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Intercity Trade and the Industrial Diversification of Cities

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Abstract

The industrial diversification of cities is explained without imposing linkages among industries. In each of two city-industries, a manufacture is produced competitively as the final good using labor and industry-specific differentiated services. Manufacturers import the services of their industry from all cities that produce them, since their technology favors variety. In specialized cities, the city-industry is large and many services are locally available but the two manufactures have to be traded among cities. In diversified cities the two manufactures are produced in the same city, and each industry crowds out half the local services of the other, but manufactures need not be imported. A lower cost of trading manufactures (e.g. railroads and intercity highways) favors a system of specialized cities, while a lower cost of trading services (e.g. telephone, the Internet) favors a system of diversified cities since the latter cities rely more on imported services, having fewer locally. A larger cost-share of services favors specialization, and high intracity commuting cost and population growth favor diversification.

Intercity Trade and the Industrial Diversification of Cities[1]

Alex Anas and Kai Xiong

1. Introduction

A central issue in urban economic theory is how cities become diversified or specialized in industry mix. There are models of a system of specialized cities by Henderson [9], Helsley and Strange [11], Henderson and Abdel-Rahman [10]. In these models, there are increasing returns within individual industries in a city (localization economies) but no economies of scope across industries (urbanization economies). In Abdel-Rahman [2] and Abdel-Rahman and Fujita [3], diversification is explained by economies of scope among industries. But all these models share a common limitation: trade among cities is not modeled. In Abdel-Rahman [4], intercity trade of final goods is modeled, but the intermediate goods are not tradable between cities. On the other hand, trade and transportation cost are important components in the New Economic Geography (Krugman [17], Fujita, Krugman and Venables [15]), but there are no intermediate goods (services), only final goods. As a result, the NEG does not explain how railroads or the inter-state highway system (which reduced the transport cost of manufactures) and the Internet (which revolutionized how information in the service sector is now distributed) can have different impacts on city structure. The NEG also ignores urban land markets.

We will explain specialization or diversification by focusing on the intercity transport costs for manufactures and services, and without imposing a technological economy of scope between industries. We follow the monopolistic competition and product differentiation paradigm in modeling urban agglomeration (Dixit and Stiglitz [7], Ethier [8], Abdel-Rahman [1], Fujita [12], Rivera-Batiz [18]), but we extend this line of models in two ways.

First, differentiated intermediate goods (services) are specific to a manufacturing industry, but are tradable among different cities at some transport cost. In the literature, all models using the presence of a technological bias for the variety of intermediate inputs to explain urban agglomeration assume that these goods are not tradable among cities

(Abdel-Rahman [1], Rivera-Batiz [18]). This ignores the fact that intercity trade of intermediate goods introduces pecuniary links among cities, creating positive intercity production externalities.[2]

Second, we assume that the intercity movement of final goods (manufactures) and intermediate goods (services) incur different transport costs. While the intercity transport of manufactures requires physical movement, the intercity transport of many services can take place through telecommunication or direct face-to-face contact. The communication revolution including the Internet improved intercity information flows. Since services are information intensive, their transport cost is lowered relative to that of manufactures.

In our model, an industry consists of a manufactured good and of specialized services used as inputs to produce it. Two types of manufactures are produced, and each city can specialize in producing one of them or diversify and produce both. The two manufactures use mutually exclusive (industry-specific) sets of services and have no direct technological relationship with one another. Following Dixit and Stiglitz [7] and

Ethier [8], we treat manufacturing as competitive and constant returns with labor and industry-specific differentiated services as inputs. Services are produced under increasing returns to scale and are monopolistically competitive.

We use the model to analyze only two symmetric equilibria under the same parameters. In one of these, each city is specialized in one manufacture, importing the other from other cities. Industry-specific services are used from the same city as well as imported from all other cities producing the same manufacture. In the other equilibrium, the cities are diversified and each city produces both manufactures. In this case each city is self sufficient in consumption and manufactures are not imported. However, industry-specific services are still imported from all other cities. We make the two industries symmetric so that the number of cities and specialized versus diversified city populations are the same in each type of equilibrium. We derive the following results.

First, different transportation shocks have different effects on specialization versus diversification. When the transport cost of manufactures decreases, diversified cities do not benefit because they do not incur any intercity transport cost for manufactures in the first place. However, specialized cities gain by spending less on importing the manufacture they do not produce. Hence, such a shock favors the specialization of cities. When the intercity transport cost of services decreases, diversified cities benefit more. The reason is that the co-location of the two manufacturing industries in the same city causes the crowding out of the services of the two industries because of limits on city size while the total system wide services does not change. More precisely, each manufacturing industry is half as big and has only half as many services available locally if it locates in a city with the other industry than if it locates in a city by itself. Therefore each manufacturer in a diversified city is more reliant on services imported from other cities. Therefore, a lowering of the cost of transporting services favors the diversification of cities. Accordingly, the telecommunication revolution and the Internet favor diversification of cities.

Second, the national population growth affects whether cities are specialized or diversified. When population is larger, there are more cities. In this situation, a diversified city can import a larger variety of services and the crowding out effects between industries become less significant. Therefore, diversified cities are favored in larger economies. More generally, our model predicts that when there are pecuniary spillovers among cities, cities might switch patterns on their growth paths even when there are no technological shocks.

Third, each equilibrium may not be optimal. Using numerical methods, we show that, under the same parameter values, the equilibrium configuration can be a system of specialized cities while the optimal configuration can be one of diversified cities. The fundamental reason that equilibrium is not necessarily optimal is the pecuniary externality arising from the intercity trade of the differentiated services that causes the number of cities in the market solution to be too few relative to the optimum.

The paper is organized as follows. Section 2 presents a basic model of a representative city in a system of cities with only one industry. Sections 3 and 4 introduce two industries and derive the equilibria for a system of diversified and specialized cities respectively. Section 5 compares the utilities achieved in the two configurations and Section 6 discusses some extensions.

2. A one-industry city system

We present a simple model of a one-industry city system. The case of two industries will be analyzed in the subsequent sections by drawing on this model.

Consider an economy with a system of n cities where the exogenously given national population is free to locate in any city. Each city produces the same manufacture tradable in the international market and a set of differentiated services tradable domestically among cities. Production of the manufacture is constant returns and uses labor and all service varieties as inputs. City residents have identical Cobb-Douglas utility functions and consume the domestic manufacture and a good imported from the international market. Service production uses only labor and exhibits increasing returns internal to the firm. Production and trading in a city occur in the CBD (the center of the city) and land is not an input in production. Each city resident, on the other hand, consumes a fixed unit of land and is endowed with one unit of time. We assume that cities are set in a hypothetical space where any two cities are equidistant. This last assumption simplifies the analysis by imposing an abstract spatial symmetry.

The manufacture production function has the form proposed by Ethier [8]:

$$X = H_x{}^u \left\{ \left(\sum_{i=1}^{n} \sum_{j=1}^{m_i} z_{d_{ij}}{}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \right\}^{1-u}, \quad 0 < u < 1 \text{ and } \sigma > 1, \qquad (2.1)$$

where, X is the output of a representative manufacturer, H_x is labor, $z_{d_{ij}}$ is the quantity of the jth service purchased from the ith city, n is the number of cities in the system and m_i is the number of services produced in city i. (2.1) exhibits an extreme technological bias for variety in the use of services. Because the marginal product of a service is infinite when it is not employed, the manufacturer will use all of the services available in the city system no matter what the price.

