An Improved Panel Unit Root Test Using GLS-Detrending

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Abstract

We propose to combine recent developments in univariate and multivariate unit root testing in order to construct a more powerful panel unit root test. We extend the GLS-detrending procedure of Elliott, Rothenberg and Stock (1996) to a panel Augmented Dickey-Fuller test. The finite sample power properties of the new test demonstrate a very large gain when compared to existing tests, especially for small panels. We then investigate the topic of Purchasing Power Parity for the post Bretton-Woods period via this new test. The results show strong rejections of the unit root hypothesis.

JEL classification: C12, C15, C32, C33

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1 Introduction

Economic analysis of most time series requires stationarity of the data. Unit root tests are commonly used to address this matter. The most well-known among them is the augmented Dickey-Fuller (ADF) unit root test. Recent works, however have acknowledge the poor power properties of this test, which leads to a vast literature attempting to overcome these disadvantages. These developments have occurred at both multivariate and univariate levels.

At the multivariate level, authors such as Levin, Lin and Chu (LLC) (2002), Im, Pesaran and Shin (1997) and Maddala and Wu (1996) offer excellent alternatives to the ADF test by combining time-series information with cross-sectional variability. The panel approach appears extremely appealing for two reasons. First, the inclusion of a limited amount of cross-sectional information induces significant improvement in term of power. Second, the data needed for this type of analysis is increasingly available.

The LLC test and more specifically the LLC hypotheses are widely used. Indeed, several works propose enhanced versions of this test, producing data specific estimations. Papell (1997) suggests accounting for heterogeneous serial correlation, while O'Connell (1998) demonstrates the necessity of allowing

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for the cross-sectional dependence in the estimation procedure.[2,3] Papell and Theodoridis (2001) incorporate both by considering a panel version of the ADF test, using the LLC hypotheses and allowing for heterogeneous serial and contemporaneous correlation.

At the univariate level, Elliott, Rothenberg and Stock (1996) develop a GLS-detrended/demeaned version of the ADF test. Running the ADF test on the GLS-transformed data leads to one of the most powerful univariate test, which they call the DF-GLS test.[4] While these innovations deliver substantial gains in power over the univariate ADF test, they still demonstrate limited performance when applied to economic time-series data. Indeed, most of the data sets available have a limited length.

The present work intends to develop a new panel unit root test, offering satisfying performance especially in case of highly persistent series and limited amount of data. Seeking a significant increase in power over existing tests, we combine the GLS-detrending of Elliott, Rothenberg and Stock (1996) with the panel ADF test, using Levin, Lin, and Chu's (2002) hypothe-

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[2]Papell (1997) shows a strong relation between the size of the panel and the rate of rejection of the unit root hypothesis.

[3]O'Connell (1998) points out the sizeable bias induced by the neglect of contemporaneous correlation when estimating cross-correlated data.

[4]Hansen (1995) proposes a more powerful alternative to the DF-GLS test by including covariates to the test, at the univariate level.

ses. We analyze the behavior of our panel unit root test for various sample sizes, panel widths, and degrees of persistence with a Monte Carlo experiment. The main result is that our panel unit root test displays significantly better finite sample power than existing univariate and panel unit root tests.

We illustrate the test with an application to the Purchasing Power Parity (PPP) query within industrialized countries. We focus on the post Bretton-Woods period because neither the panel ADF test nor the DF-GLS test are able to reject the existence of a unit root. The principal outcome is a robust overall support for the PPP hypothesis, independently of the width and the length of the panels considered.

The next section provides a concise review of the literature that relates to the understanding of our proposed unit root test. Section 3 develops the new panel unit root test and tabulates the finite sample critical values, while Section 4 conducts a detailed power experiment, where it is shown that the new test provides a significant increase in power over the panel ADF test. Section 5 presents an empirical application to PPP, and, finally, Section 6 summarizes our findings.

2 Existing Unit Root Tests: A Concise Review of the Literature

In this section, we present the existing tests which have motivated this work and which help to its understanding. First, the standard ADF unit root test runs the following regression:

$$y_t = d_t + \alpha y_{t-1} + \sum_{i=1}^{k} \psi_i \Delta y_{t-i} + u_t \tag{1}$$

where y_t is the tested series, d_t a set of deterministic regressors, k the lagged first difference terms allowing for serial correlation and u_t the error term of the regression. The unit root null hypothesis is that $\alpha = 1$, and the alternative of stationarity is $\alpha < 1$. This test is well-known for its poor power, and the subsequent literature suggests several solutions. In the next two subsections, we describe some recent developments at both multivariate and univariate levels.

2.1 More Powerful Unit Root Tests: Panel ADF Tests

The idea behind the panel unit root tests is to combine cross-sectional and time-series information to achieve a more efficient test. Several panel procedures have been developed to test the unit root null hypothesis against various alternative hypotheses. Levin, Lin, and Chu (2002) test the unit root null hypothesis against a homogenous alternative that every series in the panel is stationary with the same speed of reversion. Im, Pesaran and Shin (1997) and Maddala and Wu (1996) test the unit root null hypothesis against the alternative that at least one series in the panel is stationary. The new test, later proposed in this paper, focuses on the stationarity of the entire panel, which automatically leads us to concentrate on the LLC framework. The LLC test runs the following panel version of equation (1):

$$y_{jt} = d_{jt} + \alpha y_{j,t-1} + \sum_{i=1}^{k_j} \psi_{ji} \Delta y_{j,t-i} + u_{jt} \tag{2}$$

where α is the homogeneous rate of convergence of the panel. The null hypothesis is that $\alpha = 1$ and the alternative is that $\alpha < 1$. For each series j, $j = 1, ..., N$, $d_{it} = \beta_j' z_t$ is a set of deterministic regressors, which allows for heterogeneous intercepts and time trends and k_j lagged first differences term

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are included to account for serial correlation. The error terms are assumed to be contemporaneously uncorrelated, $E(u_{it}u_{jt}) = 0$ for $i \neq j$.

While the LLC test leads to substantial improvements over the ADF test in terms of power, it is based on the extremely restrictive assumption that the series in the panel are cross-sectionally uncorrelated. Maddala and Wu (1996) and O'Connell (1998) demonstrate that if the error terms in equation (2) are indeed contemporaneously correlated, the LLC test exhibits severe size distortions.[5] As an alternative, Papell and Theodoridis (2001) estimate the system of equations defined by (2) using Seemingly Unrelated Regressions (SUR). This version of the LLC test, which we refer to as the ADF-SUR test, accounts for serial and contemporaneous correlation. In the rest of this paper, we shall estimate equation (2) allowing for contemporaneous correlation.

Performing the ADF-SUR test is a two-step procedure. First, for each series j, $j = 1, ..., N$, the number of lagged first difference terms, k_j, must be selected to account for serial correlation. In this work, we use the general-to-specific (GS) lag-selection procedure of Hall (1994) and Ng and Perron (1995). Then, having selected k_j, the system of equations needs to be estimated via SUR, constraining the values of α to be identical across equations.

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[5] O'Connell (1998) imposes homogeneous serial correlation properties across the series, which results in under rejection of the null hypothesis.

