Is Investment in Environmental Quality a Solution to Recessions? Studying the Welfare Effects of Green Animal Spirits.

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Abstract

Assume that ‘green accounting’ has been adopted and implemented, does an investment in environmental quality play a role similar to the investment in capital in towing the economy out of a recession? To answer the question, we integrate ‘green accounting’ into a stochastic dynamic general equilibrium model to study the short-run consequences of investment in environmental quality and hereby addressing if there is an incentive-based fiscal environmental solution to recessions. Surprisingly and counter intuitive, we found that reducing the rate at which humans consume the environment renders a fiscal policy - that engage in environmental investment - less effective in providing a thrust out of a recession. Conditional on the proposed model and the calibrated parameters, we conclude that an increase of one percent in environmental investment will crowd out real quarterly consumption in a range from $ 102.74 billions to $ 171.11 billions, on average, in every quarter for seven years following the investment (measured in chained 2000 dollars). Therefore, we argue that investment in environmental quality is not a solution to recessions. This result is a striking contrast to the conclusion reached in Weitzman and Löfgren (1997, Journal of Environmental Economics and Management, 32 (2), 139-153).

JEL classification: E32, Q56, Q58.

Keywords: Environmental Quality, Green Accounting, Stochastic Dynamic General Equilibrium models.

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1 Introduction

First to call for the adoption and implementation of ‘green accounting’, Hicks [31, p. 172] sought to adjust national income accounting by including environmental capital in national income calculations. Over the decades that followed, a host of environmental critiques - to the system of national income accounts and what does it actually measure - came forward at a slow, incremental and varying pace.

While Hicks’ suggestion sparked a debate regarding national income accounting and ‘green accounting’, it was not until Repetto et al. [49] - wherein it was shown that after accounting for natural resources depletion, the Indonesian economic growth during the 1970s and 1980s should have been cut by half - that the quest to integrate environmental measures into standard national accounts took a vital role and a well-defined research agenda. Their book initiated a serious and an unsettled debate regarding the normative and positive dimensions of ‘green accounting’ among researchers (Ackerman et al. [1], Alonso and Starr [2], Athale [6], Bartelmus [8], Bartelmus and Seifert [10], Boskin [11], Eisner [19], Farzin [20], Harris and Fraser [25], Heal and Kriström [29], Lutz [37], Milon [39], Nordhaus [42] and Nordhaus and Tobin [43]). Such an accumulation of efforts culminated in the production of ‘satellite accounts’ that mainly reported environmental statistics by the United Nations[1] Statistical Office [57]. These efforts grew and resulted in a well defined body of research that emphasizes the importance of studying environmental quality.

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[1] The years that followed the UN publication witnessed an upsurge in ‘green accounting’ empirical and theoretical research (e.g., see the recent special issues of Environment and Development Economics: Advances in Green Accounting: February 2000 and May 2000 and BioScience Special Issue: Integrating Ecology and Economics: April 2000). In the US, calls for increased research funds to change the national accounts system went unheard (Moulton [40]).

Empirically, a body of 'green accounting' research is present [2]. Here, we do not address the [empirical] thorny issues of how to measure the stocks of natural resources, environmental quality or how to value non-market activities and assets. We start by assuming that 'green' GDP is implemented and mainly focus on the theoretical framework and its short-run implications.

A concern that we have about the present theoretical literature is its reliance on long-run models. Common in the literature, the theoretical propositions of green accounting are constantly and invariably framed within a derivative of the standard neoclassical growth model, e.g.: 1) as in Hartwick [27] and Heal [28], wherein a formal inclusion of the stock of natural resources was introduced into a standard growth model, or 2) as in Farzin [20], wherein the relationship between the current-value Hamiltonian, the net national product (NNP), and sustainability in the context of a cake-eating economy was examined. Since some patterns of economic growth are environmentally damaging, several long-run growth models with environmental assets have been proposed, examined and investigated as a medium and a justification to advance the need for 'green accounting' practices. To summarize the present status of the green-theoretical research agenda, the properties of balanced growth paths were derived, reported, and advocated as a pretext for the urgent need to seriously consider the practice of 'green accounting'.

A shortcoming of such a strategy is that one ignores the short-term allusions of a disturbance in environmental investment. For example, the implications of government fiscal policies and

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[2] Among many, we cite a few. Green cost-of-living indices have been successfully constructed and are readily operational (Banzhaf [7]). For the green indicators handbook, see Bartelmus [9]. The main point is that environmental quality is being thoroughly investigated (Amacher el al. [3] and Kotchen [34]).

benchmark standards on environmental quality are usually [mostly and primarily] addressed within a long-run context - with the exception of Bye [12], wherein the link between welfare and environmental taxes was investigated.

As portrayed in the literature, investment in environmental quality is socially desirable and aspire to a better future for the generations to come, along a well-defined balanced growth path and a computable green golden rule (Chichilnisky, Heal and Beltratti [15] and Heijdra [30]). Again in the literature, within a long-run framework and of interest here, there have been few attempts to address the welfare-green connection (Asheim and Weitzman [5], Banzhaf [7], Bartelmus and Seifert [10], Turner and Tschirhart [55], Weitzman [59] and Weitzman and Lӧfgren [60]). Briefly, the long-run effect is well known and widely accepted; spending on environmental quality increases the standard of living.

