FUZZY COMPARATIVE CONCORDANCE ANALYSIS.
Proposal and evaluation by a case study

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Abstract :

In this paper it is proposed a fuzzy multiple attribute analysis, that we have called *comparative concordance*, as a help instrument to the decision-making process in an environment of lack of precise information as it generally is the decision-making in regional planning. Through an application to the selection of proceeding programs of the Environmental Plan of Andalusia, 1995-2000, it will be compared to other methods.

1. INTRODUCTION

Many times Public Administrations have to take decisions about large investment volumes and/or normative performances in uncertain situations, knowing very little about their possible effects and having scarce or null information. To establish a minimal formalization threshold in the decision-making process, frequently there is no more option than attending to some experts' opinions to value the effects of the possible alternative performances, according to various criteria or objectives. This specially happens when the Regional Development Plans, which are obligatory in a great number of Spanish Autonomous Communities to raise funds from the European Union, include explicit objectives and performances related to the Environment. If sometimes it is difficult to reach the exact valuation of proceedings or behavior provisions from certain variable within the economic area, it is even more difficult to obtain the valuation of variables related to the natural resources, the water or the atmosphere pollution, the destruction of the landscape, etc. In such cases, the lack of quantified information can only be supplied by qualitative information which must be handled in an environment of ambiguity and vagueness. The fuzzy sets theory can be adopted to face these situations.

2. FUZZY SET THEORY

From the first time in which it is observed or measured the characteristic of a variable till the final application of the analysis methods that are considered the most appropriate, different sources of uncertainty can be detected[1]. A first source of uncertainty comes from the **variability of the data**, due to the non-deterministic nature of the social and natural facts. Another type of uncertainty is the **imprecision** that appears when observing or measuring the values of a variable, because of both the measure instrument and the observer that accomplishes it. Finally, the **vagueness** turns up when human language is used, being professional or not, to describe the observation or to measure the result of an experiment. This specially happens when it is necessary to work with experts' opinions which are translated into linguistic expressions that thereafter would be considered linguistic variables.

Obviously, the most known way to deal with uncertainty is the theory of probability. There are some people who even defend that the continuous logic, where it is included fuzzy logic, may be considered within this theory. However, there are at least two difficulties to think that this is true. Firstly, probability deals with uncertainty in the *well-defined* event occurrence, while continuous logic deals with the *degree of occurrence* of *wrong* defined events. Secondly, it is a mathematical fact that the intersection of a set with its complementary is always the empty set; on the other hand, when working with fuzzy sets this almost never happens.

In a fuzzy set the membership issue of an element to the set is not a matter of all or nothing(0 or 1), because different *degrees of membership* are allowed. Its characteristic or *membership function* can take any value in the real interval [0,1] and the fuzzy set A is defined as follows[2]:

$$A=\{ (x, m_A(x)) : x \in \mathbf{U} , m_A(x) \in [0, 1] \}$$

[1] See, Bandemer, H. y Näther, W. (1992): *Fuzzy Data Analysis.* Kluwer Academic Press, pp.1-8.

[2] Henceforth, a classic set will be written in bold-faced, while a fuzzy set will be in normal characters.

The membership function is the main component of a fuzzy set, so that operations in that kind of sets are defined through that function.

A very useful concept is the ***α-cut.*** It allows an interesting approach, since the family constituted by the α-cuts contains the whole information of the fuzzy set. It can be defined as follow :

$$\mathbf{A}_\alpha = \{ x \in U : m_A(x) \geq \alpha \}, \ \alpha \in [0, 1]$$

An ***α-cut*** is, therefore, the set that contains all the values of x with a membership value of at least α (membership is also called presumption or certainty). If only it is considered the values of x which $m_A(x) > \alpha$, it will be called: *strict or strong* α-cut, and it will be written $\mathbf{A}^{>}_{\alpha}$. The set $\mathbf{A}_{\alpha=1}$ may be called the *core* of the set A. Figure 1 shows an α-cut in the fuzzy set "real numbers close to 10". In this case the resulting set is represented by the thick line around the value 10, which is the nucleus (core) of the fuzzy set.

