Market price of risk implied by Asian-style electricity options

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Abstract

In this paper we propose a jump diffusion type model which recovers the main characteristics of electricity spot price dynamics, including seasonality, mean reversion, and spiky behavior. Calibration of the market price of risk allows for pricing of Asian-type options written on the spot electricity price traded at Nord Pool. The usefulness of the approach is confirmed by out-of-sample tests.

Key words: Power market, Electricity price modeling, Asian option, Market price of risk, Derivatives pricing

JEL classification: C51, G13, L94, Q40

1 Introduction

Dramatic changes to the structure of the power sector have taken place over the past two decades, including the deregulation and introduction of competitive markets. In a competitive market utilities cannot automatically pass costs to customers. This has the effect of increasing uncertainty and risks born by the investors. Electricity has changed from a primarily technical business, to one in which the product is treated in much the same way as any other commodity. However, already in the first years after the emergence of deregulated power markets it became obvious that for the valuation of electricity derivatives we cannot simply rely on models developed for financial or other

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commodity markets. The need for realistic models of price dynamics capturing the unique characteristics of electricity and adequate derivatives pricing techniques became apparent.

In this paper we study the Nordic power market with a special emphasis on the Asian-style options that were traded at Nord Pool. We discuss the issue of the market price of risk, which is essential for pricing derivatives written on the spot electricity price. The paper is structured as follows. In Section 2 we briefly describe the Nordic power exchange Nord Pool and its products, including the Asian-style option. In Section 3 we summarize the stylized facts of electricity markets so that we can pinpoint the essential model properties. Further, in Section 4 we review the most promising modeling approaches including fundamental and econometric spot market models and forward price dynamics models. We discuss the pros and cons of each approach and show how electricity derivatives can be priced within these models. Since the options' underlying is the spot system price, in Section 5 we propose an adequate mean reverting jump-diffusion type model of the spot price dynamics. Next, we calculate the value of the market price of risk implied by the Asian-style options up to some time point and, later, use it for pricing options after that time point. Finally, in Section 6 we conclude and summarize the results.

2 Nord Pool and Asian-style electricity options

Developed power markets typically exhibit spot and derivatives trading. In the beginning it is the spot market that dominates. With time, as more and more participants join in and start to actively hedge (and speculate), the volumes of traded derivatives outnumber those of spot transactions. However, it is the spot system price that is still the most influential. In particular, it is the underlying of a vast majority of derivatives.

The spot electricity market is actually a day-ahead market. A classical spot market would not be possible, since the system operator needs advanced notice to verify that the schedule is feasible and lies within transmission constraints. At the Nordic power exchange, Nord Pool, every day is divided into 24 hourly spot contracts. Before noon, the previous day, all participants send in their bids for each hour. The system price is calculated as the equilibrium point for the aggregated supply and demand curves and for each of the 24 hours. It is a theoretical price in the sense that it assumes that no congestions will occur and is the same in the whole Nordic area (Nord Pool, 2003). Besides the physically settled spot contracts, Nord Pool offers financially settled derivatives: futures, (standardized but not marked-to-market) forwards, options, and other specialized contracts (like the CdFs, i.e. Contracts for Difference).

A particularly interesting example of an electricity derivative instrument is the Asian-style option that was traded at Nord Pool. It is interesting for at least two reasons. Firstly, to our knowledge it is the only Asian-style exchange traded contract. Secondly, despite its appeal as a natural hedging instrument on very volatile markets, Nord Pool ceased listing it due to low liquidity. Whether this was a consequence of the pricing problems, inadequate contract specification or simply bad luck remains an open question.

Options trading at Nord Pool commenced on October 29, 1999. Two types of contracts have been offered: European-style options (EEO) written on the exchange traded standardized forward contracts and Asian-style options (AEO). By definition, an Asian-style option is exercised and settled automatically, in retrospect, against the price of the underlying instrument during a given period. AEO options are settled against the arithmetic average of the spot system price in the settlement period that starts after the option expires. This is in contrast to typical financial Asian options which are settled against the average price during the trading period. However, such a "financial" specification would not make sense in electricity markets due to the seasonality patterns.

AEO has settlement periods that correspond to the delivery period for the "underlying" futures block contract (a four week period). There are three AEO series listed for trading and clearing with the three nearest block contracts as "underlying" futures market (Eltermin) contracts. A new series is listed on the first trading day after a block contract has gone to delivery. A call (put) option is in-the-money if the difference between the average system price during the settlement period and the strike price is positive (negative). Settlement takes place the day after the last trading day in the settlement period. There is no payment if the option is at-the-money or out-of-the-money.

3 Stylized facts

In this section we review the so-called *stylized facts* of the Nordic market. Many of these characteristics are universal, in the sense that they are shared by the majority of electricity spot markets in the world. Yet a few are very specific to Nord Pool. Moreover, as will be seen below, some of the features are dramatically different from those found in the financial or other commodity markets. This section will enable us to pinpoint the essential properties of spot prices and thus give us sufficient grounds for proposing an adequate model of price dynamics.