2.2 More Powerful Univariate Unit Root Tests: The DF-GLS Test

Elliott, Rothenberg, and Stock (1996) construct an efficient univariate unit root test based on local-to-unity asymptotic theory. The DF-GLS test is an ADF test on GLS-demeaned (or GLS-detrended) data. Specifically, the DF-GLS test runs the following regression:

The demeaned case, $z_t = (1)$:

$$y_t^{\mu} = \alpha y_{t-1}^{\mu} + \sum_{i=1}^{k} \psi_i \Delta y_{t-i}^{\mu} + u_t \tag{3}$$

The detrended case, $z_t = (1, t)$:

$$y_t^{\tau} = \alpha y_{t-1}^{\tau} + \sum_{i=1}^{k} \psi_i \Delta y_{t-i}^{\tau} + u_t \tag{4}$$

where $y_t^{\mu}(y_t^{\tau})$ is the GLS-demeaned (GLS-detrended) series.

Equations (3) and (4) can be rewritten as:

$$y_t^{GLS} = \alpha y_{t-1}^{GLS} + \sum_{i=1}^{k} \psi_i \Delta y_{t-i}^{GLS} + u_t \ , \ with \ GLS = (\mu, \tau) \tag{5}$$

with $y_t^{GLS} = y_t - \tilde{\beta} z_t$. $\tilde{\beta}$ is the least-squares estimate of the regression

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of $\tilde{z}$ on $\tilde{y}$, i.e. $\tilde{\beta} = \left(\sum \tilde{z}_t^2\right)^{-1} \sum \tilde{z}_t \tilde{y}_t$. $\tilde{y}_t$ and $\tilde{z}_t$ are the quasi-differences of

y_t and z_t respectively, i.e. $\tilde{y}_t = (y_1, (y_2 - ay_1), ..., (y_T - ay_{T-1}))'$, and $\tilde{z}_t =$

$(z_1, (z_2 - az_1), ..., (z_T - az_{T-1}))'$. $a = 1 + \frac{\bar{c}}{T}$ represents the local alternative,

with $\bar{c} = -7$ when $z_t = (1)$ and $\bar{c} = -13.5$ when $z_t = (1, t)$.[6] The standard

hypotheses are tested: $H_0 : \alpha = 1$ versus $H_1 : \alpha < 1$.

The lag-selection issue in the DF-GLS regressions has received much at-

tention recently. Ng and Perron (2001) propose a new lag selection proce-

dure, the Modified Akaike Information Criterion (MAIC), that provides the

best combination of size and power in finite samples when combined with

the GLS-transformation.[7] In subsequent applications, we employ the MAIC

when performing the DF-GLS test.

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[6]$\bar{c} = $ -7 ($\bar{c} = $ -13.5) corresponds to the tangency between the asymptotic local power function of the test and the power envelope at 50% power in the case with a constant (the case with a constant and a trend).

[7]MAIC takes into account the nature of the deterministic components and the de-meaning/detrending procedure, which allows a better measurement of the cost of each lag-length choice.

3 An Improved Panel Unit Root Test : The DF-GLS-SUR Test

Both the ADF-SUR and the DF-GLS tests demonstrate higher power than the standard ADF test. They display, however, a limited ability to reject the unit root hypothesis for economic time series of the length generally encountered in practice. Consequently, we propose to combine both innovations to obtain a more powerful unit root test. The new test, which we refer to as the DF-GLS-SUR test, runs the following system of equations for $j = 1, ..., N$:

$$y_{jt}^{GLS} = \alpha y_{j,t-1}^{GLS} + \sum_{i=1}^{k_j} \psi_i \Delta y_{j,t-i}^{GLS} + u_{jt} \text{ , } with \text{ } GLS = (\mu, \tau) \qquad (6)$$

where α is the homogeneous rate of convergence of the panel. The standard hypotheses are tested, that is $H_0 : \alpha = 1$ versus $H_1 : \alpha < 1$.

The DF-GLS-SUR test requires a three-steps procedure. For each series j, the data needs first to be GLS-transformed, then k_j, the number of lagged first difference terms allowing for serial correlation, must be selected using MAIC. Finally the system of equations is estimated via SUR, constraining the values of α to be equal across equations and using the pre-selected k_j.

This procedure allows to test for the stationarity of the entire panel while accounting for data specific serial and contemporaneous correlation.

3.1 Finite Sample Critical Values

For the remaining of the paper, we consider the ADF-SUR test as benchmark. Even though Papell and Theodoridis (2001) use the ADF-SUR test, they do not report generic critical values. Therefore, we generate finite sample critical values for both the ADF-SUR and the DF-GLS-SUR tests. The Monte Carlo experiment considers panels with a length of $T = 25, 50, 75, 100,$ and 125, and with a width of $N = 5, 10, 15,$ and 20.[8] The data sets are generated under the null hypothesis as random walks without drift:

$$y_{jt} = y_{j,t-1} + u_{it} \tag{7}$$

where u_{jt}~$iidN(0,1)$ and the error terms are contemporaneously uncorrelated, $E(u_{it}u_{jt}) = 0$ for $i \neq j$.[9]

For each panel unit root test, we generate four sets of critical values. Two

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[8]We consider $T = 35, 50, 75, 100, 125$ for the case with heterogeneous constants and trends.

[9]In the subsequent empirical exercise, we shall explicitly allow for contemporaneously correlated errors.

sets for each model (regressions with a constant only, and with a constant and a trend). First, we assume that the true lag-length is known, that is we fix $k = 0$. We are also interested in the effects of lag-length selection on the finite sample distribution of the unit root test statistics. Accordingly, we generate critical values where at each iteration we select the lag length by GS for the ADF-SUR test and by MAIC for the DF-GLS-SUR test.

Our results are consistent with the fact that the inclusion of serial correlation, selected via the GS procedure, induces a strong increase in absolute value of the critical values.[10] Furthermore, the percentage change in critical values is more severe for the ADF-SUR test than for the DF-GLS-SUR test, i.e. when the lag length is selected via GS instead of via MAIC. The true value of k being 0 and the MAIC procedure providing the best estimation of the lag length, the critical values for both cases, $k = 0$ and $k = k^{MAIC}$, are relatively close.

The 1%, 5%, and 10% critical values are reported in *Tables 1* and *3* for the ADF-SUR test, and *Tables 2* and *4* for the DF-GLS-SUR test.

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[10]See, for example, Hall (1994).

4   Finite Sample Performances: Power Analysis

Being a combination of the ADF-SUR and the DF-GLS tests, we expect the DF-GLS-SUR test to be more powerful than each one of them. Therefore, we compute the power of the ADF-SUR and the DF-GLS-SUR tests.[11] Considering the same panels (N, T) than for the critical values, the power is computed via a Monte Carlo experiment with the data generated under the alternative, that is:

$$y_{it} = \rho y_{j,t-1} + u_{jt} \tag{8}$$

where $u_{jt} \sim iidN(0,1)$, $E(u_{it}u_{jt}) = 0$ for $i \neq j$ and $\rho < 1$. We consider the following alternatives $\rho = (0.99, 0.97, 0.95, 0.90, 0.85, 0.80)$, with the nominal size fixed at 5%. *Tables 5, 6, 7,* and 8 display the level of power for the ADF-SUR and the DF-GLS-SUR tests, for the $(k = 0)$ and the $(k = (k^{GS}, k^{MAIC}))$ cases using regressions with a constant only (*Tables 5 and 6*) or with a constant and a trend (*Tables 7 and 8*).

Below, we discuss three aspects of the results for both tests: a change

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[11] Elliott, Rothenberg, and Stock (1996) provide the power analysis for the DF-GLS test for both demeaned and detrended cases.

in T, the length of the panel, a change in N, the width of the panel, and a decrease in ρ, the persistence of the series.

