A distribution dynamics approach to regional GDP convergence in reunified Germany

Falko Juessen[*†]
University of Dortmund, Germany

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ABSTRACT
This paper presents an empirical study of GDP per worker (and per capita) convergence across German labour market regions during 1992 to 2002 using nonparametric techniques. There is evidence for a tendency towards convergence during the observed period, i.e. regions that were less productive in 1992 (East-German regions) established a higher relative GDP in 2002. It is an advantage of our approach that it allows to make predictions about the long run distribution of regional production. We predict a persistent inequality among German regions. This result implies that the substantial regional policy expenditures made by the German government and the EU will not achieve their aim of equalisation, and need therefore to be critically reviewed.
JEL-*classification*: C14, C23, O47, R11

Keywords: regional convergence, distribution dynamics, nonparametric econometrics, stochastic kernel, regional policy

[*]University of Dortmund, Department of Economics, LS VWL (Makroökonomie), 44227 Dortmund, Germany, Tel.: +49-231-755-3291, email: F.Juessen@wiso.uni-dortmund.de

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1 Introduction

This paper presents an empirical study of GDP convergence across German labour market regions during 1992 to 2002 using the distribution dynamics approach to economic convergence first introduced by Quah (1993, 1996a, 1996b, 1997). This methodology studies how the *entire* cross-sectional distribution of relative GDP evolves over time and is therefore not limited to an analysis of single moments of the underlying distribution as it is the case for traditional $\beta-$ and $\sigma-$convergence approaches. An advantage of the empirical strategy is that growth and distribution are considered jointly and simultaneously (Quah, 2001).

The convergence hypothesis states that poor economies catch up with rich ones. This topic is important in Germany because alleviating regional disparities is regarded as a fundamental objective for German (and European) policy, especially in light of East-West differentials in reunified Germany. At the heart of the debate about regional inequality stands a fundamental controversy about whether or not a process of economic homogenisation has taken or will take place in reunified Germany. Recently, the Federal President of Germany, Horst Köhler, initiated a lively debate about whether it is possible to equalise "living conditions" between the two parts of the country.

Obviously, differences in regional GDP do not perfectly reflect differences in living standards. Since better data are not easily available on a disaggregated regional level, this study uses GDP data to contribute statistical evidence to the debate about regional inequality in Germany. Besides the fact that almost all related studies also use GDP data, a further reason to focus on GDP convergence is that policy does formulate its aims with respect to GDP. For example, the main objective of EU regional policy is to promote the development of regions whose per capita GDP is below 75% of the EU average and approximately 70% of total EU regional expenditure is spent on this goal of equalisation (Overman and Puga, 2002).[1]

In order to mitigate regional inequalities, disadvantaged regions in Germany benefit from the European Structural Funds and the German "Gemeinschaftsauf-

[1] Since 1994, the East German federal states (excluding East-Berlin) are target-1 development areas, and they will receive subsidies totalling 19.229 million Euro until 2006 (Eckey, 2001).

gabe Verbesserung der regionalen Wirtschaftsstruktur" (GRW). The GRW is the guideline for German regional policy and advocates a supply-side policy supporting growth in order to equalise regional differences in living standards (Gerling, 2000). For the allocation of subsidies the GRW has defined 271 regional labour markets ("Arbeitsmarktregionen"). This paper addresses regional convergence on the level of these labour market regions. Therefore, our descriptive analysis of the extent of regional convergence is associated with the most important regional policy program in Germany, the GRW.

Ideally, we would like to analyze convergence of real GDP in order to assess if regional policy is likely to achieve its objective of equalisation. Unfortunately, data limitations prevent the calculation of cost-of-living adjusted variables. Nevertheless, we think that the present study provides important insights for assessing the development of the regional income distribution in reunified Germany (see the data section below).

A particular feature of our approach is that it allows to make predictions about the long run distribution of regional GDP. This is an important aspect because it gives an idea on the long run outcome *given* that convergence continues as it has in the last decade. Using the more technical terminology of the distribution dynamics approach, we investigate what will happen to the German regional GDP distribution if the observed dynamics remained unchanged. Under the assumption that the development of the regional distribution is affected by regional economic policy (i.e. the GRW and the European Structural Funds) we can provocatively reformulate the question under study: Where will German regions end up if policy is unchanged? Answering this question is of interest for policy makers but we should keep in mind that our analysis has to be interpreted with some care due to the data limitations described above.

The main results of this study are the following. We find evidence for a tendency towards convergence during the period we study, i.e. regions that were less productive in 1992 established a higher relative GDP in 2002. The convergence process is driven mainly by the catching-up of East German regions in relative terms.

Concerning the long run distribution of regional GDP this study provides discouraging evidence. The ergodic density we calculate on the basis of our estimates

is characterised by polarisation. In other words, the long run distribution shows clustering. According to our long run analysis it is unlikely that German labour market regions converge towards equality. This pattern is found to emerge for both GDP per worker and GDP per capita.

This means that on the one hand there clearly has been a catching-up process during the past period we can observe. But on the other hand, the long run estimates suggest that there will be an increased tendency towards divergence in the future. It is an alarming result of the present study that regional inequality is likely to increase rather than to decrease in the future.

These results are especially important with respect to the substantial regional policy expenditures taken in the past decade. According to our analysis, it is unlikely that German and European policy will prevent polarisation in the regional GDP distribution even if transfers and subsidies will be continued in a comparable magnitude. Because our work shows that regional policy is unlikely to achieve its aim of long run equalisation it is argued that policy measures need to be critically reviewed.

The remainder of this paper is organized as follows. The literature is briefly discussed in Section 2. Section 3 introduces the data employed. The empirical analysis is presented in Section 4 and the last section concludes.

2 Literature

As for most other regional convergence studies, the theoretical framework of the empirical analysis is the neoclassical growth model which suggests that regional per capita income within a country converge to the same long run steady-state (see Magrini, 2004, Durlauf, Johnson, and Temple, 2005, for recent surveys of the large literature). However, regions are by no means small closed economies but are highly integrated by goods and factor movement. Hence, in a regional context the neoclassical growth model for closed economies does not appear to be the best framework for convergence studies. Barro, Mankiw, and Sala-i-Martin (1995) have extended the neoclassical growth model for partial factor mobility and show that the basic prediction of convergence is not altered in this setting. For an elaborate analysis of the role of labour mobility in the convergence process we refer to Razin and Yuen (1997).

In contrast, new theories of industrial location, trade and integration (Fujita, Krugman, and Venables, 1999) and most models of the new growth theory cast doubts on the neoclassical optimistic prediction of convergence.