In this paper, we address and focus on the central question of how are welfare and environmental investment intertwined in the short-run. We question and study these effects and in a broader sense, we seek an answer to the following question: Does an investment in environmental quality have the potential of pulling the economy out of a recession? Assuming that 'green' accounting is adopted, and regardless of the [presumably clear and obvious] consequences in the long run, what are the effects of an increase in environmental investment in the short run? Can a fiscal policy that is primarily devoted to investing in environmental quality generate enough impetus to propel the economy out of a recession? If the answer is in the affirmative, then we should expect investment in the environment to show up on policy makers and officials to-do list pre- and post elections.

To answer the question, we propose a model wherein we integrate environmental quality into a 'green' stochastic dynamic general equilibrium model. Within a green GDP setup, the model emphasizes: 1) the degradation of environmental quality due to human consumption, and 2) natural resources depletion and regeneration. We solve it using numerical dynamic programming techniques, - specifically, we solve it as a rational expectations system with sensitivity to the structure, - and we seek to answer our question.

Section 2 presents the model and discusses its results. Section 3 addresses the testable implications of the model. Finally, Section 4 concludes and proposes related future research.

2 The Model

The model that we propose is a 'green' general equilibrium model in the spirit of Cairns [14, p. 67], Forester [21], John and Pecchenino [32] and Selden and Song [50]. However, we impose the minimum set of assumptions needed (see Asheim [4] for more assumptions). It is a *stochastic* dynamic general equilibrium (SDGE) model in which the representative agent accumulates both [stocks] environmental quality and physical capital. Faced with a random shock in environmental investment, we pursue and track the optimal decision paths followed by the economic agent and compute the relative change in her welfare.

The model is as follows. A representative agent[3] chooses the magnitude of natural [environ-

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[3] A note is due on the choice of representative consumer framework. If the focus is on the distributional effects of an investment in environmental quality, then the proper choice of framework ought to be overlapping generations model as in Fullerton [22] and Heijdra [30]. To study environmental issues, the chosen framework of a representative consumer have advantages, e.g., see Van der Ploeg and Withagen [47] and Thavonen and Kuuluvainen [54]. The single consumer [household] is assumed to be representative of the society as a whole. Her preferences are represented by a utility function which is time separable and state independent. The household is assumed to be a "dynasty" (see Hartley [26]) that derives utility from environmental quality. By accounting for green

mental quality] and physical capital to smooth her expected life-time consumption path. To do so, she is facing the following problem.

$$\max_{c_t, e_{t+1}, k_{t+1}} E_t \left[\sum_{t=0}^{\infty} \beta^t U(c_t, e_t) | I_t \right] \tag{1}$$

subject to

$$\begin{aligned}
y_t &= Ak_t^{\alpha} && 0 \leq \alpha \leq 1 && (2)\\
e_{t+1} &= (1-\delta_e)e_t - \phi c_t + z_t m_t && 0 \leq \phi \leq 1, \quad -1 \leq \delta_e \leq 1 && (3)\\
k_{t+1} &= (1-\delta_k)k_t + i_t && 0 \leq \delta_k \leq 1 && (4)\\
y_t &\leq c_t + i_t + m_t && && (5)\\
z_t &= \rho z_{t-1} + \varepsilon_t && 0 \leq \rho \leq 1, \quad \varepsilon_t \backsim iid \quad \Lambda(0,\ 0.01) && (6)\\
c_t &\geq 0,\ e_t \geq 0,\ k_t \geq 0 && (\forall t) && (7)\\
k_0 &> 0 \qquad e_0 > 0 && given && (8)
\end{aligned}$$

where $E_t[.|I_t]$ denotes the expectation upon the set of information I available at time t. The representative agent derives utility from consumption c_t and environmental quality e_t. α denotes the capital share in real per capita output. k_t, δ_k and n_t denote physical capital, its depreciation rate and labor, respectively. Environmental quality suffers two forms of depletion: natural and human. δ_e refers to the natural depreciation rate of environmental quality, or in other terms, it refers to environmental depletion due to non-consumption factors. ϕ refers to the fraction of environmental-quality degradation due to human consumption c_t. It captures the degradation in natural resources as outlined in Milon [39, p. 136, Table 8.2]. m_t denotes investment [improvement/maintenance] in environmental quality. Also, one can refer

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GDP (equation (5)), a change in the level of her utility reflects and is equivalent to a change in the overall level of social welfare (or what is labeled as "societal" welfare by Turner and Tschirhart [55]). That is, an increase (decrease) in her utility implies an improvement (loss) in social welfare.

to m_t as pollution abatement. i_t and y_t refer to investment in physical capital and 'green' GDP, respectively. Finally, the stochastic process z_t influences the environmental maintenance m_t to capture the variability in environmental investment policies. $\Lambda(.,.)$ refers to the lognormal probability density function.[4] The exogenous shock z_t follows an autoregressive process to capture the degree of time persistence in fiscal investment policies and programs (equation (6)).

We define e_t as a broadly defined index for environment quality. One plausible interpretation of e_t is a weighted index of: the quality of soil and groundwater, the cleanliness of rivers and oceans, an index of biodiversity, an inverse of the atmospheric concentration of: chlorofluorocarbons, greenhouse gases, and other pollutants. A common thread across these elements, is that this environmental index provides a basic biological support system that is needed by the economic agent. Empirical measurement of e_t is a contentious subject. In our model, we assume that such an index exists and that it does include all dimensions of environmental quality for which actual measurements exist.[5] To simplify and because of its natural depletion property and many non-priced benefits to society, think of e_t as the forest which is a prototypical resource for 'green' accounting.

e_t does not feature as an input in the production function (equation (2)). We assume that environmental quality is a public good that is affected by consumption externalities and it enters the utility function. No property rights are assigned over the the environmental resources but these are not part of the production process even though the representative

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[4] $\ln(\varepsilon_t) \backsim iid \quad N(0,\ 0.01)$. We thank Curtis Eberwein for suggesting the use of the lognormal distribution.