4.1 Demeaned case: *Tables 5* and *6*

A comparison between the power of the DF-GLS-SUR test and of the DF-GLS test demonstrates that the inclusion of few more series to the univariate DF-GLS test leads to drastic power improvements: for $\rho = 0.95$ and $T = 100$, the power of the DF-GLS test is equal to 0.26, while for the DF-GLS-SUR test the power is 0.98, with $N = 5$.

Commonly, the lag selection induces a uniform power loss, compared to the case where the lag length is known and equal to 0. The simulations show this expected outcome, with *Table 6* having a lower power than *Table 5*. Except for this difference, however, *Tables 5* and *6* present similar patterns: the DF-GLS-SUR test demonstrates an overall higher power than the ADF-SUR test. Therefore, unless it is clearly specified, the following performance analysis does not dissociate these two cases.

A sole increase in T leads to consistent increases in power for both tests with significantly stronger improvements for the DF-GLS-SUR test than the ADF-SUR test. For example, considering the case with no lags, a highly

persistent system of series, $\rho = 0.99$, a limited amount of series, $N = 5$, and a small increase in the length of the panel, T varies from 25 to 50 observations, the DF-GLS-SUR test produces an increase in power ten times higher than the ADF-SUR test. The same case with a wider panel ($N = 20$), demonstrates a similar outcome. For highly persistent series with a small number of observations, the DF-GLS-SUR test offers higher power than the ADF-SUR test, and takes better advantage of an increase in T.

For less persistent processes, for example $\rho = (0.97, 0.95)$, the DF-GLS-SUR test continues to present a stronger response to an increase in the number of observations than the ADF-SUR test.

A sole increase in N leads to consistent power improvements, with a stronger impact on the DF-GLS-SUR test than on the ADF-SUR test. For example, with lag-length selection, $\rho = 0.99$, and $T = 25$, an increase in the number of series, N evolving from 5 to 10, leads to a power increase for the DF-GLS-SUR test five times stronger than for the ADF-SUR test. Furthermore, if we compare the impact of a change in T with the impact of a change in N, the tables show for both tests that an increase in width has a stronger effect than an increase in length. Considering the case with no lags and $\rho = 0.95$, the DF-GLS-SUR test has a power of 0.83 for $(N, T) = (5, 50)$.

A minimum of 50 observations needs to be added to reach a power level close to 1, while the addition of only 5 series displays the similar result.

$(N, T) = (20, 50)$ presents an interesting case, especially if the processes are highly persistent. The ADF-SUR test is well-known for its power deficiency when the width and the length of the panel are too close. In the presence of lags, and with $\rho = 0.99$, this combination presents a power of 0.44 for the DF-GLS-SUR test and of 0.10 for the ADF-SUR test. If $\rho = 0.97$, the DF-GLS-SUR test reaches a power of 0.98 while the ADF-GLS test offers only a power of 0.22. The DF-GLS-SUR test demonstrates an impressive higher power than the ADF-SUR test for panels including highly persistent series and with a width close to the length.

As expected, a change in the series persistence also has a major influence on the behavior of both the DF-GLS-SUR and the ADF-SUR tests. In the case with lags, and $(N, T) = (10, 75)$, the DF-GLS-SUR test presents a power of 0.41 for $\rho = 0.99$, of 0.98 for $\rho = 0.97$ and of 1.00 for $\rho = 0.95$. More generally, the DF-GLS-SUR test has a power of 1.00 or close to 1.00 whenever $\rho < 0.90$.

4.2 Detrended case: *Tables 7 and 8*

Commonly, the addition of a trend to the regressions leads to a uniform loss in power for both tests: with $\rho = 0.97$ and $(N, T) = (15, 100)$, the DF-GLS-SUR$^\mu$ test produces a power of 1.00 while the DF-GLS-SUR$^\tau$ test reaches only a power of 0.30.[12] However, combining time-series information with cross-sectional information still provides significant improvements in the test performance. For $\rho = 0.95$ and $T = 100$, the DF-GLS$^\tau$ test reaches a power level of 0.10 while the DF-GLS-SUR$^\tau$ test, accounting for four more series $(N = 5)$, is able to achieve a power level of 0.35.

The GLS-transformation shows a similar impact on the size-adjusted power than in Section 3. If $\rho = 0.95$ and $(N, T) = (5, 125)$, the DF-GLS-SUR test has a power of 0.53 while the ADF-SUR test offers a power level of 0.24.

Furthermore, the amplitude of these enhancements varies following changes in T, in N or in ρ, as well as the test considered. For $\rho = 0.97$ and $N = 20$, a raise in T from 50 to 100 observations induces an power increase of 0.23 for the DF-GLS-SUR test and of 0.10 for the ADF-SUR test. Likewise, if N varies from 15 to 20 series, when $\rho = 0.97$ and $T = 100$, the power augments

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[12]The DF-GLS-SUR$^\mu$test refers to the demeaned case while the DF-GLS-SUR$^\tau$ refers to the detrended case.

by 0.32 for the DF-GLS-SUR test and by 0.03 for the ADF-SUR test. Finally, a decrease in the persistence, from $\rho = 0.97$ to $\rho = 0.95$, generates strong improvements in the performance for both tests: for $(N, T) = (10, 125)$, the observed power increase is of 0.45 for the DF-GLS-SUR test and of 0.25 for the ADF-SUR test.

To sum up, this analysis reveals two major outcomes. First, as expected, by incorporating cross-sectional variation we are able to significantly enhance the power of the univariate DF-GLS test. Secondly, the comparison of both test performances demonstrates strong improvements due to the GLS-transformation. Overall the DF-GLS-SUR test has a higher finite sample power than the ADF-SUR test, and for each increase in information (either N or T), the corresponding increase in power is larger for the DF-GLS-SUR test than the ADF-SUR test. Furthermore, our new test presents some interesting features: its power is attractively high for small panels and for highly persistent series.

4.3   Robustness Analysis

One obvious objection to this new test stands in the homogeneity imposed by the alternative hypothesis. This restriction seems to undermine the enhanced

power properties previously presented. Consequently, in this section, we focus on the impact of such a constraint by measuring the performance of the DF-GLS-SUR test when applied to series with heterogeneous rates of convergence. Our results show that, even under such conditions, the DF-GLS-SUR test remains one of the most powerful unit root tests available.

We proceed with the Monte Carlo experiment defined earlier for the power analysis, but allowing for the rate of convergence to vary across the generated series, i.e. the data generating process follows:

$$y_{jt} = \rho_j y_{j,t-1} + u_{jt} \tag{9}$$

where $u_{jt} \sim iidN(0,1)$, $E(u_{it}u_{jt}) = 0$ for $i \neq j$, and $\rho_j \leq 1$. Then we estimate equation (3) with $k = 1$.[13]

Due to the infinite number of cases existing, we focus on the six subsequent panels:

$N = 5$, $T = (50, 100)$ and ρ_j is divided in two groups such that $\rho_i = (1.00, 0.99, 0.97, 0.95, 0.90, 0.85, 0.80)$ for $i = 1, 2, 3$ and $\rho_l = (1.00, 0.99, 0.97, 0.95, 0.90, 0.85, 0.80)$ for $l = 4, 5$.

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[13] This allows us to compare the size-adjusted power with Bowman (1999) and Im, Pesaran and Shin (1997).

