# Tests for cointegration in panels with regime shifts

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### Abstract

In the paper we extend Gregory and Hansen's (1996) $ADF^*, Z_t^*$ and $Z_a^*$ cointegration tests to panel data, using the method proposed in Maddala and Wu (1999). We test the null hypothesis of no cointegration for all the units in the panel against the alternative hypothesis of cointegration, while allowing for a one-time regime shift of unknown timing for at least some regressions. We derive the panel tests for the $ADF^*, Z_t^*$ and $Z_a^*$ tests , and compare these tests with Pedroni's (1999) panel cointegration tests. We show that Gregory and Hansen's (1996) $ADF^*, Z_t^*$ and $Z_a^*$ panel tests have higher power to reject null when there is a structural change in the cointegration vector. We apply the statistics to the analysis of the well known Feldstein-Horioka puzzle for a sample of sixteen OCDE countries. After we allow for a structural break in the cointegration regression, we find strong evidence of cointegration between saving and investment rates.

Key words **:** Panel data, Panel cointegration tests, Structural breaks, Feldstein-Horioka puzzle.

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## 1. Introduction

Recently a number of panel cointegration tests have been proposed and widely adopted in applied research. For example, panel cointegration analysis has been used to investigate purchasing power parity (Pedroni, 2004; Basher and Mohsin, 2004), the Feldstein-Horioka puzzle (Ho, 2002; Banerjee and Zangheri, 2003) and international R&D spillovers (Kao *et al.*, 1999; Gutierrez and Gutierrez , 2003). The reasons are that panel cointegration tests, and also panel unit roots tests, have higher power than univariate tests, especially when the $T$ dimension of the sample is small, - see Baltagi and Kao (2000) for an overview.

The literature on testing for cointegration has essentially taken two directions. The first involves taking as null the hypothesis of no cointegration (Pedroni, 2004; Kao, 1999) or as null the hypothesis of cointegration, (McChoskey and Kao, 1998; Westerlund, 2005a). The second strategy, Maddala and Wu (1999), proposes using "meta-analysis" as suggested by Fisher (1932). This is based on combining the $p$-values of the test statistic for each cross sectional unit. Since the Fisher test is non-parametric, both cointegration tests which take as null the hypothesis of no cointegration, or as null the hypothesis of cointegration, can be used.

As is well known from the literature on structural breaks, tests for cointegration in the presence of a break tend to under-reject the null of no cointegration, see Gregory, Nason and Watt (1996). We show by a simple Monte Carlo simulation that panel cointegration tests suffer from the same problem. Thus, we concentrate on panel cointegration tests which allow for regime shifts.

We enlarge the methodology proposed in Gregory and Hansen (1996), which basically consists in computing standard Augmented Dickey Fuller $ADF$ **,** and Phillips and Ouliaris (1990) $Z_t$ and $Z_a$ cointegration tests, allowing for a one-time structural break of unknown timing in either the intercept alone or the intercept and slope. Specifically, we test the null hypothesis of no cointegration for all cross sectional units against the alternative of time-variant cointegration vector for at least some units in the panels. The panel tests are derived using the method proposed in Maddala and Wu (1999), i.e. combining the $p$-values of the test statistics. First, we highlight that panel cointegration tests which do not take into account regime shifts suffer from low power for both small and large numbers of units in the panel. Second, we show that panel cointegration tests which allow for possible regime shift have good size and power both for small and large $N$ and $T$ when a break is included in the regression.

Recently Westerlund (2005b) and Banerjee and Carrion-I-Silvestre (2004) proposed panel cointegration tests that allow for structural breaks. Specifically, Westerlund (2005b) proposed a panel LM cointegration test which extends McCoesky and Kao's (1998) panel test for the null hypothesis of cointegration allowing for multiple structural breaks in both level and trend of cointegration regression. Unlike Westerlund (2005b), we test the null hypothesis of no cointegration and we improve Westerlund's approach because we also allow for a slope shift in the cointegration regression. Banerjee and Carrion-I-Silvestre (2004) modify two of the seven Pedroni's (1999) panel cointegration tests allowing for one structural break when testing the null hypothesis of no cointegration. The advantage of our tests is that, unlike Pedroni's tests, they are available in all econometrics packages and that they do not require computing the mean and variance of the tests as is the case in Pedroni's tests. The disadvantage is that the *p*-values have to be computed by Monte Carlo simulation.