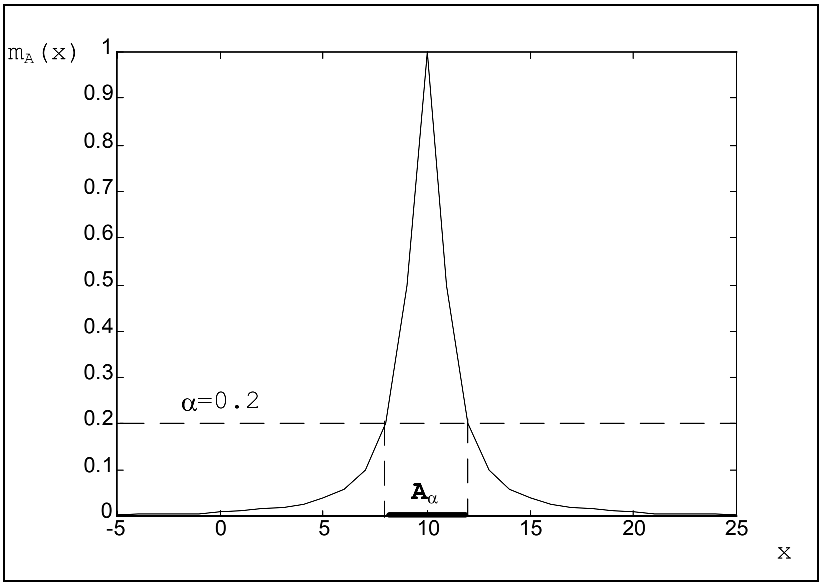

Figure 1: Real numbers close to 10

The extension principle is a fundamental idea in fuzzy set theory. It was proposed by Zadeh (1965) and it is a general method to extend, or to make possible the application of non-fuzzy mathematical concepts to the treatment of fuzzy quantities. It is especially useful in fuzzy computation aims.

If X is the Cartesian product of n universes, $X=X_1 \times X_2 \times ....\times X_n$, and $A_1, A_2, ...,$ A_n are n fuzzy sets in $X_1, X_{2,}....,X_n$, respectively. Then, if $y=f(x_{1,} x_2,...,x_n)$, the extension principle allows us to define a fuzzy set B in Y:

$$B=\{(y, m_B(y)) \mid y=f(x_1,...., x_n), (x_1,...., x_n)\in X\}$$

where

$$m_B(y) = \sup_{\substack{(x_1,.....,x_n)\in X \\ (x_1,....,x_n)\in f^{-1}(y)}} min\{m_A(x_1),.....m_A(x_n)\}, \quad si\ f^{-1}(y) \neq \varnothing$$
$$= 0, \quad otherwise$$

The extension principle has been re-elaborated using the algebraic sum instead of the minimal. However, the most used definition is the original Zadeh's proposal. The application of this principle generally implies a large volume of calculation if there are no imposed restrictions to the membership function form. Thus, for both the definition of linguistic variables and especially the representation of *fuzzy numbers*, triangular form of the membership function is generally applied, in spite of the serious warnings poured by the theory about the tremendous importance that the membership function may have.

3. FUZZY NUMBERS

A fuzzy set A in R^1 is a fuzzy number if A is convex and exits one and only one point $M \in R^1$, with $m_A(M) = 1$; ($\mathbf{A}_{\alpha=1} = M$).

The linguistic expression of that fuzzy number would be "around M". In order to a better computation, it is used to define L-R (left-right) fuzzy numbers[3]:

$$\begin{aligned} m_A(x) &= L((M - x)/l), \ \text{if } x \le M ; \ l > 0 \\ &= R((x - M)/r), \ \text{if } x \ge M; \ r > 0 \end{aligned}$$

Where L and R are decreasing functions in R^+, with $L(0) = R(0) = 1$. M is the *central value* of the fuzzy number. L and R are, respectively, the *form right and left function* while l and r are the right and left *spread.*

Although the application of the extension principle to operate with this kind of numbers is not an easy task, some algorithms have been proposed (Dubois, D. and Prade, H., 1979; pp. 333-335). That's why the use of triangular forms to define fuzzy numbers, as a particular case in which L and R functions are linear, has been generalized.

A triangular fuzzy number has, as its name says, a triangular form and can be identified by the trio (a_1, a_2, a_3). Its membership function is:

$$\begin{aligned} m_A(x) &= 0, & x < a_1 \\ &= (x-a_1) / (a_2-a_1), & a_1 \le x \le a_2 \\ &= (a_3-x) / (a_3-a_2), & a_2 \le x \le a_3 \\ &= 0, & x > a_3 \end{aligned}$$

Other way to define a triangular fuzzy number is:

$$\mathbf{A}_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}] = [a_1 + (a_2-a_1)\alpha, \ a_3 - (a_3-a_2)\alpha], \quad \forall \alpha \in [0, 1]$$

It is necessary to indicate that even though the sum, the subtraction and the multiplication of a triangular fuzzy number by a real number give a triangular fuzzy number, there are other operations as multiplication, inverse, division, maximal and minimal, which do not give as a final result a triangular fuzzy number. There have been proposed some approximations of these operations to obtain easily triangular fuzzy

[3] Dubois, D. and Prade, H. proposal. (1979, p. 340)

numbers as results (Kauffman and Gupta, ops. cit.). The operations used to carry out the application that is developed below are those proposed by these authors.

