INNOVATION AND TECHNOLOGICAL EVOLUTION IN A WESTERN EUROPEAN COUNTRY – *THE CASE OF PORTUGAL*

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Abstract:

The paper speaks of the technological evolution using a translog cost function for a western European country – Portugal. It begins by presenting the methodological framework, the estimation process based on the iterative Zellner method to estimate systems of seemingly unrelated regression equations (SURE), the empirical application to Portugal, and the interpretation of the results, namely in terms of the technological evolution.

Keywords: technological evolution, energy economics, translog cost function, SURE system.

JEL CLASSIFICATION: C0, C1, C5, C8, O33, Q4

1. INTRODUCTION

The theme that we see here is integrated in the Energy and Economic Development area, particularly in one of its points – the Production and Cost Functions with energy – of the program of an Energy Economics subject, subject that is frequently studied in undergraduate and postgraduate Economic courses.

The research has three parts, the first of which discusses the question of the introduction of the energy *input* in one of the main production functions (*putty-putty, clay-clay e putty-clay*), the second speaks of the technological evolution viewed from these functions, and the third relates an empirical application to the Portuguese reality or economy, describing successively the methodological process to follow, the preparation of the model facing its estimation, the description of the iterative estimation method, the data-base and its sources, the estimation process itself, and finally, the interpretation of the results found and its comparison with the ones obtained for other countries and other moments.

2. THE METHODOLOGICAL FRAMEWORK AND THE ESTIMATION PROCESS

A. The Methodological Framework

To appreciate the question of the Portuguese technological evolution and the position of the Portuguese economy in the international controversy or debate – if energy and capital are complementary or substitutes – we are going to use a process derived from the one initially developed by Jorgenson and Fraumeni and later used again by Berndt and Wood (1982,1985), a process followed by many authors, namely Marie N. Fagan (1997) and many other recent authors; this methodological process is based on the utilization of the translog cost function with the inputs (imported) energy, labour and capital.

The process that we are going to follow uses the aggregate translog production function. This function is twice differentiable and characterized by constant returns to scale. For the purposes we have in mind we define it as $Y = f(K,L,E,T)$ (1) where Y, K, L, E are the production output and the capital, labour and energy inputs, respectively. The inclusion of the T variable is justified with the need to represent the time shifts of the technical progress extrinsically.

Associated to the function (1) there is a unitary cost function, dual from that one, that reflects the production technology that can be represented by $c = C/Y = g(p_k, p_l, p_e, T)$ (2) where p_k, p_l, and p_e, C, and c are, respectively, the inputs prices, the total cost and the unitary cost.

Using logarithms and new parameters related to the T variable the translog cost function that we are going to use is the following

$$\ln C = \ln a_0 + g_y \ln Y + \frac{1}{2} g_{yy} \ln^2 Y + \sum_{i=1}^{3} a_i \ln p_i +$$

$$+\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} g_{ij} \ln p_i \ln p_j + \sum_i g_{iy} \ln p_i \ln Y + \frac{1}{2} g_{tt} T^2 + \qquad (3)$$

$$+ \sum_{i=1}^{3} g_{it} T \ln p_i + g_{Yt} T \ln Y + g_t T$$

where g_{ij}, g_i, α_i (i, j=1,2,...,m) and α_0 are the parameters. The linear homogeneity[1] imposes the following set of restrictions

$$\sum_{i=1}^{m} a_i = 1, \sum_{j=1}^{m} g_{kj} = 0, \sum_{j=1}^{m} g_{lj} = 0, \sum_{j=1}^{m} g_{ej} = 0, \sum_{i=1}^{m} g_{it} = 0 \;. \qquad (4)$$

The input cost share equations can be obtained by applying logarithmic derivatives to the C equation, (2), or purely deriving (3) to $\ln p_i$; using the Shepard Lemma, and admitting that both output and input prices are fixed, we have (5):

$$\frac{\P \ln C}{\P \ln p_i} = q_i = \frac{p_i X_i}{cY} = a_i + \sum_{j=1}^{m} g_{ij} \ln p_j + g_{iy} \ln Y + g_{it} T \qquad i=1,2,\ldots,m$$

The expression number (6), that we obtain deriving (3) in order to lnY,

$$\frac{\P \ln C}{\P \ln Y} = g_y + g_{yy} \ln Y + \sum_i g_{iy} \ln p_i + g_{yt} T \qquad (6)$$

can be used to study the return to scale degree and the sense carried by the technological evolution. Thus, the sign of the derivative – that varies with the time trend (T) and with the input prices (p_i) –, can be interpreted in the following way:

(a) If the sign of the derivative is negative this means the existence of positive returns to scale;
(b) If the sign of the derivative is positive then this means that there are negative returns to scale; and
(c) If the sign of the derivative is null this means that there are neither positive nor negative returns to scale.