[5] Among many, see the Grossman and Krueger [24] definition of environmental quality which includes air and water quality, nature and species diversity.

agent values the environment and contributes to its maintenance (John and Pecchenino [32], John et al. [33] and Marini and Scaramozzino [38]). Here, production exhibits constant returns to scale $y_t = Ak_t^{\alpha} n_t^{1-\alpha}$ and $n = 1$ at the steady state. Both the utility and the production functions satisfy the Inada conditions.

It is common in the literature to include environment quality in the utility function, e.g., see Cairns [14], Chichilnisky, Heal and Beltratti [15] and John and Pecchenino [32]. Here, we adopt the specific form wherein $U(c_t, e_t) = \psi \log c_t + (1 - \psi) \log e_t$, wherein $0 < \psi < 1$ denotes the weight of consumption in the utility. We will study the effects of the shock on the utility across different rates for ψ. This log-linear utility implies an intertemporal elasticity of substitution of environmental quality equal to one and that $U_{ce} = 0$. We steered away form the more general CES form - wherein $U_{ce} \geq 0$ - for the simple reason that - if adopted - it will accentuate the effects of an increase in environmental quality on consumption and will result an a larger increase in welfare[6]. Therefore, we made this restrictive assumption to provide us with the lowest conservative bound on the welfare effects of an increase in the investment of environmental quality. Since we regard e_t as a weighted index, it can take only positive values. Therefore, there is no special significance attached to $U(c_t, 0)$. Formally, $U(c_t, e_t) \in C^{(2)}$, $U_c > 0$, $U_e > 0$, $U_{cc} < 0$, $U_{ee} < 0$, i.e., continuous, twice differentiable and strictly concave.

Equation (3) describes the law of motion for environment quality. We model environmental

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[6] An increase in environmental investment m_t results in an increase in the stock of environmental quality. This increase in e_t will increase the level of utility. At the same time an increase in m_t will reduce consumption and consequently, the level of utility (welfare). The assumption of $U_{ce} \geq 0$ implies that the change in welfare resulting from environmental investment will be positively biased. See Wagner [58, p. 2, footnote 2] for an example wherein $U_{ce} \geq 0$. Also, see Chimeli and Braden [16, footnote 9, p. 373 and Corollary 1, p. 379.] wherein $U_{ce} = 0$.

quality as a discrete-time renewable resource (John and Pecchenino [32], John et al. [33], Mourmouras [41] and Ono and Maeda [45]). Note that if $\phi c_t = 0$, then $z_t m_t$ can be interpreted as gross investment in environmental quality. If $\phi = \delta_e = 0$, $m_t = 0$, $e_{t+1} = e_t$ and $A_t = z_t$ $(\forall t)$, then the model reduces to a standard real business cycle model, wherein investment in environmental quality is absent.

Capital accumulates by adding the end of period left over [after depreciation] to gross investment (i_t) (equation (4)). The resource constraint (equation (5)) emphasizes 'green' GDP on the left hand side. Investment in environmental quality (m_t) counts as part of 'green' GDP (y_t).

In the notation of the model, the social planner solves the following Bellman equation,

$$v(k, e; z) = \sup_{w \in \Gamma(x,z)} \left\{ \begin{array}{c} \psi \log\left(Ak^{\alpha} - k' + (1-\delta_k)k - z^{-1}e' + z^{-1}(1-\delta_e)e\right) \\ +(1-\psi)\log e + \beta E\left[v\left(k', e'; z'\right)\right] \end{array} \right\} \tag{9}$$

Since the objective is upper semi-continuous over a compact set, a solution exists to the model. The solution is unique given the concavity of the objective. The positiveness of the optimal solution follows from the Inada conditions. Imposing $k_0 > 0$ and $e_0 > 0$ guarantees that the optimal paths satisfy the Euler equations. This optimum is the unique competitive equilibrium allocation and supports a Pareto optimum.[7] Therefore, one can solve the social

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[7] To show that the solution to the problem exists, re-write the problem in a functional form. Let (X, ξ) and (Z, ζ) be measurable spaces, and let $(S, \vartheta) = (X \times Z, \xi \times \zeta)$ be the product space. X is defined as the set of possible values for e_t and k_t over the Borel set ξ, $X = \mathbb{R}_+^2$ and Z is the set of possible values of z_t over the Borel set ζ, $Z = \mathbb{R}_+$. S is the set of possible states for the system. The evolution of the stochastic shocks is described by the stationary transition function Q on (Z, ζ). The transition function is implicitly defined by the assumption that the shocks z_t follow the stochastic difference equation (6). The constraints are described by the correspondence $\Gamma : X \times Z \to X$. Let

planner's problem using concave programming techniques. However, and distinct from Bye [12, p. 18], we do provide numerical simulations to study the questions posed. In an infinite sequential form, the social planner solves,

$$L = \max_{c_t,k_{t+1},e_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \psi \log c_t + (1-\psi)\log e_t \\ -\lambda_t \left(c_t + k_{t+1} - (1-\delta_k)k_t + z_t^{-1}\left(e_{t+1} - (1-\delta_e)e_t + \phi c_t\right) - Ak_t^{\alpha} \right) \end{bmatrix} \tag{10}$$