$N = 15$, $T = (50, 100)$ and ρ_j is divided in three groups such that $\rho_i = (1.00, 0.99, 0.97, 0.95, 0.90, 0.85, 0.80)$ for $i = 1, 2, 3, 4, 5$, $\rho_l = (1.00, 0.99, 0.97, 0.95, 0.90, 0.85, 0.80)$ for $l = 6, 7, 8, 9, 10$ and $\rho_m = (0.80, 0.95)$ for $m = 11, 12, 13, 14, 15$.

The size-adjusted power resulting from these simulations is reported in *Figure 1*.

The 3D graphs demonstrate strong deteriorations in the DF-GLS-SUR-test performance in presence of random walks among the series. For example, if $(N, T) = (5, 50)$ and $\rho = (1.00, 0.99)$, the power level reached is 0.08 instead of 0.18 when $\rho_i = \rho_j = 0.99$. Power losses are also observed when some of the series estimated include processes more persistent than the alternative considered in the homogeneous case: if $(N, T) = (15, 50)$, the DF-GLS-SUR test achieves a power of 0.88 when $(\rho_i, \rho_j, \rho_m) = (0.97, 0.99, 0.80)$, instead of 1.00 when $\rho_i = \rho_j = \rho_m = 0.80$. The converse is also verified: if $(\rho_i, \rho_j, \rho_m) = (0.85, 0.90, 0.95)$ the power equals 1.00, while if $\rho_i = \rho_j = \rho_m = 0.95$ it equals 0.99.

The relatively poor performance of the DF-GLS-SUR test in presence of non-stationary processes encourages a comparison with the Im, Pesaran and Shin (IPS) (1997) test. *Figure 2* graphs the power simulations such that the

X-axis represents the number of stationary series among the panel and the Y-axis is the power. The panels considered have a width of $N = 5, 10, 15, 20$ and a length fixed to $T = 100$. The rates of convergence for the stationary processes are $\rho = (0.90, 0.95)$.

Overall, the DF-GLS-SUR test demonstrates a higher power than the IPS test when $\rho = 0.95$. These results imply that, in highly persistent cases, the impact of GLS-transformation prevails over the negative effect of the homogeneous alternative on the test performance. Our findings confirm that the GLS-transformation improves the finite sample power properties of the test, especially when investigating mixes of highly persistent and non-stationary series, even though the alternative hypothesis is wrong.

The DF-GLS-SUR test alternative hypothesis has a limited impact on the test performance in presence of series converging at different rates, but rather a strong and negative effect when the panel combines stationary and non-stationary processes. However, the latter observation has limited effects on the test reliability. Indeed, the DF-GLS-SUR test focuses on the stationarity of the *entire* panel: the presence of at least one unit root should lead to no rejection of the null hypothesis.[14]

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[14]Differently, the IPS test is supposed to reject the unit root hypothesis if at least one process in the panel is stationary.

To sum up, the DF-GLS-SUR test was designed to answer more accurately whether or not the panel converges. Its overall satisfying performance in presence of homogeneous or heterogeneous rates of convergence in a stationary data set confirms its accuracy. Furthermore, the relatively low power achieved in presence of random walks in the panel is not a major issue because it is still significantly higher than the nominal size (5%).[15]

5   Illustration: Purchasing Power Parity

As an illustration, we apply this new test to the Purchasing Power Parity (PPP) query. We consider quarterly CPIs and nominal exchange rates in dollars, from 1973(1), first quarter, to 1998(2), second quarter, (Source IFS, CD-Rom for 03/2002), for 21 industrialized countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, the U.K., and the U.S..We then construct the corresponding

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[15]The only issue could be to over-reject the null hypothesis but by controlling for the size, i.e. the tendency to over-reject the null, we solve this issue as long as the power is significantly higher than the size.

real exchange rate, q_j (in logarithm) follows:

$$q_j = e_j + p^* - p_j \tag{10}$$

where e_j, p_j and p^* are the logarithm of the nominal exchange rate (U.S. dollar as numeraire), the foreign CPI and the US CPI.

We first proceed with univariate estimations of the real exchange rates through the ADF and the DF-GLS tests, using as lag-length selection the GS and the MAIC procedures respectively. The results are shown in *Table 9.* Few rejections of the unit root hypothesis are observed: the ADF test never rejects while the DF-GLS test offers several rejections, varying from a 10% level for Denmark and Italy to a 5% level for Belgium, France, Germany, Greece and the Netherlands.

Next, we estimate the real exchange rates at the multivariate level with the ADF-SUR and the DF-SUR-GLS tests. The ADF-SUR test is a version of the LLC test accounting for contemporaneous correlation. The inclusion of correlation among the errors invalidates the limit distribution of the LLC test.[16] Maddala and Wu (1996) propose a bootstrapping alternative, and

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[16]Maddala and Wu (1996), Banerjee (1999), Bowman (1999), and Chang (2002) point out this issue.

demonstrate that the LLC test offers good performances when this technique is used. Therefore a Monte Carlo experiment is employed, allowing us to generate the critical values under the standard hypotheses, i.e. $H_0 : \alpha = 0$ versus $H_1 : \alpha < 0$.

Section 4, we have described the estimation process as well as the Monte Carlo experiment used to generate critical values. However, the data generating process used for the nonspecific analysis did not include cross-sectional correlation. For the data-specific critical values, we need to estimate them by estimating the non-diagonal variance-covariance matrix of the innovations.

The jth real exchange rate, $j = 1, ..., N$, follows:

$$q_{jt} = d_{jt} + \rho_j q_{j,t-1} + u_{jt} \tag{11}$$

where $u_{jt} = \beta_j u_{j,t-1} + \xi_{jt}$ with $(\xi_{1t}...\xi_{Nt})' \, N(0_N, \Omega)$ and $E(u_{it}u_{jt}) \neq 0$ for $i \neq j$.

We first run ADF regressions for each series, using Schwarz information criteria lag selection in order to estimate the characteristics of each process. Those estimates are assumed to define the true data generating processes of ξ_{jt}. Then we are able to deduce the u_{jt} and Σ, the variance-covariance

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matrix of the innovations, i.e. $(u_{1t}...u_{nt}) \sim N(0_N, \Sigma)$. The unit root is imposed in the generated process by taking partial sums. Finally, we proceed with the rest of the Monte Carlo experiment: for each process the estimation of $equation(3)((1))$ selects $k_j^{MAIC}(k_j^{GS})$, then $equation(6)((2))$ is estimated using SUR, with the pre-selected $k_j^{MAIC}(k_j^{GS})$.[17] Repeating each procedure 5000 times creates a vector of statistics. Then the critical values are calculated.

The data is grouped such that the panel of the 20 U.S.-real exchange rates (All20) includes Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, and the U. K. Then we consider the following panels: the European Community (EC), the European Monetary System (EMS), the 6 and 10 most industrialized countries (G6, G10), the Euro area as of 1999 (E10), the Euro area as of 2001 (E11), and the OECD countries (13).[18] For each panel, *Table 10* reports the esti-

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[17]We consider the case with constants only because we focus on the mean-reverting behavior of the real exchnage rates.