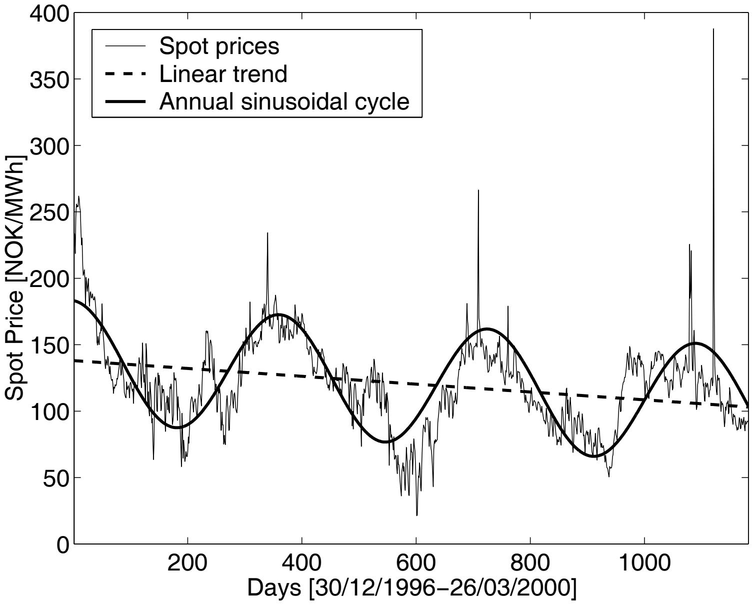

Fig. 1. Nord Pool market daily average system price since December 30, 1996 until March 26, 2000. Superimposed on the plot is an approximation of the annual cycle by a sinusoid with a linear trend.

3.1 Seasonality

It is well known that electricity demand exhibits seasonal fluctuations (Eydeland and Wolyniec, 2003, Kaminski, 1999, Pilipovic, 1998). They mostly arise due to changing climate conditions, like temperature and the number of daylight hours. In some countries also the supply side shows seasonal variations in output. Hydro units, for example, are heavily dependent on precipitation and snow melting, which varies from season to season. These seasonal fluctuations in demand and supply translate into the seasonal behavior of spot electricity prices.

In Figure 1 the Nord Pool market daily average system price since December 30, 1996 until March 26, 2000 is plotted. Superimposed on the plot is an approximation of the annual cycle by a sinusoid with a linear trend. This is in line with the approach of Pilipovic (1998) and Roncoroni and Geman (2003), who suggest fitting a proper sinusoidal function (e.g. a combination of an affine function and one or two sine functions with distinct periods) to spot prices. However, such a method could be hardly applied to some power markets, like the German one, where no clear annual seasonality is present and the spot prices behave similarly throughout the year with peaks occurring sometimes in the winter (December 2001 and December-January 2002) and sometimes in the summer (July 2002 and July-August 2003). In such a case a possible solution would be to increase the number of sine functions, at the cost reducing the tractability of the model. For example, Cartea and Figueroa (2005) used a

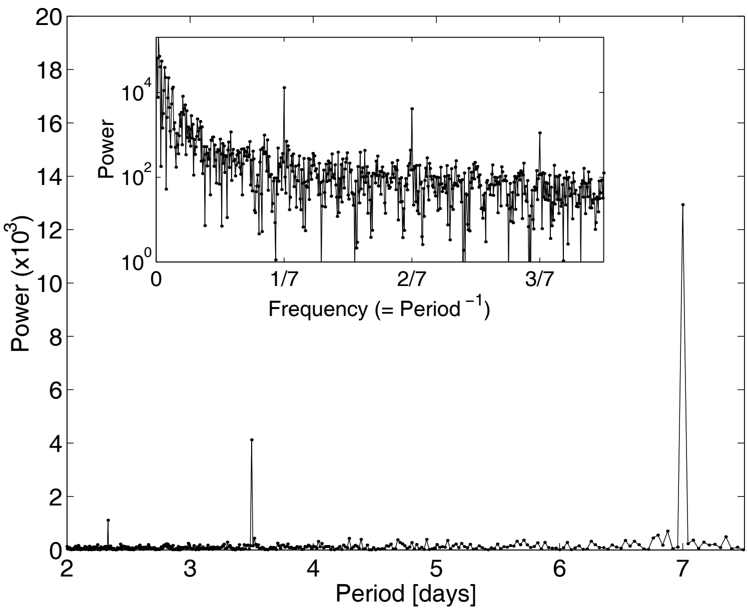

Fig. 2. Estimate of the spectral density of the system price (plotted in Fig. 1) reveals spikes at frequencies $\frac{1}{7}$, $\frac{2}{7}$, and $\frac{3}{7}$ corresponding to cycles of 7, 3.5, and 2.33 days, respectively. The inset shows the whole periodogram on a semilogarithmic scale.

Fourier series of order 5 to fit the annual seasonality pattern of the England and Wales power market. Alternatively, wavelet decomposition could be utilized, which offers yet greater flexibility (Simonsen, 2003, Stevenson, 2001). However, its forecasting applicability is very limited.