In Section 2 we briefly review the panel cointegration tests analyzed in the paper. Section 3 presents the Monte Carlo simulation study. In section 4 we use the test statistics to analyze the Feldstein-Horioka puzzle for a sample of sixteen OCDE countries. Finally, section 5 concludes.

**2. Panel cointegration tests with level and regime shift**

In this section we analyze the following system of cointegrated regressions

$$\begin{aligned} y_{it} &= \alpha_{1i} + \alpha_{2i}\varphi_{tt}^{c} + x_{it}\beta_{i1} + x_{it}\beta_{i2}\varphi_{tt}^{s} + u_{it}, \qquad \left(i = 1,\ldots,N,\ t = 1,\ldots,T\right) \\ x_{it} &= x_{it-1} + \varepsilon_{it} \end{aligned} \tag{1}$$

where $\alpha_{1i}$ and $\alpha_{2i}$ are individual constant terms, $\beta_{i1}$ and $\beta_{i2}$ are slope parameters, $u_{it}$ are stationary disturbance terms and finally, by construction, $y_{it}$ and $x_{it}$ are integrated processes of order one for all $i$.[1]

The zero mean innovation vector $w_{it} = \left(u_{it}, \varepsilon_{it}\right)'$ satisfies

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{Tr} w_{it} \Rightarrow B_i\left(\Omega\right) \text{ for all } i \text{ as } T \to \infty \tag{2}$$

where $B_i\left(\Omega\right)$ is a vector Brownian motion with asymptotic covariance $\Omega$. We assume, as do many recently proposed panel cointegration tests, that the process $w_{it}$ is independent across $i$, i.e. $E\left(w_{it}w_{js}'\right) = 0$ for all $i \neq j$ and for all $t, s$. In other words, we assume that the error terms are not cross-correlated. When relaxed, it is simple to show that the panel cointegration tests depend on nuisance parameters associated with the cross-sectional correlation properties of the data. We return to this problem in the empirical section where we use the bootstrap approach to take into account possible cross-sectional dependence.

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[1] More regressors can be included in the equation (1).

The dummy variables $j_{tt}^{c}$ and $j_{tt}^{s}$ are useful to model structural change. When there is only one level shift in the cointegration relationship we have

$$j_{tt}^{c}=\begin{cases}0 \text{ if } & t\le[Tt]\\ 1 \text{ if } & t>[Tt]\end{cases},\ j_{tt}^{s}=\mathbf{0}$$

where $t\in(0,1)$ defines the unknown (relative) time of the shifting, $[\ ]$ denotes the integer part, and, in this case, $j_{tt}^{s}$ is the null vector. In the case of a regime shift

$$j_{tt}^{c}=\begin{cases}0 \text{ if } & t\le[Tt]\\ 1 \text{ if } & t>[Tt]\end{cases},\ j_{tt}^{s}=\begin{cases}0 \text{ if } & t\le[Tt]\\ 1 \text{ if } & t>[Tt]\end{cases}.$$

Gregory and Hansen (1996) propose extensions of the well known Augmented Dickey Fuller (ADF) and Phillips and Ouliaris's (1990) $Z_a, Z_t$ tests for a single regression under the standard null hypothesis of no cointegration, i.e. $u_{it}\sim I(1)$ with $j_{tt}^{c}=j_{tt}^{s}=\mathbf{0}$ in the regression, against the alternative of cointegration with a level shift and/or a slope shift, i.e. $u_{it}\sim I(0)$ and $j_{tt}^{c}=j_{tt}^{s}\neq\mathbf{0}$ in the case of a regime shift. Their statistics are computed as the smallest values of *ADF*, and $Z_t, Z_a$ tests, across all values of $t\in T$, since small values constitute evidence against the null hypothesis. Asymptotic distributions for the test statistics are derived and presented in Gregory and Hansen (1996).

We enlarge these tests to panel data using the procedure proposed in Maddala and Wu (1999). Let $p_i$ be the asymptotic p-value of one of the three tests proposed in Gregory and Hansen (1996). We compute

$$\begin{aligned}P_l &= -\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\ln\left(p_i\right)+1\right),\\ Z &= \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\Phi^{-1}\left(p_i\right),\\ L &= \frac{1}{\sqrt{p^2N/3}}\sum_{i=1}^{N}\ln\left(\frac{p_i}{1-p_i}\right),\end{aligned} \qquad (3)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. The test statistic $P_l$ is a modification of Fisher's (1932) inverse chi-square test. The Z test is usually called the inverse normal test. Finally, the *L* test statistic is a modification of a logit test. All the corrections have been made to generate a standard normal distribution as $N\to\infty$, (see Choi (2001) for further details). Assuming cross sectional independence, under the null hypothesis of no cointegration, i.e. all $u_{it}$ are I(1) in (1), with $j_{tt}^{c}=j_{tt}^{s}=\mathbf{0}$, all the tests (3) have a standard normal distribution as $T\to\infty$ and