Assume that in equilibrium, all services are symmetric and all cities identical. Let m be the number of services produced in each city, z_{d_i} city i's demand for a service produced in city i and $z_{d_{-i}}$ city i's demand for a service produced in another city. Then (2.1) can be rewritten as

$$X = m^{\frac{\sigma(1-u)}{\sigma-1}} H_x{}^u \{[z_{d_i}{}^{\frac{\sigma-1}{\sigma}} + (n-1) z_{d_{-i}}{}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}\}^{1-u}. \qquad (2.2)$$

The intercity transport cost of a service takes the iceberg form: only a fraction τ ($0 < \tau < 1$) of the good arrives while the rest "melts" during transport. Let q be the price of a service produced and used in city i. Due to iceberg transport, the effective price of this service in any other city is $\frac{q}{\tau}$.[3]

Let the competitive wage in the city be w and the world price of the manufacture be P_x. The profit maximization problem of a manufacturer is

$$\max_{H_x, z_{d_i}, z_{d_{-i}}} \pi = P_x X - mqz_{d_i} - (n-1)m\frac{q}{\tau} z_{d_{-i}} - wH_x, \qquad (2.3)$$

subject to $H_x, z_{d_i}, z_{d_{-i}} \geq 0$, where X is given by (2.2). The first order conditions are:

$$P_x \frac{uX}{H_x} = w, \tag{2.4}$$

$$P_x \frac{(1-u)X}{z_{d_i}^{\frac{\sigma-1}{\sigma}} + (n-1)z_{d_{-i}}^{\frac{\sigma-1}{\sigma}}} z_{d_i}^{-\frac{1}{\sigma}} = mq, \tag{2.5}$$

$$P_x \frac{(1-u)X}{z_{d_i}^{\frac{\sigma-1}{\sigma}} + (n-1)z_{d_{-i}}^{\frac{\sigma-1}{\sigma}}} z_{d_{-i}}^{-\frac{1}{\sigma}} = m\frac{q}{\tau}. \tag{2.6}$$

Following Dixit and Stiglitz [7], we assume that the market structure of services is monopolistically competitive with labor the only input in production. The labor requirements for service producers are identical and technology is increasing returns:

$$H_z = f + cz_s, \tag{2.7}$$

where H_z is the total labor requirement of a service producer, f is the fixed labor requirement, c is the marginal labor requirement and z_s is the service output. Marginal cost must equal marginal revenue, and firms must make zero profit in a long run Chamberlin equilibrium with differentiated products. Let E be the price elasticity of manufacturers' demand for a service. With $E = \sigma$, [4] we can solve the price of a service given the city's wage, w:

$$q = \frac{\sigma c}{\sigma - 1} w . \tag{2.8}$$

The output of a service producer is:

$$z_s = \frac{f(\sigma-1)}{c}. \tag{2.9}$$

Substituting z_s into equation (2.7), we get the total labor demand of a service producer:

$$H_z = f\sigma. \tag{2.10}$$

In equilibrium, the number of services, m, in each city and the demand for services by manufacturers, z_{d_i} and $z_{d_{-i}}$, are such that the service markets clear and full employment is attained in all cities. Hence:

$$z_{d_i} + (n-1)\frac{z_{d_{-i}}}{\tau} = z_s\,, \tag{2.11}$$

$$H_c = H_x + mH_z\,, \tag{2.12}$$

where H_c is the city's supply of labor and is assumed given for now.

In equations (2.4)-(2.6) and (2.8)-(2.12), we have eight equations in the eight variables: w, q, z_s, z_{d_i}, $z_{d_{-i}}$, H_z, H_x and m. The number of services in a city, the city's manufacturing output and the wage implied by these equations are:

$$m = \frac{1-u}{f\sigma} H_c\,, \tag{2.13}$$

$$X = \lambda \delta^{\frac{1-u}{\sigma-1}} H_c^{\frac{\sigma-u}{\sigma-1}}\,, \tag{2.14}$$

$$w = \lambda P_x \delta^{\frac{1-u}{\sigma-1}} H_c^{\frac{1-u}{\sigma-1}}\,, \tag{2.15}$$

where, $\delta \equiv 1+(n-1)\tau^{\sigma-1}$, and $\lambda \equiv (\frac{1-u}{f\sigma})^{\frac{\sigma-u}{\sigma-1}} (\frac{u}{1-u}\frac{c\sigma}{\sigma-1})^{u} \frac{f(\sigma-1)}{c}$.

Now let us consider the consumption side. Two goods are consumed in the economy: the domestic manufacture, x, and the imported one, y. The consumer chooses quantities of x and y to maximize utility. The utility function is:

$$U = (\alpha^{-\alpha}\beta^{-\beta})x^{\alpha}y^{\beta}\,,\ 1 > \alpha\,,\ \beta > 0 \text{ and } \alpha+\beta=1, \tag{2.16}$$

Then, given the world prices P_x, P_y, and the disposable income of a city resident, $I(N)$, contingent on city population, N, the indirect utility function is:

$$U(N) = P_x^{-\alpha} P_y^{-\beta} I(N). \qquad (2.17)$$

Assume public land ownership. A city's aggregate differential rents are redistributed equally among that city's residents. Assume that the base land rent is zero and that each city resident consumes one unit of land (land demand is perfectly inelastic). With this fixed-lot-size assumption, if city population is N and the city is circular, the radius r_f is:

$$r_f = N^{\frac{1}{2}} \pi^{-\frac{1}{2}}. \qquad (2.18)$$

Assume that intracity commuting time is a linear function of the distance between a residence and the CBD. Let the time endowment for a city resident be 1. So if a city resident lives at distance r from the CBD, his labor supply after commuting is:

$$H(r) = 1 - tr, \qquad (2.19)$$

where t is the intracity time-cost of commuting per unit distance. From equations (2.18) and (2.19), the total labor supply of the city is now found as

$$H_c = \int_0^{r_f} 2\pi r H(r) dr = N(1 - kN^{\frac{1}{2}}) \quad \text{where } k \equiv \frac{2\pi^{-\frac{1}{2}} t}{3}. \qquad (2.20)$$

In equilibrium, residents achieve the same income regardless of location. Hence:

$$(1 - tr)w - R(r) = (1 - tr_f)w - R(r_f), \quad 0 \le r \le r_f. \qquad (2.21)$$

The right side of (2.21) is the net income (after transport and rent) for a resident at the fringe of the city and the left side is the net income for a resident at distance r from the CBD. At the city fringe, $R(r_f) = 0$. This gives:

$$R(r) = t(r_f - r)w. \tag{2.22}$$

Integrating this gives the total differential rent (TDR):

$$TDR = \int_0^{r_f} 2\pi r R(r)dr = \frac{kN^{\frac{3}{2}}w}{2}. \tag{2.23}$$

The disposable income for a city resident is his wage income plus his share of redistributed differential rent, less the rent he pays for his lot:

$$I(N) = (1-tr)w + \frac{TDR}{N} - R(r) = (1-kN^{\frac{1}{2}})w. \tag{2.24}$$

Substituting H_c from (2.20) into (2.15), we solve for the city wage:

$$w = \lambda P_x \delta^{\frac{1-u}{\sigma-1}} N^{\frac{1-u}{\sigma-1}} (1-kN^{\frac{1}{2}})^{\frac{1-u}{\sigma-1}}. \tag{2.25}$$

Substituting w from (2.25) into (2.24), we get:

$$I(N) = \lambda P_x [1+(n-1)\tau^{\sigma-1}]^{\frac{1-u}{\sigma-1}} N^{\frac{1-u}{\sigma-1}} (1-kN^{\frac{1}{2}})^{\frac{\sigma-u}{\sigma-1}}, \tag{2.26}$$

and substituting this into (2.17), we get the city's utility level:

$$U(N) = \lambda P_x^{1-\alpha} P_y^{-\beta} [1+(n-1)\tau^{\sigma-1}]^{\frac{1-u}{\sigma-1}} N^{\frac{1-u}{\sigma-1}} (1-kN^{\frac{1}{2}})^{\frac{\sigma-u}{\sigma-1}}. \tag{2.27}$$

From (2.27), it is easy to see that $U(N)$ first increases with N and then decreases with N. When τ increases (i.e. the transportation cost of services decreases), $U(N)$ increases. Also, $U(N)$ increases with the number of cities, n, because adding a city confers a positive externality on all other cities by increasing the diversity of services.