A different line of reasoning leads to a method of modeling seasonality by a piecewise constant function of a one year period, where for each month one tries to determine an average value out of the whole analyzed time series (Bhanot, 2000, Lucia and Schwartz, 2002). Although flexible, this method lacks smoothness, which may have a negative impact on statistical inference of the deseasonalized price process. Weron et al. (2001) proposed a method that may be viewed as an extension of this approach. It consists of fitting a function of a one year period, which is determined by taking the average (over the years in the sample) of the smoothed rolling volatility. The method yields a smooth estimate of the annual seasonal component, in the sense that the component is constant only during a one day (or one hour; depending on the sampling frequency) period.

In many cases it is also advisable to model the weekly (or even the daily) cycle. This issue may be approached by simply taking a sine function of a one week period (Borovkova and Permana, 2004), or better – a sum of three sine functions with 7, 3.5, and 2.33 day periods, as suggested by the periodogram in Fig. 2. Alternatively, we may apply the moving average technique (Brockwell and Davis, 1996), which reduces to calculating the average weekly price profile.

3.2 Price spikes

In addition to strong seasonality on the annual, weekly and daily level, spot electricity prices exhibit infrequent, but large spikes, see Fig. 1. The spot price can increase tenfold during a single hour. This is the effect of non-storability of electricity. Electricity to be delivered at a specific hour cannot be substituted for electricity available shortly after or before, since it has to be consumed at the same time as it is produced. Peaks in the spot prices are due to extreme load fluctuations, caused by severe weather conditions often in combination with generation outages or transmission failures. These spikes are normally quite short-lived, and as soon as the weather phenomenon or outage is over, prices fall back to a normal level (Kaminski, 1999, Weron et al., 2004). Modeling approaches include compound Poisson processes, regime switching mechanisms, and non-Gaussian Levy motion, for details see the Section 4.1.

3.3 Mean reversion

Energy spot prices are in general regarded to be mean reverting (Pindyck, 1999). Among all financial time series spot electricity prices are perhaps the best example of anti-persistent data, see Simonsen (2003) and Weron (2002) where Hurst R/S analysis, Detrended Fluctuation Analysis (DFA), Average Wavelet Coefficient (AWC) and periodogram regression methods were used to verify this claim. For time intervals ranging from a day to almost four years the Hurst exponent H was found to be significantly lower than 0.5, indicating mean reversion. Importantly, these results are not an artifact of the seasonality nor the spiky character of electricity spot prices. Although, the Hurst exponent generally slightly increases after removal of seasonality and/or spikes, it still is significantly lower than 0.5. For time intervals of less than 24 hours, however, H is above 0.5, suggesting persistence on the intra-daily level.

4 Modeling approaches

There have been several attempts at modeling spot electricity prices in competitive markets. The available literature has two main branches: econometric (also called: statistical) models and fundamental models. The former follow the finance tradition of modeling the stochastic processes that describe price properties. The latter build the price processes based on equilibrium models for the electricity market. For recent reviews see also Bunn and Karakatsani (2003), Eydeland and Wolyniec (2003), and Vahviläinen (2004).

4.1 Econometric models for the spot price

In the econometric approaches, modeling concentrates on the price process form and parameters. Typically the spot electricity price is assumed to follow some kind of a jump-diffusion process, obtained as a special case of the following general stochastic differential equation (SDE):

$$dS_t = \mu(S_t, t)dt + \sigma(S_t, t)dW_t + dq(S_t, t), \tag{1}$$

where W_t is a Wiener process. The drift term $\mu(S_t, t)$ usually forces mean reversion to a stochastic or deterministic long term mean at a constant rate. However, other specifications are also used. For example, Borovkova and Permana (2004) proposed the drift to be given by a potential function, which forces the price to return to its seasonal level after an upward jump. Interestingly, it allows the rate of mean-reversion to be a continuous function of the distance from this level. The volatility term $\sigma(S_t, t)$ is often, for simplicity, set to a constant. However, empirical evidence suggests that electricity prices exhibit heteroscedasticity (Karakatsani and Bunn, 2004).

The process $q(S_t, t)$ is a pure jump process (typically independent of W_t) with given intensity and severity, e.g. a compound Poisson process (Burnecki and Weron, 2005). After a jump the price is forced back to its normal level by the mean reversion mechanism or mean reversion coupled with downward jumps. Alternatively, a positive jump may be always followed by a negative jump of (approximately) the same size to capture the rapid decline – especially on the daily level – of electricity prices after a spike (Weron et al., 2004). In order to obtain pricing formulas for electricity forwards Lucia and Schwartz (2002) even dropped out the jump component and only considered one- and two-factor mean reverting diffusion models.

Instead of including the jump component $q(S_t, t)$ in eqn. (1), Deidersen and Trück (2002) suggested to model the spikes by substituting the Wiener process with a positively skewed α-stable Levy motion (Janicki and Weron, 1994). This approach is validated by a good fit of the α-stable distribution to electricity price returns (Rachev et al., 2004). Unfortunately, this approach leads to purely discontinuous price paths and limits control of the intensity of the jumps.