$N \to \infty$. Under the alternative of cointegration for at least some *i*, i.e. some or all $u_{it}$ are I(0) and $j_{tt}^{c} \neq \mathbf{0}$ in the case of a level shift, or $j_{tt}^{c} = j_{tt}^{s} \neq \mathbf{0}$ in the case of a regime shift, $P_l \to \infty$, and $Z, L \to -\infty$.

We compute the *p*-values for each $ADF^*, Z_a^*, Z_t^*$ test proposed by Gregory and Hansen (1996) under the null hypothesis of no cointegration and using response surface methodology as in MacKinnon (1991, 1994). We generate $\{y_{it}, x_{it}^{m}\}$ $I$ times at *T*=30,50,75,100,150,250 and calculate 399 equally spaced percentiles of the $ADF^*, Z_a^*, Z_t^*$ distributions, using *I*=10,000. This step has been repeated 10 times giving a $60 \times 399$ matrix of critical values $C(T, p, m)$ where *p* is the percent quantile and *m* is the number of regressors (excluding the constant) in (1) with *m*=1,..,5. We then estimate the following regression by using the GLS method

$$C(T, p, m) = y_0 + y_1 T^{-1} + y_2 T^{-2} + error. \tag{4}$$

The critical values computed using (4) are pretty similar to those presented in Gregory and Hansen (1996), Table 1. [2]

## 3. The simulation study

The data generating process used for the Monte Carlo study is based on the one proposed by Engle and Granger (1987), and used in Kao (1999) and Gutierrez (2003):

$$y_{it} - a_i - b_i x_{it} = u_{it}, \qquad u_{it} = r_i u_{it-1} + v_{it}, \tag{5.1}$$

$$\begin{bmatrix} v_{it} \\ e_{pit} \end{bmatrix} \equiv \text{iid N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & qs \\ qs & s^2 \end{pmatrix}\right].$$

$$a_1 y_{it} - a_2 x_{it} = p_{it}, \qquad p_{it} = p_{it-1} + e_{pit} \tag{5.2}$$

with $a_i \sim U[1,3]$ and $b_i = U[1,3]$. The 1000 initial observations are discarded to remove the effect of initial conditions. All random numbers are created by using GAUSS procedures. We consider all combinations of $N \in \{50, 100\}$, $T \in \{50, 100\}$. The parameters $a_i$, $q$, $s$, $a_1$ and $a_2$ are generated once, and then fixed, in all 1000 replications. In Table 1 we report the rejection frequencies of the $ADF^*, Z_a^*$ test statistics. The symbol *C* refers to a break in the intercept, and *C/S* indicates breaks in the intercept and slope. In the table we include two of Pedroni's (1999) non parametric

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[2] The critical values are freely available upon request.

panel cointegration test statistics. These are labeled Panel $t$ and Group $t$ tests.[3] The Panel $t$ test belongs to the set of Pedroni's tests that are based on pooling along what is called within-dimension, while the Group $t$ test is based on pooling along the between-dimension.

Table 1 about here

Briefly, the $ADF^*$ and $Z_a^*$ test statistics generally show correct size. Note that while the $P_l$ test statistic is moderately oversized both for small and large $N$ and $T$, Pedroni's Panel $t$ test is strongly oversized. By contrast the Group $t$ test statistic shows better properties. In terms of power, Pedroni's tests have better power both for small and large $T$,$N$ and the test statistics $ADF^*$ and $Z_a^*$ which take into account a possible level or regime shift have lower power especially for low $N$ and $T$.

In table 2 and 3 we now investigate the power of tests to detect cointegration in the presence of a level shift with $a_{i1} \sim U[1,3]$, $a_{i2} \sim U[4,8]$ and a regime shift with $a_{i1} \sim U[1,3]$, $a_{i2} \sim U[4,8]$ and $b_{i1} = U[1,3]$, $b_{i2} = [4,6]$. The breaks occur at $t = 0.25, 0.50, \text{ and } 0.75$.

