Uncertainty In Integrated Regional Models*

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Abstract

This paper examines the nature of uncertainty in integrated econometric+input-output (ECIO) regional models. We focus on three sources of uncertainty: [a] econometric model parameter uncertainty; [b] econometric disturbance term uncertainty; and [c] input-output coefficient uncertainty. Through a series of Monte Carlo simulations we analyze the relative importance of each component as well as the question of how their interaction may propagate through the integrated model to affect the distributions of the endogenous variables. Our results suggest that there is no simple answer to the question of which source of uncertainty is most important in an integrated model. Instead, that answer is conditioned upon the focus of the analysis and whether the industry specific or macro level variables are of central concerns.

Keywords: Input-output, Econometric, Integrated Model, Uncertainty

1 Introduction

The number and types of integrated regional econometric+input-output (EC+IO) models appearing in the literature have proliferated in recent years.[1] These efforts share a concern with the generation of forecasts, estimation of economic impacts and structural economic analysis.

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[1]For a recent survey see Rey (2000).

The comparison of alternative methods across these domains has been the subject of a number of recent investigations (West and Jackson, 1998; Deller and Sheilds, 1998). In this paper we add to this line of research by investigating an issue that has received only limited attention: the treatment of uncertainty in integrated regional models.

Understanding the role of uncertainty in integrated models is important for a number of reasons. In the typical use of an EC+IO model for impact analysis and forecasting the focus is often on generation of a point estimate. However, the degree of uncertainty surrounding such point estimates is rarely acknowledged. This is despite the fact that, as is outlined further below, the EC framework provides a well understood approach to generation of forecast confidence intervals. Consequently, one could argue that the full capabilities of an EC+IO model are not being fully exploited in practice.

A second main application of EC+IO models has been for the purposes of structural economic analysis. For example, the work at the Regional Economics Application Laboratory (REAL) has used integrated models to analyze the evolution of the interindustry structure of Midwestern economies (Schindler et al., 1997). Similarly, research by scholars in the Community Planning and Analysis Network (CPAN) is applying integrated models across a large number of states for the purposes of comparative economic analysis (Scott and Johnson, 1998). These efforts have provided a wealth of empirical measures regarding the nature of hollowing out, trade relations and unbundling. Here again, however, these measures are often viewed as point estimates and the underlying stochastic properties of the models generating these estimates remains relatively unexamined.

There are, of course, very good reasons why stochastic issues in integrated models have not received much attention to date. Chief among these is the discrepancy that exists between the views towards uncertainty in the literature on econometric models and the views in the input-output literature. We discuss these differences in more detail below. In one sense the treatment of uncertainty in an integrated framework can be seen as one of many areas in which modelers must make decisions regarding an integration strategy. For example, the specification of multiregional linkages can be done in a number of ways in an integrated model, as evidenced by a number of investigations (Dewhurst and West, 1991a; Rey and Dev, 1997) . By the same token, model closure in an integrated framework can be based on a host of approaches. Again, this issue has attracted attention in a number of studies (Dewhurst and West, 1991b; Rey, 1998). What distinguishes the issue of uncertainty from the treatment of spatial linkages and model closure, however, is that in the latter cases, each of the previous literatures, EC and IO, offered a wealth of approaches that could be chosen from in the development of an EC+IO model. We argue that the same can not be said for the issue of uncertainty and inference.

Given the large number of implementation decisions one has to face in integrated modeling, some guidance is clearly needed as to what aspects of an integrated model are more critical than others. While this type of question has been the focus of much attention in the regional IO literature, it is not at all clear how this question might be answered in the context of integrated models. We suggest that focusing on the underlying stochastic properties of an integrated model offers a potential route to providing some answers in this regard.

This paper attempts to highlight some of the issues that uncertainty raises for integrated modeling at the regional scale. The objectives of the paper are two-fold. First, we revisit the EC and IO literatures to outline how inference and uncertainty are treated in the individual modules of a larger EC+IO framework. We hope that by doing so we can contribute to the development of a research agenda in the integrated literature that focuses on the issues of uncertainty and inference. Our second objective is to identify a few key issues in this regard and to provide some initial insight as to their relative importance. Specifically, we focus on the issue of error/uncertainty propagation from these different modules throughout the larger integrated model. These issues are investigated using a set of Monte Carlo simulations.

The remainder of the paper is organized as follows. In section 2 we briefly discuss the treatment of uncertainty in the literatures of regional input-output modeling and regional macroeconometric modeling. Section 3 outlines the main issues that these alternative sources of uncertainty present when the IO and EC models are combined in an integrated framework. In section 4 we describe the design of a series of Monte Carlo simulations to explore some of these issues. This is followed by a presentation of the results from our simulations in section 5. The paper closes with a summary of key findings and directions for future research.

2 Existing Approaches to Uncertainty in Regional Models

2.1 Uncertainty in Input-Output Analysis

Traditionally the input-output modeling framework has omitted any sources of uncertainty. The interindustry relationships as well as those specified between imports and exports, labor/wages and output, have generally been viewed as static and deterministic. In part this reflects the process of building regional input-output models which suffers from the lack of necessary data needed to construct regional technical coefficients. Obtaining detailed survey tables of regional purchases by industry would provide ideal estimates for regional coefficients, but this approach often requires a great deal of reconciliation due to conflicting information provided by establishments, as well as the constraints on time and money required to collect the data (Miller, 1998). The often impractical process of collecting quality survey tables has led to the proliferation of various non-survey techniques for the estimation of regional coefficients (Garhart and Giarratani, 1987). Non-survey methods used to obtain regional input-output tables include approaches based on regional commodity balances, various indicators such as location quotients that identify the nature of imports and exports within a region, various iterative balance procedures such as RAS (Round, 1983), and regional purchase coefficients (RPCs) (Stevens et al., 1989).

Very often the parameterization of a regional IO model is carried out with zero degrees of freedom. Similar to the approach taken in regional CGE modeling (Harrigan and McGregor, 1988), the focus is on obtaining parameter estimates that result in a balanced and consistent model. The accuracy of these model construction approaches has sometimes been evaluated against a number of survey based regional tables (e.g. Miller and Blair, 1981; Hewings and Syversen, 1981). However, the notion of accuracy underlying these assessments has been a deterministic one.

While this is reflective of the general view in regional input output analysis, there have been a variety of attempts to address the stochastic nature of the input-output modeling framework.[2] Approaches to understanding the statistical properties range from theoretical treatments to empirically based simulations. These studies focus on a variety of input-output subjects, such as the estimation of direct and indirect coefficients, multipliers and the distribution of output.

There have been a number of efforts to place the IO models within a stochastic framework akin to that underlying most econometric modeling. Gerking (1976) represents one of the earliest attempts in this regard suggesting the estimation of interindustry coefficients using various econometric methods. One outcome of this effort is the ability to attach (estimated) standard errors to the IO coefficients. Along similar lines, West (1986) derived density functions for output multipliers for a variety of cases, and Jackson (1986) developed the full-distribution approach towards representation of uncertainty in individual IO coefficients. The stochastic nature of

[2]For overviews on stochastic IO see Jackson and West (1989); Giarratani and Garhart (1991).

the RPCs has also been acknowledged, at least implicitly, in the work on econometric modeling of these coefficients (Stevens et al., 1989). However, the resulting econometric relationships are used to parameterize the RPC component of a larger integrated model while the inherent uncertainty associated with the parameter estimates remains largely unexplored.

