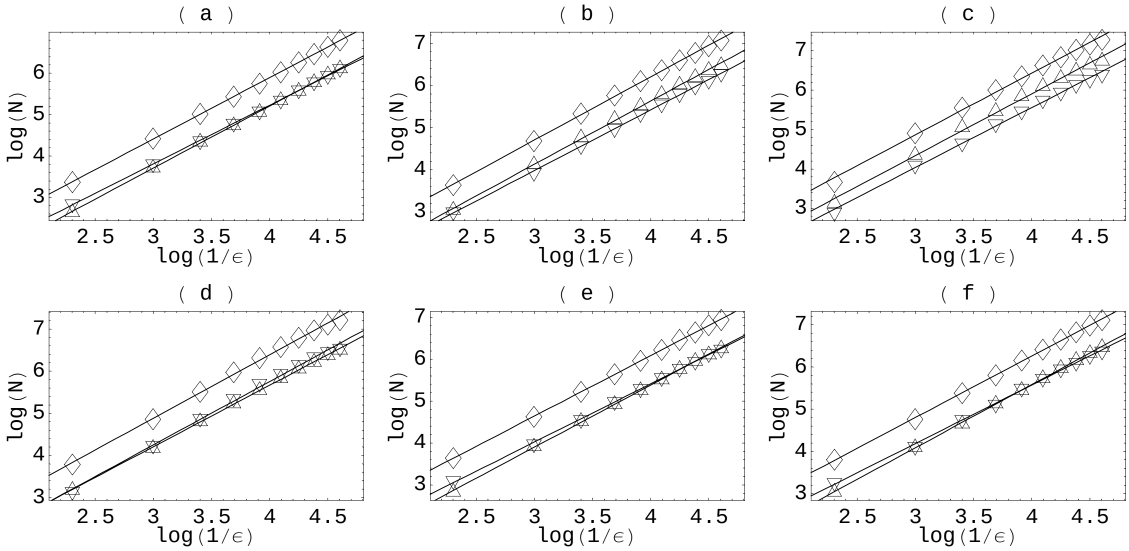
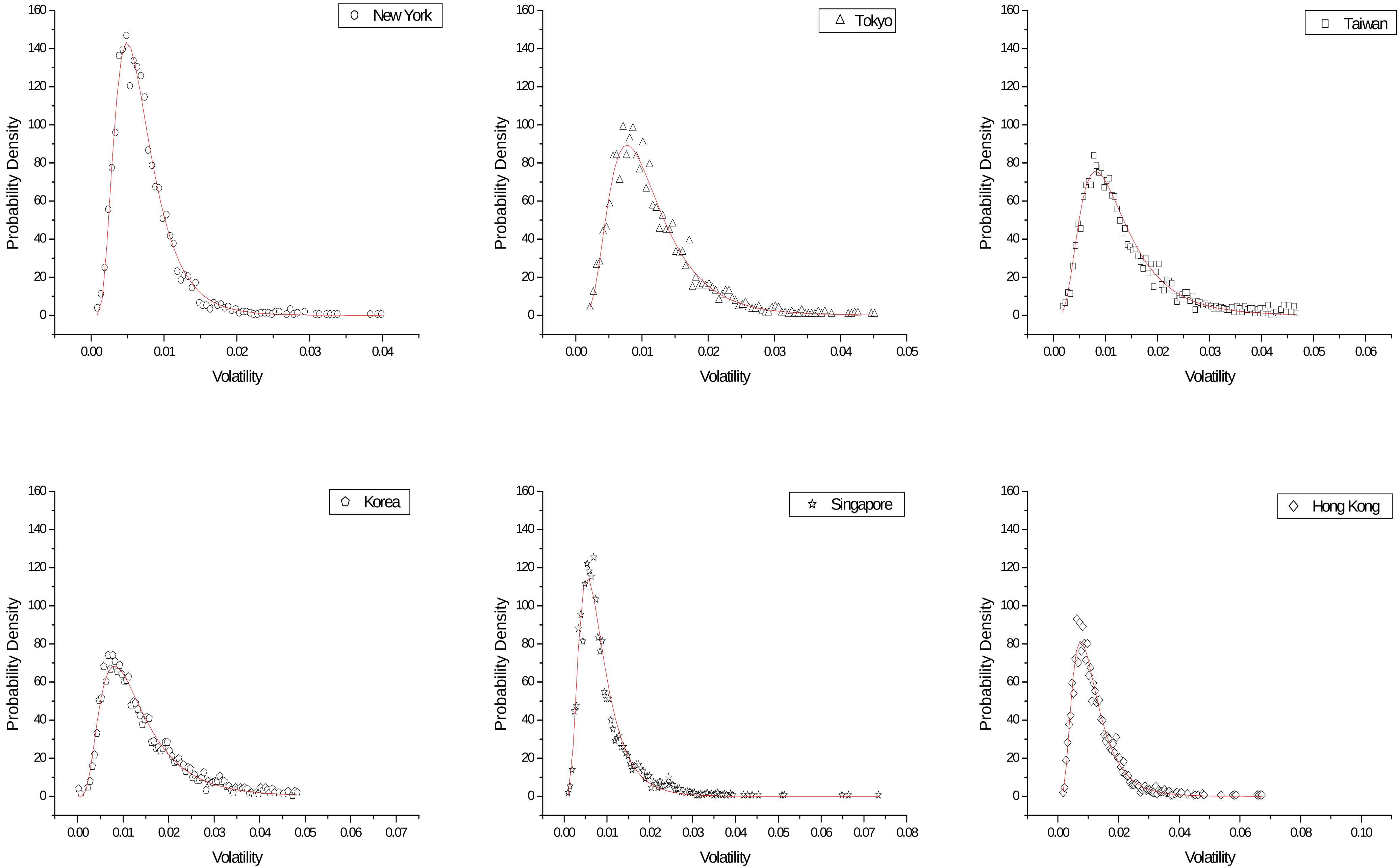
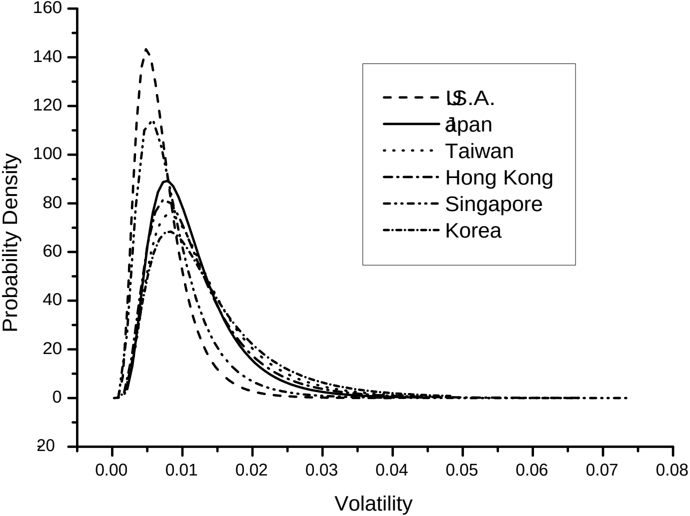