Table 2 and 3 about here

In brief, when a break in the intercept is included in the model we note first that the rejection frequencies of Pedroni's tests fall and the power is strongly affected by the location of the break in the sample, especially for small values of $N$ and $T$. The effect of a break in the intercept on the power of Pedroni's (1999) test statistics is not severe for large values of $N$ and $T$, and also when the break is located at the beginning of the period of analysis. The rejection frequencies for the tests that allow for possible shifts are always high, and are not affected by this problem. When a break is included in both the intercept and the slope the effects on the power of Pedroni's (1999) test statistics are much more severe. The tests have practically no power. By contrast, the power of our $ADF^*$ and $Z_a^*$ panel test statistics is always high for any value of $N$, $T$ and $t$. We also investigate the power of tests for $t$ when this randomly varies across the cross-section units two periods away from the breaks at $t = 0.25, 0.50, \text{ and } 0.75$. The results, not reported here for reasons of brevity, are similar to those presented in table 2 and 3: the $ADF^*, Z_a^*$ test statistics show higher power than Pedroni's test statistics.[4]

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[3] To save space, we do not report the test statistics for the $Z_t^*$. These that are quite similar to the $ADF^*$ rejection frequencies, as well as the other five test statistics presented in Pedroni (1999). Their values are available upon request.
[4] We also compute the average estimated $t$ 's, together with their standard errors. $ADF^*, Z_a^*$ tests estimate the breakpoint accurately for $t = 0.50$ and $t = 0.75$.

As previously noted, all the tests assume that there is no cross-correlation among the errors, an assumption that is almost always violated in practice. When errors are cross-correlated the distribution of these test statistics is not longer valid, because they suffer from nuisance parameter problems. In addition, Banerjee *et al*. (2005) have shown that panel cointegration tests have significant size distortions when there is cointegration between the units of panel. A suggested method to overcome these problems is computing the bootstrap distribution of the test statistics. The bootstrap method allows to take into account general forms of cross-sectional dependence. In the next section we will compute panel test statistics and their bootstrap distribution in order to analyze the Feldstein-Horioka puzzle.

**4. Empirical analysis: The Feldstein-Horioka Puzzle.**

As is well known, Feldstein and Horioka (1980) first documented the idea that international capital mobility can be inferred from the relationship between investment and the saving rates. The coefficient, labeled saving-retention coefficient, was interpreted as the proportion of the incremental saving that is invested domestically. They found a positive and not significantly different from one saving-retention coefficient. Thus the estimate suggested that an increase in domestic saving had a proportional long-term effect on domestic investment, or, to put it another way, there was little room for international capital mobility. Since then the finding of low international capital mobility and high correlation between investment and saving rates has been known as the Feldstein-Horioka puzzle. Coakley *et al*. (1996) suggest that a country's intertemporal budget constraint implies current account stationarity, or else that its saving and investment rates should cointegrate. Many studies have analyzed the cointegration properties of the saving and investment rates using different countries, sample periods and test statistics. The results are mixed but it seems that the hypothesis of cointegration is generally rejected, especially when the saving-retention coefficient is not imposed as being a unit.

Although there are many reasons that might explain lack of cointegration between saving and investment rates, we think that more attention has to be paid to changes in the saving-retention coefficient over time. For example it has been questioned whether or not cointegration between investment and saving rates emerges only during the period of fixed exchange rate regimes, due to the presence of strong capital controls which link domestic investment projects to domestic saving conditions (see Alexakis and Apergis, 1994). During a period of a floating exchange rate cointegration seems to disappear, and the outcome is usually associated with a massive reduction in capital controls. Furthermore Banerjee and Zangheri (2003) show that for fourteen EU countries the long-run relationship between saving and investment rates drops after the mid-80's, when the external accounts were fully liberalized.

As previously noted, conventional tests for cointegration have low power, especially when the *T* dimension of the sample is small. Furthermore, in the presence of a break the tests tend to under-reject the null of no cointegration. Thus we expect that using our approach, which allows testing for cointegration between saving and investment rates with powerful panel cointegration techniques and at the same time permits for endogenous regime shift in the panel data, we will be able to overcome both shortcomings.

The data set consists of a balanced panel which covers sixteen OCDE countries, Australia, Belgium, Canada, Denmark, Finland, France , Italy, Japan, Korea, the Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom, and the USA. These were observed quarterly during the period 1980-2004. Before testing for cointegration we examine whether the series are non-stationary. We use various panel unit root tests which do not take into account cross-sectional correlation of errors, such as the Im *et al.* (2003) tests, and take into account cross-sectional correlation of errors, such as the Moon and Perron (2004) and Choi (2001) tests. All the test statistics, not reported for brevity, do not reject the null hypothesis of non-stationarity. Moreover, when the Im *et al.* (2005) panel LM test is used, the previous results are robust to the introduction of a level shift in the process.