The equilibrium city size is achieved where the utility level of a city resident is maximized given the number of cities, i.e. $\frac{dU(N)}{dN} = 0$ using (2.27), and assuming large total population.[5] From this condition, we get the equilibrium city size, N^*, as

$$N^* = \frac{(1-u)^2}{k^2}\left(1+\frac{\sigma}{2}-\frac{3u}{2}\right)^{-2} = \frac{4\omega^2}{(9\omega^2+6\omega+1)}\frac{1}{k^2}, \tag{2.28}$$

where $\omega \equiv \frac{1-u}{\sigma-1}$. Given a national population, $\overline{N}$, the equilibrium number of cities is $n^* = \frac{\overline{N}}{N^*}$:

$$n^* = \frac{k^2\overline{N}}{(1-u)^2}\left(1+\frac{\sigma}{2}-\frac{3u}{2}\right)^2 = \frac{\overline{N}(9\omega^2+6\omega+1)k^2}{4\omega^2}. \tag{2.29}$$

Substituting (2.28) and (2.29) into (2.27) we could get the equilibrium utility level.[6] Now consider the socially optimal city size. As already noted, there are intercity externalities due to the trading of services, not taken into account by market agents. But suppose that a planner at the national level decides the number (and size) of cities assuming that all cities are identical. Then, the intercity externalities will be internalized. When the planner maximizes the utility level of city residents, he does not take *n*, as given but determines it as $n = \frac{\overline{N}}{N}$. So the planner considers that if there are more cities, each will be smaller. To get the optimal city size, we substitute $n = \frac{\overline{N}}{N}$ into (2.27) and maximize $U(N)$ with respect to *N*. In Xiong [19] it is shown that cities should be smaller in the optimum than they are in the equilibrium. Hence, there should be more cities in the optimum. The intuition is straightforward: positive externalities stem from the number of services. At the individual city level, when a service firm enters the city, it does not take into account that it imposes a positive externality on the manufacturers in that and in other cities. Hence, there are too few services because each firm considers the private marginal product not the social marginal product of its entry. As shown in Xiong [19], a sales subsidy on service output imposed by the planner corrects this externality and achieves the optimum. Developers who set up a new city fail to consider the positive external effect on manufacturers in other cities from the new services of their new city. Hence, the equilibrium does not have enough cities.

3. Equilibrium in a system of diversified cities

Now we introduce two industries into the city system. We assume that the imported manufactured good *y* of section 2 is no longer imported from abroad but is now also produced in the city system and that this second industry also consists of constant returns manufacturers and of differentiated services used by their industry only. In this diversified city system, all cities produce both manufactures and their respective services. Cities are identical and self sufficient in manufactures. Hence, manufactures are not traded but the differentiated services are traded within the same industry among different cities.[7] The equilibrium configuration of the diversified city system is determined in three steps. *First*, within each industry, we determine the output and wage when industry employment is given. *Second*, within each city, we determine the equilibrium city wage and the split of the total labor supply between the two industries given the city's population. *Lastly*, we determine the equilibrium city population, the number of cities and the city system's equilibrium utility level when the total population is given.

The production configuration within each industry is identical to that of the basic model described in section 2. Let the population of a representative diversified city be N. From (2.20), we can get the total labor supply of the city:

$$H_c = N(1-kN^{\frac{1}{2}}). \quad (3.1)$$

Now suppose that N_i of the city's population works in industry i, then the labor supply to industry i is: [8]

$$H_i = \frac{N_i}{N} H_c = N_i[1-kN^{\frac{1}{2}}];\ i = 1,2. \quad (3.2)$$

The total output and the wage of each industry can be derived as we did in section 2. For industry i =1,2:

$$X_i = \lambda_i \delta^{\frac{1-u_i}{\sigma-1}} N_i^{\frac{\sigma-u_i}{\sigma-1}} [1-kN^{\frac{1}{2}}]^{\frac{\sigma-u_i}{\sigma-1}}, \quad (3.3)$$

$$w_i = P_i \lambda_i \delta^{\frac{1-u_i}{\sigma-1}} N_i^{\frac{1-u_i}{\sigma-1}} [1-kN^{\frac{1}{2}}]^{\frac{1-u_i}{\sigma-1}}. \quad (3.4)$$

Where X_i and w_i are city-industry output and wage, n is the number of cities and

$$\lambda_i \equiv (\frac{1-u_i}{f\sigma})^{\frac{\sigma-u_i}{\sigma-1}} (\frac{u_i}{1-u_i} \frac{c\sigma}{\sigma-1})^{u_i} \frac{f(\sigma-1)}{c}.$$

In equilibrium, the wage rates in the two industries are the same:

$$w_1 = w_2 = w. \quad (3.5)$$

From (2.24), the income of a city resident as a function of the wage is:

$$I(N) = (1-kN^{\frac{1}{2}})w. \quad (3.6)$$

Once the income is determined, we can get the Marshallian demands for the two manufactured goods: each city resident spends a portion α of his income on one good and a portion β on the other. Since all cities are identical in equilibrium, both manufacture markets clear locally in each city (no trade) and the sum of workers in the two industries equals the city's population.[9] This gives:

$$N_1 + N_2 = N \, , \tag{3.7}$$

$$N \frac{\alpha I(N)}{P_1} = X_1 \, , \tag{3.8}$$

$$N \frac{\beta I(N)}{P_2} = X_2 \, . \tag{3.9}$$

Setting $P_2 = 1$, from equations (3.3)-(3.9) we can solve for N_1, N_2, P_1, P_2, X_1, X_2, w_1, w_2 and $I(N)$. The solutions for N_1, N_2 and P_1 are:

$$N_1 = \alpha N \, , \tag{3.10}$$

$$N_2 = \beta N \, , \tag{3.11}$$

$$P_1 = \frac{\lambda_2}{\lambda_1} \frac{\beta^{\frac{1-u_2}{\sigma-1}}}{\alpha^{\frac{1-u_1}{\sigma-1}}} \delta^{\frac{u_1-u_2}{\sigma-1}} N^{\frac{u_1-u_2}{\sigma-1}} [1 - kN^{\frac{1}{2}}]^{\frac{u_1-u_2}{\sigma-1}} \, . \tag{3.12}$$

Substituting P_1 (P_2=1) and w_1 into the indirect utility $U(N) = P_1^{-\alpha} P_2^{-\beta} (1 - kN^{\frac{1}{2}}) w_1$, we express the utility of the diversified city system in terms of city population, N:

$$U_d(N) = \lambda_1^{\alpha} \lambda_2^{\beta} \alpha^{\frac{\alpha(1-u_1)}{\sigma-1}} \beta^{\frac{\beta(1-u2)}{\sigma-1}} \delta^{\frac{1-\alpha \cdot u_1 - \beta u_2}{\sigma-1}} N^{\frac{1-\alpha \cdot u_1 - \beta u_2}{\sigma-1}} [1 - kN^{\frac{1}{2}}]^{\frac{\sigma-\alpha \cdot u_1 - \beta u_2}{\sigma-1}} \, . \tag{3.13}$$

The equilibrium city population maximizes the city utility; i.e. $\frac{dU_d(N)}{dN} = 0$.