While it is quite clear in theory what economic convergence means, measuring convergence is not a trivial task. In recent years, a number of alternative strategies have been suggested, i.e. traditional cross-sectional regressions of $\beta-$ and $\sigma-$convergence, panel data models and time series tests.[2]

While there are several studies analysing regional convergence in West Germany[3], empirical evidence regarding reunified Germany is scarce. A potential reason for this has been pointed out by Kosfeld, Eckey, and Dreger (2002) who state that regionally disaggregated data on economic growth are available only recently.

Most studies for reunified Germany are limited to an analysis of convergence between the Eastern and Western part of the country.[4] Although some authors are more pessimistic than others about convergence, the general result is that "East German labour productivity has converged on that in West Germany more slowly than was initially thought but faster than would have been expected on the basis of studies of convergence such as Barro and Sala-i-Martin (1991)" (Barell and te Velde, 2000, p. 272).

Our paper is a contribution to the literature which addresses regional convergence in reunified Germany on a disaggregated geographic level (see Kosfeld, Eckey, and Dreger, 2002, and Kosfeld and Lauridsen, 2004).

Using a spatial econometric approach to $\beta-$convergence, Kosfeld, Eckey, and Dreger (2002) find clear evidence for both per capita income and labour productivity convergence during the period 1992 to 2000. Kosfeld and Lauridsen (2004) adopt a cross-sectional spatial econometric adjustment model which bases on the concept of spatial error-correction. They find only weak evidence for conditional convergence in the year 2000.

[2]See the review of Magrini (2004) for a survey focusing on regional convergence studies.

[3]See Seitz (1995), Schalk and Untiedt (1996), Kellermann (1997), Bohl (1998), Funke and Strulik (1999) and Niebuhr (2001). In general, these studies do find evidence for both absolute and conditional income convergence in West Germany.

[4]See Hallet and Ma (1993), Burda and Funke (1995), Keller (1997), Funke and Strulik (2000) and Barrell and te Velde (2000).

A shortcoming of the $\beta-$convergence approach is that by focusing on the average behaviour of a representative region it suppresses the cross-section income dynamics one wishes to investigate (Quah, 1996a, 1997). This criticism also holds for spatial econometric extensions of the $\beta-$convergence approach. One possibility to overcome the limits of the $\beta-$convergence method is to estimate the entire GDP distribution and its dynamics over time. Only this method allows to uncover empirical regularities such as persistence, polarisation and the formation of convergence clubs. Since pronounced East-West disparities are a well-documented fact in reunified Germany (Barell and te Velde, 2000), it appears to be promising to adopt the distribution dynamics approach to Germany, which has not been done yet in the literature.[5]

3 Data

Germany's official statistics provide GDP data for disaggregated administratively defined regions (Kreise und kreisfreie Städte). A regional economic analysis based on district data can be misleading because the borders of German districts are determined by political and historical rather than economic reasons. For this reason we aggregate districts to local labour market regions which are the target areas for the German "Gemeinschaftsaufgabe Verbesserung der regionalen Wirtschaftsstruktur" (GRW). We use data for 439 German districts to define 271 labour market regions, so that center and hinterland of labour markets are adequately integrated on the basis of commuter flows.

Empirical growth studies use either GDP per capita and/or GDP per worker as a dependent variable. Since most theoretical growth models are based on production functions, their implications relate more closely to GDP per worker than GDP per capita (Durlauf, Johnson, and Temple, 2005). In general, GDP per worker is a more accurate index of average productivity than GDP per capita (Jones, 1997). Moreover, in a regional context GDP per capita data appear less informative due to possible distortions caused by commuter flows. In contrast, data on total em-

[5] Technically related studies for other countries are Andrade et al. (2004) (Brazil), Maza and Villaverde (2004) (EU), Kang (2004) (Japan), Magrini (2004) (EU), Johnson (2000) (US), Arbia, Basile, and Salvatore (2003) (Italy), Johnson (2005) (Penn World Tables), Bandyopadhyay (2002) (India), Mossi et al. (2003) (Brazil), Tortosa-Ausina et al. (2005) (Spain), Epstein, Howlett, and Schulze (2003) (OECD).

ployment (Erwerbstätige) refer to the workplace instead to the place of residence as it is the case for population data. For these reasons we decided to put our main focus on GDP per total employment.[6] In the end of the paper we briefly summarize the main results for GDP per capita.

The raw GDP data on the district level stem from the National Accounts of the States (Arbeitskreis Volkswirtschaftliche Gesamtrechnungen der Länder) compiled by the Statistical State Office Baden-Würtemberg and are measured in current prices. Regional price indices on the district level or on the level of the labour market regions are not available. Total employment and population data are reported by the Federal Office for Building and Regional Planning (Bundesamt für Bauwesen und Raumordnung).[7] The time period ranges from 1992 to 2002.[8]

The key variable in our econometric analysis is regional relative GDP per worker. Regional GDP per worker data are normalized by dividing by the labour productivity of the German economy. This allows us to abstract from the growth of the German economy during the period under study. The normalization also accounts for common changes in inflation.

Relative GDP per worker data have a natural economic interpretation as the fraction of total German production contributed by the ith region, if all regions had the same employment (Bianchi, 1997).

We do not exclude any regions from the analysis, because outlying data points represent regions which performed either extremely well or poor. From an economic point of view it is not appealing to simply delete these observations (Quah, 1997). The complete sample consists of 271 German labour market regions which are observed for 10 years.

As a starting point, we document the regional disparities in relative GDP per worker, for 1992 and 2002. Figure 1 illustrates the apparent East-West disparities in reunified Germany. It is interesting to compare incomes in labour market regions with extreme values. The average relative GDP in the ten most productive German

[6]Throughout the paper we use GDP per worker and labour productivity as synonyms for GDP per total employment.

[7]All district data have been adjusted for changes in the boundaries of the districts which occurred during the period we study, i.e. all changes caused by the various "Kreisgebietsreformen" have been accounted for. The regional borders of the districts correspond to the spatial classification (Gebietsstand) in 2001.

[8]There are no data for 1993.