The first order conditions (FOC) are,

$$\begin{aligned}
c_t: &\quad \frac{\psi}{c_t} - \lambda_t(1 + z_t^{-1}\phi) = 0 && (11)\\
k_{t+1}: &\quad -\lambda_t + \beta E_t\left[\lambda_{t+1}\left((1-\delta_k) + \alpha A k_{t+1}^{\alpha-1}\right)\right] = 0 && (12)\\
e_{t+1}: &\quad -\lambda_t z_t^{-1} + \beta E_t\left[(1-\psi)e_{t+1}^{-1} + \lambda_{t+1} z_{t+1}^{-1}(1-\delta_e)\right] = 0 && (13)\\
\lambda_t: &\quad c_t + k_{t+1} - (1-\delta_k)k_t + z_t^{-1}\left(e_{t+1} - (1-\delta_e)e_t + \phi c_t\right) - Ak_t^{\alpha} n_t^{1-\alpha} = 0 && (14)
\end{aligned}$$

B be the graph of Γ and defined as, $B = \{(x, w, z) \in X \times X \times Z : w \in \Gamma(x, z)\}$. Here,

$$\begin{pmatrix} e_t \\ k_t \end{pmatrix} \equiv \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \equiv x \in X = \mathbb{R}_+^2$$

$$\begin{pmatrix} e_{t+1} \\ k_{t+1} \end{pmatrix} \equiv \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \equiv w \in \Gamma(x, z) = \begin{pmatrix} 0, & (1-\delta_e)e - \phi c + zm \\ 0, & (1-\delta_k)k + i \end{pmatrix}$$

Let $F : B \to \mathbb{R}$ be the momentary return function.

$$F(x, w, z) = \psi \log\left(\frac{z}{z+\phi}\left(Ax_2^{\alpha} - w_2 + (1-\delta_k)x_2 - \frac{1}{z}w_1 + \frac{1}{z}(1-\delta_e)x_1\right)\right) + (1-\psi)\log x_1$$

That is $F(x, w, z)$ is the momentary utility if the current state is (x, z) and $w \in \Gamma(x, z)$ is chosen as the next period's endogenous state variables. The social planner maximizes

$$v(x, z) = \sup_{w \in \Gamma(x,z)} \left[F(x, w, z) + \beta \int v(w, z')Q(z, dz')\right]$$

Under the assumptions and $\beta \in (0, 1)$ a solution to the functional problem exists and it is the supremum function (Stokey and Lucas [52, Assumptions 9.1 and 9.2 pages 243-244, and Theorem 9.12 page 274]). Also, the associated policy correspondence G is nonempty and the plans generated by G are optimal, where G is defined as,

$$G(x, z) = \left\{w \in \Gamma(x, z) : v(x, z) = F(x, w, z) + \beta \int v(w, z')Q(z, dz')\right\}$$

Note that equations (11) and (13) are the dynamic analog of the Samuelson condition for the optimal provision of a public good. In the steady state $\overline{z} = 1$ and assuming that $\phi = 0$ in equation (11) leads to the standard asset pricing Euler Equation for consumption as in Uhlig [56, p. 51, equation (i)].

For the purposes of derivations and calibration, we specified the value of ψ to equal one half (as in Chameli and Braden [16]). Therefore, the utility is defined with equal weights allotted to consumption and environment quality. Intuitively, a higher value for ψ implies that the representative household prefers more consumption relative to environmental quality, and therefore, does not provide a reasonable ground to adopt 'green' GDP.

Solving the FOC equations and the resource constraints, the steady state of the system is as follows,

$$\overline{c} = \frac{\overline{z}(1-\beta+\beta\delta_e)\left(A\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{\alpha}{\alpha-1}} - \delta_k\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}}\right)}{\overline{z}-\phi-\beta\overline{z}+\beta\phi+2\beta\overline{z}\delta_e} \tag{15}$$

$$\overline{e} = \frac{\beta\overline{z}(\overline{z}+\phi)\left(A\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{\alpha}{\alpha-1}} - \delta_k\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}}\right)}{\overline{z}-\phi-\beta\overline{z}+\beta\phi+2\beta\overline{z}\delta_e} \tag{16}$$

$$\overline{m} = \frac{(-\phi+\beta\phi+\beta\overline{z}\delta_e)\left(A\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{\alpha}{\alpha-1}} - \delta_k\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}}\right)}{\overline{z}-\phi-\beta\overline{z}+\beta\phi+2\beta\overline{z}\delta_e} \tag{17}$$

$$\overline{\lambda} = \frac{\overline{z}-\phi-\beta\overline{z}+\beta\phi+2\beta\overline{z}\delta_e}{(1-\beta+\beta\delta_e)\left(A\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{\alpha}{\alpha-1}} - \delta_k\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}}\right)(\overline{z}+\phi)} \tag{18}$$

$$\overline{y} = A\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{\alpha}{\alpha-1}} \tag{19}$$

$$\overline{i} = \delta_k\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}} \tag{20}$$

$$\overline{k} = \left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}} \tag{21}$$

From the FOC, we log-linearize each equation around the steady state to build the system of rational expectations equations (Appendix, Subsection 5.1), and then solve it numerically with sensitivity to the structure (Uhlig [56, p. 51, equation (i)]).

Next, we study the model' properties, - specifically, the viability of the constraints imposed on the calibrated coefficients and their implications regarding the behavior of the variables. At first, we show that for the steady state variables to exist and to be non-negative, we need β to obey $\beta > \frac{1-\phi}{3-\phi}$ (as illustrated in Figure 1). Furthermore, we demonstrate that for $\overline{m}$ to be positive, β must satisfy $\beta > \frac{\phi}{\phi+\delta_e}$, and for $\overline{c}$ to be positive, β must obey $1/\beta > (1-\delta_e)$ (as illustrated in Figure 2). Therefore, to ensure that consumption is always positive, we impose the stronger condition of $\delta_e \in (0,1)$, i.e., the environment naturally depletes itself at the positive rate of δ_e. In other words, for the model to possess a non-negative steady state, the environment must be non self-healing. Interestingly, a self-healing environmental quality can generate positive consumption, only if the representative household is impatient - more specifically, if $\beta < 0.5$.