From (3.13), differentiating $U_d(N)$ with respect to N, we get equilibrium city size N_d^*:

$$N_d^{\ *} = \left\{ \frac{1-\alpha u_1 - \beta u_2}{k[1+\frac{\sigma}{2}-\frac{3}{2}(\alpha u_1 + \beta u_2)]} \right\}^2 . \tag{3.14}$$

Since the national population is $\overline{N}$, the equilibrium number of cities, $n_d^{\ *}$, is:

$$n_d^{\ *} = \frac{\overline{N}}{N_d^{\ *}} . \tag{3.15}$$

Substituting N_d^* from (3.14) and $n_d^{\ *}$ from (3.15) into (3.13), we get the equilibrium utility of a diversified city system:

$$U_d^* = \lambda_1^{\ \alpha} \lambda_2^{\ \beta} \alpha^{\frac{\alpha(1-u_1)}{\sigma-1}} \beta^{\frac{\beta(1-u2)}{\sigma-1}} [1+(n_d^{\ *}-1)\tau^{\sigma-1}]^{\frac{1-\alpha \cdot u_1-\beta u_2}{\sigma-1}} N_d^{*\ \frac{1-\alpha \cdot u_1-\beta u_2}{\sigma-1}} (1-kN_d^{*\frac{1}{2}})^{\frac{\sigma-\alpha \cdot u_1-\beta u_2}{\sigma-1}} \tag{3.16}$$

4. Equilibrium in a system of specialized cities

There are two trade patterns in this configuration. *First*, cities of different types trade manufactures with each other exporting one manufacture and importing the other. *Second*, cities of the same type trade the services of their shared service industry. We first solve the equilibrium for each type of city when the population of that type of city is given and then analyze the equilibrium size and the equilibrium number of cities of each type when the city-system population is given.

The internal structure of a representative city producing only one manufacture was fully specified in section 2. Suppose that there are n_1 type-one cities each with population N_1 and n_2 type-two cities each with population N_2. Since the two types of cities don't have any direct production relationship, we can use the results of section 2 to get the total output, the equilibrium wage, and the utility level of an individual city of each type. For a city specialized in good i, i =1,2:

$$X_i = \lambda_i \delta_i^{\frac{1-u_i}{\sigma-1}} N_i^{\frac{\sigma-u_i}{\sigma-1}} [1 - kN_i^{\frac{1}{2}}]^{\frac{\sigma-u_i}{\sigma-1}}, \tag{4.1}$$

$$w_i = P_i \lambda_i \delta_i^{\frac{1-u_i}{\sigma-1}} N_i^{\frac{1-u_i}{\sigma-1}} [1 - kN_i^{\frac{1}{2}}]^{\frac{1-u_i}{\sigma-1}}, \tag{4.2}$$

$$U_1(N_1) = \lambda_1 P_1^{1-\alpha} (\frac{P_2}{\theta})^{-\beta} \delta_1^{\frac{1-u_1}{\sigma-1}} N_1^{\frac{1-u_1}{\sigma-1}} [1 - kN_1^{\frac{1}{2}}]^{\frac{\sigma-u_1}{\sigma-1}}. \tag{4.3}$$

$$U_2(N_2) = \lambda_2 P_2^{1-\beta} (\frac{P_1}{\theta})^{-\alpha} \delta_2^{\frac{1-u_2}{\sigma-1}} N_2^{\frac{1-u_2}{\sigma-1}} [1 - kN_2^{\frac{1}{2}}]^{\frac{\sigma-u_2}{\sigma-1}} \tag{4.4}$$

Where $\delta_i \equiv 1 + (n_i - 1)\tau^{\sigma-1}$ and θ is the portion of the final good which arrives at the importing city. Let $I_i(N_i)$ be the income of a resident in a type i city. From (2.24) we know that:

$$I_i(N_i) = (1 - kN_i^{\frac{1}{2}}) w_i, \tag{4.5}$$

For the two types of specialized cities to coexist, utility must be equal across cities, and the national labor market must clear, while the population constraint is satisfied. So we have the equilibrium conditions:

$$U_1(N_1) = U_2(N_2), \tag{4.6}$$

$$n_1 N_1 + n_2 N_2 = \overline{N}, \tag{4.7}$$

$$n_1 N_1 \frac{\alpha I_1(N_1)}{P_1} + n_2 N_2 \frac{\alpha I_2(N_2)}{P_1} = n_1 X_1 \tag{4.8}$$

$$n_1 N_1 \frac{\beta I_1(N_1)}{P_2} + n_2 N_2 \frac{\beta I_2(N_2)}{P_2} = n_2 X_2 \tag{4.9}$$

We can solve $w_1, w_2, I_1, I_2, X_1, X_2, U_1, U_2, P_1, P_2, n_1, n_2$ from the twelve equations (4.1)-(4.9) by setting $P_2 = 1$. The solutions for P_1 and the number of cities are:

$$P_1 = \frac{\lambda_2}{\lambda_1} \frac{\delta_2^{\frac{1-u_2}{\sigma-1}}}{\delta_1^{\frac{1-u_1}{\sigma-1}}} \theta^{(\alpha-\beta)} \frac{N_2^{\frac{1-u_2}{\sigma-1}}[1-kN_2^{\frac{1}{2}}]^{\frac{\sigma-u_2}{\sigma-1}}}{N_1^{\frac{1-u_1}{\sigma-1}}[1-kN_1^{\frac{1}{2}}]^{\frac{\sigma-u_1}{\sigma-1}}}, \tag{4.10}$$

$$n_1 = \frac{\overline{N}}{N_1} \frac{1}{1+\frac{1-\alpha}{\alpha}\theta^{(\alpha-\beta)}}, \tag{4.11}$$

$$n_2 = \frac{\overline{N}}{N_2} \frac{\frac{1-\alpha}{\alpha}\theta^{(\alpha-\beta)}}{1+\frac{1-\alpha}{\alpha}\theta^{(\alpha-\beta)}}. \tag{4.12}$$

Substituting P_1 from (4.10) into (4.3) and (4.4), we get the utility for s =1,2:

$$U_s(N_1, N_2) = \lambda_1^{\alpha} \lambda_2^{\beta} \theta^{2\alpha\beta} \delta_1^{\frac{\alpha(1-u_1)}{\sigma-1}} \delta_2^{\frac{\beta(1-u_2)}{\sigma-1}} N_1^{\frac{\alpha(1-u_1)}{\sigma-1}} [1-kN_1^{\frac{1}{2}}]^{\frac{\alpha(\sigma-u_1)}{\sigma-1}} N_2^{\frac{\beta(1-u_2)}{\sigma-1}} [1-kN_2^{\frac{1}{2}}]^{\frac{\beta(\sigma-u_2)}{\sigma-1}} \tag{4.13}$$