To measure the technical progress in quantitative terms we derive the expression number (3) in order to the T variable to obtain equation number (7):

$$\frac{\P \ln C}{\partial T} = g_t + \sum_{i=1}^{3} g_{it} \ln p_i + g_{yt} \ln Y + g_{tt} T \qquad (7)$$

It's worth to note that although the bias parameters of the technical progress, g_{it} (i =1,2,...,m), are constants for the i-th input, the variation of the average cost, c, $\P \ln c_t / \P T = \P c_t / c_t$, is endogenous once the input prices vary with time. As is referred by Makota Ohta[2], with increasing returns to scale, the dual of the increasing cost rate, $\partial \ln c/\partial T$, is the negative of the primal of the growing rate of the multifactor productivity, $\partial \ln Y/\partial T$, obtained from the production function (1). This is the reason why the expression (7) relates the growing multifactor productivity with the input prices evolution (p_i) and with time (T).

Representing by b_i the rate of growth of the cost share of the i-th factor, we can write

$$b_i = \frac{\P q_i}{\P t} \cdot \frac{1}{q_i} = \frac{\P q_i}{q_i} \qquad i=1,2,\ldots,m \;. \qquad (8)$$

Based on this result we can establish the following grid that inter-relates the possible signs of the coefficients b_i and the technological change measured in terms of the intensity utilization of the i-th input (i=K, L, E):

a) If $b_i < 0 \Leftrightarrow$ then the technical progress is said to be input i saving;

[1] Related to this function see, for instance, Manso, J. R. Pires (1991) - "A Substituição de Factores na Economia Portuguesa - Uma Aplicação da Função Translog", T. M., ISEG, or Vilares (1989).
[2] Makota Ohta (1974) - "A Note on the Duality between Production and Cost Function: Rate of Returns to Scale and Rate of Technical Progress ", *Economic Studies Quarterly*, v. 25, 12, pp 63-65.

b) If $b_i = 0 \Leftrightarrow$ then the technical progress is said to be input i neutral; and

c) If $b_i > 0 \Leftrightarrow$ then the technical progress is said to be input i using.

But as $\frac{\partial q_i}{\partial T}=g_{it}$ (9) then the increasing rate b_i (i=K, L, E) can be written as

$$b_i=g_{it}\cdot\frac{1}{q_i}=\frac{g_{it}}{q_i} \qquad i=1,2,\ldots,m \qquad (10).$$

From the input cost share (5) we see that as q_i (i=1,2,...,m) is the cost share, and it is always positive, then the sign of b_i depends on the sign of g_{it} and, under these conditions, as is said by K. Sato, we can classify the technical progress according to the following rules:

a) The economy is input i-saving if g_{it} is null and

b) The economy is input i-using if the parameter g_{it} is positive[3].

According to this approach the sign of the parameter that is associated with the T variable of each one of the input cost share, (3), gives also qualitative information of the technical progress. It's worth to refer that the interpretation of the coefficients g_{it} must be done very carefully. The reasons for this, were inventoried by Berndt and Wood[4] (1985):

1. First of all, as the g_{it} parameters are constant, they don't vary in consequence of changes in the relative input price levels; consequently these coefficients g_{it} can't be used to test the validity of innovation inducted hypotheses, particularly, in the terms referred by Binswanger[5], when we put the hypotheses that the performance of the technical progress reflects either the relative input scarcity or the variation of the input prices themselves;

2. Secondly, the g_{it} coefficients represent more the relative effect than the absolute effect of technical change on the demand of the input production factors. If, for instance, the technological variation were Hicks neutral ($g_{it}=0$ for i = 1, 2, ..., m) then this case is equivalent to admit that the absolute demand of the production factors would be reduced a common proportion, letting the relative demand of the production factors unchanged; the same happens when $g_{it}>0$, case in which the technological change is factor i-using; nevertheless this doesn't imply, necessarily, that the technological change increases the absolute value of the demand of the input i. In such a case the reduction of the demand of the ith production factor individually may not be affected in the same way as the others as a consequence of technical progress;

3. Third, once the equation relative to the growing rate of unitary cost shows that the cost is affected by the input prices, then the sign of the g_{it} parameter gives a qualitative information about the changing of p_i on the cost variation (or on the multifactor growing productivity rate); when there is technical progress then $\partial lnc/\partial T$ (the cost increasing rate) is negative. If p_i increases and g_{it} is positive then the effect of this change on the price level is to relent the reduction of the unitary cost rate of growth. On the other side if p_i increases and g_{it} is negative then the cost-changing rate (in the reduction sense) accelerates. This is the reason why when the technical change is input i using and p_i increases, then, *ceteris paribus*, the increasing multifactor productivity rate reduces; on the other side when the technical change is input i-saving and p_i augments, then, *ceteris paribus*, the increasing rate of the multifactorial productivity increases.

B. The Estimation Process

For estimation purposes we are going to use the translog cost function given by the equation (3). This function when developed for three inputs (i=1,2,3), becomes

[3] It should be noted the correspondence between the values b_i and g_{it} with i=1,2,...,m.