2.2 Uncertainty in Econometric Models

In contrast to the IO literature, the role of uncertainty features prominently in the econometric literature, at least at the national and international scales. Fundamentally, the specification of an econometric model rests on the notion of an underlying data generating process (DGP):

$$y = f(Z, \beta, \epsilon) \tag{1}$$

where a dependent variable y is related to a set of right hand side determinants Z, some of which may also be endogenous, via a linear or non-linear function $f()$ which is parameterized by the vector of coefficients β and a random error term ϵ. From a classical (i.e. non-Bayesian) perspective, the choice of an estimator for the unknown parameters β is based on the estimator's repeated sampling properties which in turn will be dependent on the form of the DGP.[3]

The properties of the DGP and those of the estimator also provide a formal framework for hypotheses testing and, perhaps more importantly in the current context, developing confidence intervals for forecasts of the dependent variables. The latter represents a clear recognition of the inherent uncertainty of an econometric model. Analytical approaches towards the development of such intervals can become intractable as the size (as represented by the number of equations) of a macroeconometric system increases, or if there are non-linearities in the system. In these cases alternative approaches towards estimating the stochastic nature of the econometric model are required.

Stochastic simulation (Fair, 1994) is a technique for empirically generating the distribution of the endogenous variables based on estimated or assumed distributions for the parameter and/or error terms. For example:

$$\hat{y}_r = f(Z, \hat{\beta}_r, \hat{\epsilon}_r) \tag{2}$$

where $\hat{\beta}_r$ and $\hat{\epsilon}_r$ represent vectors of randomly generated variates from the estimated or assumed

[3] For Bayesian perspectives on integrated modeling at the regional scale see LeSage and Magura (1991); Rickman (2001) and LeSage and Rey (2002).

distributions for the parameters and error terms, respectively. Given a realization r on these parameters and errors, the larger econometric structure is solved to generate a realization for the dependent variables $\hat{y}_r$. This process is repeated for a large number of realizations to generate empirical distributions for each dependent variable. These distributions can then be used for constructing confidence intervals for point forecasts and impact estimates.[4]

An examination of the literature on structural[5] regional macroeconometric modeling, however, reveals that it is the first interpretation of uncertainty that has been exploited in applied work. Very seldom are measures of uncertainty attached to point forecasts or impact estimates generated by structural macroeconometric models at the regional level. This raises an interesting dichotomy between the regional IO literature and the regional EC literature. In the former, uncertainty has been largely ignored, yet regional scientists have been at the forefront of the recent efforts to introduce uncertainty into the analytical framework. In the latter, a wealth of methods for treating uncertainty in EC models have been developed over the years, yet regional scientists have tended to utilize only a small subset of these methods in applied macroeconometric modeling.

3 Uncertainty in Integrated Models

As discussed in the previous section each of the core components (EC or IO) can be based on one or more approaches to inference. In the context of an integrated model this raises an interesting combinatorial problem. At the same time, there is the related questions of how the inferential properties of the larger integrated model are shaped by the specific choice to this combinatorial problem.[6]

This issue can be explored more fully by considering a stylized integrated model of a regional economy with n industries which begins with the familiar IO identity:

$$X = AX + Y \tag{3}$$

where X is an n by 1 vector of industry output, Y is an n by 1 vector of final demands and A

[4]Stochastic simulation is similar to bootstrapping of an econometric model (Vino, 1993), with the exception that in the bootstrap the resampling is done with respect to the endogenous and exogenous variables to repeat the estimation process with a focus on the distribution of model parameters.

[5]Structural is taken to exclude a-theoretic time-series approaches such as Box-Jenkins and VAR modeling.

[6]This section draws heavily on Rey (2000).

is an n by n regional input-output coefficients matrix with a typical element:

$$a_{ij} = \frac{x_{ij}}{X_i}. \tag{4}$$

It is important to emphasize the role of aggregation in the integration. At the macroeconomic level, there are m elements of aggregate final demand: personal consumption $\mathbf{C}$; investment $\mathbf{I}$; government expenditures $\mathbf{G}$; and net exports ($\mathbf{NE} = \textit{Exports} - \textit{Imports}$). Each of these aggregate components is obtained as the sum of the industry specific values, for example:

$$\mathbf{C} = \sum_{i=1}^{n} C_i \tag{5}$$

and total gross regional product is:

$$\mathbf{Y} = \mathbf{C} + \mathbf{I} + \mathbf{G} + \mathbf{NE}. \tag{6}$$

$$\mathbf{C} = Z_C \beta_C + \epsilon \tag{7}$$

where Z_C is a vector of determinants of consumption with associated parameters β_C and ϵ is a stochastic error term. Estimates for β_C are obtained through the application of an appropriate econometric method to (7).

Moving from the macroeconomic level of the GRP components in (6) to final demand at the industry level is accomplished using fixed distributional shares for each component of GRP:

$$Y_j = h_{Cj}\mathbf{C} + h_{Ij}\mathbf{I} + h_{Gj}\mathbf{G} + h_{NEj}\mathbf{NE} \tag{8}$$

where $\sum_{j=1}^{n} h_{Cj} = 1 = \sum_{j=1}^{n} h_{Ij} = \sum_{j=1}^{n} h_{Gj} = \sum_{j=1}^{n} h_{NEj}$.

The closure of the integrated model can be accomplished in a number of ways. In this paper, we explore a simple consumption related closure by specifying (7) as:

$$\mathbf{C} = Z_C \beta_C + VA\beta_{VA} + \epsilon \tag{9}$$

where VA is value added obtained as:

$$VA = i_n' \hat{V} X \tag{10}$$

where i_n is a unit vector and $\hat{V}$ is a diagonal matrix of value added coefficients.[7]

The econometric closure of the integrated model introduces a number of complications that can be highlighted by re-expressing the basic input-output identity from (3) as:

$$X = AX + h_c(Z_C\beta_C + \beta_{VA}VA + \epsilon) + h_{\hat{F}}\hat{F}i_{m-1} \tag{11}$$

where h_c is an n by 1 vector of share coefficients, $h_{\hat{F}}$ is an n by $m-1$ matrix of share coefficients and:

$$\hat{F} = \begin{bmatrix} I & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & NE \end{bmatrix}. \tag{12}$$

This structural equation is further modified by taking into account the linkage between value added and industry output from (10):

$$X = AX + h_c\left[Z_C\beta_C + \beta_{VA}i_n'\hat{V}X + \epsilon\right] + h_{\hat{F}}\hat{F}i_{m-1} \tag{13}$$

where i_n is an n *by* 1 unit vector. The reduced form for this system is:

$$X = \Gamma^{-1}h_cZ_c\beta_C + h_{\hat{F}}\hat{F} + \Gamma^{-1}h_c\epsilon \tag{14}$$

where:

$$\Gamma = \left(I - A - h_c\beta_{VA}i_n'\hat{V}\right). \tag{15}$$

The integrated multiplier matrix Γ^{-1} poses some difficulties from an inferential perspective. The first is that even if unbiased estimates for β_{VA} could be obtained for (9), their insertion into (15) would not likely lead to an unbiased estimate for the multiplier matrix, since it is well

[7] It should be noted that the coupled models often decompose aggregate VA as determined by (10) using econometric equations that permit factor substitution (i.e., between labor and capital) in function of relative prices (Treyz, 1993).

known that for a non-linear function:

$$E\Big[\Gamma^{-1}\Big] \neq E\Big[\Gamma\Big]^{-1}. \tag{16}$$

The second difficulty relates to the expected value of the third term in (14) which, in general, will not be equal to zero even though $E[\epsilon] = 0$. This is because some of the elements of Γ will be stochastic. As a result of these complications, the estimates of the policy multipliers of the exogenous variables Z_c are likely to be biased as are the estimates of gross sectoral output X.

As a consequence the standard methods for impact analysis using either a stand alone IO model or an EC model cannot be directly applied to the case of an integrated EC+IO model. While IO models are deterministic, their integration with a stochastic EC model results in a stochastic EC+IO model. On the other hand, when using EC models for impact analysis, the property of unbiasedness typically holds and this simplifies the development of confidence intervals to attach to estimated impacts. In the case of the EC+IO model, however, one can no longer rely on the unbiasedness property.