A more realistic (and technical) approach was taken by Burger et al. (2004) who incorporated a SARIMA forecast of the system load into the spot price formula. This technique is justified by the fact that the spot price is heavily dependent on the system load as a result of the supply stack structure (Bunn, 2004). Davison et al. (2002) modeled the average power demand and generation capacity. Based on the ratio between demand and capacity, in their model the electricity price switches randomly between two price levels. This is equivalent to saying that a jump in the electricity price is simply a change to another regime (the spike regime) that follows a different stochastic process than the so-called base regime. The switching mechanism can be assumed to be governed by a random variable that follows a Markov chain with two (or more) possible states (Bierbrauer et al., 2004, Huisman and de Jong, 2003, Huisman and Mahieu, 2003). Clearly the probability of being (and also staying) in the base regime is supposed to be much higher than that for the spike regime. Regime switching models are also able to consider spikes that last for more than just one time period (an hour, a day), without the disadvantage of slow mean reversion after a jump.

4.2 Fundamental models

Models that include fundamental factors are more tractable than econometric ones. Economic reasoning can be used to deduct properties of the factors modeling the supply-demand balance which determines electricity prices. Yet many approaches supplement fundamental models with econometric processes. Johnsen (2001) presented a supply-demand model for the hydro-dominant Norwegian power market from a time before the common Nordic market had started. He used hydro inflow, snow, and temperature conditions to explain spot price formation. Skantze and Ilic (2001) considered a fundamental model for the electricity price dynamics that incorporated the seasonality of prices, stochastic supply outages, and mean reversion. Barlow (2002) used a mean-reverting process for the demand and a fixed supply function to end up with a mean-reverting process for the spot price. Based on stochastic climate factors (temperature and precipitation), Vahviläinen and Pyykkönen (2004) modeled hydrological inflow and snow-pack development that affect hydro power generation, the major source of electricity in Scandinavia. Their model was able to capture the observed fundamentally motivated market price movements.

Unfortunately, fundamental models possess one serious flaw that disqualifies them in some situations. Namely, they rely on a number of external data (like plant availability, load forecasts, temperature, precipitation, snow pack) being available for the analysis. We do not possess such data.

4.3 Derivative pricing

Despite differences, both econometric and fundamental models share a common feature that makes derivatives pricing problematic. Namely, when calibrating these models we are using real world data and not the "riskless world" prices. In order to price derivatives we need to take into account the risk premium observable in the market. Since the convenience yield argument cannot be applied – due to the non-storability of electricity (Eydeland and Wolyniec, 2003, Roncoroni and Geman, 2003) – we are left with two alternatives: (i) model the forward price dynamics instead of the spot prices or (ii) calibrate the models also to derivative prices and in this way infer the market price of risk. The former approach has been advocated in a number of papers and is reviewed in the next subsection. However, its application to pricing derivatives written on the spot electricity price is questionable as the classical spot-forward price relationship is violated, again due to the non-storability of electricity (Vahviläinen, 2002). Since such derivatives constitute a considerable (if not the major) part of the market, in Section 5 we present a model in line with the latter modeling approach.

4.4 Forward price dynamics

The relation between electricity spot and forward prices is more complicated than in most financial and commodity markets. Short-term supply-demand equilibrium determines the electricity spot prices. As a result, the current spot price does not necessarily have anything to do with the spot price at some future time point. Similarly, no explicit connections exist between forward prices of different maturities. Moreover, the dynamical structure of the electricity forward curve is very complicated. Koekebakker and Ollmar (2001) concluded that two stochastic factors are unable to explain its dynamics as well as in interest rate markets.

The benefit from modeling the forward curve directly is that, unlike with the spot models, there is no problem fitting the model to the current forward curve or pricing derivatives written on this curve. Clewlow and Strickland (2000) proposed a multi-factor model for the electricity forward curve and showed how very general European-style derivatives, as well as, path-dependent and American-style options can be priced within this model. Audet et al. (2004) considered modeling the electricity forward curve dynamics with parameterized volatility and correlation structures. Eydeland and Geman (1999) argued that the forward curve model should include fundamental factors and incorporated the actual supply stack, fuel price, and load forecasts into their model.

5 Empirical analysis

Having described the stylized facts, the pricing approaches proposed in the literature and the AEO option contract specifications we are ready to put forward a pricing model. For the empirical analysis we have chosen the Nord

Pool market daily average system prices (denoted by P_t) from December 30, 1996 until March 26, 2000. The choice of this particular time period is not incidental – 1996 was a dry year with exceptionally high electricity prices and the first part of 2000 is used for testing the model. March 26, 2000 is the last day of the four week settlement period for the AEO-GB0300 options (i.e. AEO options for which the "underlying" futures block contract was GB0300 with delivery between February 28 and March 26). We could not use data beyond March 26, 2000 because later that year trading was very scarce with the last transaction involving AEO options taking place on February 2, 2001. Due to the very low frequency of the spikes, we will use the whole time period for calibration of the jump components and, as a result of the estimation scheme, of the seasonal components as well. The stochastic part X_t will be calibrated using data up to December 10, 1999. The remaining period will be used for out-of-sample pricing of AEO options.