Table 4 reports the values of the two previously analyzed Pedroni's tests and the values of the $ADF^*, Z_t^*, Z_a^*$ test statistics when only the intercept is allowed to change and when changes in both the intercept and slope are permitted. When computing the $ADF^*$ test statistics the order of the autoregressive correction is chosen by using Ng and Perron's (2001) criterion with lag max equal to five. The long run variances for the $Z_t^*$ and $Z_a^*$ test statistics are computed using a Parzen window with $T^{1/2}$ autocovariances.

Table 4 about here

Note that, as previously reported, all the test statistics might be influenced by possible cross-sectional correlation among errors or, as highlighted in Banerjee and Zangheri (2003), by possible cointegration across the units in the panel. To control whether the errors are cross-sectional dependent, we compute a recent test proposed by Pesaran (2004) which has been proved to be valid for a variety of linear panel data models, including stationary and unit root dynamic heterogeneous panels. The test consists of computing under the null hypothesis of cross-sectional independence the following statistic

$$\sqrt{\frac{2T}{N(N-1)}}\left(\sum_{i=1}^{N}\sum_{j=i+1}^{N}\hat{r}_{ij}\right) \quad (6)$$

where $\hat{r}_{ij}$ is the sample estimate of the pair-wise correlation of the OLS residuals. Pesaran (2004) shows that the test has a standard normal distribution. We compute (6) for the sample of OCDE countries, obtaining a value for the Pesaran's test of -15.74. Thus we reject the null hypothesis of cross-sectional independence of the errors.

As previously reported, one approach to overcoming this problem is computing the bootstrap distribution of the test statistics. We use, as in Banerjee and Carrion-i-Silvestre (2004), the sieve bootstrap method proposed by Chang, Park and Song (2004), with 5,000 bootstrap replications.

Looking at the values of the test statistics many interesting results emerge. First it is easy to infer from the critical values of the Normal distribution that both Pedroni's tests reject the null hypothesis of no cointegration when assuming cross-section independence, but that this conclusion is not robust in the presence of cross-section dependence, since the bootstrap critical values do not allow rejection of the null hypothesis of no cointegration.

By contrast, the $ADF^*, Z_t^*, Z_a^*$ test statistics strongly reject the null hypothesis of no cointegration both when only the intercept, and when the intercept and slope, are permitted to change. As is easy to infer from the bootstrapped critical values, now the null hypothesis is rejected by all the test statistics at the 5% level of significance. The only exceptions are in the case of a change in the intercept and slope, where the $ADF^*_{P_l}$, $Zt^*_{P_l}$ and $Za^*_{P_l}$ test statistics now allow rejecting the null at the 10% level of significance. Thus, by allowing the intercept and/or the slope to change we find strong evidence that investment and saving rates are cointegrated.

## 5. Conclusions

We have extended Gregory and Hansen's (1996) cointegration tests to panel data using the method proposed in Maddala and Wu (1999). We tested the null hypothesis of no cointegration for all the units in the panel against the alternative hypothesis of cointegration while allowing for a one-time regime shift of unknown timing. We compared these tests with standard panel cointegration tests and with Pedroni's (1999) cointegration tests. We show that Gregory and Hansen $ADF^*, Z_t^*, Z_a^*$ panel tests have higher power to reject null when there is a break in the intercept and/or slope in the cointegrating vector. Thus if standard panel cointegration tests that do not take into account regime shift do not reject the null and $ADF^*, Z_t^*, Z_a^*$ panel tests do, this may imply that structural changes are important in the cointegrating vector for at least some units in the panel. We apply the statistics to the analysis of the well known Feldstein-Horioka puzzle for a sample of sixteen OCDE countries. After we allow for a structural break in the cointegration regression, we are able to reject the null hypothesis of no cointegration between saving and investment rates.