[18]EC includes Belgium, Denmark, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain, and the U.K. EMS includes Belgium, Denmark, France, Germany, Ireland, Italy, and the Netherlands. G6 includes Canada, France, Germany, Italy, Japan, and the U.K. For G10, Belgium, the Netherlands, Sweden, and Switzerland are added. E11 includes Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, and Spain. E10 does not include Greece. 13 includes Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Sweden,

mated α, the t-statistic and the corresponding half-life (HL_α) for the period

$1973(1) - 1998(2)$.[19]

The panels considered vary in size with a width including between 6 and 20 US-real exchange rates and a length of 102 observations. As shown in the performance analysis, the DF-GLS-SUR test demonstrates an high power for these specific cases (a minimum power level of 90%) while the ADF-SUR test behaves poorly, at least for the small panels (a power level of 20%). For the studied panels, the bias of the DF-GLS-SUR test is negligible compared to the bias of the ADF-SUR test. Furthermore, the high power observed for the DF-GLS-SUR test combined with a size fixed at 5% implies that the results strongly reflect the information available in the data.

The DF-GLS-SUR test demonstrates uniformly stronger rejections, with 7 rejections at 1% and 1 at 5% while the ADF-SUR test shows a majority of rejection at 5% or less.[20] By using a more powerful alternative to the existing tests, we are able to produce the strongest evidence of PPP for the floating period.

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and the U.K.

[19] We calculate the half-life based on α, that is $HL_\alpha = \frac{\ln 0.5}{\ln \alpha}$.

[20] However, the DF-GLS-SUR test generates larger half-lives than the ADF-SUR test. Studies such as Murray and Papell (2002), and Lopez, Murray and Papell (2003) produce similar results.

6 Conclusion

The literature already provides several more powerful alternatives to the ADF unit root test. However, all of them demonstrate limited ability to reject correctly the unit root hypothesis when applied to highly persistent time series with a limited span. This paper attempts to produce a more efficient panel unit root test allowing a more reliable analysis of such data sets. Our new test, the DF-GLS-SUR test, is an extension of Elliott, Rothenberg, and Stock's (1996) GLS-transformation to a version of the Levin, Lin and Chu's (2002) test. The use of Monte Carlo simulations allow us to show the interesting behavior of this new test. For both the demeaned and detrended cases, the DF-GLS-SUR test offers a uniformly higher finite-sample power than the ADF-SUR test. Furthermore, the DF-GLS-SUR-test performance remains attractive when studying a data with heterogeneous rates of convergence across the series.

The most pertinent feature of the DF-GLS-SUR test stands in its satisfying power when applied to highly persistent processes with limited amount of observations. Indeed, it is always a challenge to increase significantly the time-series dimension of economic data while the cross-sectional dimension is easily extendable.

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Table 1: Finite Sample Critical Values for the ADF-SUR Test, $z=(1)$

N	T	1%		5%		10%	
		$k=0$	$k=k^{GS}$	$k=0$	$k=k^{GS}$	$k=0$	$k=k^{GS}$
5	25	-5.6707	-7.2479	-4.8651	-6.1155	-4.4903	-5.5355
	50	-5.2441	-5.8627	-4.5867	-5.1077	-4.2463	-4.6495
	75	-5.0430	-5.4978	-4.4447	-4.7705	-4.1315	-4.4324
	100	-5.0184	-5.3612	-4.4298	-4.6821	-4.1170	-4.3136
	125	-4.9788	-5.2008	-4.3978	-4.5648	-4.1079	-4.2351
10	25	-7.6518	-9.3829	-6.6882	-8.1226	-6.2560	-7.4331
	50	-6.6553	-7.3540	-6.0092	-6.5780	-5.6690	-6.1478
	75	-6.4209	-6.8628	-5.7586	-6.1366	-5.4237	-5.7433
	100	-6.2951	-6.5348	-5.6789	-5.9484	-5.3702	-5.6094
	125	-6.2544	-6.5169	-5.6366	-5.8687	-5.3230	-5.5030
15	25	-10.4550	-11.5438	-9.1043	-10.1968	-8.3837	-9.5094
	50	-7.9136	-8.6581	-7.2513	-7.8649	-6.8559	-7.4345
	75	-7.5525	-8.0338	-6.8655	-7.2639	-6.5137	-6.8718
	100	-7.2861	-7.6344	-6.7148	-7.0169	-6.3795	-6.6551
	125	-7.2301	-7.5393	-6.7228	-6.8976	-6.3569	-6.5463
20	25	-	-	-	-	-	-
	50	-9.3641	-10.2436	-8.4795	-9.1318	-8.0607	-8.6413
	75	-8.5003	-8.9527	-7.8859	-8.2686	-7.5369	-7.8973
	100	-8.2865	-8.7111	-7.7323	-8.0245	-7.3488	-7.6499
	125	-8.1448	-8.4751	-7.5264	-7.7635	-7.1730	-7.3898

Table 2: Finite Sample Critical Values for the DF-GLS-SUR Test, $z=(1)$

N	T	1%		5%		10%	
		$k=0$	$k=k^{MAIC}$	$k=0$	$k=k^{MAIC}$	$k=0$	$k=k^{MAIC}$
5	25	-3.1576	-3.5736	-2.3489	-2.6501	-1.9262	-2.1687
	50	-2.9406	-2.9617	-2.2707	-2.3190	-1.8473	-1.9156
	75	-2.8451	-2.8570	-2.2057	-2.2081	-1.8601	-1.8896
	100	-2.8089	-2.8238	-2.1433	-2.1820	-1.7854	-1.8192
	125	-2.7992	-2.7987	-2.1297	-2.1258	-1.7666	-1.7813
10	25	-3.6376	-3.9895	-2.7942	-3.0074	-2.2337	-2.4728
	50	-3.1048	-3.2857	-2.4295	-2.5088	-2.0350	-2.1203
	75	-3.1182	-3.1464	-2.3658	-2.4636	-1.9687	-2.0469
	100	-2.9855	-2.9702	-2.3049	-2.3375	-1.8906	-1.9548
	125	-2.9169	-2.9290	-2.2864	-2.3249	-1.9417	-1.9620
15	25	-4.5752	-4.7473	-3.3616	-3.5269	-2.7309	-2.9017
	50	-3.4798	-3.5937	-2.6615	-2.7977	-2.2484	-2.3617
	75	-3.3142	-3.3346	-2.5281	-2.6392	-2.1707	-2.2413
	100	-3.1826	-3.2989	-2.4966	-2.5599	-2.0692	-2.1302
	125	-3.1182	-3.1900	-2.3693	-2.3957	-2.0246	-2.0595
20	25	-	-	-	-	-	-
	50	-3.8437	-3.9963	-2.9495	-3.1214	-2.4988	-2.6501
	75	-3.5204	-3.6599	-2.7059	-2.8150	-2.3362	-2.4182
	100	-3.3528	-3.4175	-2.5408	-2.6331	-2.1907	-2.2403
	125	-3.1817	-3.2041	-2.5374	-2.6082	-2.1950	-2.2489

Table 3: Finite Sample Critical Values for the ADF-SUR Test, z=(1,t)