4. FUZZY MULTIPLE CRITERIA ANALYSIS

Since the 1960s it began to be developed the multiple objective programming and the multiple attribute methods. This growing field of help to the decision-making has become one of the most active and interdisciplinary in the economic and business administration area and in operational research. A recent study on the subject, including some commentaries on the fuzzy multiple criteria methodologies, can be found in Korhonen, Moskowitz and Wallenius (1992), and a classic synthesis effort is found in Vincke (1989). In Seo and Sakawa (1988) it is also made a wide exposition of the basic topics, including the fuzzy approach, emphasizing its application to the regional planning. In fuzzy multiple criteria methods framework, the first Bellman and Zadeh`s article (1970) has encouraged a vast and growing literature: Baas and Kwakernaaak (1977) or Yager (1977, 1978) among others. A good critical synthesis , which establishes limitations and possibilities of these approaches, can be found in Kickert (1978). The volume edited by Zimmermann, Zadeh and Gaines (1984), which has an introducing article written by the publishers on the perspectives of the methodological joint of fuzzy sets and decision analysis, is also very interesting, as well as the one edited by Chen and Hwang (1992). There are other works which try to make fuzzy so well-known crisp methods as ELECTRE o PROMETHEE[4], introducing fuzzy approximations to dominance, concordance or discordance relations[5].

Fuzzy decision making methods basically consist of two phases:

- The aggregation of the performance scores with respect to all the attributes for each alternative (rating).
- The ranking of the alternatives according to the aggregated scores (rates).

It is not the purpose of this paper to make a theoretical valuation of the different existing methods[6], but to redefine some of them (and, finally, to made a proposal) to value the suitability of fuzzy set theory to real problems as the one which we expose here: the ranking of the performance programs in the Environmental Plan of Andalusia, taking as decision criteria the goals fixed in it.

In all the accomplished applications the valuation of the possible courses of action, for every attribute (goal) with which alternative performances are measured as well as the relative importance of these attributes, which is also called weight, has been represented by linguistic terms. A group of technical personnel in the Environment Ministry of the Andalusian Government evaluated the contribution of each alternative to the achievement of each objective (the valuation or rate for each criteria) as well as the weights for each objective. These variables have been called valuations and weights,

[4] See Singh, Rao and Alam (1989), Brans and Mareschal (1990). Furuta (1993) or Munda, Nijkamp and Rietveld (1995).

[5] These procedures have been described in several papers. See B. Mareschal (1989) or Roy (1985), for example.

[6] Some commentaries and critics can be seen in Kickert (1978; op. cit. pp. 60-77), Tong and Bonissone (1984) or Zimmermann (1990).

respectively. The adjectives that have been proposed for each variable are in the Table 1.

Table 1
LINGUISTIC VARIABLES AND THEIR ADJECTIVES

Valuations	Weights
Very Negative (VN) Negative (N) Fairly Negative (FN) Indifferent (I) Fairly Positive (FP) Positive (P) Very Positive (VP)	Very low (VL) Low (L) Medium (M) High (H) Very high (VH)

In Table 1 it is shown that the variable "valuations" has seven different expressions. Since in a so probably contradictory context as Environment performances it goes from the evaluation of an alternative as very negative for any criteria, to the evaluation of it as very positive for others. The variable "weights" has five possible expressions and it gives the relative importance of the different objectives.