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**Table 1. Testing for cointegration with no regime shifts :**

$$y_{it} - a_i - b_i x_{it} = u_{it}, \quad u_{it} = r_i u_{it-1} + v_{it}, \quad a_i \sim U[1,3]; b_i \sim U[1,3];$$

$$a_1 y_{it} - a_2 x_{it} = p_{it}, \quad p_{it} = p_{it-1} + e_{it} \quad \begin{bmatrix} v_{it} \\ e_{it} \end{bmatrix} \equiv \text{iid } \mathrm{N}(\mathbf{0}, \mathbf{I}).$$

| | T/N | $r_i = 1.0$ | | $r_i \sim U[0.8,1]$ | |
|---|---|---|---|---|---|
| | | 50 | 100 | 50 | 100 |
| $ADF^*_{P_l^C}$ | 50 | 0.082 | 0.076 | 0.358(0.283) | 0.478(0.403) |
| | 100 | 0.077 | 0.068 | 0.980(0.963) | 1.000(1.000) |
| $ADF^*_{Z^C}$ | 50 | 0.050 | 0.050 | 0.326(0.324) | 0.464(0.463) |
| | 100 | 0.057 | 0.052 | 0.985(0.981) | 1.000(1.000) |
| $ADF^*_{L^C}$ | 50 | 0.051 | 0.052 | 0.335(0.334) | 0.486(0.485) |
| | 100 | 0.064 | 0.059 | 0.982(0.973) | 1.000(1.000) |
| $Z^*_{a\,P_l^C}$ | 50 | 0.071 | 0.069 | 0.424(0.360) | 0.571(0.515) |
| | 100 | 0.082 | 0.069 | 0.993(0.991) | 1.000(1.000) |
| $Z^*_{a\,Z^C}$ | 50 | 0.047 | 0.045 | 0.424(0.426) | 0.604(0.627) |
| | 100 | 0.065 | 0.058 | 0.996(0.993) | 1.000(1.000) |
| $Z^*_{a\,L^C}$ | 50 | 0.050 | 0.045 | 0.431(0.419) | 0.604(0.627) |
| | 100 | 0.060 | 0.058 | 0.995(0.992) | 1.000(1.000) |
| $ADF^*_{P_l^{C/S}}$ | 50 | 0.070 | 0.062 | 0.289(0.250) | 0.605(0.624) |
| | 100 | 0.075 | 0.065 | 0.958(0.933) | 1.000(1.000) |
| $ADF^*_{Z^{C/S}}$ | 50 | 0.051 | 0.047 | 0.323(0.318) | 0.410(0.374) |
| | 100 | 0.060 | 0.053 | 0.968(0.962) | 1.000(1.000) |
| $ADF^*_{L^{C/S}}$ | 50 | 0.055 | 0.045 | 0.321(0.312) | 0.439(0.444) |
| | 100 | 0.060 | 0.053 | 0.967(0.960) | 1.000(1.000) |
| $Z^*_{a\,P_l^{C/S}}$ | 50 | 0.060 | 0.062 | 0.341(0.300) | 0.519(0.581) |
| | 100 | 0.077 | 0.068 | 0.980(0.968) | 1.000(1.000) |
| $Z^*_{a\,Z^{C/S}}$ | 50 | 0.043 | 0.038 | 0.392(0.405) | 0.519(0.544) |
| | 100 | 0.065 | 0.052 | 0.992(0.984) | 1.000(1.000) |
| $Z^*_{a\,L^{C/S}}$ | 50 | 0.045 | 0.045 | 0.387(0.399) | 0.474(0.451) |
| | 100 | 0.069 | 0.056 | 0.986(0.975) | 1.000(1.000) |
| Panel *t* (non parametric) | 50 | 0.104 | 0.147 | 0.999(0.998) | 1.000(1.000) |
| | 100 | 0.095 | 0.100 | 1.000(1.000) | 1.000(1.000) |
| Group *t* (non parametric) | 50 | 0.053 | 0.065 | 0.995(0.994) | 1.000(1.000) |
| | 100 | 0.037 | 0.029 | 1.000(1.000) | 1.000(1.000) |

Rejection frequencies at the 5% level of significance using critical values computed following Mackinnon (1991,1994) for $ADF^*, Z_a^*$ and the standard normal distribution for Pedroni's (1999) tests statistics, in 1000 replications. In parentheses are the size-adjusted rejection frequencies based on estimated critical values with $r_i = 1$.