N	T	1%		5%		10%	
		$k=0$	$k=k^{GS}$	$k=0$	$k=k^{GS}$	$k=0$	$k=k^{GS}$
5	35	-6.9862	-9.9797	-6.3261	-8.6032	-5.9711	-7.9052
	50	-6.7121	-7.8731	-6.1253	-7.1001	-5.7737	-6.6813
	75	-6.4433	-7.2291	-5.9397	-6.5950	-5.6530	-6.2486
	100	-6.4847	-7.0777	-5.9368	-6.4079	-5.6393	-6.0715
	125	-6.3935	-6.9050	-5.8432	-6.2775	-5.5704	-5.9402
10	35	-9.3890	-12.9343	-8.6539	-11.4676	-8.3111	-10.7788
	50	-8.8541	-10.4200	-8.2646	-9.5357	-7.9407	-9.0788
	75	-8.5459	-9.4320	-7.9619	-8.7541	-7.6502	-8.4123
	100	-8.4465	-9.1381	-7.8478	-8.4828	-7.5839	-8.1482
	125	-8.2961	-8.9418	-7.7640	-8.2326	-7.4833	-7.9433
15	35	-11.6544	-16.3897	-10.7663	-14.5835	-10.4220	-13.7604
	50	-10.7395	-12.3456	-10.1020	-11.5615	-9.7999	-11.0971
	75	-10.2063	-11.3239	-9.6669	-10.5951	-9.3861	-10.2325
	100	-10.0315	-10.8169	-9.4767	-10.1849	-9.1507	-9.8115
	125	-9.8206	-10.4819	-9.3496	-9.8900	-9.0727	-9.5505
20	35	-	-	-	-	-	-
	50	-12.5861	-14.5574	-11.9055	-13.6222	-11.5147	-13.1007
	75	-11.7610	-12.9922	-11.2252	-12.2593	-10.8897	-11.8815
	100	-11.4545	-12.2848	-10.8657	-11.6550	-10.5851	-11.2859
	125	-11.3289	-11.9535	-10.7024	-11.3388	-10.4153	-11.0085

Table 4: Finite Sample Critical Values for the DF-GLS-SUR Test, $z=(1,t)$

N	T	1%		5%		10%	
		$k=0$	$k=k^{MAIC}$	$k=0$	$k=k^{MAIC}$	$k=0$	$k=k^{MAIC}$
5	35	-5.6655	-6.1837	-6.0890	-5.6667	-5.7906	-5.3943
	50	-5.4555	-5.7931	-5.3722	-4.8815	-5.0888	-4.6203
	75	-5.1615	-5.5635	-5.0796	-4.7250	-4.8120	-4.4606
	100	-5.0973	-5.4424	-4.9069	-4.6111	-4.6513	-4.3884
	125	-5.0596	-5.3895	-4.8384	-4.5767	-4.5512	-4.3201
10	35	-7.4675	-8.1771	-8.3094	-7.7066	-8.0535	-7.4251
	50	-7.0952	-7.6604	-7.1533	-6.5814	-6.9261	-6.3123
	75	-6.7551	-7.2354	-6.7399	-6.2724	-6.4708	-6.0011
	100	-6.5987	-6.9806	-6.4973	-6.1126	-6.2131	-5.8673
	125	-6.4887	-6.8641	-6.3360	-5.9645	-6.0661	-5.7220
15	35	-9.2049	-9.8818	-9.2049	-9.3956	-8.3701	-9.1696
	50	-8.4095	-9.1628	-8.6821	-7.9467	-8.4536	-7.6964
	75	-7.9856	-8.6046	-8.0940	-7.4962	-7.8286	-7.2521
	100	-7.7685	-8.1982	-7.7327	-7.2597	-7.4862	-7.0184
	125	-7.6625	-8.0551	-7.5585	-7.1493	-7.2943	-6.8944
20	35	-	-	-	-	-	-
	50	-10.5111	-9.7688	-10.0582	-9.2706	-9.8306	-9.0317
	75	-9.7579	-9.1550	-9.2963	-8.6639	-9.0418	-8.4335
	100	-9.3108	-8.8193	-8.8340	-8.3454	-8.5908	-8.1195
	125	-9.0688	-8.6311	-8.5981	-8.1462	-8.3455	-7.9277

Table 5: Size-Adjusted Power for the ADF-SUR and the DF-GLS-SUR Tests, z=(1), no lags

N	T	$\rho = 0.99$		0.97		0.95		0.90		0.85		0.80	
		ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR
5	25	0.0588	0.1246	0.0726	0.2836	0.0870	0.4434	0.1660	0.7968	0.3036	0.9462	0.4998	0.9906
	50	0.0646	0.1782	0.1002	0.5596	0.1668	0.8286	0.5156	0.9968	0.8618	0.9998	0.9884	1.0000
	75	0.0848	0.2814	0.1810	0.8032	0.3562	0.9752	0.8812	1.0000	0.9984	1.0000	1.0000	1.0000
	100	0.0868	0.3766	0.2310	0.9422	0.5342	0.9980	0.9894	1.0000	1.0000	1.0000	1.0000	1.0000
	125	0.1082	0.4718	0.3426	0.9700	0.7400	1.0000	0.9994	1.0000	1.0000	1.0000	1.0000	1.0000
10	25	0.0622	0.1686	0.0750	0.4646	0.1022	0.7112	0.2132	0.9708	0.4422	0.9980	0.7128	1.0000
	50	0.0720	0.3460	0.1410	0.8894	0.2790	0.9944	0.7956	1.0000	0.9914	1.0000	1.0000	1.0000
	75	0.1028	0.5052	0.2688	0.9860	0.5876	1.0000	0.9942	1.0000	1.0000	1.0000	1.0000	1.0000
	100	0.1160	0.6710	0.4136	0.9996	0.8474	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	125	0.1440	0.7832	0.5970	1.0000	0.9664	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	25	0.0542	0.2198	0.0674	0.5932	0.0778	0.8280	0.1454	0.9954	0.2866	0.9998	0.5420	1.0000
	50	0.0794	0.4666	0.1718	0.9730	0.3614	0.9998	0.9148	1.0000	0.9990	1.0000	1.0000	1.0000
	75	0.1154	0.6906	0.3624	0.9996	0.7494	1.0000	0.9994	1.0000	1.0000	1.0000	1.0000	1.0000
	100	0.1456	0.8178	0.5812	1.0000	0.9528	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	125	0.1556	0.9350	0.9904	1.0000	0.9938	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	25	-	-	-	-	-	-	-	-	-	-	-	-
	50	0.0896	0.5578	0.1860	0.9912	0.4044	1.0000	0.9538	1.0000	1.0000	1.0000	1.0000	1.0000
	75	0.1284	0.7998	0.4342	1.0000	0.8456	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	100	0.1506	0.9350	0.6366	1.0000	0.9808	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	125	0.2092	0.9772	0.7294	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 6: Size-Adjusted Power for the ADF-SUR and the DF-GLS-SUR Tests, z=(1), with lags