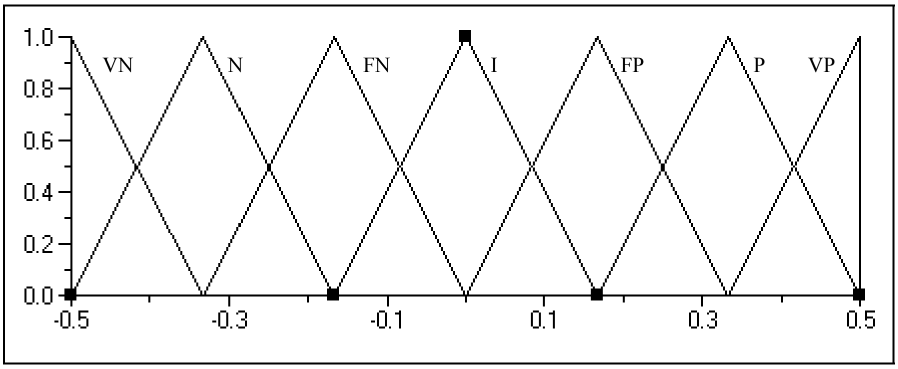

Figure 4: Linguistic terms (adjectives) of variable "valuations"

Each adjective for both variables constitute a fuzzy number, whose graphical representations are shown in Figures 4 and 5.

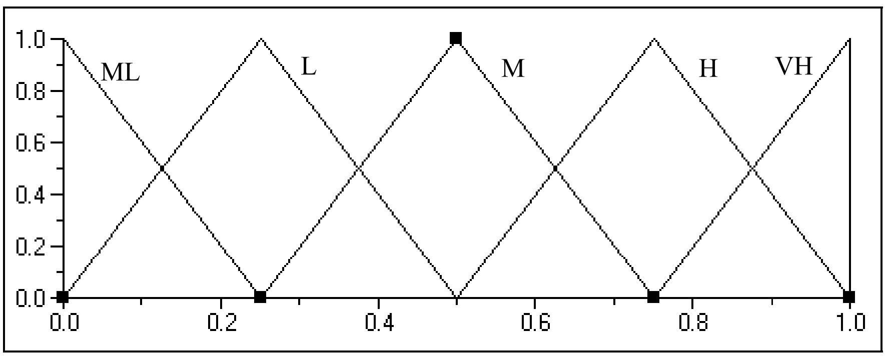

Figure 5: Linguistic terms (adjectives) of variable "weights".

TABLE 2
ANDALUSIAN ENVIRONMENTAL PLAN

Programs		**Objectives**	
Num.	**Title**	**Num.**	**Title**
1	Air quality	1	Urban environment improvement.
2	Noise prevention and noise reduction		
3	Residues	2	Nature conservation.
4	Environment improvement.		
5	Habitat conservation	3	Coastal environmental quality improvement.
6	Flora and fauna conservation.		
7	Fight against erosion and desert.		
8	Forest fire prevention and extinction.	4	A sustainable hydrologic model attainment.
9	Defense of vegetation from plagues and diseases		
10	Protected natural spaces management.		
11	Natural resources management.	5	Economical activities related to the Environment promoting.
12	Public use.		
13	Cattle ways.		
14	Coastal water quality.		
15	Hydraulic resources sustainable use.		
16	Continental water quality.		
17	Energy and Environment.		
18	Nature integral development.		
19	Economy and Environment.		
20	Participation		
21	I + D		
22	Resource formation and qualification.		
23	Environmental education and communication.		
24	International cooperation.		

The performance programs and the general objectives which are proposed in the Environmental Plan of Andalusia are those in Table 2.

The adjectives for each of the linguistic variables which have been defined above, are fuzzy sets. These fuzzy sets are the values to be introduced into the impact matrix (V). Here, V_{ij} is a fuzzy set representing the contribution of the i^{th} program to the j^{th} goal. The impact matrix for n programs and k goals would be as follows:

$$V = \begin{bmatrix} V_{11} & \cdots & V_{1j} & \cdots & V_{1k} \\ \vdots & \ddots & & \iddots & \vdots \\ V_{i1} & & V_{ij} & & V_{ik} \\ \vdots & \iddots & & \ddots & \vdots \\ V_{n1} & \cdots & V_{nj} & \cdots & V_{nk} \end{bmatrix}$$

In the same way, the weight of the j^{th} goal will be a fuzzy set representing an adjective for the linguistic variable "weighting". We will call the weight vector as:

$$\mathbf{W} = [W_1\ W_2\ \ldots\ W_j\ \ldots\ W_k].$$

We will present several alternative approaches on the impact matrix. The methods we have used are the following:

A.- The first method consists of the fuzzy simple additive weighting method to find the fuzzy utilities (fuzzy final ratings) of the different programs. It is a very classical and frequently used method that aggregates the alternatives for each one of the goals in a very simple way:

$$V_i = \sum_{j=1}^{k} W_j \cdot V_{ij}$$

Some fuzzy applications were, as we said, conducted by Baas and Kwakernaak (1977) and Yager (1978). We will apply this method in a different way, using fuzzy arithmetic, and considering both the rating of alternatives and the goal weights as fuzzy numbers. It will also be used the triangular approach of membership functions for fuzzy arithmetic operations which give non triangular results[7]. That rating vector will be used to rank alternatives later.