**Table 2. Level shift in the intercept**

$$y_{it} - a_{it} - b_i x_{it} = u_{it}, \quad \begin{bmatrix} a_{it} = a_{i1}, \ t \le [t\mathrm{T}] \\ a_{it} = a_{i2}, \ t > [t\mathrm{T}] \end{bmatrix} \qquad u_{it} = r_i u_{it-1} + v_{it}, \quad r_i \sim U[0.8,1]; \ b_i \sim U[1,3]$$

$$a_1 y_{it} - a_2 x_{it} = p_{it}, \qquad p_{it} = p_{it-1} + e_{it} \qquad \begin{bmatrix} v_{it} \\ e_{it} \end{bmatrix} \equiv \text{iid } \mathrm{N}(\mathbf{0}, \mathbf{I}).$$

| Test statistics | T | $a_{i1} \sim U[1,3], \ a_{i2} \sim U[4,8]$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | N=50 | | | N=100 | | |
| | | $t$ | | | $t$ | | |
| | | 0.25 | 0.50 | 0.75 | 0.25 | 0.50 | 0.75 |
| $ADF^*_{P_l^C}$ | 50 | 0.869(0.815) | 0.923(0.894) | 0.885(0.855) | 0.978(0.966) | 0.992(0.989) | 0.989(0.979) |
| | 100 | 1.000(0.998) | 0.999(0.999) | 0.999(0.998) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $ADF^*_{Z^C}$ | 50 | 0.854(0.854) | 0.925(0.924) | 0.892(0.890) | 0.971(0.971) | 0.995(0.995) | 0.988(0.987) |
| | 100 | 0.999(0.998) | 0.999(0.999) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $ADF^*_{L^C}$ | 50 | 0.849(0.849) | 0.929(0.928) | 0.898(0.898) | 0.969(0.969) | 0.994(0.994) | 0.988(0.988) |
| | 100 | 0.999(0.998) | 1.000(0.998) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $Z^*_{a\,P_l^C}$ | 50 | 0.940(0.925) | 0.961(0.944) | 0.938(0.916) | 0.994(0.988) | 0.997(0.996) | 0.997(0.996) |
| | 100 | 1.000(0.999) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $Z^*_{a\,Z^C}$ | 50 | 0.925(0.925) | 0.961(0.962) | 0.918(0.918) | 0.990(0.991) | 0.996(0.996) | 0.992(0.992) |
| | 100 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $Z^*_{a\,L^C}$ | 50 | 0.933(0.924) | 0.962(0.959) | 0.930(0.925) | 0.992(0.994) | 0.997(0.997) | 0.993(0.993) |
| | 100 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| Panel $t$ (nonparametric) | 50 | 0.924(0.817) | 0.715(0.560) | 0.715(0.535) | 0.995(0.970) | 0.898(0.753) | 0.903(0.751) |
| | 100 | 1.000(0.999) | 1.000(0.999) | 1.000(0.999) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| Group $t$ (nonparametric) | 50 | 0.767(0.745) | 0.477(0.448) | 0.252(0.236) | 0.942(0.924) | 0.644(0.576) | 0.316(0.264) |
| | 100 | 1.000(1.000) | 0.998(0.999) | 0.993(0.997) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |

Rejection frequencies at the 5% level of significance using critical values computed following Mackinnon (1991,1994) for $ADF^*, Z^*_a$ and the standard normal distribution for Pedroni's (1999) tests statistics, in 1000 replications. In parentheses are the size-adjusted rejection frequencies based on estimated critical values with $r_i = 1$.

**Table 3. Regime shift in the intercept and slope**

$$y_{it} - a_{it} - b_{it}x_{it} = u_{it}, \quad \begin{bmatrix} a_{it} = a_{i1},\ \mathrm{t} \le [t\mathrm{T}];\ \ b_t = b_{i1},\ \mathrm{t} \le [t\mathrm{T}] \\ a_{it} = a_{i2},\ \mathrm{t} > [t\mathrm{T}];\ \ b_t = b_{i2},\ \mathrm{t} > [t\mathrm{T}] \end{bmatrix} \qquad u_{it} = r_i u_{it-1} + v_{it},\ \ r_i \sim U[0.8,1]$$

$$a_1 y_{it} - a_2 x_{it} = p_{it}, \qquad p_{it} = p_{it-1} + e_{it} \qquad \begin{bmatrix} v_{it} \\ e_{it} \end{bmatrix} \equiv \text{iid N}(\mathbf{0},\mathbf{I}).$$