N	T	ρ = 0.99 ADF-SUR	DF-GLS-SUR	0.97 ADF-SUR	DF-GLS-SUR	0.95 ADF-SUR	DF-GLS-SUR	0.90 ADF-SUR	DF-GLS-SUR	0.85 ADF-SUR	DF-GLS-SUR	0.80 ADF-SUR	DF-GLS-SUR
5	25	0.0544	0.0710	0.0570	0.1682	0.0864	0.2330	0.1272	0.4470	0.1788	0.6152	0.2140	0.7126
	50	0.0622	0.1478	0.0846	0.4582	0.1514	0.6782	0.3442	0.9306	0.7360	0.9734	0.7362	0.9804
	75	0.0970	0.2520	0.1472	0.7412	0.2934	0.9206	0.6760	0.9932	0.9044	0.9992	0.9684	0.9994
	100	0.0856	0.3194	0.1938	0.8820	0.4288	0.9824	0.8842	0.9988	0.9832	1.0000	0.9976	1.0000
	125	0.1092	0.4350	0.3070	0.9278	0.6314	0.9970	0.9748	1.0000	0.9986	1.0000	1.0000	1.0000
10	25	0.0654	0.1204	0.0666	0.4734	0.1064	0.5102	0.1816	0.7976	0.2712	0.9138	0.3360	0.9552
	50	0.0810	0.2856	0.1202	0.8658	0.2522	0.9526	0.6022	0.9920	0.8646	1.0000	0.9522	1.0000
	75	0.1034	0.4154	0.2278	0.9836	0.4946	0.9972	0.9282	1.0000	0.9960	1.0000	1.0000	1.0000
	100	0.1230	0.5450	0.3660	1.0000	0.7360	1.0000	0.9954	1.0000	0.9980	1.0000	1.0000	1.0000
	125	0.1396	0.7264	0.5070	1.0000	0.8902	1.0000	0.9994	1.0000	0.9996	1.0000	1.0000	1.0000
15	25	0.0750	0.1762	0.0992	0.4898	0.1200	0.6716	0.2304	0.9192	0.2310	0.9828	0.4714	0.9938
	50	0.0956	0.3716	0.1860	0.9340	0.3276	0.9994	0.7592	1.0000	0.7594	1.0000	0.9920	1.0000
	75	0.1208	0.5854	0.3330	0.9972	0.6418	1.0000	0.9850	1.0000	0.9850	1.0000	1.0000	1.0000
	100	0.1508	0.7538	0.5168	1.0000	0.8828	1.0000	0.9996	1.0000	0.9998	1.0000	1.0000	1.0000
	125	0.1782	0.9086	0.6838	1.0000	0.9912	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	25	-	-	-	-	-	-	-	-	-	-	-	-
	50	0.1004	0.4450	0.2204	0.9890	0.4038	0.9986	0.8496	1.0000	0.8498	1.0000	0.9992	1.0000
	75	0.1450	0.7216	0.4224	1.0000	0.7728	1.0000	0.9804	1.0000	0.9974	1.0000	1.0000	1.0000
	100	0.1674	0.9080	0.5950	1.0000	0.9414	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	125	0.2202	0.9582	0.8168	1.0000	0.9970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

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Table 7: Size-Adjusted Power for the ADF-SUR and the DF-GLS-SUR Tests, z=(1,t), no lags

N	T	ρ = 0.99		0.97		0.95		0.90		0.85		0.80	
		ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR
5	35	0.0504	0.0520	0.0530	0.0638	0.0652	0.0804	0.1050	0.1730	0.2290	0.3490	0.3996	0.5748
	50	0.0510	0.0544	0.0616	0.0690	0.083	0.1012	0.2850	0.2906	0.4598	0.6318	0.8452	0.9064
	75	0.0514	0.0576	0.0830	0.0984	0.1412	0.1880	0.4960	0.6540	0.8896	0.9676	0.9952	0.9998
	100	0.0594	0.0638	0.0958	0.1418	0.2098	0.3330	0.7544	0.9144	0.9926	0.9986	1.0000	1.0000
	125	0.0596	0.0640	0.1400	0.1860	0.3412	0.4740	0.9486	0.9866	1.0000	1.0000	1.0000	1.0000
10	35	0.0542	0.0522	0.0550	0.0576	0.0762	0.0894	0.1518	0.2502	0.3516	0.5524	0.6372	0.8398
	50	0.0550	0.0524	0.0774	0.0874	0.1080	0.1452	0.3400	0.5238	0.7486	0.9178	0.9630	0.9964
	75	0.0556	0.0598	0.0964	0.1176	0.2050	0.2884	0.7604	0.9142	0.9952	0.9990	1.0000	1.0000
	100	0.0598	0.0644	0.1456	0.2002	0.3670	0.5412	0.9724	0.9966	1.0000	1.0000	1.0000	1.0000
	125	0.0668	0.0724	0.2058	0.3076	0.5768	0.7840	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	35	0.0540	0.0548	0.0636	0.0732	0.0808	0.1150	0.1762	0.3654	0.4250	0.7356	0.7506	0.9606
	50	0.0538	0.0568	0.0748	0.0896	0.1204	0.1796	0.4466	0.6810	0.8728	0.9794	0.9416	1.0000
	75	0.0556	0.0588	0.1096	0.1530	0.2542	0.4002	0.8896	0.9838	1.0000	1.0000	1.0000	1.0000
	100	0.0588	0.0678	0.1656	0.2744	0.4608	0.7232	0.9966	1.0000	1.0000	1.0000	1.0000	1.0000
	125	0.0704	0.0844	0.2582	0.4304	0.7178	0.9100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	35	-	-	-	-	-	-	-	-	-	-	-	-
	50	0.0508	0.0550	0.0688	0.0970	0.0084	0.2078	0.4596	0.7866	0.9104	0.9956	0.1000	1.0000
	75	0.0538	0.0586	0.1110	0.1778	0.2736	0.4850	0.9396	0.9960	1.0000	1.0000	1.0000	1.0000
	100	0.0648	0.0740	0.2008	0.3454	0.5688	0.8350	0.9990	1.0000	1.0000	1.0000	1.0000	1.0000
	125	0.0750	0.0920	0.3216	0.5276	0.8248	0.9738	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 8: Size-Adjusted Power for the ADF-SUR and the DF-GLS-SUR Tests, z=(1,t), with lags

N	T	$\rho=0.99$		0.97		0.95		0.90		0.85		0.80	
		ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR	ADF-SUR	DF-GLS-SUR
5	35	0.0488	0.0498	0.0524	0.0576	0.0540	0.0790	0.0630	0.1596	0.0692	0.2938	0.0820	0.4546
	50	0.0492	0.0530	0.0556	0.0714	0.0676	0.1134	0.1214	0.3060	0.2074	0.5918	0.3204	0.7756
	75	0.0512	0.0552	0.0724	0.0980	0.1082	0.1932	0.2768	0.6234	0.5146	0.8838	0.7344	0.9658
	100	0.0578	0.0612	0.0938	0.1564	0.2854	0.3564	0.4902	0.8770	0.7802	0.9854	0.9310	0.9962
	125	0.0532	0.0662	0.1148	0.2164	0.2388	0.5332	0.6996	0.9694	0.9396	0.9970	0.9860	0.9994
10	35	0.0492	0.0514	0.0540	0.0620	0.0560	0.0828	0.0656	0.2226	0.0866	0.4398	0.1116	0.6506
	50	0.0522	0.0528	0.0678	0.0806	0.0888	0.1426	0.1776	0.4616	0.3254	0.7856	0.5200	0.9262
	75	0.0552	0.0570	0.0840	0.1274	0.1500	0.3124	0.4738	0.8600	0.7908	0.9882	0.9408	0.9992
	100	0.0588	0.0678	0.1160	0.2228	0.2590	0.5636	0.6460	0.9856	0.9718	0.9998	0.9976	1.0000
	125	0.0632	0.0766	0.1810	0.3864	0.4314	0.8360	0.9442	1.0000	0.9996	1.0000	0.9998	1.0000
15	35	0.0502	0.0516	0.051	0.0656	0.057	0.0944	0.0724	0.2610	0.0978	0.5284	0.1304	0.7502
	50	0.0500	0.0522	0.0706	0.0908	0.1028	0.1736	0.2426	0.5902	0.4510	0.7856	0.6872	0.9818
	75	0.0570	0.0622	0.0954	0.1668	0.1902	0.4128	0.5906	0.9508	0.9042	0.9884	0.9850	1.0000
	100	0.0592	0.0656	0.1362	0.3090	0.3264	0.7452	0.8952	0.9984	0.9962	1.0000	1.0000	1.0000
	125	0.0648	0.0858	0.2208	0.4956	0.5534	0.9364	0.9904	1.0000	1.0000	1.0000	1.0000	1.0000
20	35	-	-	-	-	-	-	-	-	-	-	-	-
	50	0.0516	0.0560	0.0704	0.0998	0.0978	0.1972	0.2606	0.6760	0.8550	0.9428	0.7438	0.9952
	75	0.0574	0.0656	0.1098	0.1806	0.2198	0.4744	0.6836	0.9816	0.9560	0.9996	0.9968	1.0000
	100	0.0648	0.0730	0.1712	0.3558	0.4192	0.8272	0.9612	1.0000	1.0000	1.0000	1.0000	1.0000
	125	0.0732	0.0960	0.2536	0.5914	0.6512	0.9748	0.9988	1.0000	1.0000	1.0000	1.0000	1.0000