5.1 Data preprocessing

As stated previously (see also Fig. 1), the annual cycle can be quite well approximated by a sinusoid of the form:

$$S_t = A \sin\left(\frac{2\pi}{365}(t+B)\right) + Ct. \tag{2}$$

The parameters were estimated through a two step procedure. First, a least squares fit is used to obtain initial estimates of all three parameters. Then the time shift parameter B is chosen such as to maximize the p-value of the Bera-Jarque test for normality applied the deseasonalized and spikeless log-prices (see below) yielding: $\hat{A} = 45.19$, $\hat{B} = 94.8$ and $\hat{C} = -0.0295$. Like demand, spot electricity prices are not uniform throughout the week. The intra-week and intra-day variations of demand caused by different level of working activities translate into periodical fluctuations in electricity prices. However, in the present analysis we do not address the issue of intra-day variations and analyze only daily average prices as the AEO options are settled against the average system price during a four week period. We deal with the intra-week variations by preprocessing the data using the moving average technique, which reduces to calculating the weekly profile s_t, i.e. an average week, and subtracting it from the spot prices (Brockwell and Davis, 1996, Weron et al., 2004). In what follows we model the logarithm of the deseasonalized prices (with respect to the weekly and annual cycles; in short: deseasonalized log-prices):

$$d_t = \log(P_t - s_t - S_t). \tag{3}$$

The time series d_t is plotted in Fig. 3.

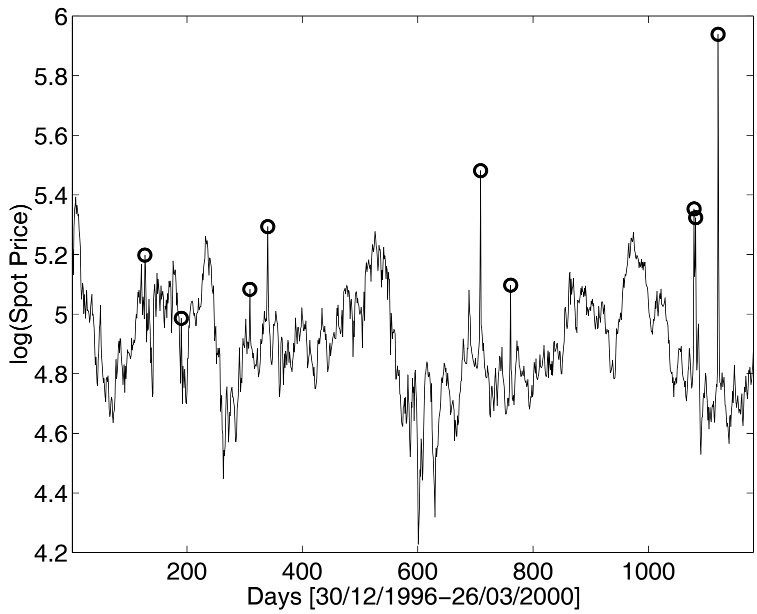

Fig. 3. Logarithm of the deseasonalized, with respect to the annual and weekly cycles, spot price (plotted in Fig. 1). Circles denote spikes; for spike definition see text.

5.2 Jump-diffusion model

Despite their rarity, price spikes are the very motive for designing insurance protection against electricity price movements. This is one of the most serious reasons for including jump components in realistic models of electricity price dynamics. Reflecting the fact that on the daily scale spikes typically do not last more than one time point (i.e. one day), like in Weron et al. (2004), we let a positive jump be always followed by a negative jump of about the same magnitude. This is achieved by letting d_t be a sum of a mean reverting stochastic part X_t and an independent jump component. Following the standard approach put forward in Section 4.1, the jump component is modeled by a compound Poisson process of the form $J_t dq_t$, where J_t is a random variable responsible for the spike severity and q_t is a Poisson process with intensity κ.

The choice of J_t and κ depends on the definition of the spike. We adopt the following: a spike is an increase in the price (formally: an increase in d_t) exceeding $H = 2.5$ standard deviations of all price changes (i.e. $d_t - d_{t-1}$) followed by a decrease in the price. The threshold level is set arbitrarily. The usual threshold $H = 3$ results in only six spikes in the whole series, while $H = 2.5$ yields nine spikes and captures all "obvious" peaks seen in the plot of d_t, see Fig. 3. Like in Cartea and Figueroa (2005) the extraction of the spikes from the original series is performed iteratively – the algorithm filters the series and removes all price changes greater than H standard deviations of all price changes at that specific iteration. The algorithm is repeated until no further spikes can be filtered. After the spikes are extracted, the price d_t at these time points is replaced by the arithmetic average of the two neighboring prices yielding the deseasonalized and "spikeless" log-prices X_t.

The extracted nine spikes do not allow for a sound statistical analysis of the spike severity nor intensity. For the sake of simplicity we let J_t be a lognormal random variable $\log J_t \sim N(\mu, \rho^2)$, although the empirical spike size distribution seems to have heavier tails. Since J_t represents the size of the logarithm of the spike magnitude it is truncated at the maximum price attainable in the market (10000 NOK) to ensure a finite mean of the price process P_t. Moreover, we let q_t be a homogeneous Poisson process with intensity κ. Again the sample suggests that this may not be the best choice – six spikes were observed in winter and only one in each of the other seasons. However, estimating a periodic intensity function (of a non-homogeneous Poisson process) using only nine time points would lead to large estimation errors. Concluding, maximum likelihood estimation of the jump component parameters yields $\hat{\mu} = -1.2774$, $\hat{\rho} = 0.65124$, and $\hat{\kappa} = 0.0076207$.