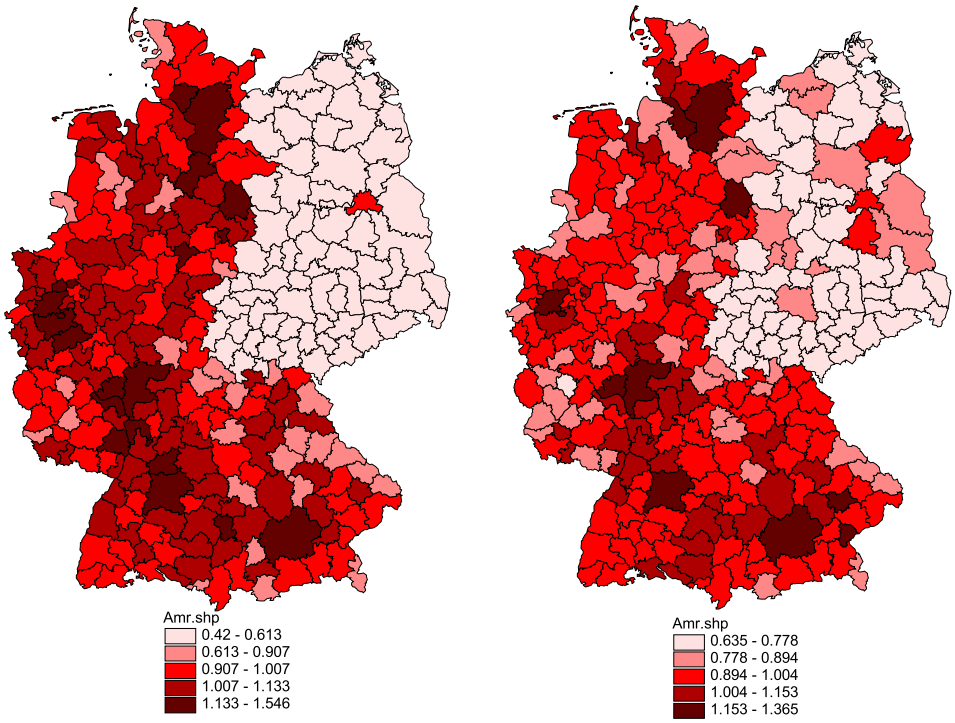

Figure 1: GDP per worker across German labour market regions, relative to the national average. Left: 1992, right: 2002.

regions is 1.31 in 1992 and 1.28 a decade later. The ten poorest regions had an average of only 0.45 in 1992 and 0.65 times the German average in 2002. These numbers clearly illustrate that regional disparities are very pronounced in reunified Germany.

In order to get a first impression of the dynamics of regional inequalities we constructed a table (not reported) showing relative GDP per worker, for 1992 and 2002, and ranked the regions in descending order in terms of their 1992 position. Then, we calculated the Spearman rank correlation coefficient in order to assess if the position in the league table of GDP in 1992 is a good predictor of that position a decade later. The coefficient takes a value of 0.86 indicating that the dominant pattern is persistence but there is also some mobility in relative positions.

In the remainder of the present paper it is analyzed if regional inequalities continue to persist, particularly if they do so after a decade of substantial regional

policy expenditures.

4 Empirical analysis

The outline of the empirical analysis is as follows. In a first step we estimate density functions of relative GDP per worker for different years. This procedure is a way to test the convergence hypothesis by evaluating whether unimodality in the distributions is present or not. For example, if we start with a bimodal (or multimodal) density in a given point in time, convergence implies a tendency of the distribution to move towards unimodality (Bianchi, 1997).

In a second step we estimate transition probabilities to analyze mobility within the GDP distributions. This means, we examine how a given individual of the distribution at a given point in time transits to another part of the distribution in the future. In other words, we analyze if regions move up and down in the ranking of labour productivities.

After this, we calculate the ergodic or invariant density of relative GDP per worker implied by the estimated transition probabilities. This allows to make long run predictions on the GDP distribution in reunified Germany.

4.1 Density functions of relative GDP

Nonparametric kernel techniques (Silverman, 1986) are used to analyze the external shape of the distribution of German relative labour productivity for two different years, 1992 and 2002.

We employ adaptive kernel methods with flexible bandwidth, which are especially useful to estimate multi-modal densities when a fixed bandwidth estimation may lead to undersmoothing in areas with only sparse observations and to oversmoothing in others.[9]

A two-step procedure is used to estimate the adaptive kernels. First, an initial (or pilot) estimate of the probability density function with fixed bandwidth is computed. Then, this pilot estimate is used to adapt the size of the bandwidth over the data points when computing the final kernel density. To illustrate, let λ_i denote the local bandwidth factor at each sample point which are proportional to

[9]The following brief description of adaptive kernel methods is based on Van Kerm (2003).

the square root of the underlying density functions at the sample points:

$$\lambda_i = \lambda(x_i) = \left(G/\tilde{f}(x_i)\right)^{0.5} \tag{1}$$

where x_i are the data points, G is the geometric mean over all i of the pilot density estimate $\tilde{f}(x)$. The local bandwidths are then computed as $h_i = h \cdot \lambda_i$, where h is the fixed bandwidth of the pilot estimate.

A crucial point in nonparametric econometrics is the choice of the (pilot) bandwidth h. The larger the value of h, the smoother is the density estimate. Among several possibilities to select, we choose the smoothing parameter to be

$$h = 0.9An^{-1/5} \tag{2}$$

where $A = \min(\text{standard deviation, interquartile range}/1.34)$. This bandwidth criterion has been recommended by Silverman (1986) and has been used by almost all related studies.[10]

Figure 2 shows the estimated density functions of relative GDP per worker for the initial and final year of the sample period using a Gaussian kernel. The densities are normalized so that the areas under the graphs integrate to unity.

In 1992, the distribution is clearly bimodal. The first mode is at 0.51 and the second is at 1.01 times the German labour productivity. Regions in the productive cluster have twice the income of those in the other group.[11] In our setting, we suggest that the cluster of poor regions in 1992 is mainly formed by regions located in the new federal states because of their low GDP levels after German reunification. We will examine this issue below.

A decade later, in 2002, the density has changed substantially. The two peaks of the distribution correspond to 0.76 and 0.93 times the German GDP per worker. Possibly, the left peak is not a significant mode in the distribution anymore. Anyway, there is considerably weaker evidence for a clustering of the poor regions in comparison to 1992. It seems as if most poor regions have increased their relative

[10] To check the robustness of our results, we alternatively employ the '2 stage solve the equation' bandwidth selection method for the estimation of our preferred specification in Section 4.3.

[11] One can perform bootstrap multimodality tests as in Bianchi (1997) to formally test for two peaks, but we believe the figure speaks for itselves and there are no doubts about the presence of *exactly* two peaks.