Finally, we study the effect of the stochastic shock on consumption. Under standard conditions - such as $\beta < 1$ and $\phi > 0$, we demonstrate that consumption will always decrease following an investment in environmental quality. That is, such a 'green animal spirit' will crowd out consumption.

Proposition 1 *For the model to possess non-negative steady states values in $\overline{c}$, $\overline{e}$ and $\overline{m}$, we must have $\beta > \frac{1-\phi}{3-\phi}$.*

Proof. *See the Appendix.* □

The proposition emphasizes that for the steady state to exist and to be non-negative, β

$\in \{0,1\}$ must satisfy $\beta > \frac{1-\phi}{3-\phi}$. Figure 1 illustrates the area wherein the steady state variables exist and are non-negative. Within the standard and acceptable range of $\beta \in (0.9, 1)$, ϕ can take on any value.

Proposition 2 *If* $(\beta = 0,\ \phi = 0$ *and* $\forall \delta_e \in (-1,1))$ *or* $(\beta = 1,\ \delta_e = 0$ *and* $\forall \phi \in (0,1))$ *or* $(\beta = 0.5,\ \forall\ \phi = \delta_e \in (0,1))$ *then* $\overline{m} = 0$.

Proof. *See the Appendix.* □

Even if ‘green’ GDP is adopted as a national accounting system, the proposition describes a situation wherein there will not be any investment in environmental quality, if one of the following is true:

(1) the representative agent is impatient (i.e., rate of time preference equals to infinity) and does not consume the environment, regardless of the value of the environmental degradation. This is a situation where she mainly depends on physical capital.

(2) the representative agent is very patient (i.e., rate of time preference equals to zero) and the environment does not degrade, nor heal, regardless of the rate at which humans consume the environment.

(3) the representative agent holds a rate of time preference of 100 percent and the rate at which human consume the environment equals to the rate at which the environment degrades.

In our quarterly calibration, we use $\beta = 0.99$, which implies a positive amount of investment in environmental quality.

Proposition 3 *To ensure that* $\overline{c} \neq 0$, *impose the stronger assumption that* δ_e *should be nonnegative.*

Proof. *See the Appendix.* $\square$

This proposition states that for consumption to be positive at the steady state, the stock of environmental quality must be non self-healing. Given that there is no point estimate for δ_e, we investigated the range over which consumption could be negative. Figure 2 illustrates the area wherein consumption is positive as function of the calibrated parameters β and δ_e.

What is the effect of the shock on consumption? The effect of the shock is captured in equations (22) and (23).

$$\frac{\partial \overline{c}}{\partial z} = \frac{(1-\beta+\beta\delta_e)\left(A\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{\alpha}{\alpha-1}} - \delta_k\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}}\right)\phi\,(-1+\beta)}{\left(\overline{z}-\phi-\beta\overline{z}+\beta\phi+2\beta\overline{z}\delta_e\right)^2} \tag{22}$$

$$\frac{\partial \overline{e}}{\partial z} = \frac{\beta\left(A\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{\alpha}{\alpha-1}} - \delta_k\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}}\right)\left(\overline{z}^2-\beta\overline{z}^2+2\beta\overline{z}^2\delta_e-\phi^2+\beta\phi^2-2\overline{z}\phi+2\overline{z}\beta\phi\right)}{\left(\overline{z}-\phi-\beta\overline{z}+\beta\phi+2\beta\overline{z}\delta_e\right)^2} \tag{23}$$

Proposition 4 *If* $\beta < 1$, $\phi > 0$, $\forall\delta_e \in (0,1)$ *and* $z \neq \frac{\phi(1-\beta)}{1-\beta+2\beta\delta_e}$ *then* $\frac{\partial\overline{c}}{\partial z} < 0$.

Proof. *See the Appendix.* $\square$

That is, under the regular assumptions of: 1) $\beta < 1$, which is needed for the convergence of the summation of momentary utility, and 2) $\phi > 0$, i.e., a positive percentage of environmental quality is consumed, a stochastic increase in environmental investment crowds out consumption. This effect is illustrated by the impulse response in Figure 5 and Figure 6 for different values of δ_e and ϕ. That is, 'green' animal spirits crowd out consumption.

So what happens after an investment in environmental quality? Under regular conditions and as long as the environment suffers a natural decay (i.e., $\delta_e > 0$), the previous proposition argues that the economy could face a recession. How steep will the decline be? The following proposition shows that the answer depends on the estimated values of the parameters β, δ_e and ϕ.

Proposition 5 *If* $\beta < 1$, *and* $\overline{y} > \overline{i} \implies \frac{\partial \overline{c}}{\partial \delta_e} < 0$.

Proof. *See the Appendix.* $\square$

The novelty in this model is that we study the dynamics of how environmental degradation reduces consumption and consequently, reduces 'green' GDP. To emphasis this issue, let's take the known Brazilian example. If Brazil decided to cut down its entire rainforest and sell it, standard [non-green] GDP would increase due to the market value of the trees. In this case, environmental degradation increases standard [non-green] GDP. The same point was made in Repetto et al. [49] for the Indonesian economy. In the model presented here, environmental degradation subtracts from consumption and consequently, from 'green' GDP. It reduces steady state consumption $\left(\frac{\partial \overline{c}}{\partial \delta_e} < 0 \text{ from Proposition 5}\right)$ and increases steady state environmental investment $\left(\frac{\partial \overline{m}}{\partial \delta_e} > 0\right)$ by the same amount. Therefore, the total effect on 'green' GDP is non-positive.