[4] Berndt, E. R e Wood,D. O. - "Concavity and the Specification of Technical Progress", in Fericelli e Lésourd (Editores) (1985) - "Enérgie: Modélisation et Econométrie", Economica, Paris, pp 444-471.

[5] See Hans P. Binswanger (1974) and this same author with Vernon W. Ruttan (1978).

$$
\begin{aligned}
\ln C = {} & \ln a_0 + g_y \ln Y + \frac{1}{2} g_{yy} \ln^2 Y + a_1 \ln p_1 + a_2 \ln p_2 + \\
& a_3 \ln p_3 + \frac{1}{2} g_{11} \ln^2 p_1 + g_{12} \ln p_1 \ln p_2 + g_{13} \ln p_1 \ln p_3 + \\
& g_{21} \ln p_2 \ln p_1 + \frac{1}{2} g_{22} \ln^2 p_2 + g_{23} \ln p_2 \ln p_3 + \\
& g_{31} \ln p_3 \ln p_1 + g_{32} \ln p_3 \ln p_2 + \frac{1}{2} g_{33} \ln^2 p_3 + \\
& + g_{1y} \ln p_1 \ln Y + g_{2y} \ln p_2 \ln Y + g_{3y} \ln p_3 \ln Y + \\
& + \frac{1}{2} g_{tt} T^2 + g_{1t} T.\ln p_1 + g_{2t} T.\ln p_2 + g_{3t} T.\ln p_3 + \\
& g_{yt} T.\ln Y + g_t .T
\end{aligned}
\tag{11}
$$

where g_{ij}, g_i, α_i and α_0 are the parameters of the model and i, j= 1 (capital), 2 (labour) and 3 (energy imports) inputs. As we know the equation (11) substitutes, with the referred advantages, the production function that has the same name (translog).

The linear homogeneity imposed to this function conducts to the following restrictions on the parameters

$$
\begin{aligned}
& a_1 + a_2 + a_3 = 1, \qquad g_{11} + g_{12} + g_{13} = 0, \\
& g_{21} + g_{22} + g_{23} = 0, \qquad g_{31} + g_{32} + g_{33} = 0, \\
& g_{1t} + g_{2t} + g_{3t} = 0
\end{aligned}
\tag{12}
$$

On the other side the production homoteticity still imposes the conditions

$$
g_{1t} = 0, \qquad g_{2t} = 0, \qquad g_{3t} = 0 . \tag{13}
$$

The cost share equations of the factors can be obtained by applying logarithmic derivatives to the C expression; using the Shepard Lemma, assuming that the *output* and the factor prices are fixed we have for a general factor i (i=1,2,3) (14):

$$
\frac{\P \ln C}{\P \ln p_i} = q_i = \frac{p_i X_i}{cY} = a_i + g_{ii} \ln p_i + g_{i2} \ln p_2 + g_{i3} \ln p_3 + g_{iy} \ln Y + g_{it} T
$$

As is referred by E. Berndt and D Wood a useful interpretation of the g_j parameters of the translog cost function is related to the definition of share elasticity. As we saw earlier the first partial derivative of lnC in relation to lnpi is the cost share equation of the i-th input. On the other side the derivative of the cost share equation in order to $\ln p_j$ is equal to g_{ij} what is equivalent to say that the parameters of the translog cost function $g_{ij} = \frac{\P^2 \ln c}{\P \ln p_i \P \ln p_j}$ (15) mesaures the response of the demand of the ith factor (in terms of the i-th cost share) to changes in $\ln p_j$.

The Allen (partial) elasticities of substitution (AES) between the ith and jth *inputs* (i, j = 1, 2, 3) can be estimated, recurring to the coefficients of the translog cost function, using the equations number (16), the first ones for the own elasticity of substitution, the seconds for the cost share elasticities:

$$
\begin{aligned}
& s_{ii} = \frac{g_{ii} + q_i^2 - q_i}{q_i^2} = 1 + \frac{g_{ii} - q_i}{q_i^2} \qquad i = 1,2,3 \\
& s_{ij} = 1 + \frac{g_{ij}}{q_i q_j} \qquad i, j = 1,2,3 \qquad i \neq j
\end{aligned}
\tag{16}
$$

The demand elasticity of the ith input (here represented by X_i) in order to the jth input price, which can be defined as

$e_{ij}=\frac{\partial \ln X_i}{\partial \ln p_j}$ (17) can be estimated, using the translog cost function, by the equation $e_{ij}=q_j.s_{ij} \quad i,j=1,2,3$ (18) with $\sum_j e_{ij}=0 \quad i=1,2,\ldots$ (19).