In order to exploit the inferential capabilities of integrated EC+IO models alternative approaches need to be considered. There are two general possibilities. The first would be to rely on asymptotic theory to provide analytical results that could form the basis for developing confidence intervals (Goldberger, 1990). However, such an approach may be difficult to implement in practice due to the nonlinearity of (14) and the question of how relevant large sample results would be in the finite sample situations facing most regional modelers.

The second alternative approach could be based on a resampling strategy such as a bootstrap (Efron and Tibshirani, 1993; Jeong and Maddala, 1993) or a stochastic simulation perspective (Fair and Taylor, 1990; Murinde, 1992). In these approaches one creates an artificial sampling distributions for the $\beta_{\mathbf{C}}$ and/or ϵ, based on their estimated variance-covariance matrices and assumed parametric densities, and substitutes realizations from these distributions into (14) to generate sampling distributions for the industry outputs X. Theses distributions can be examined to construct empirical confidence intervals for the estimated impacts and forecasts generated by the integrated model. A variation on this resampling theme could also be based on a direct examination of the conditional distributions for the model parameters by employing Markov Chain Monte Carlo sampling (Chib and Greenberg, 1996).[8]

[8]We thank an anonymous referee for this suggestion.

Additionally, there is the issue of the specification of A. Originally, this was treated as known (deterministic). Specifying this as stochastic would further complicate (15). Moreover, additional sources of uncertainty arise when regionalizing the A matrix. Typically, regional purchase coefficients (RPCs) are used to transform the direct requirements table (A) to adjust for leakages in the form of imports into the region. The RPC coefficients often are unknown and require estimation.[9]

As a precursor to developing methods for inference in integrated models, what is required is an understanding of the data generating process (DGP) that an integrated framework offers. More specifically, we need to understand how various sources of uncertainty in an integrated framework affect the distribution of the endogenous variables that are then modeled with empirical integrated models. As outlined above there are several sources of uncertainty associated with integrated models. While these have been identified by a number of researchers, we have limited evidence on the relative importance of these different sources of uncertainty in the context of integrated models. This is the focus of the remainder of the paper.

4 Monte Carlo Simulations

To investigate the nature of uncertainty in integrated models we develop a set of Monte Carlo simulations using a fairly simple example of a coupled model. We rely on simulations so as to provide for the examination of a wide set of model DGPs. For each DGP we generate a large number of realizations for the endogenous variables in the model resulting in empirical distributions for these variables. We are generally interested in how the different sources of uncertainty in the DGP impact these distributions. The general structure that generates the alternative DGPs we examine is given by equations (3)-(14). Using this structure, we examine three sources of uncertainty

1. Error Uncertainty
2. EC Parameter Uncertainty
3. IO Parameter Uncertainty.

[9] RPCs represent the proportion of goods and services of a particular industry that are supplied by regional industries, rather than imported. See Stevens et al. (1989) for more details.

The first two sources of uncertainty are introduced in the macro consumption function which takes on the following specification in our simulation model:

$$\mathbf{C} = \beta_0 + \beta_1 \mathbf{GDP} + \epsilon. \tag{17}$$

Both the error term, ϵ and the β parameters are viewed as stochastic within the context of our model. Realizations for the error terms are generated as:

$$\epsilon \sim N(0, \sigma^2 I) \tag{18}$$

where σ^2 takes on two alternative values to allow for variations in the level of uncertainty in this model component. EC parameter uncertainty is treated in a similar fashion:

$$\beta \sim N(\beta^*, \Sigma_\beta). \tag{19}$$

The values for β^* and Σ_β were based on empirical estimates of a macro consumption function reported in Pindyck and Rubinfeld (1997).

The coupled model is closed through the income-consumption feedback loop as:

$$\mathbf{GDP} = \iota' Q X \tag{20}$$

where:

$$Q = \hat{V} + A - R.A \tag{21}$$

and $R.A$ represents the dot multiplication of the A matrix of technical IO coefficients and the R matrix of RPCs.

The structural form for our model can be viewed in one of two ways. In the imports-explicit representation we have:

$$X = AX + C + F - M \tag{22}$$

where:

$$M = AX - R.AX \tag{23}$$

with C being personal consumption and F is other final demand. In the imports-implicit representation we have:

$$X = R.AX + C + F. \tag{24}$$

The reduced form, based on (24) is obtained by noting that:

$$C = h_c\mathbf{C} \tag{25}$$

and

$$F = h_f\mathbf{F}. \tag{26}$$

Substitution of (17) into (25) and solving for the reduce form yields:

$$X = \left[I - R.A - h_c\beta_1\iota' Q\right]^{-1}\left[h_c\beta_0 + h_c\epsilon + h_f\mathbf{F}\right]. \tag{27}$$

A vast literature has focused on the estimation of a regional multipliers in the input-output model framework (Miller and Blair, 1985; Garhart and Giarratani, 1987; Lahr, 1993). We take the regional technical coefficients to be identical to the national values[10] and introduce the uncertainty through the RPCs. This representation of the regional IO coefficient as the product of a RPC and a national technical coefficient reflects common practice in applied regional IO analysis (Miller, 1998).

We allow for four variations in the construction of the RPC matrix in order to identify the relative importance of uncertainty in this component. We first specify RPC values for each industry in the model:

$$r = \begin{bmatrix} r_1 \\ r_2 \\ . \\ . \\ r_n \end{bmatrix} \tag{28}$$

where $n = 7$ in our simulation.

[10]This study incorporates the 1967 U.S. national direct requirements table.

The first method in generating RPCs requires the mean and variance of r:

$$\mu_r = \sum_{i=1}^{n} r_i/n, \tag{29}$$

$$\sigma_r^2 = E[r_i^2] - \mu_r^2. \tag{30}$$

By incorporating a finding by Garhart and Giarratani (1987), we assume each element in the full n by n RPC matrix is generated randomly from a beta distribution with a mean of μ_r and a variance of σ_r^2:

$$R_{ij} \sim \text{Beta}\left(\mu_r, \sigma_r^2\right). \tag{31}$$

We will term this randomly generated RPC matrix as R_A.

The second variation in the RPC portion of the simulation process calculates the mean of each row in R_A, and distributes the value in a uniform fashion across the rows of R_B:

$$R_B = R_A \iota / \left(\iota' \iota\right) I \iota \iota', \tag{32}$$

where ι is a unit vector. Both methods for obtaining R_A and R_B were implemented by Garhart and Giarratani (1987). This study incorporates two additional methods for obtaining the RPC matrix which allow for additional heterogeneity across industries.

The third approach to obtaining the RPC matrix is completed entirely on an industry specific basis. That is, we assume that each row in the full RPC matrix is generated from a separate Beta distribution with industry specific means and variances. The mean for each industry is merely the corresponding coefficient in the original r vector from (28) with each element of the RPC matrix taken as:

$$R_{C,ij} \sim \text{Beta}\left(r_i, \sigma_{r_i}^2\right). \tag{33}$$

The last RPC matrix generation method calculates the mean of each row in the R_C, and distributes it uniformly across the rows of R_D:

$$R_D = R_C \iota / \left(\iota' \iota\right) I \iota \iota'. \tag{34}$$

Similar to R_B, the coefficients in R_D vary from industry to industry, but remain constant for each row.

The values of the parameters in the DGP under the alternative scenarios we simulate are listed in Table 1.

[Table 1 about here.]

Each possible combination of parameter values from Table 1 represents an individual DGP. We generate 1,000 realizations of all the dependent variables in the system for each DGP.