The Figure 6 shows the fuzzy final rating for each program. Looking at the figure, the intuitive result is to consider the fuzzy numbers on the right as the better ones.

Afterwards we have applied three different fuzzy ranking methods to determine mathematically the ranking order of alternatives:

a) ***Fuzzy semiordering method:***

To determine the ranking, the fuzzy semiordering notion is used. The semiorder between two fuzzy numbers X_i and $X_{i'}$ is defined as follows:

$$X_i < X_{i'} \Leftrightarrow X_{i'} = \max.(X_i, X_{i'}) \text{ and } X_i = \min.(X_i, X_{i'})$$

If the semiorder $X_i < X_{i'}$ is satisfied for a pair of X_i and $X_{i'}$, it can be said that the ith alternative is better than the i'^{th} alternative. We have found that with this method most of the alternatives can not be ordered, and that is why many times it will be useful the introduction of different α-cuts, so as to re-order the initially not ordered set of programs. An example of this can be seen in Figure 7.

Once we have fixed an α-cut, we can build an 0-1 matrix which shows the dominance or non dominance relations between alternatives in accordance with the fuzzy semiordering concept. A fuzzy graph will show the dominance relations. Obviously, when increasing α-cuts the preference order will become more exhaustive (and *crispier* too).

The application of this method to the Environmental Plan of Andalusia, provides the results in the Table 3.

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[7] Kauffmann and Gupta (1988; pp.55-67). See Bonissone's approach, in Chen and Hwang (1992).

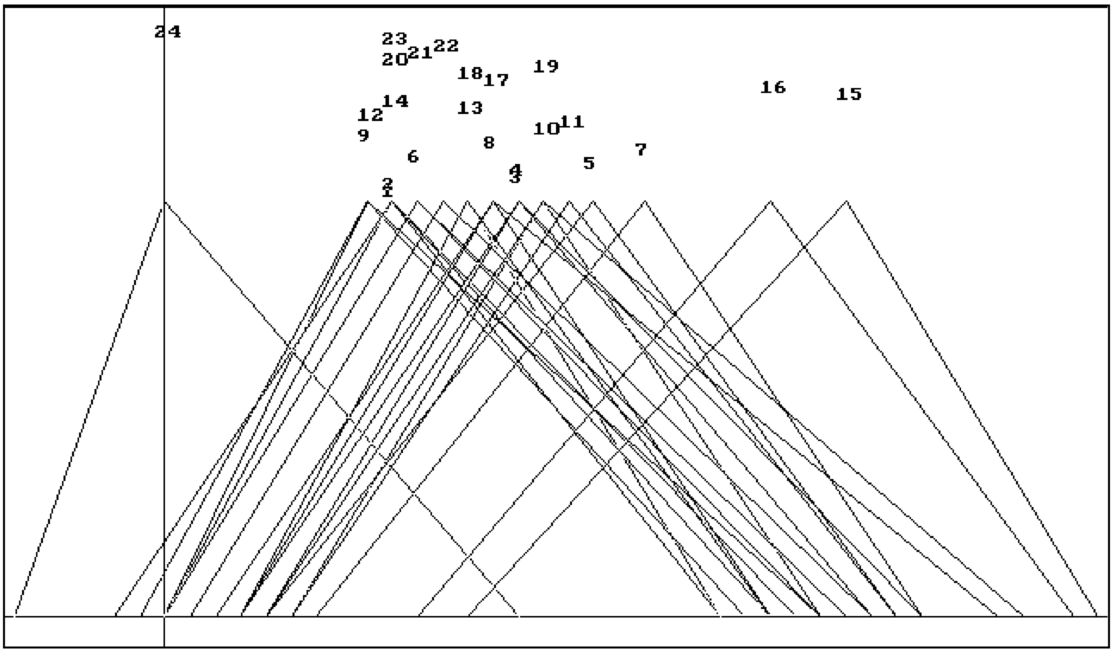

Figure 6: Fuzzy rating with fuzzy simple additive weighting method.