Equilibrium city size is achieved at sizes where $\frac{\partial U_s(N_1, N_2)}{\partial N_1} = 0$ and $\frac{\partial U_s(N_1, N_2)}{\partial N_2} = 0$, taking the numbers of each type of city, n_1 and n_2 as given. From these two conditions we can easily get the equilibrium size of each type of city. For i =1,2:

$$N_i^* = [\frac{1-u_i}{k(1+\frac{\sigma}{2}-\frac{3}{2}u_i)}]^2 = \frac{4\omega_i^2}{(9\omega_i^2+6\omega_i+1)} \frac{1}{k^2} \tag{4.14}$$

where $\omega_i \equiv \frac{1-u_i}{\sigma-1}$. Substituting (4.14) into (4.11) and (4.12), we get the equilibrium number of cities:

$$n_1^* = \frac{\overline{N}}{1+\frac{1-\alpha}{\alpha}\theta^{(\alpha-\beta)}} \frac{k^2(1+\frac{\sigma}{2}-\frac{3}{2}u_1)^2}{(1-u_1)^2}, \tag{4.15}$$

$$n_2^* = \frac{\frac{1-\alpha}{\alpha}\theta^{(\alpha-\beta)}\overline{N}}{1+\frac{1-\alpha}{\alpha}\theta^{(\alpha-\beta)}} \frac{k^2(1+\frac{\sigma}{2}-\frac{3}{2}u_2)^2}{(1-u_2)^2}. \tag{4.16}$$

Substituting n_1^*, n_2^*, N_1^*, N_2^* into (4.13), we get the equilibrium utility level:

$$\begin{aligned} U_s^* = {} & \lambda_1^{\alpha}\lambda_2^{\beta}\theta^{2\alpha\beta}[1+(n_1^*-1)\tau^{\sigma-1}]^{\frac{\alpha(1-u_1)}{\sigma-1}}[1+(n_2^*-1)\tau^{\sigma-1}]^{\frac{\beta(1-u2)}{\sigma-1}} \times \\ & N_1^{*\frac{\alpha(1-u_1)}{\sigma-1}}[1-kN_1^{*\frac{1}{2}}]^{\frac{\alpha(\sigma-u_1)}{\sigma-1}} N_2^{*\frac{\beta(1-u_2)}{\sigma-1}}[1-kN_2^{*\frac{1}{2}}]^{\frac{\beta(\sigma-u_2)}{\sigma-1}} \end{aligned} \tag{4.17}$$

Now let us briefly compare the equilibrium utility levels of the diversified and specialized equilibra, (3.16) and (4.17), respectively. Equation (3.16) has a leading term $\alpha^{\frac{\alpha(1-u_1)}{\sigma-1}}\beta^{\frac{\alpha(1-u_2)}{\sigma-1}}$, which is less than 1. At the same time, equation (4.17) has a leading term $\theta^{2\alpha\beta}$, which is also less than one (since $\theta<1$). In the next section, we will show that these terms capture the benefits of diversification and specialization respectively and that they play a critical role in determining the relative utilities of the two configurations.

5. Efficiency of diversified and specialized city systems

In the above sections we derived the equilibria for a system of specialized and a system of diversified cities when cities are symmetric with one another. We will now compare the two equilibra and see how the exogenous parameters determine which is more efficient.[10]

We will consider two situations. First, we will examine efficiency under the assumption that the agents who set up a city (e.g., a developer such as the one of the

Appendix) do not take into account how the presence of that city affects other cities. Next, we will reexamine efficiency under the assumption that the central planner determines the number of cities and, hence, also the size of each city.

To simplify the analysis, we will assume that $u_1 = u_2 = u$ and $\alpha = \beta = 0.5$, making the two industries symmetric.[11] Substituting these into (3.14) and (4.14), we get that city sizes in the diversified and specialized equilibria are the same:

$$N_d{}^* = N_1{}^* = N_2{}^* = [\frac{1-u}{k(1+\frac{\sigma}{2}-\frac{3}{2}u)}]^2 = \frac{4\omega^2}{(9\omega^2+6\omega+1)}\frac{1}{k^2}, \quad (5.1)$$

recalling $\omega \equiv \frac{1-u}{\sigma-1}$. Substituting $u_1 = u_2 = u$ and $\alpha = \beta = 0.5$ into (3.15), (4.15) and (4.16), we get:

$$n_d{}^* = n_1{}^* + n_2{}^* = \overline{N}\cdot[\frac{1-u}{k(1+\frac{\sigma}{2}-\frac{3}{2}u)}]^{-2} = \frac{\overline{N}(9\omega^2+6\omega+1)k^2}{4\omega^2}, \quad (5.2)$$

that the number of cities and the city size (hereafter n^* and N^*) are the same whether the city system is diversified or specialized.

<u>Proposition 1:</u>(*Diversification crowds out services*). *Assume* $u_1 = u_2 = u$ *and* $\alpha = \beta = 0.5$. *Let* m_s *be the industry-specific services produced in a city that is specialized in either manufacture. Let* $m_{d1} = m_{d2} = m_d$ be *the industry-specific services in a diversified city producing both manufactures. Then,* $m_d = 0.5\, m_s$: *diversification reduces by half the number of services locally available to each industry. Meanwhile, the city-system-wide services in any one industry is unaffected by diversification or specialization and is* $\frac{\overline{N}}{2N^*}m_s$.

Proof: Recall that under the symmetry assumed, we have already shown that specialized and diversified cities are of the same total population, *N* (see (5.1)). Since the city labor supply is $H_c = N(1 - kN^{\frac{1}{2}})$, it follows that the total labor supply is the same in each type of city. In a specialized city, the labor market clears by $H_c = H_{xs} + m_s H_z$ where H_{xs} is the labor employed in manufacturing. In a diversified city, the labor market clears by $H_c = 2H_{xd} + 2m_d H_z$ where H_{xd} is the labor demand by each of the two manufacturing industries and $H_z = f\sigma$ is the labor demanded by each service producer. $H_{xs} = \frac{uP_x X_s}{w_s}$ where X_s and w_s are given by (4.1) and (4.2) respectively. Using these, $H_{xs} = uH_c$ and, hence, $m_s = \frac{(1-u)}{f\sigma} N(1 - kN^{\frac{1}{2}})$. To clear the labor market in a diversified city, $m_d =$ $\frac{(1-u)}{2f\sigma} N(1 - kN^{\frac{1}{2}}) = 0.5m_s$. To see that the total number of services is unaffected by the type of equilibrium (diversified or specialized cities) define the total industry specific services in the two cases as M_s and M_d. Note that $M_s = n_s m_s$ and $M_d = n_d m_d$. But we have shown that $n_d = 2n_s$ and that $m_d = 0.5m_s$. Hence, $M_s = M_d = \frac{\overline{N}}{2N^*} m_s$. ■

We will hereafter refer to this result as the "crowding out effect". It is crucial in the next results we will derive because it shows that manufacturers located in diversified cities have fewer services locally and are more reliant on services produced in other cities and thus, more sensitive to changes in the intra-city cost of trading services, than are manufacturers located in specialized cities.