Our experiments are designed to analyze the relative importance of the three sets of uncertainty introduced above. Here we focus on the distribution of the dependent variables and how these distributions are affected by turning various source of uncertainty on or off in the DGP. It is important to note that this design differs from that used in traditional Monte Carlo studies of parameter estimators, where the focus is on the properties of the distribution of the estimator given different specifications for the error terms of a model. We focus on the distribution of the dependent variable rather than that of an estimator.

Our approach also differs from that adopted in the stochastic simulation literature where the emphasis is on developing estimators of the model uncertainty. We are simulating the uncertainty directly so as to uncover the relative importance of the different sources, rather than trying to estimate those sources given a realization from the model.

To examine the relative importance of the alternative sources of uncertainty we rely on measures of variation (coefficient of variation and variance) for each of the endogenous variables. We can then compare the values of each of these measures for the same dependent variable *across* different DGPs reflecting alternative sources of uncertainty.

5 Results

A summary of the results for the EC+IO simulations are reported in Tables 2-11. The tables provide insight into the relative importance of each source of uncertainty as well as the issue of propagation and distribution of the various sources of uncertainty in integrated models. Furthermore, these issues have to be addressed for both the macro and micro (i.e., industry specific) endogenous variables.

5.1 Individual Sources of Uncertainty

At the macro level it is apparent that the estimates for total consumption $\mathbf{C}$ and total final demand $\mathbf{Y}$ are primarily affected by error uncertainty, and to a lesser degree by the uncertainty in the β coefficients. From a stand alone perspective, Table (2) identifies a zero coefficient of variation for estimates of **C** and **Y** due to the RPC method chosen. This is directly attributed to the nature of equation (17), which specifies **C** as a function of β and ϵ, and **Y** is a function of consumption and the identity equation for other final demand (26). Furthermore, the structural coefficient (β) produced relatively small coefficients of variation for **C** and **Y**, while the error term introduced the most ($\mathbf{C}(\epsilon_{low}) = .0523,\ \mathbf{C}(\epsilon_{high}) = .1065,\ \mathbf{Y}(\epsilon_{low}) = .0326,\ \mathbf{Y}(\epsilon_{high}) = .0662$).

The remaining macro variables (**X** and **VA**) are affected in various ways by the error term and the RPC method chosen. Focusing on the error term, it is clear that the coefficients of variation (CV) for the two variables are similar (see Table 2). When the error is introduced with a low level of variance the CV values are $\mathbf{VA}(\epsilon_{low}) = .0325,\ \mathbf{X}(\epsilon_{low}) = .0327$. The corresponding CV values roughly double when the error term is introduced with a higher level of variation. Returning to Table (2), at an aggregate level, variations in the estimates for **VA** and **X** are higher in the RPC case A method (.0700, .0804) and are at their least when RPC case D is introduced (.0278, .0258).

[Table 2 about here.]

[Table 3 about here.]

Turning to the micro scale, we make use of the $\%R$ and $\%\epsilon$ statistics, which identify the percentage of variation in individual variable estimates attributed to the RPC and ϵ terms. It is apparent that individual industries are affected differently by the error term and RPC method introduced. Some industries are highly sensitive to the choice of RPC method, while other remain relatively stable. For example, when error uncertainty is introduced at a low level, industry 3 registered a $\%R$ value as high as .45 (Table 4), to as low as .14 (Table 7), while industry 1 remained highly affected by every RPC method (.99 in Table 4 to .97 in Table 5). Industry 2 and 7 also exhibit high $\%R$ coefficients when initial error variance is at a minimum, and they do not vary significantly over RPC methods (see Tables 4-7). Industries 4-6 registered lower $\%R$ statistics which varied over the same simulations.

While the RPC method chosen seems to pull the $\%R$ statistics for individual industries in a particular direction, there are exceptions worth noting. Moving from RPC case B to C the $\%R$ for industry 4 declined while the corresponding coefficients for all other industries increased (see Tables 5, 6). A comparison of RPC methods A and C yields a similar result. While the majority of industry specific $\%R$ statistics drop from case A to C, the coefficients for industries

6 and 7 actually rise (see Tables 4, 6).

Lastly, an increase in the initial variance of the error term will have the most profound effect on the $\%R$ values for each industry. By paring the results tables by RPC method (i.e. Table 4 with Table 8 and so forth), it is clear that increased error variance will begin to outweigh the uncertainty caused by each RPC method. Every $\%R$ value for each industry will drop when additional error variance is introduced.

[Table 4 about here.]

[Table 5 about here.]

[Table 6 about here.]

5.2 Propagation of Uncertainty

The propagation issues are analyzed by calculating the ratio of the cumulative singular variation and the joint variation of all sources of uncertainty (CJVR - Cumulative and Joint Variation Ratio). In short, the cumulative variation is computed as the sum of all variation attributed by each source of uncertainty when introduced alone, and the joint variation captures the variation in endogenous variable estimates when the all three sources of uncertainty are introduced simultaneously. A CJVR value of 1 would signify the absence of propagation among the various sources of uncertainty.

In general terms, the larger the variance of uncertainty in the ϵ and β terms in the specified model, the greater the level of propagation between sources of uncertainty. RPC case C with low model variance (Table 6) yielded CJVR values for **VA** and **X** near 1.0, but when higher initial variance is introduced (Table 10) the CJVR values drop significantly to .80 for total output, and .79 for total value added. While both the level of cumulative and joint variation rise when additional model variance is introduced the joint variation seems to increase at a higher rate. RPC case A is the only outlier in this respect, in that while the CJVR for **VA** and **X** are consistently below 1.0, the measure actually rose when additional variance was examined (see Tables 4, 8). Overall RPC case D delivers the lowest CJVR values with low model variance, and coupled with increased model variance the CJVR values plummet further, demonstrating a significant amount of propagation among the three sources of uncertainty (see Tables 7, 11).

A further look at the effects of increased model variance indicates that some specific industry variables are more subject to propagation than others. For example, industry 2 had the same

CJVR values for RPC case C with low and high model variance (1.05 in Tables 6, 10), while industry 4 fell from 1.04 to .79 over the same two simulations. In RPC case D, industry 1 had a CJVR of .99 with low model variance and its corresponding CJVR was .86 when model variance increased, while the CJVR for industry 7 only changed by .01 over the same two models (see Tables 7, 11).

On an industry specific basis, the choice of RPC method will have an effect on its CJVR. Recalling that at the macro level, RPC case D had the highest degree of propagation, it is interesting to note that industry 3 was relatively high for that case (1.05 in Table 7), and in fact RPC case B with low model variance produced the lowest CJVR (.91 in Table 5). While there may be a few contradictions, the degree of propagation seems to be at its greatest in RPC case D and at its least in method C.

[Table 7 about here.]

[Table 8 about here.]

[Table 9 about here.]

[Table 10 about here.]

[Table 11 about here.]

6 Conclusion

Our analysis represents an initial foray into the question of uncertainty in EC+IO models and we acknowledge that the model employed in our simulation was very simple by applied standards. That simplicity allowed us to focus on three specific sources of uncertainty and how they impact the distributions of the dependent variables in an integrated model.

We first focused on the relative importance of individual sources of uncertainty. Due to the nature of the consumption function we noted that the estimates for total consumption (**C**) and total final demand (**Y**) are primarily affected by the error term and to a small extent by the β structural component. It was then shown that the coefficients of variation for total value added (**VA**) and total output (**X**) are very similar, and respond in an according fashion over various model simulations. A further look into the variation of estimates for **VA** and **X** demonstrated that they are at their highest with RPC method A, and at their lowest with RPC case D.