On its side the return to scale economies can be calculated or measured recurring to the (Caves, Christensen e Swanson, 1981) expression

$$RTS=\frac{1-\frac{\partial \ln C}{\partial \ln K}}{\frac{\partial \ln C}{\partial \ln Y}} \qquad (20)$$

Before we obtain these estimations it's worth to say that the parameters' estimation process is not an easy task taking in account the fact that it is necessary to respect all the restrictions on the parameters and also the fact that the random errors are interrelated. According to these restrictions the system of equations has three equations, by integration of some of these restrictions, and adding to each expression the error term of the econometric models, converts itself to:

$$q_1=a_1+g_{11}\ln\left(\frac{p_1}{p_3}\right)+g_{12}\ln\left(\frac{p_2}{p_3}\right)+g_{1t}T+u_1 \qquad (21)$$

for the first input (for instance, capital), and to

$$q_2=a_2+g_{21}\ln\left(\frac{p_1}{p_3}\right)+g_{22}\ln\left(\frac{p_1}{p_3}\right)+g_{2t}T+u_2 \qquad (22)$$

for the input number 2 (labor). Relatively to the third input – the imports of energy – the values of its coefficients are estimated by recurring to the following equation system (23):

$$\begin{aligned} &a_3 = 1 - a_1 - a_2, \qquad g_{31} = -(g_{11} + g_{21}),\\ &g_{32} = -(g_{12} + g_{22}) \qquad g_{33} = g_{11} + g_{22} + 2g_{12}, \qquad (23)\\ &g_{3t} = -(g_{1t} + g_{2t}) \end{aligned}$$

The technical progress effect on the translog cost function can be analyzed by deriving the expression of lnC to the T variable:

$$\frac{\partial \ln C}{\partial T}=g_i+g_{1t}\ln p_1+g_{2t}\ln p_2+g_{3t}\ln p_3+g_{Yt}\ln Y+g_{it}T \qquad (24)$$

This equation gives the cost-increasing rate of variation.

The effect of the technical progress on the input demands is given by the evolution of the increasing cost rate of the respective input, i. e., for a general i factor (i=1,2,3), by $b_i=\frac{\partial \ln q_i}{\partial t}=\frac{\partial \ln q_i}{q_i}$ (25). As these rates can be written for the same factor as $b_i=\frac{\partial q_i}{\partial t}.\frac{1}{q_i}$ (26), then deriving the three cost-share equations in order to the T variable we have $\frac{\partial q_i}{\partial t}=g_{it}$ (27) and introducing (27) in (26) we arrive to $b_i=g_{it}.\frac{1}{q_i}=\frac{g_{it}}{q_i}$ (28).

The variances of the parameters of the third equation can be obtained using the expressions (29)

$$\begin{aligned} &\mathrm{var}(a_3) = \mathrm{var}(a_1) + \mathrm{var}(a_2) + 2\,\mathrm{cov}ar(a_1, a_2)\\ &\mathrm{var}(g_{31})=\mathrm{var}(g_{11})+\mathrm{var}(g_{21})+2\mathrm{cov}ar(g_{11},g_{21})\\ &\mathrm{var}(g_{32})=\mathrm{var}(g_{12})+\mathrm{var}(g_{22})+2\mathrm{cov}ar(g_{12},g_{22}) \qquad (29)\\ &\mathrm{var}(g_{33})=\mathrm{var}(g_{11})+\mathrm{var}(g_{22})+4\mathrm{var}(g_{12})+2\mathrm{cov}ar(g_{11},g_{22})+\\ &+4\mathrm{cov}ar(g_{11},g_{12})++4\mathrm{cov}ar(g_{12},g_{22})\\ &\mathrm{var}(g_{3t}) = \mathrm{var}(g_{1t}) + \mathrm{var}(g_{2t}) + 2\,\mathrm{cov}ar(g_{1t}, g_{2t}) \end{aligned}$$

The estimation process that we are going to adopt is known as the Zellner method; this method needs some adaptations to the translog cost function. In our presentation we are going to restrict to the case in which we have only the three referred inputs.

Let u_t and u_{1t} be expressed as (30) $u_t = E\lfloor u_{jt} \rfloor \wedge u_{1t} = 1 - u_{2t} - u_{3t}$ (30); it can be demonstrated that the u_i (i=1,2,3 or i=K,L,E) variance/covariance matrix that we are going to represent by Ω, is singular, i. e., that its determinant is null ($|\Omega| = 0$ (31)). Under these conditions we can eliminate one equation – for instance the third one – and write the equations system of share equations as (32)

$$q_{1t} = a_1 + g_{11} \ln\left(\frac{p_{1t}}{p_{3t}}\right) + g_{12} \ln\left(\frac{p_{2t}}{p_{3t}}\right) + g_{1t}T + u_{1t}$$

$$q_{2t} = a_2 + g_{21} \ln\left(\frac{p_{1t}}{p_{3t}}\right) + g_{22} \ln\left(\frac{p_{2t}}{p_{3t}}\right) + g_{2t}T + u_{2t}$$

with t=1,2,...,n. These equations still have to verify the condition $g_{12} = g_{21}$ (33).

To derive the third equation estimators of the parameters we use (i) the condition $a_1 + a_2 + a_3 = 1$, from which we derive the estimator (34) $\hat{a}_3 = 1 - \left(\hat{a}_1 + \hat{a}_2\right)$ (34).