3 Results

Given the absence of empirical evidence on the estimated values for the δ_e and ϕ parameters, we perform sensitivity analysis. We restrict the space of $\phi \in (0,1)$ and $\delta_e \in (-1,1)$. Indeed, the parameter δ_e can be calibrated to a negative value. The interpretation of a negative value for δ_e is that the environment heals itself absent of pollution. The calibrated parameters are chosen to ensure that the capital to output steady state value matches the sample data. The following Table reports the calibrated parameters used to generate the impulse responses.

[**Insert Table 1 here**]

For a self-healing (non-healing) environment, welfare increases (decreases) with δ_e (Figure 3). Following a shock to the investment of environmental quality, consumption decreases and environmental quality increases. From the utility, the sum of these effects dictates the change in welfare. In all our simulations, the increases in environmental quality outweighed the decrease in consumption and therefore welfare improved. Figure 4 illustrates the effect of the shock on the utility $(\partial U/\partial z)$ as function of the parameters ϕ and δ_e. In response to a shock, $\partial U/\partial z$ is decreasing in ϕ. As ϕ increases, the effect of the shock on the utility diminishes. The larger is the consumption dependence on environmental quality, the smaller is the effect on welfare from the shock.

Following a shock z in environmental investment, the impulse response for consumption are illustrated in Figures 5 and 6. The larger is the value of δ_e (natural depletion), the more pronounced is the effect on consumption. A high rate of degradation forces a larger decrease in consumption to smooth the representative agent' utility with the effect persisting for at least 28 quarters. From Figure 5, with an 80 percent depletion rate, a positive shock to environmental quality forces consumption to decrease by 10 percent relative to its steady state value. Across all values of δ_e, the range in consumption drop is from 6 percent to 10 percent. From Figure 6, faced with a stochastic shock in environmental investment, consumption decreases in an inverse relation to the value of ϕ. A higher value of depletion due to consumption implies a lower drop in consumption. The effect of a one percent increase in environmental investment is less pronounced on consumption, the higher is the value of consumption related depletion. Across all values of ϕ, the decrease in consumption varies from 6 percent to 17 percent with the effect persisting for at least 28 quarters.

Finally, we investigate if the results are robust to the relative weights of consumption and environmental quality in the utility function, specifically, ψ. Figures 7 and 8 depict the influence of the ϕ and δ_e parameters on the utility at the steady state. Figure 7 shows that the weight of consumption in the utility function plays a crucial role at lower rates for environmental depletion. At a higher level of environmental depletion ($\delta_e = 80$ percent), utility is symmetric in ψ with a minimum at ψ=0.5. Figure 8 illustrates utility as function of the consumption weight and the rate ϕ at which humans consume the environment. As people put more weight on consumption (higher ψ), the parameter ϕ becomes important and yields a higher utility, the higher is its value.

To summarize, the decrease in consumption ranges from 6 percent to 17 percent following a one percent increase in environmental investment. The effect of the shock is immediately felt, reaches a trough within a year of the shock, and lasts for at least 7 years. The larger (smaller) is the value of consumption related environmental degradation, the weaker (stronger) is the effect of the shock on consumption. The latter result is surprisingly counter intuitive.

So what happens to consumption following a one percent increase in the investment of environmental quality? Evaluated in billions of chained 2000 dollars, a one percent decrease in 2005 total real personal consumption expenditures amounts to $ 19.41 billions per quarter.[8] Conditional on the calibrated parameter values, the model suggests that an increase of one percent in environmental investment will lead to a range from $ 102.74 billions to $ 171.11 billions in lost real quarterly consumption, on average, in every quarter for seven years following the investment.

4 Conclusions

Does an investment in environmental quality have the potential of pulling the economy out a recession? In other words, assuming that 'green' accounting is adopted, can a fiscal policy that is primarily devoted to investing in environmental quality [green animal spirits] generate enough impetus to propel the economy out of a recession? We conclude that the answer is no.

Following a one percent increase in the investment of environmental quality, we found that the effect of the shock on consumption is immediately felt, reaches a trough within a year, and lasts for at least 7 years. The larger (smaller) is the value of consumption related environmental degradation, the weaker (stronger) is the effect of the shock on consumption.

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[8] Source: U.S. Department of Commerce: Bureau of Economic Analysis. The series ID: PCECC96 refers to the seasonally adjusted annual rate (SAAR) quarterly real personal consumption expenditures in billions of chained 2000 Dollars. During the first quarter of 2005, the SAAR real quarterly personal consumption stood at $ 7,764.9 billions.

This result is surprisingly counter intuitive. Whereas most studies encourage a reduction in environmentally-based human consumption, here we found that reducing it renders a fiscal policy - that engages in environmental investment - less effective in providing a thrust out of a recession.

In a business-cycle context, we found that *eliminating* these short-run fluctuations (in environmental investment) results in a range of *gains* in consumption from six percent to seventeen percent. This range is well within the band suggested in the business cycle literature.[9]

A host of empirical applications and theoretical extensions of the model could be considered as an agenda for future research. The results of the model are empirically testable hypotheses. Conditional on data availability - specifically a well-developed measure of environmental quality - such an empirical investigation should lead to a greater insight into how investment in environmental quality influences the economy in the short run. Theoretical extensions could to be pursued along the lines of integrating the production of environmental quality, pollution emissions in the production process, and the policy choice that might lead to a Pareto-improving equilibrium.