Turning to micro variable simulation, it was shown that some estimates for industry specific variables are greatly affected by the choice of RPC, while other remain relatively constant over all cases. Furthermore, while their are a few exceptions, the amount of variation in industry variables due to the RPC source of uncertainty move in a similar direction. Lastly, it was found that as initial model variance increases, the uncertainty attributed by the ϵ term will begin to outweigh the uncertainty introduced by the RPC matrix

We then examined how the sources of uncertainty in integrated models propagate. It was noted that the choice of RPC alters the propagation of uncertainty. More specifically, in the presence of low model variance, RPC case D has the highest degree of propagation, and RPC method C propagates the least. The introduction of additional variance to the ϵ and β terms causes a significant increase in the propagation of uncertainty for RPC cases C and D, has little effect on RPC case B, and actually decreases in RPC case A. It was also shown that some industries are affected by the propagation of the uncertainty terms, while others are not.

Our results suggest that there is no simple answer to the question of which source of uncertainty is most important in an integrated model. Instead, that answer is conditioned upon the focus of the analysis and whether the industry specific or macro variables are of central concern. In some ways this relates to the issues of holistic versus partitive accuracy that have attracted much attention in the IO literature (Jensen, 1980). Given the higher dimensionality of integrated models, however, this issue appears to be more complex and requires further attention.

In future work, we intend to increase the complexity of the model so as to examine a wider set of issues. More specifically, we specified each source of uncertainty as independent in our experiments, yet in the practice of implementing operational integrated models it is likely that there are multiple forms of interdependence between these sources of uncertainty.[11] For example, a misspecification of a final demand equation could lead to errors in other econometric components of an integrated model when the equations are specified as a simultaneous system of equations. This would introduce non-orthogonal sources of uncertainty that need to be examined.

A related issue concerns the extension of integrated models for multiregional systems. Clearly, uncertainty about intraregional purchase coefficients (as we have examined here) implies uncertainty about interregional flows and coefficients. How these spatially interdependent sources

[11] We thank an anonymous referee for this point.

of uncertainty would influence the behavior of an integrated model becomes a pressing issue. Approaches towards treating this problem in a two-region setting suggested by Round (1983) would need to be extended to integrated models of larger systems.

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List of Tables

Table 1: Uncertainty Combinations in Simulations

Parameter	Low Variance	High Variance
$E[\beta_0]$	22750.47	22750.47
$E[\beta_1]$	0.59	0.59
$E[\epsilon]$	0.0	0.0
σ_{β_0}	16.0	32.0
σ_{β_1}	0.000128	0.000257
ρ_{β_0,β_1}	-0.74	-0.74
σ_ϵ^2	10000.0	20000.0

Table 2: Coefficient of variation of endogenous variables associated with a single source of uncertainty.

	β Low	β High	ϵ Low	ϵ High	R_a	R_b	R_c	R_d
C	0.0004	0.0009	0.0523	0.1065	0.0000	0.0000	0.0000	0.0000
Y	0.0003	0.0006	0.0326	0.0662	0.0000	0.0000	0.0000	0.0000
VA	0.0003	0.0006	0.0325	0.0660	0.0700	0.0399	0.0427	0.0278
X	0.0003	0.0006	0.0327	0.0664	0.0804	0.0448	0.0417	0.0258
X_1	0.0002	0.0004	0.0257	0.0522	0.3096	0.1629	0.1925	0.1062
X_2	0.0003	0.0005	0.0307	0.0624	0.3521	0.1731	0.2652	0.1232
X_3	0.0003	0.0005	0.0316	0.0642	0.0283	0.0153	0.0221	0.0130
X_4	0.0003	0.0006	0.0338	0.0686	0.1531	0.0850	0.0530	0.0275
X_5	0.0002	0.0005	0.0273	0.0555	0.0531	0.0389	0.0550	0.0397
X_6	0.0003	0.0006	0.0357	0.0725	0.0788	0.0533	0.0849	0.0609
X_7	0.0002	0.0005	0.0289	0.0586	0.2365	0.1420	0.2875	0.1700
VA_1	0.0002	0.0004	0.0257	0.0522	0.3096	0.1629	0.1925	0.1062
VA_2	0.0003	0.0005	0.0307	0.0624	0.3521	0.1731	0.2652	0.1232
VA_3	0.0003	0.0005	0.0316	0.0642	0.0283	0.0153	0.0221	0.0130
VA_4	0.0003	0.0006	0.0338	0.0686	0.1531	0.0850	0.0530	0.0275
VA_5	0.0002	0.0005	0.0273	0.0555	0.0531	0.0389	0.0550	0.0397
VA_6	0.0003	0.0006	0.0357	0.0725	0.0788	0.0533	0.0849	0.0609
VA_7	0.0002	0.0005	0.0289	0.0586	0.2365	0.1420	0.2875	0.1700

Table 3: Coefficient of variation of endogenous variables associated with all sources of uncertainty.

	β, ϵ, R_a Low	β, ϵ, R_a High	β, ϵ, R_b Low	β, ϵ, R_b High	β, ϵ, R_c Low	β, ϵ, R_c High	β, ϵ, R_d Low	β, ϵ, R_d High
C	0.0526	0.1053	0.0530	0.1090	0.0516	0.1124	0.0516	0.1124
Y	0.0327	0.0656	0.0330	0.0679	0.0321	0.0698	0.0321	0.0698
VA	0.0796	0.0962	0.0525	0.0785	0.0513	0.0815	0.0430	0.0745
X	0.0889	0.1038	0.0566	0.0813	0.0504	0.0811	0.0418	0.0740
X_1	0.3135	0.3118	0.1634	0.1664	0.1912	0.2051	0.1076	0.1207
X_2	0.3546	0.3573	0.1801	0.1816	0.2597	0.2655	0.1337	0.1409
X_3	0.0420	0.0693	0.0365	0.0672	0.0378	0.0711	0.0337	0.0690
X_4	0.1560	0.1643	0.0923	0.1101	0.0589	0.0874	0.0435	0.0759
X_5	0.0597	0.0771	0.0472	0.0688	0.0615	0.0822	0.0489	0.0712
X_6	0.0886	0.1074	0.0639	0.0924	0.0897	0.1131	0.0700	0.0968
X_7	0.2446	0.2507	0.1511	0.1613	0.2919	0.3086	0.1772	0.1870
VA_1	0.3135	0.3118	0.1634	0.1664	0.1912	0.2051	0.1076	0.1207
VA_2	0.3546	0.3573	0.1801	0.1816	0.2597	0.2655	0.1337	0.1409
VA_3	0.0420	0.0693	0.0365	0.0672	0.0378	0.0711	0.0337	0.0690
VA_4	0.1560	0.1643	0.0923	0.1101	0.0589	0.0874	0.0435	0.0759
VA_5	0.0597	0.0771	0.0472	0.0688	0.0615	0.0822	0.0489	0.0712
VA_6	0.0886	0.1074	0.0639	0.0924	0.0897	0.1131	0.0700	0.0968
VA_7	0.2446	0.2507	0.1511	0.1613	0.2919	0.3086	0.1772	0.1870

Table 4: Uncertainty propagation for low values of variance, RPC case A.