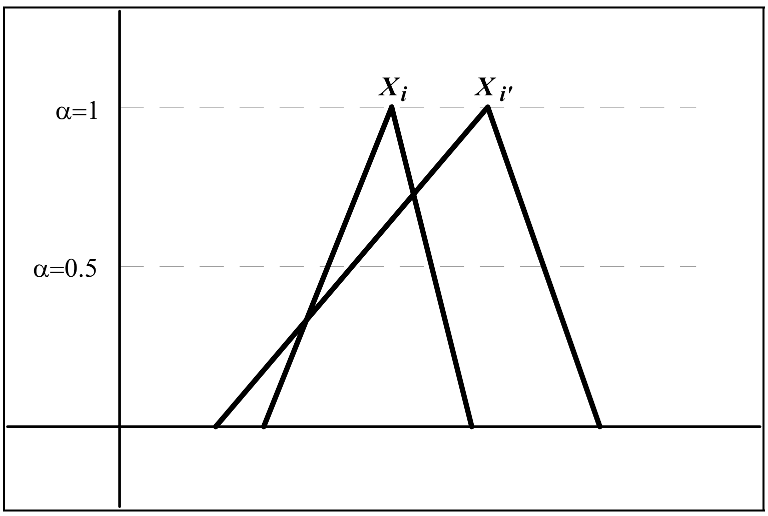

Figure 7: Fuzzy semiordering using different α-cuts.

Program number	Ranking α=0	Ranking α=0.5	Ranking α=0.8
1	15	15	15
2	16	16	16
3		7	7
4			5
5	R	R	
6	E	E	10, 11, 19
7	S	S	
8	T	T	
9			3, 4, 8, 17
10			
11	O	O	
12	F	F	
13			13, 18, 22
14	P	P	
15	R	R	6, 21
16	O	O	
17	G	G	
18	R	R	
19	A	A	1, 2, 14, 20, 23
20	M	M	
21	S	S	
22			9, 12
23			
24	24	24	24

Table 3: Resultant hierarchies with different α-cuts using fuzzy semiordering method.

b) Linear ordering of fuzzy numbers:

Kaufmann and Gupta proposed a linear ranking of fuzzy numbers which consists of the application of three criteria, in such a way that if the first one does not give out an unique ranking, the second criterion will be applied, and if this one does not either, the third one will be used. Those criteria are: the removal, the mode and the divergence. The removal with respect to $k \in R$ for a L-R fuzzy number A can be expressed as follow:

$$R(A,k) = \int_{k}^{M} dx + \frac{1}{2}\left\{ \int_{M}^{max(x)} R(x)\, dx - \int_{min(x)}^{M} L(x)\, dx \right\}, \quad with \quad k \le min(x) \le M \le max(x)$$

Its value for triangular fuzzy number is much more easy[8] to obtain. The second criteria, to apply to classes having the same removal, is the mode (M) and then, if there are any other classes left, the divergence criteria, $\{max(x)\text{-}min(x)\}$, will be hold.

The application of the linear ordering method produce these results:

Program number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Ranking	15	16	7	19	5	11	4	10	17	3	8	18	22	13	21	6	23	20	14	12	1	2	9	24

[8] See Kauffman and Gupta (1985; pp. 37-44)

Table 4: Resultant hierarchy using fuzzy linear ranking method.

c) Chen´s approach: ordering using left and right scores:

Given fuzzy numbers X_i , . . . , X_n , the left and right scores refer to the intersections of a fuzzy number X_i with the fuzzy min and the fuzzy max respectively[9]. Figure 8 illustrates the aforementioned notions.

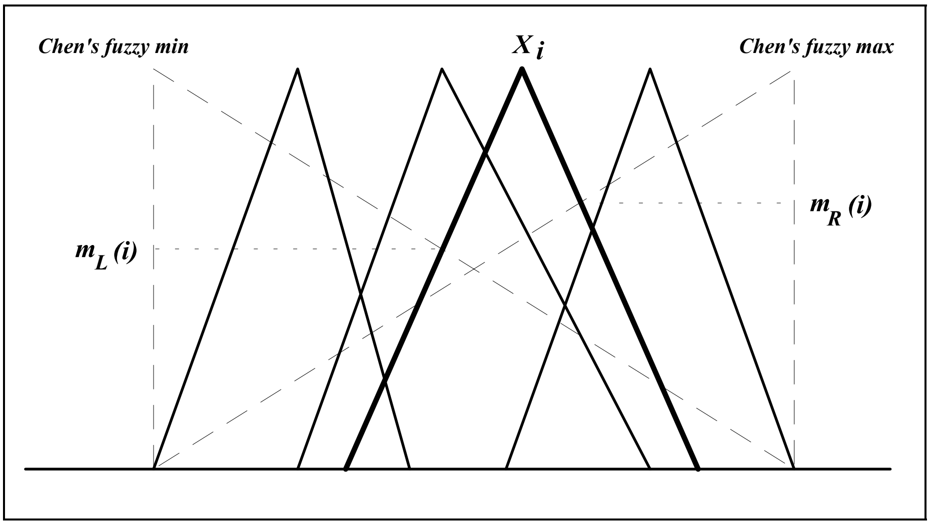

Figure 8: The left and right scores by Chen's method.