Putting all the facts together, the jump diffusion model has the following form:

$$d_t = J_t dq_t + X_t \qquad \text{or} \qquad P_t = s_t + S_t + e^{J_t dq_t + X_t}, \tag{4}$$

where X_t is the stochastic component. The exponent in the last term of eqn. (4) reflects the fact that the marginal distribution of X_t is approximately Gaussian, whereas the deseasonalized, with respect to the weekly and annual cycles, and "spikeless" spot prices can be very well described by a lognormal distribution, i.e. their logarithms are approximately Gaussian. The fit is surprisingly good, the Bera-Jarque test for normality (Spanos, 1993) yields a p-value of 0.97, see also Fig. 4. For comparison, the p-value for the "spiky" deseasonalized log-prices d_t is less than 0.0001, allowing us to reject normality at any reasonable level. Since the the marginal distribution of X_t is approximately Gaussian we are tempted to propose the simplest mean reverting model with Gaussian marginals, i.e. the Vasicek model.

The Vasicek (1977) model, also referred to as an arithmetic Ornstein-Uhlenbeck process, was originally proposed for describing interest rate dynamics. It is governed by the following SDE:

$$dX_t = (\alpha - \beta X_t)dt + \sigma dW_t = \beta(L - X_t)dt + \sigma dW_t, \tag{5}$$

where W_t is a Wiener process. This is a one-factor model that reverts to the mean $L = \frac{\alpha}{\beta}$ with β being the magnitude of the speed of adjustment. The second term is responsible for the volatility of the process. The conditional distribution of X at time t is normal with mean $E[X_t] = \frac{\alpha}{\beta} + (X_0 - \frac{\alpha}{\beta})e^{-\beta t}$ and

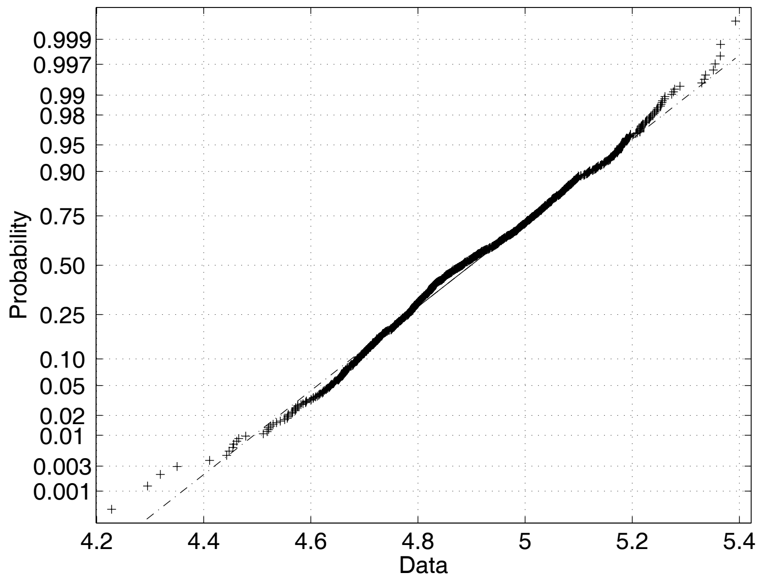

Fig. 4. The normal probability plot of the stochastic part X_t of the deseasonalized log-price d_t in eqn. (4). The crosses form a straight line indicating a Gaussian distribution.

variance $Var[X_t] = \frac{\sigma^2}{2\beta}(1 - e^{-2\beta t})$. These relations imply that $E[X_t] \to L = \frac{\alpha}{\beta}$ as $t \to \infty$. Starting at different points the Vasicek model trajectories tend to reverse to the long run mean and stabilize in the corridor defined by the standard deviation of the process. The parameters of the mean reverting process (5) can be estimated using the Generalized Method of Moments (GMM), for details see Cliff (2000) and Hansen (1982). For example, GMM estimation for data up to December 10, 1999 yields: $\hat{\alpha} = 0.2760$, $\hat{\beta} = 0.0560$, and $\hat{\sigma} = 0.0459$.

5.3 Market price of risk

Since we calibrate the stochastic part, and hence the whole model, using real world data we need to include the risk premium before we start pricing options or other derivatives. For simplicity, we assume that the market price of risk λ is a deterministic constant and, hence, a predictable process. By virtue of the Girsanov theorem there exists a probability measure $\mathcal{P}^\lambda$, equivalent to the original "risky" probability measure $\mathcal{P}$, such that the process

$$W_t^\lambda \equiv W_t + \int_0^t \lambda(s)ds = W_t + \lambda t, \tag{6}$$

is a Wiener process under $\mathcal{P}^\lambda$ (Musiela and Rutkowski, 1997). Using Itô calculus we can write:

$$
\begin{aligned}
dX_t &= \beta(L - X_t)dt + \sigma dW_t = \beta(L - X_t)dt + \sigma d(W_t^\lambda - \lambda t) \\
&= \beta\left(\frac{\alpha - \lambda\sigma}{\beta} - X_t\right) dt + \sigma dW_t^\lambda. \qquad (7)
\end{aligned}
$$

Under the new measure X_t follows the same Vasicek-type of SDE with the same speed of mean reversion β and the same volatility σ, but a different long-term mean $\tilde{L} = \frac{\alpha-\lambda\sigma}{\beta}$.