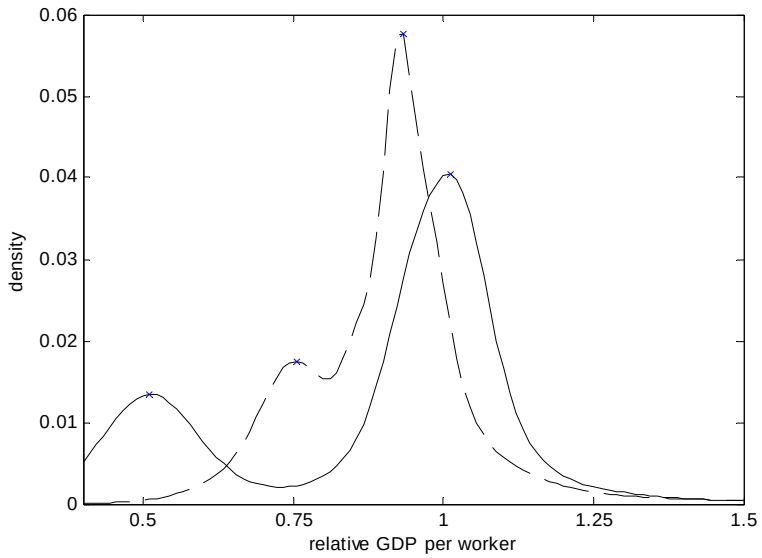

Figure 2: Densities of relative GDP per worker across 271 German labour market regions, 1992 and 2002.

GDP. The apparent convergence across German regions is reflected by the distance between the peaks, which decreases from 0.50 in 1992 to 0.18 in 2002.

To illustrate further the convergence between German labour market regions, we compute the variances of the two estimated distributions as

$$variance = \sum \left(u - \sum u \cdot f(u) \right)^2 \cdot f(u), \tag{3}$$

where u is the grid on the x-axis and $f(u)$ is the normalized density of u. If there is convergence, the dispersion of the density will tend to fall over time.

In 1992, the variance of the distribution is 0.051 and it decreases to 0.018 a decade later. Hence, we conclude that the relative GDP per worker distribution has become more equal over time.

Since we suppose that the observed tendency towards convergence in reunified Germany is primarily driven by the catching-up process of East German regions, it is interesting to analyze the shape of the GDP distribution for West- and East Germany separately. This experiment allows us to assess if there is also convergence

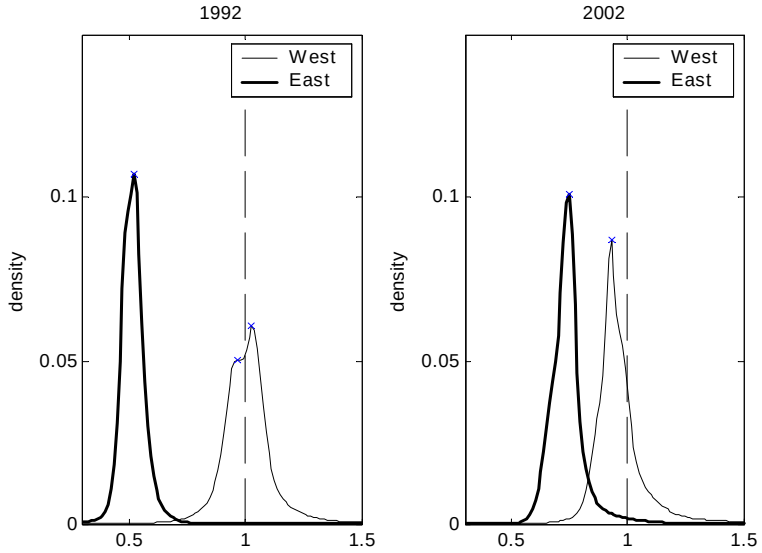

Figure 3: Densities of relative income per worker for West- and East German regions separately, 1992 and 2002.

within the Western part of Germany.

Figure 3 shows the kernel-smoothed densities of relative GDP per worker for the Western and Eastern part of Germany separately. There are 204 West German and 67 East German labour market regions. We evaluated the density for West and East German regions at the same values, so that the two graphs can be compared in one figure.

Consider the densities for East German regional labour productivities first. In 1992, the peak of the distribution is at 0.52 times the German average. As expected, this mode roughly corresponds to the left peak in the distribution for all German labour market regions as displayed in figure 2 and provides evidence that the cluster of poor regions in figure 2 is mainly formed by East German regions. In 2002, the distribution has shifted to the right; the peak is now at 0.75 times the German average. East German labour market regions have increased their relative productivity over time. However, most East German regions still have considerably lower productivity levels than the national average. This finding is well compatible with other studies, such as Barell and te Velde (2000).

The density for West German regions did not change very much. The peak with the highest density corresponds to 1.03 times the German average in 1992 and it decreases to 0.93 in 2002. Nearly the same peaks were obtained for the GDP distribution of all German regions as displayed in figure 2. This confirms the presumption that the West German regions cluster together in the center of the distribution of all German regions.[12]

4.2 Transition dynamics

So far we have analyzed the (external) shapes of relative labour productivity distributions for two different years. Obviously, the densities have fluctuated but we could not say anything on movements of individual regions in the distribution. However, for describing convergence it is important to have information on how units move within the distribution. Generally, a broad range of intra-distribution dynamics is possible, for example, over time there are some initially rich regions falling behind; poor regions overtaking the rich; and groups of regions, beginning at similar levels of development, eventually diverging (Quah, 1996a).

In this section we analyze intra-distribution mobility by developing a probability model of transitions which captures the distribution's law of motion. This means, we examine how a given individual of the distribution at time t (e.g. 1992 or some other year) transits to another part of the distribution by the time $t + \tau$ (e.g. 2002).

One possibility to examine transition probabilities is to discretise the income space and then count the observed transitions out of and into distinct discrete cells of a Markov transition probability matrix (Quah, 1993). However, Bulli (2001) has shown that any arbitrary discretisation of the state space alters the probabilistic properties of the data. A better approach is not to use a discretisation at all but instead allowing the number of cells of the Markov transition probability matrix to tend to infinity (Quah, 1997). In this continuous case, the transition probability "matrix" becomes a stochastic kernel. In simple words, the kernel is a big non-negative matrix whose rows sum to unity, satisfying regularity conditions to ensure that a limiting distribution exists (Quah, 2001).

[12] Again, the apparent convergence between the East and the West of Germany is reflected by the declining distance between the peaks in the distributions.

Assuming that the process describing the evolution of the distribution is time-invariant and first-order Markov, the relationship between the two distributions can be written as[13]

$$f_{t+\tau}(z) = \int_0^\infty g_\tau(z|x) f_t(x) dx \tag{4}$$

where $g_\tau(z|x)$ is the $\tau-$period ahead density of z conditional on x. In other words, the stochastic kernel $g_\tau(z|x)$ maps the distribution from period t to period $t+\tau$. Note that the distribution in time $t+\tau$ depends only on t and not on the distribution prior to t. The kernel shows the probability that a given region transits to a certain state (rich or poor) of relative GDP *given* that it is in a certain state of relative GDP in the starting period.