	$Var\,(\beta)$	$Var\,(\epsilon)$	$Var\,(R)$	$Var\,(\sum)$	$Var\,(Joint)$	$CJVR$	β %	ϵ %	R %
C	39.18	569514.99	0.00	569554.17	569329.95	1.00	0.00	1.00	0.00
Y	39.18	569514.99	0.00	569554.17	569329.95	1.00	0.00	1.00	0.00
VA	20.74	301433.87	1503404.77	1804859.37	1936210.29	0.93	0.00	0.17	0.83
X	75.92	1103701.48	7232046.91	8335824.32	8797647.97	0.95	0.00	0.13	0.87
X_1	0.04	544.24	93356.29	93900.56	96473.29	0.97	0.00	0.01	0.99
X_2	0.01	114.78	18406.22	18521.00	18573.47	1.00	0.00	0.01	0.99
X_3	0.59	8649.19	7045.68	15695.46	15405.76	1.02	0.00	0.55	0.45
X_4	13.71	199339.81	4594353.39	4793706.91	4739987.24	1.01	0.00	0.04	0.96
X_5	1.55	22574.55	88787.62	111363.73	111902.28	1.00	0.00	0.20	0.80
X_6	6.95	100960.16	519383.62	620350.73	652721.58	0.95	0.00	0.16	0.84
X_7	0.01	82.14	6312.15	6394.30	6841.51	0.93	0.00	0.01	0.99
VA_1	0.01	88.49	15179.69	15268.18	15686.51	0.97	0.00	0.01	0.99
VA_2	0.00	45.11	7233.48	7278.59	7299.21	1.00	0.00	0.01	0.99
VA_3	0.12	1691.76	1378.12	3069.99	3013.32	1.02	0.00	0.55	0.45
VA_4	2.24	32604.96	751474.23	784081.44	775294.79	1.01	0.00	0.04	0.96
VA_5	0.76	11098.33	43650.68	54749.77	55014.54	1.00	0.00	0.20	0.80
VA_6	2.77	40301.45	207328.45	247632.67	260554.52	0.95	0.00	0.16	0.84
VA_7	0.00	26.08	2003.82	2029.90	2171.87	0.93	0.00	0.01	0.99

Notes

$Var(\beta)$: Variance of endogenous variable associated with β uncertainty only.

$Var(\epsilon)$: Variance of endogenous variable associated with ϵ uncertainty only.

$Var(R)$: Variance of endogenous variable associated with R uncertainty only.

$Var\left(\sum\right)$: Sum of variances of endogenous variable from single source of uncertainty simulations.

$Var\,(Joint)$: Variance of endogenous variable when all three sources of uncertainty are present.

$CJVR$: $VAR\left(\sum\right)/VAR\,(JOINT)$

β %: $Var(\beta)/VAR\left(\sum\right)$

ϵ %: $Var(\epsilon)/VAR\left(\sum\right)$

R %: $Var(R)/VAR\left(\sum\right)$

All values for variance are in thousands

Table 5: Uncertainty propagation for low values of variance, RPC case B

	$Var(\beta)$	$Var(\epsilon)$	$Var(R)$	$Var\left(\sum\right)$	$Var(Joint)$	$CJVR$	β %	ϵ %	R %
C	39.18	569514.99	0.00	569554.17	582295.27	0.98	0.00	1.00	0.00
Y	39.18	569514.99	0.00	569554.17	582295.27	0.98	0.00	1.00	0.00
VA	20.74	301433.87	479384.22	780838.82	831863.09	0.94	0.00	0.39	0.61
X	75.92	1103701.48	2199367.54	3303144.94	3508883.90	0.94	0.00	0.33	0.67
X_1	0.04	544.24	25836.40	26380.68	26253.33	1.00	0.00	0.02	0.98
X_2	0.01	114.78	4288.23	4403.01	4655.90	0.95	0.00	0.03	0.97
X_3	0.59	8649.19	2055.69	10705.47	11712.20	0.91	0.00	0.81	0.19
X_4	13.71	199339.81	1357238.84	1556592.36	1608688.55	0.97	0.00	0.13	0.87
X_5	1.55	22574.55	47293.12	69869.22	69882.63	1.00	0.00	0.32	0.68
X_6	6.95	100960.16	235846.82	336813.92	339154.04	0.99	0.00	0.30	0.70
X_7	0.01	82.14	2235.98	2318.13	2573.15	0.90	0.00	0.04	0.96
VA_1	0.01	88.49	4200.99	4289.49	4268.78	1.00	0.00	0.02	0.98
VA_2	0.00	45.11	1685.24	1730.35	1829.73	0.95	0.00	0.03	0.97
VA_3	0.12	1691.76	402.09	2093.96	2290.87	0.91	0.00	0.81	0.19
VA_4	2.24	32604.96	221996.42	254603.63	263124.73	0.97	0.00	0.13	0.87
VA_5	0.76	11098.33	23250.73	34349.82	34356.41	1.00	0.00	0.32	0.68
VA_6	2.77	40301.45	94145.74	134449.96	135384.10	0.99	0.00	0.30	0.70
VA_7	0.00	26.08	709.82	735.90	816.86	0.90	0.00	0.04	0.96

Notes
$Var(\beta)$: Variance of endogenous variable associated with β uncertainty only.
$Var(\epsilon)$: Variance of endogenous variable associated with ϵ uncertainty only.
$Var(R)$: Variance of endogenous variable associated with R uncertainty only.
$Var\left(\sum\right)$: Sum of variances of endogenous variable from single source of uncertainty simulations.
$Var(Joint)$: Variance of endogenous variable when all three sources of uncertainty are present.
$CJVR$: $VAR\left(\sum\right)/VAR(JOINT)$
β %: $Var(\beta)/VAR\left(\sum\right)$
ϵ %: $Var(\epsilon)/VAR\left(\sum\right)$
R %: $Var(R)/VAR\left(\sum\right)$
All values for variance are in thousands

Table 6: Uncertainty propagation for low values of variance, RPC case C

	$Var\,(\beta)$	$Var\,(\epsilon)$	$Var\,(R)$	$Var\left(\sum\right)$	$Var\,(Joint)$	$CJVR$	β %	ϵ %	R %
C	39.18	569514.99	0.00	569554.17	548263.37	1.04	0.00	1.00	0.00
Y	39.18	569514.99	0.00	569554.17	548263.37	1.04	0.00	1.00	0.00
VA	20.74	301433.87	650285.23	951739.84	934642.88	1.02	0.00	0.32	0.68
X	75.92	1103701.48	2301405.70	3405183.10	3358167.17	1.01	0.00	0.32	0.68
X_1	0.04	544.24	67237.54	67781.81	68147.15	0.99	0.00	0.01	0.99
X_2	0.01	114.78	14168.36	14283.14	13552.42	1.05	0.00	0.01	0.99
X_3	0.59	8649.19	4178.25	12828.03	12190.45	1.05	0.00	0.67	0.33
X_4	13.71	199339.81	713398.95	912752.47	880902.23	1.04	0.00	0.22	0.78
X_5	1.55	22574.55	101345.46	123921.56	125309.73	0.99	0.00	0.18	0.82
X_6	6.95	100960.16	665681.11	766648.21	742559.44	1.03	0.00	0.13	0.87
X_7	0.01	82.14	8530.76	8612.91	8774.41	0.98	0.00	0.01	0.99
VA_1	0.01	88.49	10932.79	11021.29	11080.69	0.99	0.00	0.01	0.99
VA_2	0.00	45.11	5568.04	5613.15	5325.98	1.05	0.00	0.01	0.99
VA_3	0.12	1691.76	817.25	2509.13	2384.42	1.05	0.00	0.67	0.33
VA_4	2.24	32604.96	116686.92	149294.12	144084.55	1.04	0.00	0.22	0.78
VA_5	0.76	11098.33	49824.49	60923.58	61606.05	0.99	0.00	0.18	0.82
VA_6	2.77	40301.45	265727.73	306031.95	296416.16	1.03	0.00	0.13	0.87
VA_7	0.00	26.08	2708.13	2734.21	2785.48	0.98	0.00	0.01	0.99

Notes

$Var(\beta)$: Variance of endogenous variable associated with β uncertainty only.

$Var(\epsilon)$: Variance of endogenous variable associated with ϵ uncertainty only.

$Var(R)$: Variance of endogenous variable associated with R uncertainty only.