Standard arbitrage arguments with two derivative assets allow us to conclude that $\mathcal{P}^\lambda$ can be treated as the risk-adjusted or risk-neutral measure (Lucia and Schwartz, 2002). This means that if we estimated the market price of risk then we would know the dynamics of the stochastic component X_t in the riskless world and, hence, we could price any derivatives on the spot electricity price. We have to mention, though, that no analytical formulas are known for the Nord Pool variant of the Asian option. In what follows we will thus use Monte Carlo simulations. The pricing of a particular option for a given day will be based on the average payout from five thousand simulated price trajectories of the price process P_t. The seasonal and spike components' parameters are estimated from the whole time period and the stochastic component's parameters are estimated from a time series ending on the previous day.

One way of finding the market price of risk is to imply it from option prices. This technique resembles recovery of the implied volatility in the Black-Scholes model. The procedure consists of finding λ^* such that it minimizes the mean squared error between the market and model option prices. The market prices are in fact averages of the bid and ask offers. We could not use transaction data since on some days no transactions took place. We start with $\lambda = 0$ and then run a simplex minimization routine. This procedure is time consuming since at every minimization time step the option price has to be evaluated using Monte Carlo simulations. The results for AEO call options are shown in Fig. 5. Evidently the implied market price of risk is not constant but can be very well approximated with a linear function $\lambda(t)$. In a similar study Cartea and Figueroa (2005) and Lucia and Schwartz (2002) calibrated and used for pricing derivatives a constant λ. Our results show that using the simplified constant form of the market price of risk is too restrictive and may lead to large pricing errors.

The fit to the implied market price of risk for the first 31 trading days (i.e. until December 10, 1999): $y_1 = -0.0075x + 8.15$ is remarkably similar to the one for the whole time period shown in Fig. 5: $y_2 = -0.0074x + 7.98$. Hence, it an be used to forecast future (after December 10, 1999) values of λ. These values, in turn, let us price options using the risk-adjusted probabilities. Sample results of such a procedure are shown in Fig. 6. The earliest day in this sample (November 2, 1999) is an in-sample verification, but the remaining three days are out-of-sample (at least as far as the market price of risk is concerned)

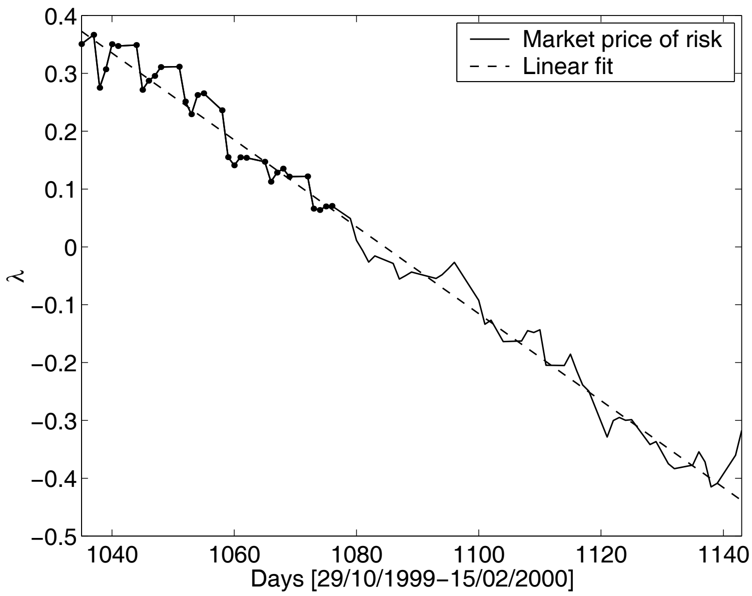

Fig. 5. The market price of risk λ implied from AEO call option prices and a linear fit to the first 31 values (black dots), i.e. until December 10, 1999. The linear fit is remarkably similar to the one for the whole time period (not shown here).