As a starting point, we set $\tau = 10$ and estimate $g_{10}(z|x)$ using adaptive kernel methods. This means, x is relative productivity in 1992 and z is the same variable a decade later. To estimate the kernel, we first estimate the joint density of z and x. Then, we compute the marginal density of x by integrating over z. The ratio of the joint density to the marginal density provides the estimate of $g_{10}(z|x)$.

Figure 4 shows a three-dimensional plot of the estimated conditional density $g_{10}(z|x)$. The vertical axis measures the density for each pair of points in the 1992-2002 space. The lines that run parallel to the 2002 axis reveal the probability of transiting from the corresponding point in the 1992 axis to any other point ten years ahead. Note that the Markov interpretation is possible because on both axes the same possible values (the income states) are reported.

The dynamics of the regional distribution can be seen more clearly from a contour plot of the surface of the bidimensional density. The lines in Figure 5 connect points at the same density on the three-dimensional graph.

To interpret the figure recall that one can recover the probability density function associated with any point in the 1992 axis by slicing across the figure from this specific point, parallel to the 2002 axis. This projection is similar to one single row of a Markov transition matrix in which all entries sum up to one (Andrade et al., 2004). If all mass were concentrated around the 45°-diagonal there would be complete persistence (no mobility) in the distribution because different parts

[13] This simplified presentation of Quah's (1997) methodology was proposed by Johnson (2000, 2005). It can also be found in Durlauf, Johnson, and Temple (2005).

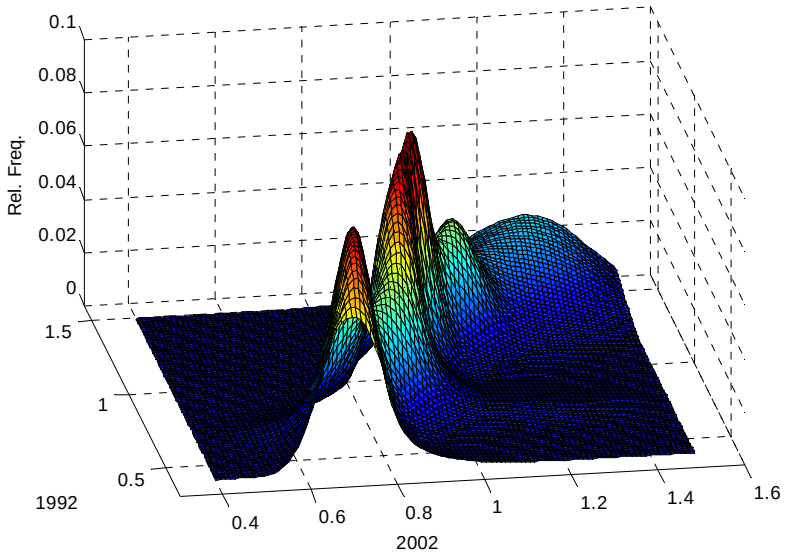

Figure 4: Bidimensional conditional density, $g_{10}(y|x)$, transition 1992-2002.

of the distribution remain where they begin. In contrast, convergence manifests in the kernel if most probability mass were concentrated parallel to the 1992 axis.

Figure 5 tells that most of the mass of the conditional distribution lies *below* the 45° line for values of relative productivity *less* than 1 and *above* the line for values *greater* than 1. This means that regions with GDP below the German average in 1992 tend to have increasing relative GDP over the 10-year horizon. Similarly, regions with incomes above the average tend to have decreasing relative incomes. This pattern is consistent with a tendency towards convergence and confirms the results of our previous univariate density analysis.

The interesting question is to which level of relative GDP per worker the regions are likely to converge. Examining the local maxima in the conditional distribution (indicated with stars) we suppose that most regions tend to congregate at lower relative GDP levels than the national average. In order to perform a more formal analysis of the long run distribution of regional GDP we calculate the ergodic density implied by our estimates.[14]

[14]The only studies we are aware of which also employ the continous kernel approach and calculate the implied ergodic density are those of Johnson (2000, 2005).

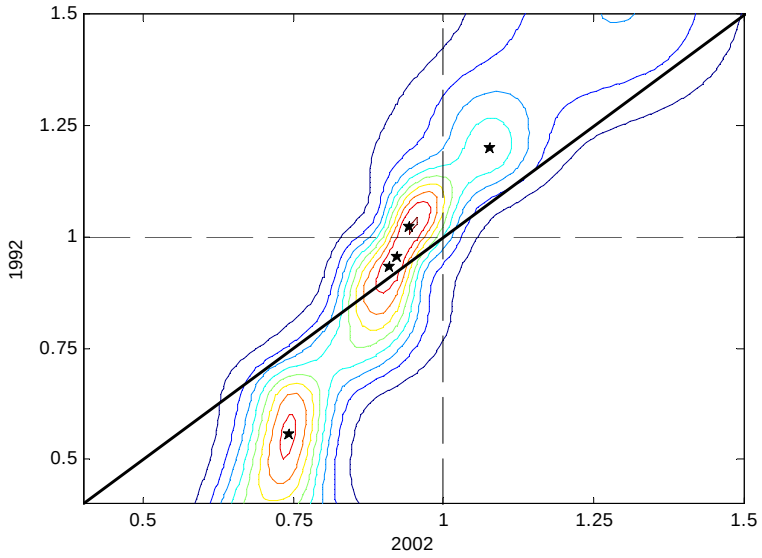

Figure 5: Contour plot of $g_{10}(y|x)$, transition 1992-2002.

4.3 Long run predictions

This far we have analyzed the distribution dynamics only inside the sample considered. Now we turn to a an out-of-sample extrapolation.

Given an estimate for $g_\tau(z|x)$ one can compute the implied long run or ergodic density of relative GDP (so long as it exists). The long run density, $f_\infty(z)$, is the solution to

$$f_\infty(z) = \int_0^\infty g_\tau(z|x) f_\infty(x) dx. \tag{5}$$

If there is long run convergence towards German GDP per worker the ergodic density is strongly unimodal with a mean close to one. In contrast, multiple peaks in the ergodic distribution provide evidence for the persistence of convergence clubs in the long run. In this case, some regions catch up with one another but only within particular subgroups (Baumol, 1986).

This paper suggests two methods to solve for the ergodic density, $f_\infty(z) = f_\infty(x)$. An intuitive approach is to multiply the transition probability kernel $g_\tau(z|x)$ many times with itself until the density has converged. Using this iterative procedure, observed transition probabilities are projected further into the future.

The second way is related to an eigenvector and eigenvalue problem. From inspection of (5) it can be seen that the stationary distribution can be represented as an eigenvevtor of $g_\tau(z|x)$ corresponding to the eigenvector one.[15] We checked that both ways to compute the ergodic density, yield the same result in our application.