Now define $Q^* = \frac{U_d{}^*}{U_s{}^*}$. Substitute $u_1 = u_2 = u$ and $\alpha = \beta = 0.5$ into $U_d{}^*$ given by (3.16) and $U_s{}^*$ given by (4.17). Using (5.1) and (5.2), Q^* is

$$Q^* = \frac{[1+(n^*-1)\tau^{\sigma-1}]^{\frac{1-u}{\sigma-1}}(\frac{1}{2})^{\frac{1-u}{\sigma-1}}}{[1+(\frac{n^*}{2}-1)\tau^{\sigma-1}]^{\frac{1-u}{\sigma-1}}\theta^{\frac{1}{2}}} = \left(\frac{n^* A(\tau,\sigma)+1}{n^* A(\tau,\sigma)+2}\right)^{\frac{1-u}{\sigma-1}}\left(\frac{1}{\theta^{\frac{1}{2}}}\right), \tag{5.3}$$

where, $A(\tau,\sigma) \equiv \frac{\tau^{\sigma-1}}{1-\tau^{\sigma-1}}$ and $n^* = f(\overline{N}, k, \frac{1-u}{\sigma-1})$ as given by (5.2). Note that in the denominator of (5.3), n^* is divided by two because in the system of specialized cities, each city imports services only from cities of the same type.

Q^* in equation (5.3) is the ratio of the utility of the diversified to the specialized city system with n^* cities in each case. When $Q^* > 1$ (< 1), a system of diversified (specialized) cities is the higher utility equilibrium. If $Q^*=1$, then the two configurations have equal utility.

<u>**Proposition 2**</u>: *Under symmetry ($u_1 = u_2 = u$ and $\alpha = \beta = 0.5$) either specialization or diversification can be the higher utility equilibrium. The following conditions cause a diversified system of cities to attain a higher utility than a system of specialized cities*:

- **(a)** *Sufficiently low cost share of services (1-u);*
- **(b)** *Sufficiently high intercity transport cost for manufactures (1/θ);*
- **(c)** *Sufficiently high intracity commuting cost (k);*
- **(d)** *Sufficiently low intercity transport cost for services (1/τ);*
- **(e)** *Sufficiently high national population ($\overline{N}$);*
- **(f)** *Sufficiently high elasticity of substitution among services (σ).*

<u>**Proof:**</u> For diversification to be the higher utility equilibrium, Q^* must be larger than 1.

Note that the numerator of (5.3) is not affected by θ. A sufficiently small θ guarantees that $Q^* > 1$, whereas a θ close to 1 guarantees $Q^* <1$. To see that specialization can generate higher utility, let θ be large and τ be small. In this case, the denominator of (5.3) is larger and the numerator of (5.3) becomes relatively smaller and we can easily have $Q^* < 1$ (for example, when $\theta =1$ and $\tau < 1$, Q^* is always smaller than one). Claims (a)-(e) of the Proposition directly follow from taking derivatives to show that $\frac{\partial Q^*}{\partial(1-u)} < 0$, $\frac{\partial Q^*}{\partial\theta} < 0$, $\frac{\partial Q^*}{\partial k} > 0$, $\frac{\partial Q^*}{\partial\tau} > 0$, $\frac{\partial Q^*}{\partial\overline{N}} > 0$. To see claim (f), we examine the two extremes. Note that as $\sigma \to \infty$ the service varieties become perfect substitutes and in fact $N^* \to 0$ from (5.1). Since, in a world of perfect substitutes, the smallest city possible should be at least large enough to produce one service variety and some manufactures using only that variety, it follows that the number of cities also remains finite. Meanwhile, $A(\tau,\sigma) \to 0$ in (5.3) while the exponent $\frac{1-u}{\sigma-1} \to 0$. Hence, as $\sigma \to \infty$, $Q^* \to \frac{1}{\theta^{\frac{1}{2}}} > 1$ and diversified cities achieve higher utility. At the other extreme, as $\sigma \to 1$ from above, varieties are viewed as highly distinct and each as extremely valuable. From (5.1), $N^* \to \frac{1}{k^2}$: a very large city that contains a lot of varieties. From (5.3), since $\frac{1-u}{\sigma-1} \to \infty$, the leading parenthesis goes to zero (because it is a fraction) and, hence, $Q^* \to 0$. Hence, specialized cities achieve higher utility. ■

To grasp the intuition behind the results of this Proposition, let's look at the fundamental drivers that determine the city utility levels under the different configurations. *First*, the positive externalities within industries stem from the technological need for the variety of industry-specific services, and these externalities are higher when the cost share of services (1-u) is higher in the city or when service varieties are viewed as highly unique (σ close to 1). *Second*, there are no positive externalities between the two industries. Since the equilibrium city size is the same whether the city has one industry or two industries (see (5.1)), the positive externalities within each industry when the two industries coexist in a city are smaller than when only one industry exists in the city because a larger variety of industry-specific services is produced in the latter case ("crowding out effect" of Proposition 1). *Third*, the industry-specific services are tradable among different cities. Therefore, an industry in one city can benefit from the varieties of services in other cities the source of intercity externalities. However, these externalities are mediated by the intercity transport cost of these industry-specific services. At one extreme, if the intercity transport cost of services is infinite, these externalities disappear at the other extreme if transport costs are zero, the externalities are at their strongest. *Fourth*, the city system is closed. Therefore, if a city produces just one manufacture it must import the other from the rest of the city system. If the intercity transport cost of manufactures is high, then it is highly inefficient to import the manufacture from other cities. These four features drive the claims of Proposition2, and we are now ready to discuss the intuition.

First, let's see why specialization gives higher utility when externalities within industries are high (claim(a)). In our model, when services have a larger share in final goods production relative to labor (high 1- u) the production externalities within industries are higher. This favors a specialized city system with larger city-industries and, hence, a higher variety of services. A diversified city would realize fewer of these industry-specific externalities because of the crowding out of local services (the leading term $\alpha^{\frac{\alpha(1-u_1)}{\sigma-1}}\beta^{\frac{\alpha(1-u_2)}{\sigma-1}}$ in (3.16) decreases when u_1 and u_2 are smaller).

That a lower intercity transport cost of manufactures increases the efficiency of a specialized city system (claim (b)) follows from $\frac{\partial Q^*}{\partial \theta} < 0$, where θ is the portion of manufactures which arrive at the destination. When θ increases, a diversified city system doesn't benefit because it does not incur any intercity transportation costs for manufactures in the first place. However, a specialized city system gains because transportation cost is saved when manufactures are imported from cities of the other type (in (4.17), the leading term $\theta^{2\alpha\beta}$ increases with θ).

k measures the unit time-cost of intracity commuting. Since $\frac{\partial Q^*}{\partial k} > 0$, we conclude that a lower intracity commuting cost increases the efficiency of a specialized system relative to a diversified one (claim(c)). Note that when k decreases, the equilibrium city size grows and there are fewer cities. In this situation, specialized cities gain a lot of efficiency because more services are available in the city. However, for diversified cities, the efficiency gain from becoming larger is partially offset by the crowding out effect, therefore the utility gain is relatively smaller.

In claim (d) τ measures the intercity transport cost for services and a large τ implies a smaller transport cost. Since $\frac{\partial Q^*}{\partial \tau} > 0$, we conclude that a shock that increases τ favors diversification. From Proposition 1, in diversified cities, a city-industry is half as big as it would be in a specialized city and relies more heavily on the services produced in other cities. Therefore, when the intercity transport cost of services decreases, diversified cities benefit more from spending less on importing services than do specialized cities. Thus, when cities are diversified, the crowding out effects become less important since now the services produced in other cities are cheaper to import.