Table 9: Univariate Unit Root Tests

	ADF			DF-GLS		
	α	t_α	k^{GS}	α	t_α	k^{MAIC}
Australia	-0.3014	-0.3014	0	-0.0055	-0.2447	1
Austria	-0.0638	-2.0407	0	-0.0343	-1.3519	6
Belgium	-0.0440	-1.5798	0	-0.0538	-1.9745**	1
Canada	-0.0083	-0.6009	4	0.0037	0.3204	4
Denmark	-0.0421	-1.4943	0	-0.0418	-1.7438*	1
Finland	-0.0541	-1.7942	0	-0.0341	-1.4104	0
France	-0.0549	-1.7576	0	-0.0720	-2.3718**	1
Germany	-0.0549	-1.7720	0	-0.0699	-2.0650**	6
Greece	-0.0628	-1.9407	0	-0.0649	-2.1180**	5
Ireland	-0.0946	-2.4182	0	-0.0658	-1.9506**	0
Italy	-0.0647	-1.9340	0	-0.0646	-1.9398*	0
Japan	-0.0442	-1.8689	5	-0.0138	-0.9673	1
Netherlands	-0.0585	-1.8615	0	-0.0603	-2.0685**	1
Norway	-0.0419	-1.3657	0	-0.0442	-1.7194	1
New Zealand	-0.0271	-0.9547	0	-0.0491	-1.7777	1
Portugal	-0.0494	-1.7244	0	-0.0366	-1.4140	0
Spain	-0.0508	-1.8404	0	-0.0360	-1.7018	2
Sweden	-0.0066	-0.2838	0	-0.0168	-0.7311	1
Switzerland	-0.0657	-2.1218	0	-0.0273	-1.4288	1
U.K	-0.0889	-2.2337	5	-0.0471	-1.6549	0

Table 10: Multivariate Unit Root Tests

	ADF-SUR			DF-GLS-SUR		
	α	t_α	HL_α	α	t_α	HL_α
All20	0.941	-8.396**	11.398	0.974	-5.222****	26.311
EC	0.928	-6.791***	9.276	0.959	-5.074****	16.557
EMS	0.931	-5.249**	9.695	0.953	-4.484****	14.398
G6	0.949	-4.513	13.242	0.973	-3.043***	25.324
G10	0.954	-5.504*	14.719	0.972	-3.981****	24.407
Euro10	0.945	-5.738***	12.253	0.982	-2.878**	38.161
Euro11	0.944	-6.043**	12.028	0.981	-3.127***	36.134
13	0.934	-7.275***	10.152	0.97	-4.771****	22.757

Figure 1: Robustness Analysis

a. T = 50, N = 5

b. T = 50, N = 15, $\rho_m = 0.80$

c. T = 50, N = 15, $\rho_m = 0.95$

d. T = 100, N = 5
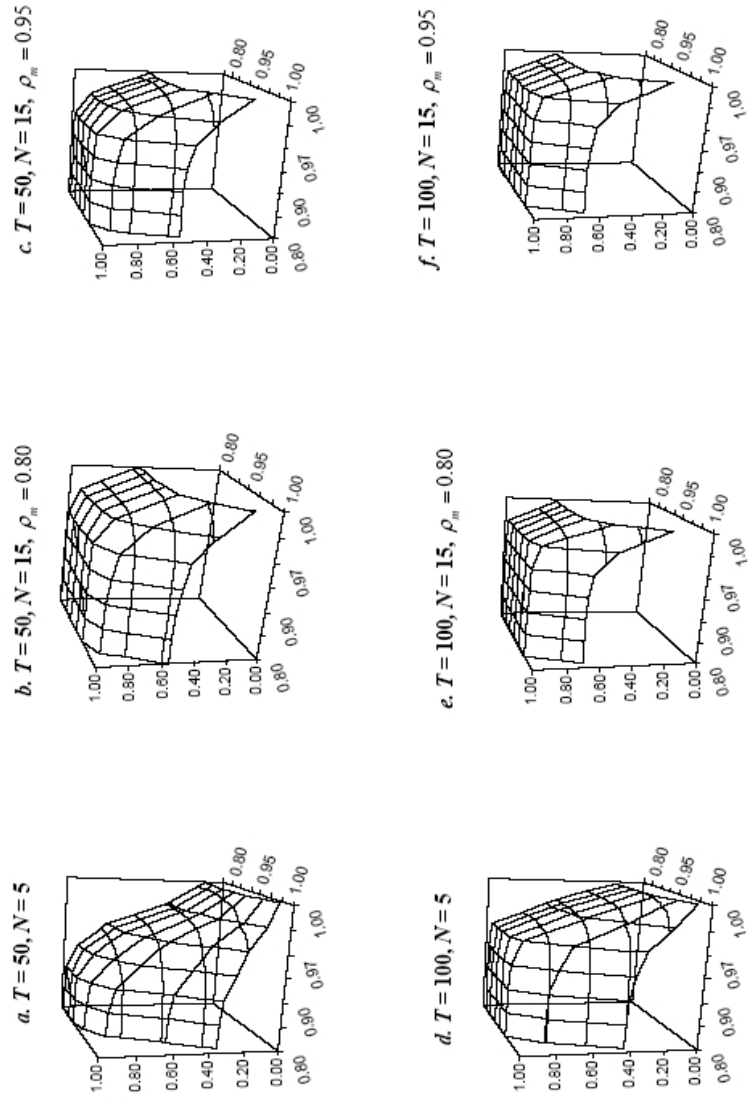
e. T = 100, N = 15, $\rho_m = 0.80$

f. T = 100, N = 15, $\rho_m = 0.95$

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Figure 2: The DF-GLS-SUR Test (—) Versus The IPS Test (−−)

N=5, $\rho_j = 0.90$, $j = 1,2,...,5$

N=5, $\rho_j = 0.95$, $j = 1,2,...,5$

N=10, $\rho_j = 0.90$, $j = 1,2,...,10$

N=10, $\rho_j = 0.95$, $j = 1,2,...,10$

N=15, $\rho_j = 0.90$, $j = 1,2,...,15$

N=15, $\rho_j = 0.95$, $j = 1,2,...,15$

N=20, $\rho_i = 0.90$, $j = 1,2,...,20$

N=20, $\rho_i = 0.95$, $j = 1,2,...,20$