(ii) the conditions $g_{i1} + g_{i2} + g_{i3} = 0 \qquad i = 1,2,3$, (35), from which we derive (36)

$$\begin{aligned} \hat{g}_{13} &= -\left(\hat{g}_{11} + \hat{g}_{12}\right) \\ \hat{g}_{23} &= -\left(\hat{g}_{21} + \hat{g}_{22}\right) \\ \hat{g}_{33} &= 2\hat{g}_{12} + \hat{g}_{11} + \hat{g}_{22} \end{aligned} \qquad (36)$$

and (iii) the condition $g_{1t} + g_{2t} + g_{3t} = 0$ from which we derive $\hat{g}_{3t} = -\hat{g}_{1t} - \hat{g}_{2t}$ (36').

The variances of the new estimators of the parameters we came to derive are the following:

$$\begin{aligned} & v\left(a_3\right) = v\left(a_1\right) + v\left(a_2\right) + 2\operatorname{cov}\left(a_1, a_2\right) \\ & v\left(g_{13}\right) = v\left(g_{11}\right) + v\left(g_{12}\right) + 2\operatorname{cov}\left(g_{11}, g_{12}\right) \\ & v\left(g_{23}\right) = v\left(g_{12}\right) + v\left(g_{22}\right) + 2\operatorname{cov}\left(g_{12}, g_{22}\right) \\ & v\left(g_{33}\right) = 4v\left(g_{12}\right) + v\left(g_{11}\right) + v\left(g_{22}\right) + 4\operatorname{cov}\left(g_{11}, g_{13}\right) + \\ & + 2\operatorname{cov}\left(g_{11}, g_{22}\right) + 4\operatorname{cov}\left(g_{12}, g_{22}\right) \\ & v(g_{3t}) = v(g_{1t}) + v(g_{2t}) + 2\operatorname{cov}(g_{1t}, g_{2t}) \end{aligned} \qquad (37)$$

To apply this method to the estimation of the translog cost function we begin by constructing the following matrices and vectors:

$$\begin{aligned} q_1 &= \left(q_{11} \quad q_{12} \quad \cdots \quad q_{1n}\right) \quad nx1 \\ q_2 &= \left(q_{21} \quad q_{22} \quad \cdots \quad q_{2n}\right) \quad nx1 \end{aligned} \qquad (38)$$

$$\begin{aligned} z_1 &= \left(\ln\frac{p_{21}}{p_{31}} \quad \ln\frac{p_{22}}{p_{32}} \quad \cdots \quad \ln\frac{p_{2n}}{p_{3n}}\right) \quad nx1 \\ z_2 &= \left(\ln\frac{p_{11}}{p_{31}} \quad \ln\frac{p_{12}}{p_{32}} \quad \cdots \quad \ln\frac{p_{1n}}{p_{3n}}\right) \quad nx1 \end{aligned} \qquad (39)$$

$$\begin{aligned} u_1 &= \left(u_{11} \quad u_{12} \quad \cdots \quad u_{1n}\right) \quad nx1 \\ u_2 &= \left(u_{21} \quad u_{22} \quad \cdots \quad u_{2n}\right) \quad nx1 \end{aligned} \qquad (40)$$

$$X_1=\begin{bmatrix}1 & 1 & z_{21}\\ 1 & 2 & z_{22}\\ \dots & \dots & \dots\\ 1 & T & z_{2n}\end{bmatrix} \qquad X_2=\begin{bmatrix}1 & 1 & z_{11}\\ 1 & 2 & z_{12}\\ \dots & \dots & \dots\\ 1 & T & z_{1n}\end{bmatrix} \tag{41}$$

$$\begin{aligned} B_1 &= \begin{pmatrix}a_1 & g_{1t} & g_{11}\end{pmatrix}\\ B_2 &= \begin{pmatrix}a_2 & g_{2t} & g_{22}\end{pmatrix}\end{aligned} \tag{42}$$

$$\begin{aligned} b_1 &= \begin{bmatrix}g_{12}\end{bmatrix}\\ b_2 &= \begin{bmatrix}g_{21}\end{bmatrix}\end{aligned} \tag{43}$$

Then we use the general model $Y=XB+U$ (44), where Y is a column vector composed by the two column sub-vectors q_1 and q_2, X is a rectangular matrix such that in the initial iteration is $X=\begin{bmatrix}X_1 & z_1 & 0 & 0\\ 0 & 0 & z_2 & X_2\end{bmatrix}$, but from the 2nd iteration on becomes $X=\begin{bmatrix}X_1 & z_1 & 0\\ 0 & z_2 & X_2\end{bmatrix}$; B is a column vector that is initially (1st iteration) $B=\begin{pmatrix}B_1 & b_1 & b_2 & B_2\end{pmatrix}$ and later (in the other iterations) becomes $B=\begin{pmatrix}B_1 & b & B_2\end{pmatrix}$ and u is a column vector composed $U=\begin{pmatrix}u_1 & u_2\end{pmatrix}$.