When the national population becomes larger, there are more cities. The share of local services in any one city becomes a smaller fraction of the total services in the city system. But since diversified cities produce half as many services as specialized cities, a diversified city imports a higher share of its services from other cities and relies less on services produced locally than does a specialized city. Hence, population growth weakens the crowding out effect. As a result, a system of diversified cities gains more than a system of specialized cities (claim(e)).

And the last claim (f) is explained as follows. When the services are perfect substitutes ($\sigma \to \infty$) variety has no value and all manufacturers use only one local service since imported services are more expensive. In this situation, since variety has no value all positive externalities vanish and cities are diversified because there is no need to incur costs of trading manufactures. On the other hand, when the service variety is extremely important, manufacturers want maximum variety and this is accomplished in large specialized cities that have a large number of services. In this case manufactures are traded between cities but the cost of doing so is well worth it.

Historically, the development of the railroad system and the interstate highway system significantly reduced intercity transportation cost for manufactures. And the invention of the automobile and the development of public transportation systems lowered intracity commuting cost. According to Proposition 2, these technological advances favored a specialized city system, *ceteris paribus*, allowing cities to realize larger scale economies, growing and becoming specialized. There is not much literature on how the more recent telecommunication revolution is affecting the efficiency of diversification. A telecommunication advance makes one city benefit more from other cities and thus increases positive intercity externalities. Proposition 2 indicates that such an advance favors the efficiency of a diversified city system since, because of the crowding out effect, a diversified city relies more heavily on imported services and services are information intensive.

In the literature (e.g. Henderson [9], Abdel-Rahman and Fujita [3]), the effect of population growth on the specialization or diversification of cities is not examined. We showed that population growth favors diversification, an important finding since world population is growing.

Proposition 2 is testable. We can construct different time paths of urban development by positing different sequences of exogenous shocks. An interesting and realistic path might be a diversification-specialization-diversification path. In stage one of such a path, a system of small diversified cities is the equilibrium. This stage lasts for a long time with the industries exhibiting low scale economies, high intracity commuting cost and high intercity transportation cost for manufactures. At the next stage, there are shocks that increase intraindustry externalities (industrial revolution), reduce the intercity transportation cost for manufactures (e.g. the development of railways, trucking and highways) and intracity commuting cost (e.g. invention of automobiles). With these shocks, the city system should switch from many small and diversified cities to larger and specialized ones. In the last stage, the national urban population size becomes very large and there are shocks that greatly reduce the intercity transportation cost of services (e.g. telecommunication improvements such as Internet, e-mail and eventually teleconferencing). In this stage, the city system should become diversified again.

Recall that, in the analysis so far, market agents (or city developers) ignored the fact that if their city got larger the number of cities would also change. A central planner on the other hand, could directly take into account the interdependence of city sizes. For the regime of diversified cities, the city size under planning is obtained after substituting $n = \frac{\overline{N}}{N}$ into δ in equation (3.13) and solving the maximization problem with respect to city size, *N,* to get an interior solution. This optimal city size is:

$$N_d^{\ *} = \left\{ \frac{\omega + \sqrt{\omega^2 - A(\tau,\sigma)(1+\omega)(1+3\omega)k^2\overline{N}}}{k(1+3\omega)} \right\}^2, \qquad (6.1)$$

where $A(\tau,\sigma) \equiv \frac{\tau^{\sigma-1}}{1-\tau^{\sigma-1}}$ and $\omega = \frac{1-u}{\sigma-u}$. For a specialized city system, the planner would substitute (4.11) and (4.12) into δ_1 and δ_2 in (4.13) and maximize $U_S(N_1, N_2)$ with respect to N_1 and N_2. Optimal city sizes (again interior solutions) for both types of cities under specialization, N_1^* and N_2^*, are:[12]

$$N_s^* = N_1^* = N_2^* = \left\{ \frac{\omega + \sqrt{\omega^2 - \frac{1}{2} A(\tau,\sigma)(1+\omega)(1+3\omega) k^2 \overline{N}}}{k(1+3\omega)} \right\}^2 . \tag{6.2}$$

Define the utility of the diversified city system relative to the utility of the specialized city system under planning as $\overline{Q^*}$. Substituting city size from (6.1) into (3.16) and from (6.2) into (4.17) and using $u_1 = u_2 = u$ and $\alpha = \beta = 0.5$:

$$\overline{Q^*} = \frac{U_d^*}{U_s^*} = \frac{(\frac{1}{2})^{\frac{1-u}{\sigma-1}}}{\theta^{\frac{1}{2}}} \cdot \frac{[1+(\frac{\overline{N}}{N_d^*}-1)\tau^{\sigma-1}]^{\frac{1-u}{\sigma-1}} N_d^{*\frac{1-u}{\sigma-1}} [1-kN_d^{*\frac{1}{2}}]^{\frac{\sigma-u}{\sigma-1}}}{[1+(\frac{\overline{N}}{2N_s^*}-1)\tau^{\sigma-1}]^{\frac{1-u}{\sigma-1}} N_s^{*\frac{1-u}{\sigma-1}} [1-kN_s^{*\frac{1}{2}}]^{\frac{\sigma-u}{\sigma-1}}} \tag{6.3}$$

Using numerical examples, we can show that the value of $\overline{Q^*}$ can either be larger or smaller than one. Most importantly, our simulation results show that $\overline{Q^*} > Q^*$ for all exogenous variables, where Q^* is defined by (5.3). Therefore, we could have $\overline{Q^*} > 1$ and $Q^* < 1$. This is expressed by the following proposition.

Proposition 3: *It is possible that the market equilibrium is a system of specialized cities while an equilibrium system of diversified cities would yield higher utility.*

The intuition behind Proposition 3 is as follows. In our model, intercity trade of services generates positive externalities, which are not taken into consideration by the market agents. The higher these intercity externalities, the farther the equilibrium is from the optimum. In a diversified city system, there is more intercity trade of services since city-industries are half as big by Proposition 1. Hence, a diversified city relies more on the services imported from other cities. As a result, the diversified city system is more distorted in equilibrium than is the specialized system. So, it is possible that in equilibrium the utility level in a diversified system is smaller than that in the specialized system, but after all the market inefficiencies are corrected,[13] the diversified system achieves a higher utility level. In the literature, the departure of the equilibrium from the optimum in the context of specialization versus diversification is not analyzed.[14]

6. Some Extensions

We have analyzed the relative efficiency of city systems with specialized or diversified cities when both manufactures and services are tradable among different cities. Throughout the analysis, however, we concentrated on the case when cities and the two industries are symmetric. One direct extension of our model is to consider the existence of a mixed equilibrium in which some cities are diversified and some are specialized. It would be interesting to see whether such a mixed city system is a stable equilibrium when total population is large and whether it yields higher utility than the equilibria examined here.

Another interesting extension of this paper is that we can consider cities producing only manufactures or only services. In Anas and Xiong [6], this approach is used to analyze the emergence of new cities with one industry composed of a manufacture and its related services. With two industries, can we have a hierarchical city system with a few big diversified cities producing both manufactures and their related services, few more specialized and smaller cities producing only one manufacture and its related services, and many smaller cities producing only manufactures or only services? These are some of the interesting questions our modeling approach has the potential to answer.