An important aspect of the limiting distribution is that it is independent of initial conditions (Quah, 2001). Hence, the stationary distribution shows the probability of becoming a poor, middle or rich region independent of the starting value of relative productivity. Therefore, one has to keep in mind that the ergodic density does not allow inference which labour market regions form the different clusters (if there are any).

Moreover, we have to assume that the observed law of motion is stable over time. We should be aware of the fact that we can interpret the long run density only as showing the likely outcome given that the realized transitions characterise future developments. Under the assumption that the law of motion is affected by regional policy we suggest to interpret the time-invariance assumption as "unchanged" regional policy expenditures. Alternatively, we can relax this assumption and interprete the ergodic density more technically as the likely long run outcome given that the observed dynamics (which may be influenced by various factors) remain unchanged. Even this more technical view allows important insights for assessing the long run development of regional inequality in reunified Germany.

As a starting point, we calculate the ergodic density implied by $g_{10}(z|x)$ (transition between 1992 and 2002).

The bold line in Figure 6 displays this ergodic density. The distribution looks unimodal and the peak is at 0.91 times the German GDP per worker. This means, the probability is highest to become a below-average income region. The shape of the ergodic distribution provides evidence for long run convergence of relative productivities in Germany because there are no convergence clubs apparent.

The variance of the ergodic density implied by $g_{10}(z|x)$ is 0.009. Since the variance of the actual density of relative GDP per worker in 2002 is 0.018 we could be tempted to conclude that there will be more equality among German

[15] For a more elaborate presentation of this idea see the webappendix of Johnson (2005), downloadable from http://irving.vassar.edu/faculty/pj/pj.htm.

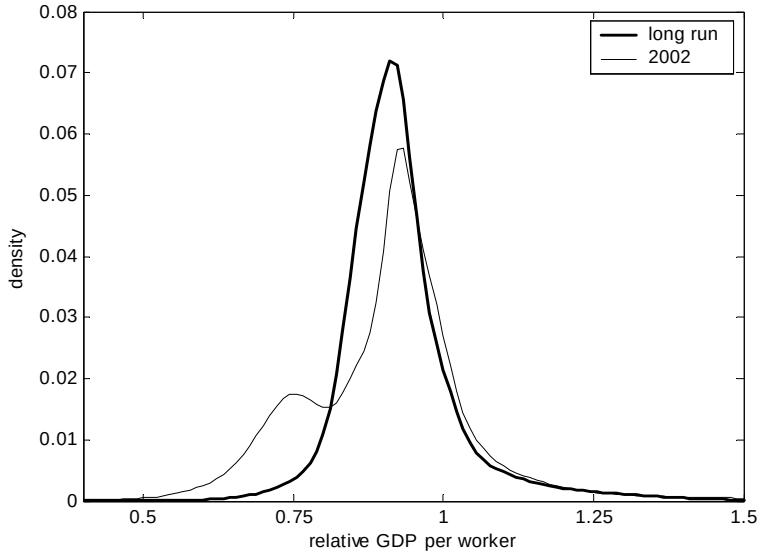

Figure 6: Bold line: Ergodic density of relative GDP per worker, calculated on the basis of $g_{10}(z|x)$ (transition between 1992-2002). Thin line: Actual density of relative GDP per worker in 2002.

labour market regions in the long run than today. This preliminary impression is confirmed by comparing the ergodic density with the actual distribution of relative productivities in 2002, which is displayed as the thin line in Figure 6.

Unfortunately, the computed long run distribution is unlikely to be a realistic forecast because the estimation suffers from two major problems.

First, the time-invariance assumption is unlikely to be fulfilled during the transition period 1992-2002. It is well-known that the catching-up process of East German regions slowed down considerably in the second half of the last decade (see Eckey, 2001 and Barell and te Velde, 2000 for a detailed discussion). For example, Kosfeld and Lauridsen (2004) attribute the lack of significant conditional convergence obtained in their cross-sectional study refering to the year 2000 to the apparent convergence slowdown.

Consequently, a long run forecast based on the transition period 1992-2002 is likely to overstate the extent of convergence because the comparatively fast initial catching-up process of East-German regions in the first years after German

reunification is unlikely to be representative for future periods. It is therefore more convincing to exclude the first years after German reunification from the analysis, which were turbulent years after the political turn. The period 1994-2002 is characterised by more stability and hence observed transitions during this time should allow more realistic predictions about the long run.

The second problem is that we have computed the ergodic density on the basis of one observed transition for a longer time period (10 years) but with only relatively few observations (271). Obviously, it is difficult to detect multiple modes in the distribution if the sample size is comparatively small. To increase the efficiency of the estimation, we now vary the assumed frequency of transitions from multiple years to annual and then pool the observed transitions. This procedure is strongly recommended by Quah (2001, 308) because taking transition steps with long time intervals instead of annual frequencies is likely to be "correspondingly noisy, with even fewer observations informing the estimate". Note that the pooling of transitions is possible because the probabilities of transitions only depend on the length of the time step and not on the point in time when this step is taken.

In order to obtain a more realistic estimate for the long run distribution of relative GDP, we estimate the law of motion for $g_1(z|x)$ based on one-year transitions during 1994 to 2002, which means we pool the transitions 1994-1995, 1995-1996, and so on. The sample now consists of 2168 observations (271 labour market regions times 8 observed transitions).

The thick line in Figure 7 displays this ergodic density evaluated at the same values as the ergodic density based on $g_{10}(z|x)$.

The important finding is that *not* all regions become eventually equal to one another in the long run. In contrast to the implied ergodic density reported before, our preferred specification as displayed in Figure 7 lacks the optimistic view of long run convergence towards a unimodal GDP distribution. We find a tendency of the cross-regional income distribution to converge to a long run distribution having two clusters, an outcome which can be called polarisation (Quah, 1997). The left mode in the distribution is at a relative labour productivity of only 0.73 times the national average. The peak with the highest density corresponds to 92% of the German average.

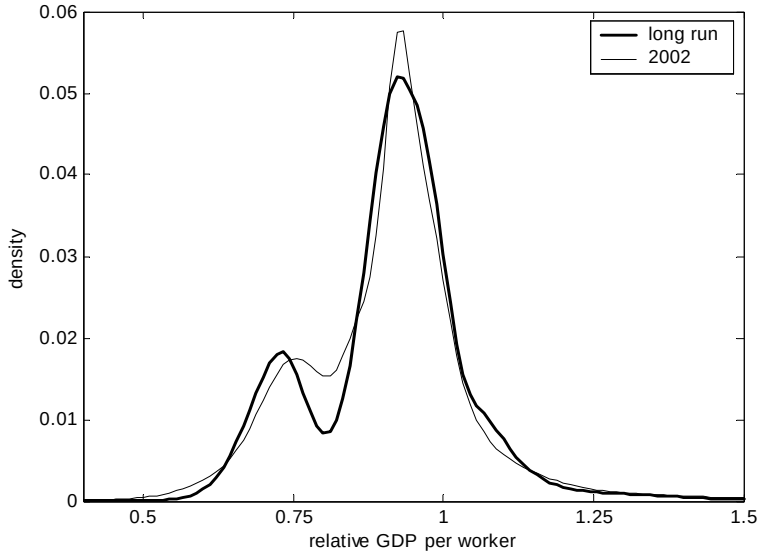

Figure 7: Bold line: Ergodic density of relative GDP per worker, calculated on the basis of $g_1(z|x)$ (yearly transitions between 1994-2002). Thin line: Actual density of relative GDP per worker in 2002.