The next step is to put the system (44) as is shown by (45)

$$\begin{bmatrix}q_1\\ q_2\end{bmatrix}=\begin{bmatrix}X_1 & z_1 & 0\\ 0 & z_2 & X_2\end{bmatrix}\begin{bmatrix}B_1\\ b\\ B_2\end{bmatrix}+\begin{bmatrix}u_1\\ u_2\end{bmatrix} \tag{45}$$

The variance/covariance matrix of the residuals can be expressed as $\Omega_u=\left[\Omega\otimes I_n\right]$ (46), where

$$\Omega=\begin{bmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{bmatrix} \tag{47}$$

The estimator of the B vector of the parameters of the translog model is

$$\hat{B}=\left[X'\Omega^{-1}\otimes I_n X\right]^{-1} X'\Omega^{-1}\otimes I_n Y \tag{48}$$

This system, when decomposed in the components of each matrix, is equivalent to (49)

$$\begin{bmatrix}\hat{B}_1\\ \hat{b}\\ \hat{B}_2\end{bmatrix}=\begin{bmatrix} s^{11}X_1'X_1 & \Sigma s^{1j}X_1'z_j & s^{12}X_1'X_2\\ \Sigma s^{j1}z_j'X_1 & \Sigma\Sigma s^{ij}z_i'z_j & \Sigma s^{j2}z_j'X_2\\ s^{21}X_2'X_1 & \Sigma s^{2j}X_2'z_j & s^{22}X_2'X_2\end{bmatrix}\cdot\begin{bmatrix}\Sigma s^{1j}X_1'Y_j\\ \Sigma\Sigma s^{ij}z_i'Y_j\\ \Sigma s^{2j}X_2'Y_j\end{bmatrix}$$

where σ^{ij} is the ij-th element of the inverse matrix Ω. The estimator of the variance/covariance matrix of the parameters is $\Omega_{\hat{B}}=\left[X'\Omega^{-1}\otimes I_n X\right]^{-1}$.

As the variance/covariance matrix of the $u_i u_j$ errors is unknown we can't use directly the process we came to refer, reason why we use the Zellner iterative process for systems of seemingly unrelated regression equations (SURE). The steps of this process are the following: 1st we use the OLS method to estimate the two equations parameters of the model; 2nd we estimate the residuals $e_1^{(0)}$, $e_2^{(0)}$ using the following equations $e_i^{(0)}=q_i-X_i\hat{b}_i-z'_i b_i \quad i=1,2$; 3rd we estimate the variance-covariance matrix of the residuals u_i, $\Omega^{(0)}$, using the relation $\hat{s}_{ij}^{(0)}=\frac{1}{n-4}e_i'^{(0)}e_j^{(0)} \quad i.j=1,2$; 4th we estimate the inverse matrix of $\Omega^{(0)}$; 5th we estimate the vectors $B_1^{(0)}$, $b^{(0)}$, $B_2^{(0)}$, $\Omega_{\hat{B}}^{(0)}$ using the relation $\hat{B}=\left[X'\Omega^{-1}\otimes I_n X\right]^{-1}\left[X'\Omega^{-1}\otimes I_n Q\right]$ and $\Omega_{\hat{B}}=\left[X'\Omega^{-1}\otimes I_n X\right]^{-1}$; 6th we obtain the new estimate $\Omega^{(1)}$ for the residual matrix Ω using the expression $s_{ij}^{(1)}=\frac{1}{n}\left(q_i-X_iB_i^{(1)}-z_ib^{(1)}\right)\left(q_j-X_jB_j^{(1)}-z_jb^{(1)}\right)$

where the (1) indicates the first iteration; 7[th] we repeat all the process from the 4[th] step on putting (1) instead of (0).

The process ends when the two following conditions are filled: (a) the difference between the bigger element of the matrix $\Omega^{(s)} x \left[\Omega^{(s-1)}\right]^{-1}$ (where s identifies the iteration) and the corresponding value of the identity matrix with 2 columns, I_2, is in absolute values, inferior to 0,001, i. e., when $\left\|\Omega^{(s)} x \left[\Omega^{(s-1)}\right]^{-1} - I_2\right\| < 0{,}001$; (b) the relative variation between two consecutive estimates for each one of the eight parameters $a_1, a_2, g_{11}, g_{12}, g_{22}, g_{23}$, g_{1t} and g_{2t} is, in absolute value, inferior to 0,01, i. e., when

$$\left|\frac{k_j^{(s)} - k_j^{(s-1)}}{k_j^{(s-1)}}\right| < 0{,}01 \qquad j = 1, 2, \ldots, 8, \tag{50}$$

where $k_1 = a_1, k_2 = a_2, k_3 = g_{11}, k_4 = g_{12}, k_5 = g_{22}$, $k_6 = g_{23}$, $k_7 = g_{1t}$, and $k_8 = g_{2t}$; *s* indicates the order of the iteration.