Finally, an important area of inquiry centers on how the diversification versus specialization of city systems influences growth and development and how growth, in turn, induces specialization or diversification. There has been some empirical work on this topic (Glaeser, Scheinkman and Schleifer [16]) but it is based on no formal theory of how or why diversification occurs. Our model has the potential to inform the specification of empirical models.

References

1. H. M. Abdel-Rahman, Product Differentiation, Monopolistic Competition and City Size, *Regional Science and Urban Economics*, 18, 69-86 (1988).

2. H. M. Abdel-Rahman, Shareable Inputs, Product Variety and City Size, *Journal of Regional Science*, 30, 359-374 (1990).

3. H. M. Abdel-Rahman and M. Fujita, Specialization and Diversification in a System of Cities, *Journal of Urban Economics*, 33, 189-222 (1993).

4. H. M. Abdel-Rahman, When do cities specialize in production?, *Regional Science and Urban Economics*, 26, 1-22 (1996).

5. A. Anas, On the Birth and Growth of Cities: Laissez and Planning Compared, *Regional Science and Urban Economics*, 22, 243-258 (1992).

6. A. Anas and K. Xiong, The Formation and Growth of Specialized Cities, Working Paper, State University of New York at Buffalo (1999).

7. A. K. Dixit and J. E. Stiglitz, Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, 67(3), 297-308 (1977).

8. W. Ethier, National and International Returns to Scale in the Modern Theory of International Trade, *American Economic Review*, 72, 389-405 (1982).

9. J. V. Henderson, “Urban Development: Theory, Fact and Illusion”, Oxford, UK: Oxford University Press (1988).

10. J. V. Henderson and H. M. Abdel-Rahman, Efficiency through Decentralization with Product Diversity, *Regional Science and Urban Economics*, 21, 491-509 (1991).

11. R. W. Helsley and W. C. Strange, Matching and Agglomeration Economies in a System of Cities, *Regional Science and Urban Economics*, 20, 189-212 (1990).

12. M. Fujita, A Monopolistic Competition Model of Spatial Agglomeration: Differentiated Product Approach, *Regional Science and Urban Economics*, 18, 87-124 (1988).

13. M. Fujita, “Urban Economic Theory: Land Use and City Size”, Cambridge, UK: Cambridge University Press (1989).

14. M. Fujita, On the Self-Organization and Evolution of Economic Geography, *The Japanese Economic Review*, Vol. 47, No. 1, 1996, 34-61 (1996).

15. M. Fujita, P. Krugman and A. J. Venables, “The Spatial Economy”, Cambridge, MA: The MIT Press (1999).

16. E. L. Glaeser, J. Scheinkman and A. Schleifer, Economic growth in a cross-section of cities, *Journal of Monetary Economics* 36, 117-143 (1995).

17. P. Krugman, Increasing Returns and Economic Geography, *Journal of Political Economy*, 99, no. 3, 483-99 (1991).

18. F. Rivera-Batiz, Increasing Returns, Monopolistic Competition, and Agglomeration Economies in Consumption and Production, *Regional Science and Urban Economics*, 18, 125-153 (1988).

19. K. Xiong, "Intercity and Intracity Externalities in a System of Cities: Equilibrium, Transient Dynamics and Welfare Analysis", Unpublished Ph.D. Dissertation, Department of Economics, State University of New York at Buffalo (1998).

Footnotes

[1] This paper is based on chapters 2 and 3 of Kai Xiong's Ph.D. Dissertation. We thank Vernon Henderson, two reviewers and the editor for useful comments.

[2] Consider the growth in the intercity trading of financial services of all kinds in recent years, especially via the Internet.

[3] Here we assume that monopolistically competitive service producers use the mill-pricing strategy. Buyers of the services (i.e., manufacturers) bear all the transport cost.

[4] See Dixit and Stigliz [7] or Fujita [13] for the proof.

[5] There are various market mechanisms to support this equilibrium notion. The new economic geography (Krugman [18]) stresses atomistic defection of firms, while the public finance literature assumes utility-taking competitive city developers (Henderson

[9]). Xiong [19] assumes profit maximizing developers who operate in a contestable development market. See Appendix to see how this mechanism supports the equilibrium.

[6] Xiong [19] provides a detailed discussion of the properties of this equilibrium and does a comparative statics analysis.

[7] We only analyze the case when all the diversified cities are symmetric. Because of this symmetry, each city is self-sufficient in its manufactures and there is no intercity trade of manufactures.

[8] The residential choice of a city resident is independent of the industry he works in because residents have identical tastes and earn the same wage in equilibrium. So each industry's share of labor supply is equal to the share of workers in that industry.

[9] The service markets are already cleared in the manner implied in section 2.

[10] We intentionally avoided modeling the complex asymmetric configurations in which diversified and specialized cities can coexist. It can be gleaned from prior work (e.g. Anas [5]) that when total population is sufficiently large, symmetric equilibria dominate asymmetric ones and that symmetric equilibria are stable while asymmetric ones are unstable. In Anas and Xiong [6], we show that this basic insight survives in more complex models similar to the one developed here: although the transition path can involve prolonged periods in which an asymmetric configuration exists, in the long run when total population is sufficiently large, a symmetric equilibrium yields higher welfare. Armed with this perspective, it is reasonable to conjecture, that a system of cities

that is disturbed by shocks will - in the long run - tend to settle on a symmetric equilibrium.

[11] In Xiong [19], the case of two asymmetric industries is also analyzed. It is shown quantitatively that the qualitative conclusions of the symmetric case all survive the asymmetry.

[12] For interior solutions, assume that the expressions under the square root signs in (6.1) and (6.2) are positive.

[13] See Xiong [19] for the first-best policy that makes the equilibrium optimal.

[14] Anas [5] analyzes a simple one-industry city system and shows that optimum and equilibrium do not generally coincide with respect to the number of cities and that the optimum city-system configuration is not always stable. When more than one industry exists, our model shows that the number of cities may or may not be optimal and that the optimal structure of industries within a city can also depart from the equilibrium.

Appendix: Equilibrium City Size

Suppose that cities are formed not by atomistic action of firms and workers but by city developers operating in a contestable market. A developer is an entrepreneur who sets up a new community and rents out land to producers and consumers in the national economy. Suppose that a city developer charges a fee that is proportional (at the rate a) to the disposable income of a city resident. The disposable income after paying this fee is $(1-a)I(N)$ where $I(N)$ is given by (2.26). Assume that the city developer's marginal cost of attracting an additional resident is *d*. The developer's profit function is $\pi = aI(N)N - dN$. The first order condition is $aN\frac{\partial I(N)}{\partial N} + aI(N) - d = 0$. Since the development market is contestable, in equilibrium all city developers can only earn zero profit. Setting $\pi = 0$ we get $aI(N)N - dN = 0$ or $a = \frac{d}{I(N)}$. Plugging this into the first order condition, we get $\frac{\partial I(N)}{\partial N} = 0$, which give the same solution for efficient city size as $\frac{\partial U(N)}{\partial N} = 0$. If the size of a city is smaller than the efficient size, the city developer can always earn a temporary profit by attracting more renters and the city grows. If the city is larger than efficient, then it is unstable since new cities of more efficient size could attract renters, leading to a decrease in the population of this city and an increase in the number of cities. An individual developer takes *n* (the number of cities) as given when he sets his city's size, since he cannot control the number of cities in the economy. Although in the model we presented *a*=*d*=0 was assumed, all of the qualitative results would be unchanged if we were to introduce developers as described in this Appendix.