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[9] See Campbell and Cochrane [13], Dolmas [18], Krusell and Smith [35], Lucas [36, Chapter III], Obstfeld [44], Otrok [46] and Tallarini [53], wherein they found that the elimination of business cycle fluctuations resulted in gains ranging from one to twenty percent in terms of real consumption.

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5 Appendix

5.1 *The Log-linearized steady state*

This is the FOC log-linearized system, where x denotes the deviation from the steady state.

$$\begin{aligned}
y_t &= \alpha k_t \qquad (24)\\
-\overline{C}^{-1}\psi c_t &= \left(\overline{\Lambda} + \phi\overline{\Lambda Z}^{-1}\right)\lambda_t - \phi\overline{\Lambda Z}^{-1} z_t \qquad (25)\\
\overline{\Lambda}\lambda_t &= \beta\overline{\Lambda}(1-\delta_k)\lambda_{t+1} + \beta\alpha A\overline{\Lambda K}^{\alpha-1}\left(\lambda_{t+1} + (\alpha-1)k_{t+1}\right) \qquad (26)\\
\overline{\Lambda Z}^{-1}\lambda_t - \overline{\Lambda Z}^{-1} z_t &= -\beta\overline{E}^{-1}(1-\psi)e_{t+1} + \beta(1-\delta_e)\overline{\Lambda Z}^{-1}\left(\lambda_{t+1} - z_{t+1}\right) \qquad (27)\\
\overline{Y}y_t &= \overline{C}c_t + \overline{I}i_t + \overline{M}m_t \qquad (28)\\
\overline{K}k_{t+1} &= (1-\delta_k)\overline{K}k_t + \overline{I}i_t \qquad (29)\\
\overline{E}e_{t+1} &= (1-\delta_e)\overline{E}e_t + \overline{ZM}\,\left(z_t + m_t\right) - \phi\overline{C}c_t \qquad (30)
\end{aligned}$$

5.2 *Mathematical Proofs of the Propositions (Detailed)*

5.2.1 *Proof of Proposition 1*

The proof of Proposition 1 will be carried over the following steps.

Lemma 1 *Given equation (7), the steady state equations, the assumptions made on the utility function, the production function and the Inada conditions, at the steady state:* $\overline{y} \neq \bar{i}$.

Proof. From the steady state equations 19 and 20, $\left(\overline{y} = \bar{i}\right) \implies [(\alpha = 1) \wedge (A = \delta_k)]$. Using the contrapositive, this is equivalent to $[(\alpha \neq 1) \vee (A \neq \delta_k)] \implies \left(\overline{y} \neq \bar{i}\right)$. However, from the assumptions on the production function and the Inada conditions, we know that $\alpha \neq 1$. Finally, use detachment to get $\overline{y} \neq \bar{i}$. □

Definition 1 *Let D denotes the denominator $z - \phi - \beta z + \beta\phi + 2\beta z\delta_e$ at the steady state, i.e., when $\overline{z} = 1$. $D_{\phi,\delta_e,\beta} : \mathbb{R}_{(0,1)} \times \mathbb{R}_{(-1,1)} \times \mathbb{R}_{(0,1)} \to \mathbb{R}$ and $D_{\phi,\delta_e,\beta} \equiv \{1 - \phi - \beta + \beta\phi + 2\beta\delta_e; \phi \in$*

$(0,1), \delta_e \in (-1,1), \beta \in (0,1)\}$.

Definition 2 *The denominator* $D_{\phi,\delta_e,\beta}$ *could be negative, positive or zero. At the steady state* $\overline{z} = 1$, *let's define* $\delta_e < \frac{1}{2\beta}[\beta + \phi - \phi\beta - 1]$ *as (≡P1), and* $\delta_e > \frac{1}{2\beta}[\beta + \phi - \phi\beta - 1]$ *as (≡P2).*

Lemma 2 $D_{\phi,\delta_e,\beta} \nrightarrow \infty$.

Proof. By Definition 1, $\nexists (\phi, \delta_e, \beta) \subseteq \mathbb{R}_{(0,1)} \times \mathbb{R}_{(-1,1)} \times \mathbb{R}_{(0,1)} \in dom(D_{\phi,\delta_e,\beta})$ such that $D_{\phi,\delta_e,\beta} \rightarrow \infty$. □

Lemma 3 *For the steady state to exist (wherein* $\overline{z} = 1$*), then* $\delta_e \gtrless \frac{1}{2\beta}[\beta + \phi - \phi\beta - 1]$*,i.e.,* $P1 \vee P2$ *(from Definition 2).*

Proof. If $\delta_e = \frac{1}{2\beta}[\beta + \phi - \phi\beta - 1]$, then $\overline{c} \rightarrow \infty$, $\overline{e} \rightarrow \infty$ and $\overline{m} \rightarrow \infty$ (i.e., the denominator of equations (15), (16), and (17) goes to zero). □

Lemma 4 $\delta_e > \frac{1}{2\beta}[\beta + \phi - \phi\beta - 1] \implies \beta > \frac{1-\phi}{3-\phi}$ *or using Definition 2,* $P2 \implies \beta > \frac{1-\phi}{3-\phi}$

Proof. $\delta_e > \frac{1}{2\beta}[\beta + \phi - \phi\beta - 1]$ and $\delta_e > -1$ (from Definition 1), therefore, $\frac{1}{2\beta}[\beta + \phi - \phi\beta - 1] > -1$, multiplying all by 2β yields and knowing that β is positive (from Definition 1) $[\beta + \phi - \phi\beta - 1] > -2\beta$, solving yields $\beta > \frac{1-\phi}{3-\phi}$ □

Lemma 5 *For the model to possess non-negative steady states values in* $\overline{c}$, $\overline{e}$ *and* $\overline{m}$, *we must have* $\delta_e \nless \frac{1}{2\beta}[\beta + \phi - \phi\beta - 1]$, *i.e.,* $\sim P1$.