(a)
(b)
(c)
(d)
(e)
(f)
volatility
day

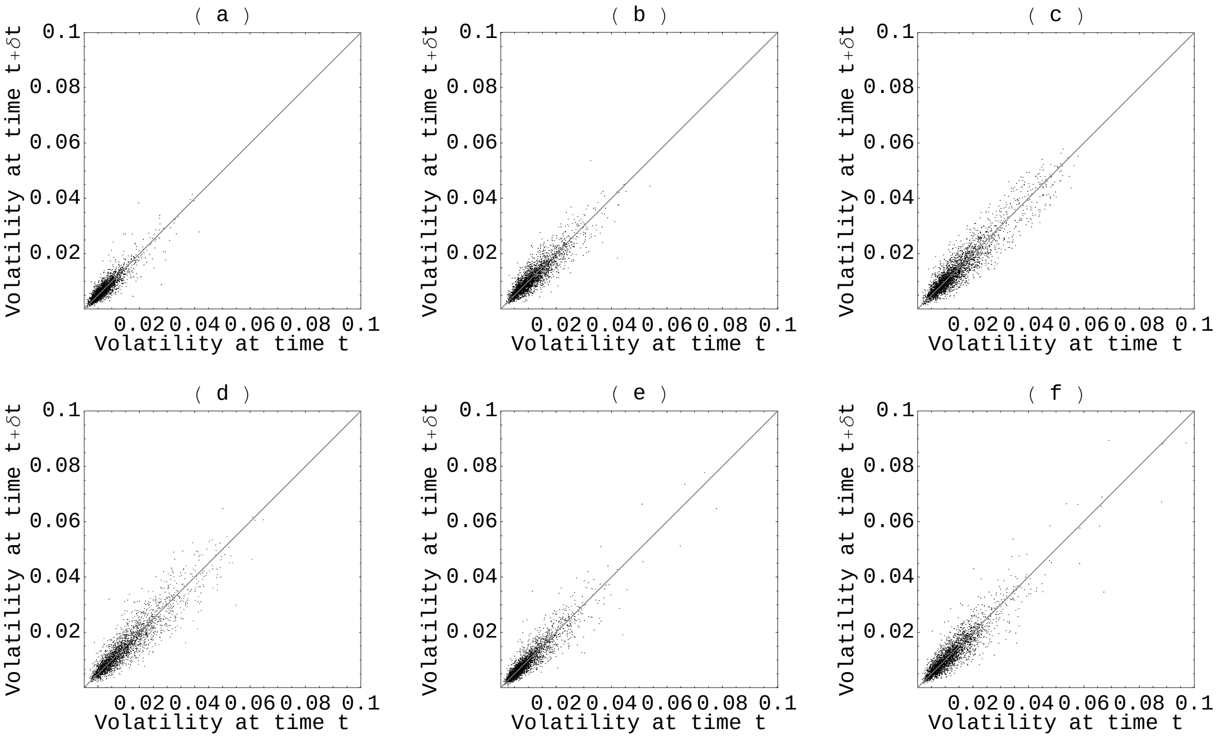


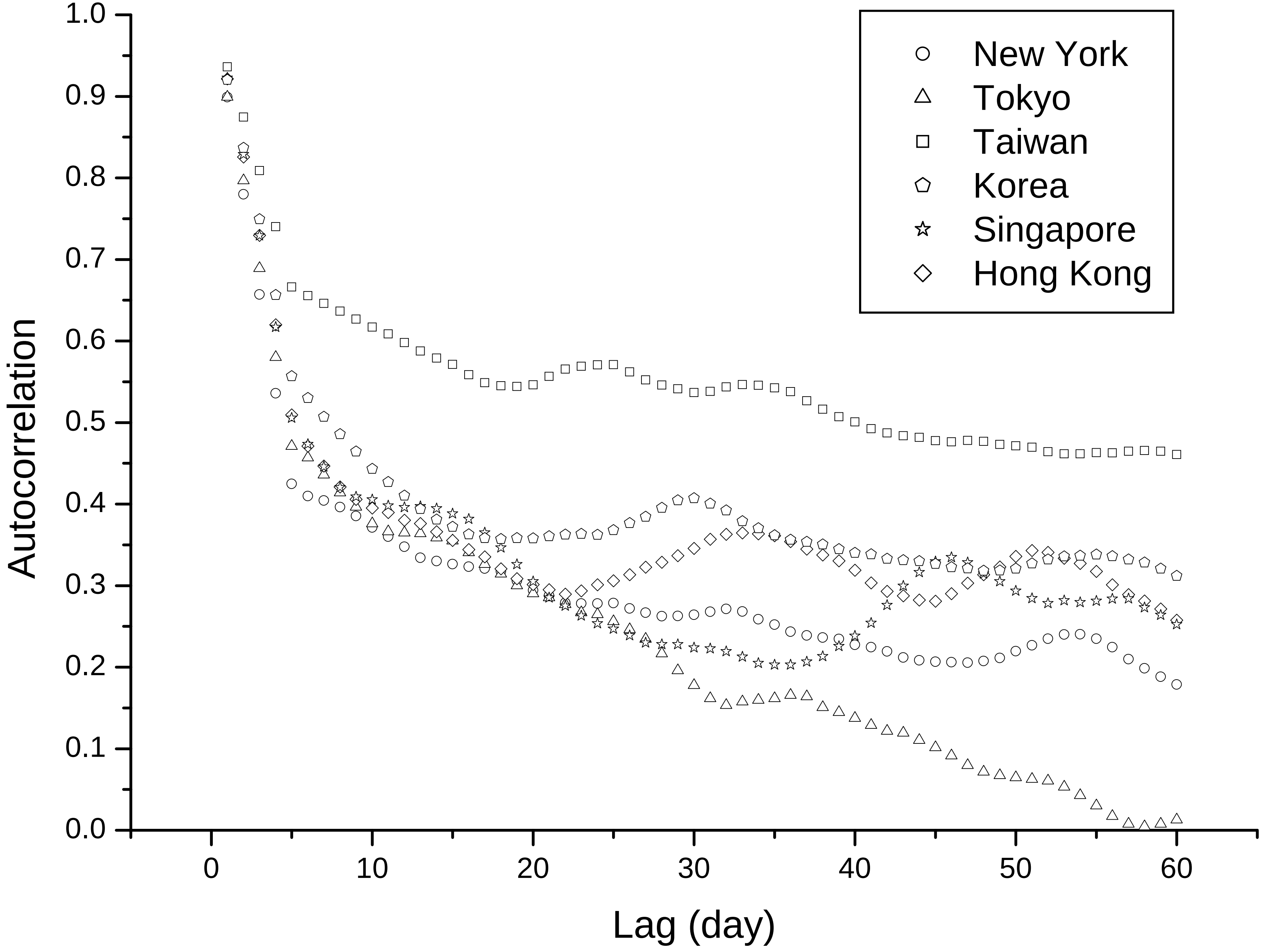
(a)
(b)
(c)
(d)
(e)
(f)
log(N)
log(1/ϵ)


New York
Tokyo
Taiwan
Korea
Singapore
Hong Kong
Probability Density
Volatility
0
20
40
60
80
100
120
140
160
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.10



U.S.A.
Japan
Taiwan
Hong Kong
Singapore
Korea
Probability Density
160
140
120
100
80
60
40
20
0
-20
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Volatility


(a)
(b)
(c)
(d)
(e)
(f)
Volatility at time t+δt
Volatility at time t
0.02
0.04
0.06
0.08
0.1



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Autocorrelation
0
10
20
30
40
50
60
Lag (day)
New York
Tokyo
Taiwan
Korea
Singapore
Hong Kong