When this process is finished we can estimate the following results, including the own and cross demand-price elasticities and the own and cross too Allen elasticities of substitution.

3. EMPIRICAL APPLICATION TO THE PORTUGUESE ECONOMY

A. The Database and its Origin

The data values for the variables capital, labor and energy were, all of them, extracted from the book "Long Series for the Portuguese Economy"[6] edited by the Portuguese Central Bank. Following the methodology described by Vilares (1989) and other authors, and by Manso (1991), given the absence of secure time series relative to the stock *inputs* of capital and labor used in the production process, we identified the first time series with the gross production surplus[7] and the second with the remunerations[8]. For the energy variable we identified it with the imported energy (more or less 80% of all the primary energy needs).

All the analysis uses deflated values, what is equivalent to say that all the variables are put in constant values. Following the authors referred above and also Berndt and Wood and Gregory and Griffin, to cite only the more important, for the prices of the inputs we used their price deflators; thus for the input capital, we used the investment deflator (FBCF), for the labor input, the deflator of the private consumption, and for the imported energy input, the deflator of imports.

The time period is 1974 (the year in which the Portuguese democracy began) to 1995 (the last year covered by the data base), 21 years of the last quarter of the 20[th] century. It's worth to say that besides the fact that the last year of the series (1995) is already an old one, we thought it would be good to use it, instead of trying to up-to-date it attending to compatibility and coherency questions; the rationality of this option is that these data were prepared by the Portuguese Central Bank, and are usually accepted as of great scientific rigor, reason why we should wait that they conduct to credible results and conclusions, quality that we could not guarantee if we used data from several origins (INE, DPP and/or Eurostat) trying to obtain more up-to-dated series.

[6] "Séries Longas da Economia Portuguesa", Banco de Portugal
[7] It's the part of the product that goes to the capital input.
[8] It's the part of the product that goes to labour input.

B. The Empirical Results Found and its Interpretation

Using the iterative Zellner method to estimate seemingly unrelated regression equations (SURE) as we told before we found, for the initial iteration, the following results: (1) for the input capital (i=1), the expression (51):

$$\hat{q}_1 = 0{,}2553 + 0{,}0119T + 0{,}0382\ln\left(\frac{p_1}{p_3}\right) - 0.1435\ln\left(\frac{p_2}{p_3}\right)$$
$$(t) \quad (5.28559) \quad (6.79267) \quad (0.237777) \qquad (-1.23487) \tag{51}$$
$$R^2 = 0.921925 \quad P(F)=0.00000$$

and (2) for the input labor (i=2) the equation:

$$\hat{q}_2 = 0{,}7708 - 0{,}0094T - 0.1527\ln\left(\frac{p_1}{p_3}\right) + 0.1610\ln\left(\frac{p_2}{p_3}\right)$$
$$(t) \quad (13.23531) \quad (-4.42175) \quad (-0.78862) \quad (1.148985) \tag{52}$$
$$R^2 = 0.891127 \quad P(F)=0.0000$$

Following the iterative process described earlier we achieve, after several iterations, to the results that we present in the table n. 1. This table also contains the estimates of the parameters of the input energy, that were estimated using the expressions (34), (35), (36) and (36’). The same table besides presenting the estimates also presents the standard deviations of these estimates, and the t-values to verify their statistical significance.

Table n. 1: the estimates and its significance

Parameters	Estimates	Standard deviations	t-Values
α_1	0,219353	0,146586	1,496408
g_{1t}	0,010973	0,01308	0,838907
g_{11}	-0,023074	0,323302	-0,713702
g_{12}	0,161331	0,297264	0,542719
α_2	0,770983	0,123519	6,241803
g_{2t}	-0,009335	0,004482	-2,083048
g_{22}	-0,153384	0,410705	-0,373464
α_3	0,995048	0,121622	8,814690
g_{3t}	-0.001637	0,012769	-0.002788
g_{13}	0.0006941	0,248969	-0,000597
g_{23}	-0,0000795	0,133212	-0,002597
g_{33}	-0,0000614	0,236654	-10,66997

It’s worth to note that some of the estimates are not significant in statistical terms at the 5% level of significance.

After optimizing the results we estimated the following Allen (partial) elasticities of substitution (AES): (i) σ_{11}= -3,513466, σ_{22}= -0,99107, σ_{33}= -86,45692, for the own elasticities, and (ii) σ_{12}= 1,711481, σ_{13}= 1,159286, σ_{23}= 0,989428 for the cross elasticities of substitution.

In terms of interpretation of these values we can say that the 1st group of values shows a particularly high sensitivity to the own substitution elasticity of imported energy. The same conclusion, but with less sensitive elasticity, can be maintained either with capital or with labor (this one the less sensitive).

The cross Allen elasticities of substitution found for the binomial capital-labor, capital-energy and labor-energy show that the for the Portuguese economy of the years 1974 to 1995, the relations among these three combinations of two inputs are of substituibility, more important in the first case (1.711481), less important in the second (1.159286) and much less in the third (0.989428).