$Var\left(\sum\right)$: Sum of variances of endogenous variable from single source of uncertainty simulations.

$Var\,(Joint)$: Variance of endogenous variable when all three sources of uncertainty are present.

$CJVR$: $VAR\left(\sum\right)/VAR\,(JOINT)$

β %: $Var(\beta)/VAR\left(\sum\right)$

ϵ %: $Var(\epsilon)/VAR\left(\sum\right)$

R %: $Var(R)/VAR\left(\sum\right)$

All values for variance are in thousands

Table 7: Uncertainty propagation for low values of variance, RPC case D

	$Var(\beta)$	$Var(\epsilon)$	$Var(R)$	$Var\left(\sum\right)$	$Var(Joint)$	$CJVR$	β %	ϵ %	R %
C	39.18	569514.99	0.00	569554.17	548263.37	1.04	0.00	1.00	0.00
Y	39.18	569514.99	0.00	569554.17	548263.37	1.04	0.00	1.00	0.00
VA	20.74	301433.87	275347.78	576802.38	653552.79	0.88	0.00	0.52	0.48
X	75.92	1103701.48	880102.47	1983879.88	2297316.51	0.86	0.00	0.56	0.44
X_1	0.04	544.24	20334.51	20878.78	21035.17	0.99	0.00	0.03	0.97
X_2	0.01	114.78	3063.33	3178.12	3560.95	0.89	0.00	0.04	0.96
X_3	0.59	8649.19	1452.89	10102.67	9659.24	1.05	0.00	0.86	0.14
X_4	13.71	199339.81	191564.19	390917.71	476994.29	0.82	0.00	0.51	0.49
X_5	1.55	22574.55	52600.44	75176.55	79300.31	0.95	0.00	0.30	0.70
X_6	6.95	100960.16	341577.61	442544.72	449393.74	0.98	0.00	0.23	0.77
X_7	0.01	82.14	2898.83	2980.98	3140.42	0.95	0.00	0.03	0.97
VA_1	0.01	88.49	3306.38	3394.88	3420.31	0.99	0.00	0.03	0.97
VA_2	0.00	45.11	1203.86	1248.97	1399.42	0.89	0.00	0.04	0.96
VA_3	0.12	1691.76	284.18	1976.05	1889.32	1.05	0.00	0.86	0.14
VA_4	2.24	32604.96	31333.15	63940.36	78019.45	0.82	0.00	0.51	0.49
VA_5	0.76	11098.33	25859.97	36959.06	38986.43	0.95	0.00	0.30	0.70
VA_6	2.77	40301.45	136351.54	176655.76	179389.77	0.98	0.00	0.23	0.77
VA_7	0.00	26.08	920.25	946.33	996.94	0.95	0.00	0.03	0.97

Notes
$Var(\beta)$: Variance of endogenous variable associated with β uncertainty only.
$Var(\epsilon)$: Variance of endogenous variable associated with ϵ uncertainty only.
$Var(R)$: Variance of endogenous variable associated with R uncertainty only.
$Var\left(\sum\right)$: Sum of variances of endogenous variable from single source of uncertainty simulations.
$Var\,(Joint)$: Variance of endogenous variable when all three sources of uncertainty are present.
$CJVR$: $VAR\left(\sum\right)/VAR\,(JOINT)$
β %: $Var(\beta)/VAR\left(\sum\right)$
ϵ %: $Var(\epsilon)/VAR\left(\sum\right)$
R %: $Var(R)/VAR\left(\sum\right)$
All values for variance are in thousands

Table 8: Uncertainty propagation for high values of variance, RPC case A

	$Var\,(\beta)$	$Var\,(\epsilon)$	$Var\,(R)$	$Var\,(\sum)$	$Var\,(Joint)$	$CJVR$	β %	ϵ %	R %
C	168.82	2331448.68	0.00	2331617.49	2298749.55	1.01	0.00	1.00	0.00
Y	168.82	2331448.68	0.00	2331617.49	2298749.55	1.01	0.00	1.00	0.00
VA	89.35	1233993.14	1503404.77	2737487.26	2831005.91	0.97	0.00	0.45	0.55
X	327.16	4518271.47	7232046.91	11750645.55	11994634.80	0.98	0.00	0.38	0.62
X_1	0.16	2227.97	93356.29	95584.41	98504.80	0.97	0.00	0.02	0.98
X_2	0.03	469.86	18406.22	18876.11	19414.79	0.97	0.00	0.02	0.98
X_3	2.56	35407.55	7045.68	42455.80	42105.83	1.01	0.00	0.83	0.17
X_4	59.09	816046.17	4594353.39	5410458.64	5220442.69	1.04	0.00	0.15	0.85
X_5	6.69	92414.43	88787.62	181208.75	185381.06	0.98	0.00	0.51	0.49
X_6	29.93	413305.07	519383.62	932718.62	969859.63	0.96	0.00	0.44	0.56
X_7	0.02	336.28	6312.15	6648.45	6977.95	0.95	0.00	0.05	0.95
VA_1	0.03	362.27	15179.69	15541.98	16016.83	0.97	0.00	0.02	0.98
VA_2	0.01	184.65	7233.48	7418.15	7629.85	0.97	0.00	0.02	0.98
VA_3	0.50	6925.62	1378.12	8304.24	8235.78	1.01	0.00	0.83	0.17
VA_4	9.66	133476.38	751474.23	884960.28	853880.37	1.04	0.00	0.15	0.85
VA_5	3.29	45433.72	43650.68	89087.69	91138.93	0.98	0.00	0.51	0.49
VA_6	11.95	164983.83	207328.45	372324.22	387150.24	0.96	0.00	0.44	0.56
VA_7	0.01	106.75	2003.82	2110.58	2215.18	0.95	0.00	0.05	0.95

Notes
$Var(\beta)$: Variance of endogenous variable associated with β uncertainty only.
$Var(\epsilon)$: Variance of endogenous variable associated with ϵ uncertainty only.
$Var(R)$: Variance of endogenous variable associated with R uncertainty only.
$Var\,(\sum)$: Sum of variances of endogenous variable from single source of uncertainty simulations.
$Var\,(Joint)$: Variance of endogenous variable when all three sources of uncertainty are present.
$CJVR$: $VAR\,(\sum)\,/VAR\,(JOINT)$
β %: $Var(\beta)/VAR\,(\sum)$
ϵ %: $Var(\epsilon)/VAR\,(\sum)$
R %: $Var(R)/VAR\,(\sum)$
All values for variance are in thousands

Table 9: Uncertainty propagation for high values of variance, RPC case B

	$Var\,(\beta)$	$Var\,(\epsilon)$	$Var\,(R)$	$Var\left(\sum\right)$	$Var\,(Joint)$	$CJVR$	β %	ϵ %	R %
C	168.82	2331448.68	0.00	2331617.49	2463753.67	0.95	0.00	1.00	0.00
Y	168.82	2331448.68	0.00	2331617.49	2463753.67	0.95	0.00	1.00	0.00
VA	89.35	1233993.14	479384.22	1713466.71	1849941.82	0.93	0.00	0.72	0.28
X	327.16	4518271.47	2199367.54	6717966.18	7199591.56	0.93	0.00	0.67	0.33
X_1	0.16	2227.97	25836.40	28064.53	26587.32	1.06	0.00	0.08	0.92
X_2	0.03	469.86	4288.23	4758.12	4714.55	1.01	0.00	0.10	0.90
X_3	2.56	35407.55	2055.69	37465.81	39605.69	0.95	0.00	0.95	0.05
X_4	59.09	816046.17	1357238.84	2173344.10	2267605.24	0.96	0.00	0.38	0.62
X_5	6.69	92414.43	47293.12	139714.24	147808.26	0.95	0.00	0.66	0.34
X_6	29.93	413305.07	235846.82	649181.81	705085.44	0.92	0.00	0.64	0.36
X_7	0.02	336.28	2235.98	2572.28	2952.68	0.87	0.00	0.13	0.87
VA_1	0.03	362.27	4200.99	4563.28	4323.09	1.06	0.00	0.08	0.92
VA_2	0.01	184.65	1685.24	1869.90	1852.78	1.01	0.00	0.10	0.90
VA_3	0.50	6925.62	402.09	7328.21	7746.76	0.95	0.00	0.95	0.05
VA_4	9.66	133476.38	221996.42	355482.47	370900.27	0.96	0.00	0.38	0.62
VA_5	3.29	45433.72	23250.73	68687.74	72667.01	0.95	0.00	0.66	0.34
VA_6	11.95	164983.83	94145.74	259141.51	281457.22	0.92	0.00	0.64	0.36
VA_7	0.01	106.75	709.82	816.58	937.34	0.87	0.00	0.13	0.87

Notes

$Var(\beta)$: Variance of endogenous variable associated with β uncertainty only.