| Test statistics | T | $a_{i1} \sim U[1,3],\ \ a_{i2} \sim U[4,8];\ b_{i1} \sim U[1,3],\ b_{i2} \sim U[4,6].$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | N=50 | | | N=100 | | |
| | | $t$ | | | $t$ | | |
| | | 0.25 | 0.50 | 0.75 | 0.25 | 0.50 | 0.75 |
| $ADF^*_{P_l^{C/S}}$ | 50 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| | 100 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $ADF^*_{Z^{C/S}}$ | 50 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| | 100 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $ADF^*_{L^{C/S}}$ | 50 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| | 100 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $Z^*_{a\,P_l^{C/S}}$ | 50 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| | 100 | 1.000(0.999) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $Z^*_{a\,Z^{C/S}}$ | 50 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| | 100 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| $Z^*_{a\,L^{C/S}}$ | 50 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| | 100 | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) | 1.000(1.000) |
| Panel $t$ (nonparametric) | 50 | 0.021(0.007) | 0.002(0.000) | 0.000(0.000) | 0.057(0.007) | 0.000(0.000) | 0.000(0.000) |
| | 100 | 0.156(0.066) | 0.002(0.001) | 0.000(0.000) | 0.370(0.152) | 0.001(0.000) | 0.000(0.000) |
| Group $t$ (nonparametric) | 50 | 0.096(0.075) | 0.000(0.000) | 0.000(0.000) | 0.141(0.101) | 0.000(0.000) | 0.000(0.000) |
| | 100 | 0.255(0.301) | 0.000(0.001) | 0.000(0.000) | 0.468(0.612) | 0.000(0.000) | 0.000(0.000) |

Rejection frequencies at the 5% level of significance using critical values computed following Mackinnon (1991,1994) for $ADF^*, Z^*_a$ and the standard normal distribution for Pedroni's (1999) tests statistics, in 1000 replications. In parentheses are the size-adjusted rejection frequencies based on estimated critical values with $r_i = 1$.

| Table 4. Panel cointegration statistics with regime shift, 16 OCDE countries 1980.1 – 2004.4 (a) | | | | | |
|---|---|---|---|---|---|
| **Model** | **Tests** | **Bootstrap distribution (b)** | | | |
| | | 1% | 2.5% | 5% | 10% |
| **Pedroni (1999,2004) tests** | | | | | |
| Panel *t* (nonparametric) | -5.333(0.00) | -7.180 | -6.807 | -6.422 | -5.877 |
| Group *t* (nonparametric) | -5.270(0.00) | -8.630 | -8.101 | -7.654 | -7.166 |
| **Change in the intercept** | | | | | |
| $ADF^*_{P_l}$ | 7.583(0.00) | 4.697 | 4.008 | 3.349 | 2.596 |
| $ADF^*_Z$ | -4.225(0.00) | -0.754 | -0.167 | 0.316 | 0.890 |
| $ADF^*_L$ | -4.997(0.00) | -1.230 | -0.460 | 0.084 | 0.815 |
| $Zt^*_{P_l}$ | 9.916(0.00) | 10.771 | 9.734 | 9.091 | 8.302 |
| $Zt^*_Z$ | -4.971(0.00) | -4.310 | -3.705 | -3.281 | -2.781 |
| $Zt^*_L$ | -6.427(0.00) | -5.794 | -5.060 | -4.472 | -3.902 |
| $Za^*_{P_l}$ | 10.241(0.00) | 10.942 | 10.030 | 9.264 | 8.362 |
| $Za^*_Z$ | -4.524(0.00) | -3.814 | -3.249 | -2.815 | -2.270 |
| $Za^*_L$ | -6.256(0.00) | -5.322 | -4.578 | -4.030 | -3.349 |
| **Change in the intercept and slope** | | | | | |
| $ADF^*_{P_l}$ | 4.349(0.00) | 6.506 | 5.724 | 5.022 | 4.242 |
| $ADF^*_Z$ | -2.808(0.00) | -1.783 | -1.277 | -0.761 | -0.193 |
| $ADF^*_L$ | -3.117(0.00) | -2.518 | -1.879 | -1.193 | -0.480 |
| $Zt^*_{P_l}$ | 9.832(0.00) | 11.483 | 10.834 | 10.182 | 9.393 |
| $Zt^*_Z$ | -4.374(0.00) | -4.689 | -4.270 | -3.911 | -3.422 |
| $Zt^*_L$ | -5.492(0.00) | -6.268 | -5.723 | -5.295 | -4.663 |
| $Za^*_{P_l}$ | 10.101(0.00) | 12.327 | 11.296 | 10.546 | 9.748 |
| $Za^*_Z$ | -4.464(0.00) | -4.635 | -4.077 | -3.617 | -3.058 |
| $Za^*_L$ | -5.947(0.00) | -6.283 | -5.653 | -5.051 | -4.338 |
| (a) in parenthesis asymptotic p-values; (b) The bootstrap is based on 5,000 replications. | | | | | |