The fact that capital and energy inputs are substitutes corroborates the conclusions extracted by some international authors that affirm that these two factors are substitutes in most countries of the Occidental Europe, as is defended by the Gregory and Griffin school, and contradicts what is defended by the Berndt and Wood school and by ourselves in an earlier study[9], authors that say that energy and capital are complements[10]. As a reason for the changing of our position we can only give the one given by the reconciliation school that says that the number of factors considered in the analysis, and the fact of considering explicitly the T variable as an independent one, can introduce great changes either in terms of the results or in terms of the extracted conclusions.

The (average) elasticities of demand in relation to price (also named the demand-price elasticities) are (i') ε_{11} = -1,273854, ε_{22} = -0,619832, ε_{33} = -1,03912, for the own elasticities, and (ii') ε_{12} = 1,070391, ε_{13} = 0,013933, ε_{23} = 0,011892, ε_{21} =0,6205204, ε_{31} = 0,4203145 and ε_{32} =0,6188055, for the cross elasticities.

The results found for the own demand-price elasticities show that price increases (reductions) of each one of the *inputs* correspond to reductions (increases) of their own demands and that these changes are bigger, in absolute values, for capital, for energy and finally for labor. These values show that the Portuguese economy of the period 1974-1995 is very *price responsive* particularly in the case of imported energy and also of capital.

The values of the cross demand-price elasticities found permits to conclude that: (1) if we increase the price of the capital input the demand for labor and energy increase too as a reaction to the reduction of the demand of the capital input; (2) if we increase the hourly price of labor we increase the demand for both the inputs capital and energy; and (3) if we increase the price of energy we increase the demand for capital and labor.

The value found for the elasticity of capital to labor shows a certain sensibility of the demand for capital corresponding to price variations of labor.

The quality of the initial regressions done to use the iterative Zellner method for SURE models is good, based on the determinant coefficients of the two initial regressions (84,99% (R=92,19%) and 79,41% (R=89,11%)) and on the global statistical significance of these regressions. The optimized and final solution encountered confirms the validity of these results.

To appreciate the characteristics of the technical evolution verified along the 21 years of our analysis we substitute the values of the coefficients of the time variable, g_{it}, by their estimates to obtain the values of b_i. Thus, for the input capital, we have $\hat{b}_1 = 0{,}03026$. This value, being a positive one, translates, for this time period, a technical evolution relatively capital-using. This is related to the fact that in this period there were low rates of interest in real terms, when compared to the higher value of the labor price.

For the labor input, we have $\hat{b}_2 = -0{,}014926$; this result being negative denotes a technological evolution relatively laborsaving. The reason for this fact is related to the rigidity of the Portuguese labor legislation – an argument frequently used by the corporate associations – and with the not so low hourly costs of labor, in real terms, specially when compared to the price of capital input – the interest rate.

For energy we have $\hat{b}_3 = -0{,}13624$; this result, being negative, translates a technological evolution relatively energy-saving; this situation is not strange to the following facts: (1) the high price of this factor – comparatively, with the interest rate (the price of money or capital), (2) the higher intensity of utilization of the other inputs, fact that could conduct to a decrease in the intensity of utilization of the energy inputs; and (3) the fact that during a substantial part of this time period the country had lack of gold and foreign currency, the two indispensable payment means to pay the imports of energy.

4. CONCLUSIONS

To speak only of the fundamental of this empirical study we can say that the technological evolution in Portugal during the 21 years of our study was characterized by the following: (1) by being *capital-using*, fact that is related to the not so high rate of interest, in real terms, and to the rigidity of the labor laws; (2) by being *labor and energy saving*, i. e., by using more rationally the scarce resources in labor and energy –, what has to see with the relatively high

[9] Manso (1991) (V. Manso, J. R. Pires (1991) – "A Substituição entre Factores na Economia Portuguesa – Uma Aplicação da Função Translog Considerando a Energia como Factor de Produção Autónomo", TM, ISEG, UTL, Lisboa.
[10] Conclusions extracted with data from the manufact uring industry of the USA.

prices of these two inputs when compared to the interest rate, and, too, with the absence of gold and foreign currencies during a great period of time (at least till the Portuguese adhesion to the European Union); (3) by the *substituibility of the two inputs* capital and labor, and labor and energy; (4) by confirming a greater alignment of the Portuguese economy with the ones of the western Europe in what respects the capital and energy substituibility, in accordance with the school of Gregory and Griffin and (5) by being *price-responsive* – a conclusion that could be extracted using the Allen Elasticities of substitution and the demand-price elasticities, what means that the demand is sensitive to price variations of the inputs capital and labor and especially of the imp orted energy; (6) by being very sensitive – based on the cross elasticities of substitution –, to variations of the concurrent inputs, with the exception of labor to variations of the prices of the *input* energy, and, finally, (7), taking in account the values of the demand-price elasticities found, by being low *price-responsive* (the only exception is the demand for capital services in order to changes in the price of the hourly labor).

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