$Var(\epsilon)$: Variance of endogenous variable associated with ϵ uncertainty only.

$Var(R)$: Variance of endogenous variable associated with R uncertainty only.

$Var\left(\sum\right)$: Sum of variances of endogenous variable from single source of uncertainty simulations.

$Var\,(Joint)$: Variance of endogenous variable when all three sources of uncertainty are present.

$CJVR$: $VAR\left(\sum\right)/VAR\,(JOINT)$

β %: $Var(\beta)/VAR\left(\sum\right)$

ϵ %: $Var(\epsilon)/VAR\left(\sum\right)$

R %: $Var(R)/VAR\left(\sum\right)$

All values for variance are in thousands

Table 10: Uncertainty propagation for high values of variance, RPC case C

	$Var(\beta)$	$Var(\epsilon)$	$Var(R)$	$Var\left(\sum\right)$	$Var(Joint)$	$CJVR$	β %	ϵ %	R %
C	168.82	2331448.68	0.00	2331617.49	2576846.83	0.90	0.00	1.00	0.00
Y	168.82	2331448.68	0.00	2331617.49	2576846.83	0.90	0.00	1.00	0.00
VA	89.35	1233993.14	650285.23	1884367.73	2346838.69	0.80	0.00	0.65	0.35
X	327.16	4518271.47	2301405.70	6820004.34	8627334.25	0.79	0.00	0.66	0.34
X_1	0.16	2227.97	67237.54	69465.66	76664.68	0.91	0.00	0.03	0.97
X_2	0.03	469.86	14168.36	14638.25	13942.48	1.05	0.00	0.03	0.97
X_3	2.56	35407.55	4178.25	39588.37	42820.95	0.92	0.00	0.89	0.11
X_4	59.09	816046.17	713398.95	1529504.20	1924965.90	0.79	0.00	0.53	0.47
X_5	6.69	92414.43	101345.46	193766.58	223109.94	0.87	0.00	0.48	0.52
X_6	29.93	413305.07	665681.11	1079016.10	1177467.32	0.92	0.00	0.38	0.62
X_7	0.02	336.28	8530.76	8867.06	9104.77	0.97	0.00	0.04	0.96
VA_1	0.03	362.27	10932.79	11295.08	12465.64	0.91	0.00	0.03	0.97
VA_2	0.01	184.65	5568.04	5752.71	5479.27	1.05	0.00	0.03	0.97
VA_3	0.50	6925.62	817.25	7743.37	8375.66	0.92	0.00	0.89	0.11
VA_4	9.66	133476.38	116686.92	250172.96	314856.55	0.79	0.00	0.53	0.47
VA_5	3.29	45433.72	49824.49	95261.50	109687.58	0.87	0.00	0.48	0.52
VA_6	11.95	164983.83	265727.73	430723.51	470023.43	0.92	0.00	0.38	0.62
VA_7	0.01	106.75	2708.13	2814.89	2890.35	0.97	0.00	0.04	0.96

Notes

$Var(\beta)$: Variance of endogenous variable associated with β uncertainty only.

$Var(\epsilon)$: Variance of endogenous variable associated with ϵ uncertainty only.

$Var(R)$: Variance of endogenous variable associated with R uncertainty only.

$Var\left(\sum\right)$: Sum of variances of endogenous variable from single source of uncertainty simulations.

$Var(Joint)$: Variance of endogenous variable when all three sources of uncertainty are present.

$CJVR$: $VAR\left(\sum\right)/VAR(JOINT)$

β %: $Var(\beta)/VAR\left(\sum\right)$

ϵ %: $Var(\epsilon)/VAR\left(\sum\right)$

R %: $Var(R)/VAR\left(\sum\right)$

All values for variance are in thousands

Table 11: Uncertainty propagation for high values of variance, RPC case D

	$Var(\beta)$	$Var(\epsilon)$	$Var(R)$	$Var\left(\sum\right)$	$Var(Joint)$	$CJVR$	β %	ϵ %	R %
C	168.82	2331448.68	0.00	2331617.49	2576846.83	0.90	0.00	1.00	0.00
Y	168.82	2331448.68	0.00	2331617.49	2576846.83	0.90	0.00	1.00	0.00
VA	89.35	1233993.14	275347.78	1509430.27	1955022.40	0.77	0.00	0.82	0.18
X	327.16	4518271.47	880102.47	5398701.11	7176465.54	0.75	0.00	0.84	0.16
X_1	0.16	2227.97	20334.51	22562.63	26382.67	0.86	0.00	0.10	0.90
X_2	0.03	469.86	3063.33	3533.23	3950.94	0.89	0.00	0.13	0.87
X_3	2.56	35407.55	1452.89	36863.01	40332.67	0.91	0.00	0.96	0.04
X_4	59.09	816046.17	191564.19	1007669.45	1447349.55	0.70	0.00	0.81	0.19
X_5	6.69	92414.43	52600.44	145021.57	167539.21	0.87	0.00	0.64	0.36
X_6	29.93	413305.07	341577.61	754912.61	856935.79	0.88	0.00	0.55	0.45
X_7	0.02	336.28	2898.83	3235.13	3365.55	0.96	0.00	0.10	0.90
VA_1	0.03	362.27	3306.38	3668.67	4289.81	0.86	0.00	0.10	0.90
VA_2	0.01	184.65	1203.86	1388.53	1552.69	0.89	0.00	0.13	0.87
VA_3	0.50	6925.62	284.18	7210.30	7888.96	0.91	0.00	0.96	0.04
VA_4	9.66	133476.38	31333.15	164819.19	236735.36	0.70	0.00	0.81	0.19
VA_5	3.29	45433.72	25859.97	71296.98	82367.34	0.87	0.00	0.64	0.36
VA_6	11.95	164983.83	136351.54	301347.32	342073.10	0.88	0.00	0.55	0.45
VA_7	0.01	106.75	920.25	1027.01	1068.41	0.96	0.00	0.10	0.90

Notes

$Var(\beta)$: Variance of endogenous variable associated with β uncertainty only.

$Var(\epsilon)$: Variance of endogenous variable associated with ϵ uncertainty only.

$Var(R)$: Variance of endogenous variable associated with R uncertainty only.

$Var\left(\sum\right)$: Sum of variances of endogenous variable from single source of uncertainty simulations.

$Var(Joint)$: Variance of endogenous variable when all three sources of uncertainty are present.

$CJVR$: $VAR\left(\sum\right)/VAR(JOINT)$

β %: $Var(\beta)/VAR\left(\sum\right)$

ϵ %: $Var(\epsilon)/VAR\left(\sum\right)$

R %: $Var(R)/VAR\left(\sum\right)$

All values for variance are in thousands