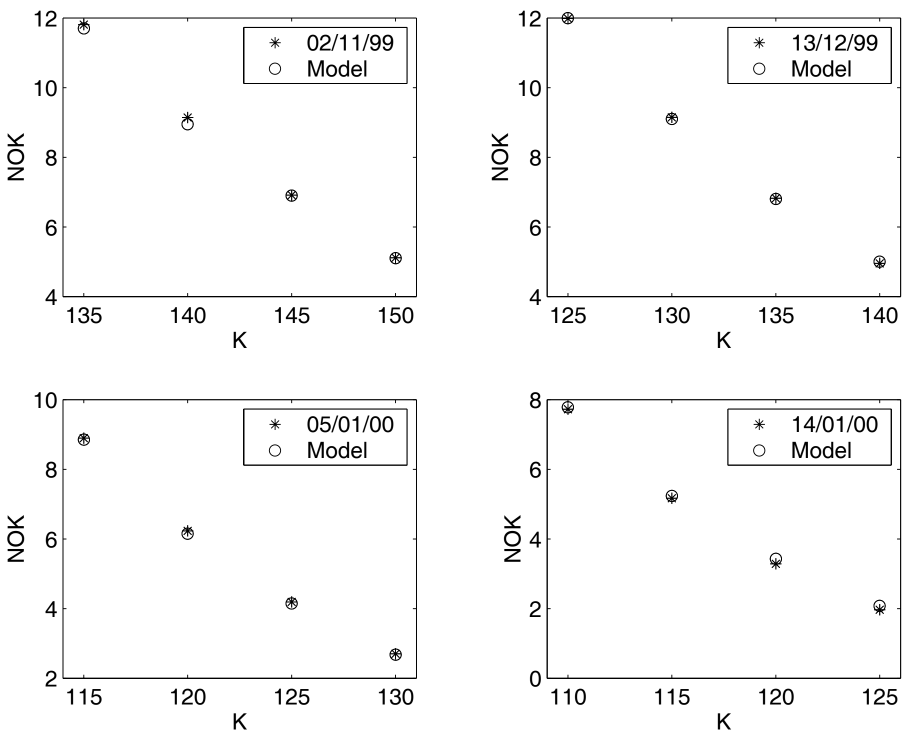

Fig. 6. Market ($*$) and model ($\circ$) prices of AEO call options. The fit is remarkably good, even for the out-of-sample dates, i.e. after December 10, 1999.

confirmations of the usefulness of the approach.

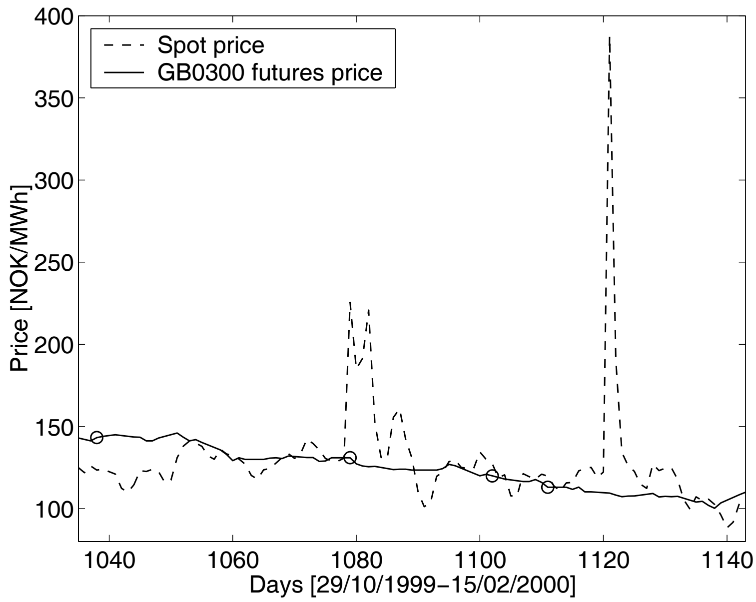

Fig. 7. Spot electricity and GB0300 futures prices during the period October 29, 1999 – February 15, 2000. Compare with Fig. 5 and note the similarity between the market price of risk and the futures price. Circles denote the four days for which the pricing results are presented in Fig. 6.

Interestingly, the plot of the market price of risk closely resembles the "underlying" futures price, compare Figs. 5 and 7. In fact, linear regression of λ on the futures price yields a very good fit with $R^2 = 0.9764$. This is probably due to the fact that λ changes the long-term mean of the stochastic component and the GB0300 futures price is a forecast of the spot electricity price during delivery, which coincides with the settlement period of the option. When the fundamental factors move the futures prices, the option prices have to adjust accordingly.

6 Conclusions

In this paper we propose a model which recovers the main characteristics of electricity spot price dynamics, including seasonality, mean reversion, and spiky behavior. The seasonality is modeled by a sinusoidal function coupled with a moving average technique for the weekly cycle. Mean reversion is achieved through a Vasicek type stochastic differential equation (7). Finally, the spike formation mechanism is an independent compound Poisson process. After calibration of the market price of risk the model allows for pricing of Asian-type options written on the spot electricity price. The usefulness of the approach is confirmed by out-of-sample tests.

We also note that there is a strong linear relationship between the market price of risk and the futures price, at least during the period studied in this paper. This relation suggests an alternative model calibration method. Instead of implying λ's from option prices for an initial part of the trading interval, we may simply substitute the market price of risk in (7) with the current futures price adjusted by a regression coefficient. This latter approach has yet to be studied and is left for future research. In particular, it would be interesting to see whether this relationship holds throughout the year, also in the periods when the futures price does not decline steadily. Unfortunately, the limited amount of data may severely hamper sound statistical analysis.

Finally, in Figure 5 we can observe that the market price of risk changes sign and varies significantly during the study period. As in our simple model λ is related to the market expectation of future spot prices, this variation of λ could support the observation that the electricity forward risk premia vary in time (Longstaff and Wang, 2004).

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