One can examine the long run distribution only visually because the continuous approach provides no formal statistical tests (Quah, 2001). This means, there is "as yet" no theory of inference for testing hypotheses on the distributions (Quah, 2001). It could be that the data are also consistent with other limiting descriptions, but the important insight of our study is that we do *not* find evidence for long run equalisation of relative GDP, which can be regarded as one important objective of regional policy in Germany.[16]

As a robustness test of our results we smooth the data by using a logarithmic transformation of the relative GDP per worker variable. The log transformation, which is frequently applied in related studies, affects the shape of the density distribution of the original data. The ergodic density computed for the transformed data still exhibits twin-peaked polarisation and the results are very similar.[17]

Moreover, we checked if our results are sensitive to the assumed frequency

[16]The reader is reminded that the analysis cannot account for regional cost-of-living differences.

[17]The left peak corresponds to 0.73 and the right peak to 0.93 times the average. The figure is delegated to the referee appendix.

of transitions. Again, it turned out that the results are robust. We tried using annual transitions beginning in 1995 instead of 1994 and two-year instead of one-year transitions. In both cases, the ergodic density shows the same pattern of polarisation and the estimated peaks do not differ from the values of our preferred specification reported above.[18]

Lastly, we tested if the preferred specification of the ergodic density is exceedingly sensitive to the employed bandwidth selection method. A more sophisticated '2 stage solve the equation' estimate of the pilot bandwidth does not significantly change our results.[19]

The thin line in figure 7 shows the actual density of relative GDP in 2002 evaluated at the same grid points. Comparing the ergodic density with the actual one gives us interesting insights: for relative GDP greater than 0.92 times the German average (the right peak) the two densities look very similar. In this part of the distribution there will not be much of a change in the future. The most striking difference to the actual distribution in 2002 is that the long run distribution shows a more pronounced clustering of regions with below average productivity, which means that multimodality is less pronounced in the actual point-in-time distribution of the year 2002. It is an alarming result of our study that we predict an increased tendency towards divergence again in the future.

We also performed the whole analysis for relative GDP per capita instead of GDP per worker. Due to space limitations we only replicate the most important results.

Figure 8 shows the ergodic density of relative GDP per capita estimated with the same specification as used above. This means, the law of motion is estimated for $g_1(z|x)$ based on one-year transitions during 1994 to 2002. The three peaks are at 0.60, 0.78 and 0.90. As for GDP per worker data, the long run estimates suggest a more pronounced clustering as in comparison to the actual distribution in 2002. In accordance with our previous results, this pattern is particularly apparent for low relative GDP per capita levels.

We conclude that equalisation of GDP per capita is unlikely to be achieved in

[18] The figures are delegated to the referee appendix.

[19] The figure is delegated to the referee appendix.

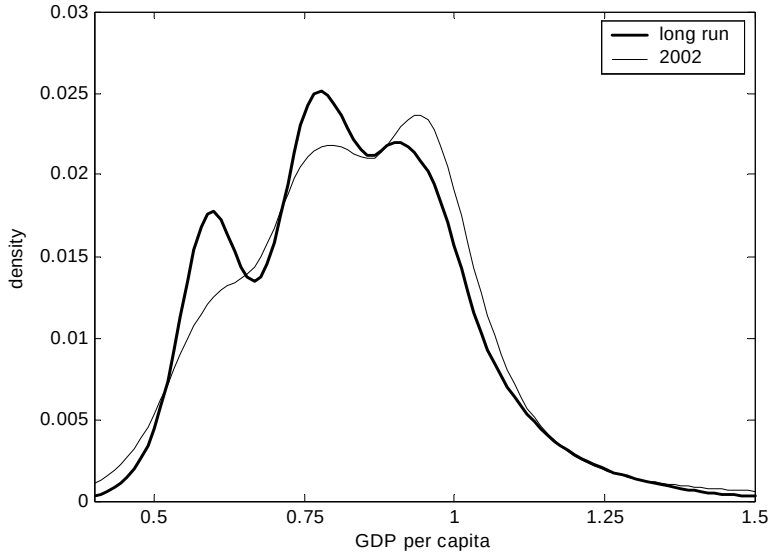

Figure 8: Bold line: Ergodic density of relative GDP per capita, calculated on the basis of $g_1(z|x)$ (yearly transitions between 1994-2002). Thin line: Actual density of relative GDP per capita in 2002.

the future, if the observed law of motion of the distribution remains stable over time. The findings for GDP per capita are well compatible with the previous results obtained using GDP per total employment data.

5 Conclusion

Our analysis provides evidence for convergence across German labour market regions during 1992 to 2002. Extremely poor regions in 1992 tend to have increasing GDP while extremely rich regions tend to have decreasing GDP per worker. The convergence process is driven mostly by the catching-up of East German regions in relative terms. We derived our results using a continuous state-space approach to Markov transition functions.

However, German regions are unlikely to converge towards *equality* in the long run. This means, that regions will not become equal to one another in terms of GDP per worker. Rather, our long run estimates predict a persistent inequality for the future. A similar result of long run stratification is obtained using GDP

per capita data.

Taking the most benevolent view of German regional policy, one can interpret the evidence for convergence during 1992 to 2002 as a success. Yet, our study suggests that the aim of long run equalisation will not be achieved given that the observed law of motion of the regional GDP distribution remains unchanged. According to our analysis convergence is unlikely to proceed further in the future. Quite the contrary, there will be an increased tendency towards polarisation.

Under the assumption that policy has an influence on the the law of motion of the distribution, one can either conclude that a continuation of past and current efforts is not sufficient or that policy expenditures are ineffective. In both cases, the substantial regional policy expenditures made by the German government and the EU need to be reviewed critically.

As mentioned in the beginning of the paper one has to keep in mind that differences in GDP are only imperfect proxies for differences in living standards. Obviously, GDP does not correspond to income and in particular it does not correspond to consumption. A promising but difficult task for future research is to to find better proxies for real disposable income on the regional level. Comparing the results obtained with income or consumption data with the results of the present study allows to link the distribution dynamics approach to theories of consumption risk-sharing, which has not been done yet in the literature.