Those intersections, $m_R(i)$ and $m_L(i)$, are the left and right scores respectively. They together guarantee the full utilization of information contained in X_i. Since the higher $m_R(i)$ values indicate better fuzzy numbers, and higher $m_L(i)$ values indicate worse fuzzy numbers, the total score of X_i can be defined as:

$$m_M(i) = (m_R(i) + 1 - m_L(i)) / 2$$

The higher $m_M(i)$ value determines the preferred fuzzy number X_i. The results reached are those in Table 5.

Program number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Ranking	15	16	7	5	19	11	10	4	17	3	8	18	22	13	21	6	23	20	14	12	1	2	9	24

Table 5: Results using Chen's method.

B.- Method based on the *concordance* and *discordance* concepts, a well-known subject in multiple criteria analysis done by pairwise comparison of all individual alternatives. We have applied it incorporating fuzzy evaluations and fuzzy arithmetic to compare alternatives and to obtain the respective indices and matrices.

The *fuzzy concordance* is defined as:

9 See Chen and Hwang (1992).

$$c_{ii'} = \sum_{j \in C_{ii'}} W_j$$

$$where\ C_{ii'} = \left\{ j \mid V_{ij} \geq V_{i'j} \right\}$$

The concordance reflects the preference of plan i with respect to i'. A concordance matrix C including all concordance values $c_{ii'}$ can be calculated.

Fuzzy Concordance Matrix:

$$C = \left[c_{ii'}\right] = \begin{bmatrix} - & c_{12} & \cdots & \cdots & \cdots & c_{1n} \\ c_{21} & - & \ddots & & & c_{2n} \\ \vdots & \ddots & - & \ddots & & \vdots \\ \vdots & & \ddots & - & \ddots & \vdots \\ \vdots & & & \ddots & - & \vdots \\ c_{n1} & c_{n2} & \cdots & \cdots & \cdots & - \end{bmatrix}$$

The *fuzzy discordance* is defined by the expression:

$$d_{ii'} = \max_{j \in D_{ii'}} \left\{ W_j \cdot \left| V_{ij} - V_{i'j} \right| \right\}$$

$$with\ D_{ii'} = \left\{ j \mid V_{ij} < V_{i'j} \right\}$$

and the Fuzzy Discordance Matrix will be:

$$D = \left[d_{ii'}\right] = \begin{bmatrix} - & d_{12} & \cdots & \cdots & \cdots & d_{1n} \\ d_{21} & - & \ddots & & & d_{2n} \\ \vdots & \ddots & - & \ddots & & \vdots \\ \vdots & & \ddots & - & \ddots & \vdots \\ \vdots & & & \ddots & - & \vdots \\ d_{n1} & d_{n2} & \cdots & \cdots & \cdots & - \end{bmatrix}$$

The concordance and the discordance relations must be aggregated to get the outranking relation. We have made it in two phases. Firstly, we have changed these matrices of triangular fuzzy numbers into two matrices where each one of their elements are a membership function value meaning the comparative strength or presumption level of the concordance and discordance between alternatives. The procedure used for this change is the application of Chen's approach, used to rank all the fuzzy numbers into each one of the matrices, which has been described above. We have called these new matrices as the ***Fuzzy Comparative Concordance Matrix*** and the ***Fuzzy Comparative Discordance Matrix***, respectively. The elements in these matrices could be called ***comparative concordance and discordance indices***, going from 0 to 1. Secondly, to obtain a singular dominance matrix, we have take into account the fact that when the divergence is too high for two alternatives it produces phenomena of incomparability, introducing for this reason a veto threshold (d_v) to get the aggregated dominance matrix: the fuzzy comparative discordance indices higher than this threshold will not allow comparison between two alternatives.