The aim of this study was to describe the extent of regional convergence in Germany. An obvious next step in the analysis is to try to explain the observed polarisation outcome in Germany. As Quah (1997) and others have shown it is possible to extent the distribution dynamics approach to allow for conditioning variables. A first step would be to stick to the usual suspects of growth such as human capital and investment. It remains to be investigated if the polarisation outcome can be explained by conditioning the GDP distributions on variables which account for differences in steady states. Moreover, controlling for spatial spillover effects may improve our understanding of the polarisation outcome in Germany. We leave these interesting questions for further research.

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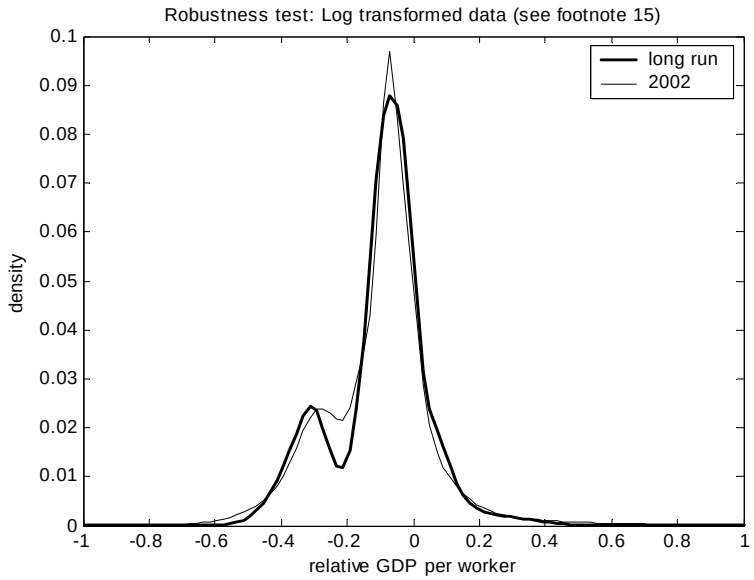
A Appendix for the referee

This appendix is not intended for publication.

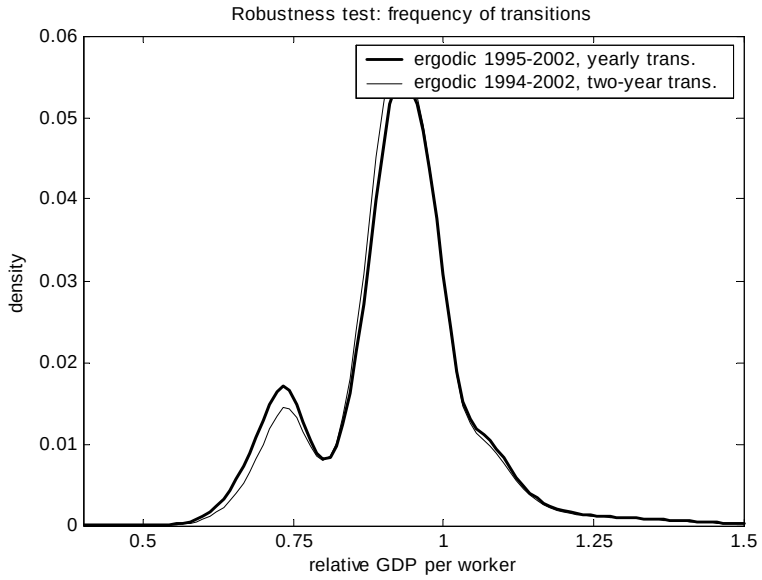
A.1 Robustness test: log transformed data



Bold line: Ergodic density of log relative GDP per worker, calculated on the basis of $g_1(z|x)$ (yearly transitions between 1994-2002). Thin line: Actual density of log relative GDP per worker in 2002.

For the log transformed data, the peak of the ergodic distribution correspond to $\exp(-0.3131) = 0.7312$ and $\exp(-0.0707) = 0.9317$.

A.2 Robustness test: frequency of transitions

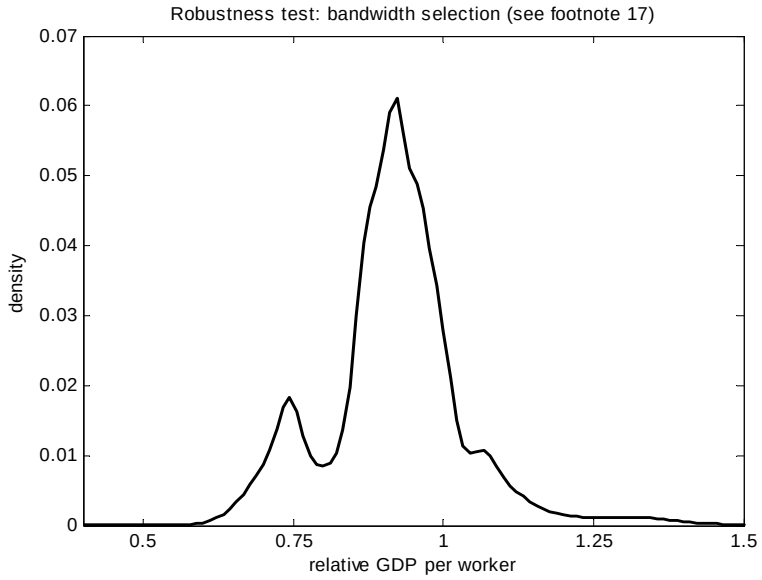


Bold line: Ergodic density of relative GDP per worker, calculated on the basis of $g_1(z|x)$ (yearly transitions between 1995-2002). Thin line: Ergodic density of relative GDP per worker, calculated on the basis of $g_2(z|x)$ (two-year transitions between 1994-2002).

The first alternative specification uses one-year transitions beginning in 1995 instead of 1994. There are 1897 observed transitions. The peaks correspond to 0.7333 and 0.9222.

The second alternative specification uses two-year transitions between 1994 and 2002, which means we pool the transitions 1994-1996, 1995-1997, and so on. There are 1897 observed transitions. The peaks correspond to 0.7333 and 0.9222.

A.3 Robustness test: bandwidth selection



Ergodic density of relative GDP per worker, calculated on the basis of $g_1(z|x)$ (yearly transitions between 1994-2002). Bandwith selected according to '2-stage solve the equation' method.

This specification uses an alternative bandwidth selection method ('2 stage solve the equation' estimate). The peaks are at 0.7444 and 0.9222 (the peak at 1.0667 is probably insignificant and should not be over-interpreted).