In this way, we defined a ***fuzzy quantified dominance matrix*** which establishes the fuzzy relations between alternatives, as follows[10]:

$$M=\left[m_{ii'}\right]=\begin{bmatrix} - & m_{12} & \cdots & \cdots & \cdots & m_{1n} \\ m_{21} & - & \ddots & & & m_{2n} \\ \vdots & \ddots & - & \ddots & & \vdots \\ \vdots & & \ddots & - & \ddots & \vdots \\ \vdots & & & \ddots & - & \vdots \\ m_{n1} & m_{n2} & \cdots & \cdots & \cdots & - \end{bmatrix}$$

where

$$m_{ii'}=\begin{cases} c_{ii'} & \text{if } d_{ii'}<d_v \\ 0 & \text{if } d_{ii'}\geq d_v \\ & \forall\ i\neq i' \end{cases}$$

This matrix will be used in order to develop a fuzzy graph with each program as a node and each $m_{ii'}$ as a weight (membership function value) assigned to the arc emanating from node i and terminating at node i'. Each one of the $m_{ii'}$ elements are the grade of "credibility" of the dominance relation of program i on the program i'. Finally, this fuzzy graph will be analyzed by giving a credibility threshold (c_t) and remaining only the relations which weights are higher than it.

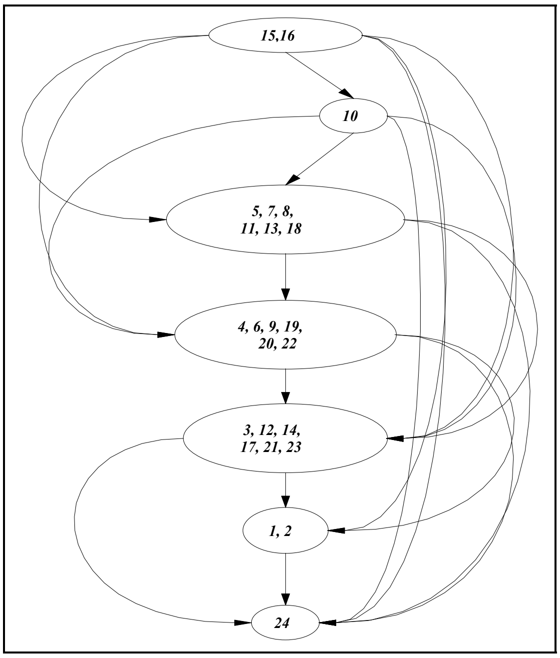

Figure 9: Reduced graph with d_v=0.39 and c_t=0.46.
(Note: The weights are not in the figure)

The application of the mentioned method giving the mean values both to the veto threshold and to the credibility threshold are those in figure 9. This figure comes from

[10] Singh, D., Rao, J. R. and Alam, S. (1989), used $m_{ii'}$ = min { $c_{ii'}$, (1-$d_{ii'}$)}.

the reduced graph and the strong connected components analysis. It would be possible to study new dominance relations into a block increasing the threshold level into its relations and making again the same analysis.

5. CONCLUSIONS.

Given the public administrations interest for making decisions in an objective way, even if there is a lack of information to afford it, it seems that fuzzy multiple criteria techniques can be a good instrument to do it. Working with triangular fuzzy numbers and fuzzy arithmetic, as rating alternatives as ranking them, is recommendable, specially if we do not have any good reason to believe in any other possibility about the membership function shapes.

Kaufmann and Gupta fuzzy ordering method is very easy to apply in that case and yields a very exhaustive ranking. Actually, just the removal concept approximately gets the ordering by it self. So does Chen´s approach with very similar results. However, the fuzzy semiordering method has more problems in order to classify the alternatives because its concept supposes a very hard condition. Certainly this is the aim of fuzzy theory against the crispy thinking but actually, in our example at least, only it is possible to differentiate the two programs ahead and the last one. Nevertheless, we can relax it by applying successive α-cuts (really we can do it in any other situation) making crispier our decision.

The three ranking methods mentioned above yield similar results except in the middle of the ranking, because it is more difficult to distinguish between very close alternatives, as it is well known.

The fuzzy comparative concordance method we have redefined gives similar results for the two first positions and for the last one too, but it is quite different in the rest of the ordering. Obviously it comes from the fact it uses a very different aggregation method. Anyway, pairwise comparisons method, even much more complex, uses more information and furthermore has stronger possibilities to reach full meaning results by using graph theory.

Finally, we have to say the principal program in the Environment Plan of Andalusia should be the named “Hydraulic resources sustainable use” (number 15 comparative concordance index is greater than number 16) which certainly matches to the Andalusian reality.

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