Proof. The proof is carried using Indirect proof, Modus Ponens, Modus Tollens, and simplification. If $P1$ is true, then the numerator of each one of the following equations [equations (15), (16), and (17)] must be negative to reach positive steady states values. For equation (15), this means that $(1 - \beta - \beta\delta_e) < 0$, dividing by β and rearranging yields, $1/\beta < 1 - \delta_e$.

Since β is the discount factor and $\beta < 1$, this implies that $1/\beta > 1$, i.e., $1 < 1/\beta < 1 - \delta_e$, therefore δ_e must be negative, $\delta_e < 0$ ($\equiv$Q1). For equation (16), $\beta(1+\phi) < 0$, and since $\beta > 0$, this implies that $\phi < -1$ ($\equiv$Q2). For equation (17), $(-\phi + \beta\phi + \beta\delta_e) < 0$, simplifying yields $\delta_e < \phi(1-\beta)/\beta$ ($\equiv$Q3). Using Modus Ponens $[P1 \wedge (P1 \implies (Q1 \wedge Q2 \wedge Q3))] \implies (Q1 \wedge Q2 \wedge Q3)$. Since $Q2$ is a $\rightarrow\leftarrow$ (contradiction) to Definition 1, we use Modus Tollens $[\sim(Q1 \wedge Q2 \wedge Q3) \wedge (P1 \implies (Q1 \wedge Q2 \wedge Q3))] \implies \sim P1$, therefore we conclude $\sim P1$. To explain, by assuming $P1$, we get a $\rightarrow\leftarrow$ (contradiction), $Q2 \notin D_{\phi,\delta_e,\beta}$. In other words, to have non-negative steady state values using a negative denominator, we need $\phi < -1$ (Q2), which is not feasible in economic terms. ϕ is the fraction of environment quality used for consumption, i.e., it is positive and it is less than one in value. $\square$

Proposition 1 *For the model to possess non-negative steady states values in* $\overline{c}$, $\overline{e}$ *and* $\overline{m}$, *we must have* $\beta > \frac{1-\phi}{3-\phi}$.

Proof. *From Lemmas 3, 4 and 5, the result is obtained using detachment.* $\square$

5.2.2 Proof of Proposition 2

Lemma 6 $\overline{m} = 0 \iff \left(\beta = \frac{\phi}{\phi+\delta_e}\right) \vee (D_{\phi,\delta_e,\beta} \to \infty) \vee (\overline{y} = \overline{i}) \iff \left(\beta = \frac{\phi}{\phi+\delta_e}\right)$

Proof. Given Lemma 1 and Lemma 2, the last $\iff$ is achieved using contradictive addition and simplification. First, let's demonstrate that $\left(\beta = \frac{\phi}{\phi+\delta_e}\right) \implies \overline{m} = 0$. Solving $\left(\beta = \frac{\phi}{\phi+\delta_e}\right)$ yields $-\phi + \beta\phi + \beta\delta_e = 0$. Given that at the steady state $\overline{z} = 1$, $-\phi + \beta\phi + \beta\delta_e = 0$ implies that $\overline{m} = 0$ ($\equiv$M1). Now, let's demonstrate that $\overline{m} = 0 \implies \left(\beta = \frac{\phi}{\phi+\delta_e}\right)$. For $\overline{m} = 0$, it must be that $(-\phi + \beta\phi + \beta\delta_e = 0) \vee (D_{\phi,\delta_e,\beta} \to \infty) \vee (\overline{y} = \overline{i}) \iff (-\phi + \beta\phi + \beta\delta_e = 0)$ from Lemma 1 and Lemma 2, therefore $\overline{m} = 0 \implies \left(\beta = \frac{\phi}{\phi+\delta_e}\right)$ ($\equiv$M2). The last $\iff$

is carried using contradictive addition, since $D_{\phi,\delta_e,\beta} \nrightarrow \infty$ from Lemma 2 and $\overline{y} \neq \overline{i}$ from Lemma 1. Combine M1 and M2, $\overline{m} = 0 \iff \left(\beta = \frac{\phi}{\phi+\delta_e}\right)$. □

Proposition 2 *If* $(\beta = 0,\ \phi = 0$ *and* $\forall \delta_e \in (-1,1))$ *or* $(\beta = 1,\ \delta_e = 0$ *and* $\forall \phi \in (0,1))$ *or* $(\beta = 0.5,\ \forall\ \phi = \delta_e \in (0,1))$ *then* $\overline{m} = 0$.

Proof. The result is a corollary from Lemma 6 and P1. □

5.2.3 Proof of Proposition 3

Lemma 7 $\overline{c} = 0 \iff ((1-\delta_e) = 1/\beta) \vee (D_{\phi,\delta_e,\beta} \to \infty) \iff (1-\delta_e) = 1/\beta \implies \delta_e \in (-1,0)$.

Proof. Given Lemma 2, the last $\iff$ is achieved using contradictive addition and simplification. $\overline{c} = 0 \implies \delta_e = 1 - (1/\beta) = (\beta - 1)/\beta$

$$\implies \begin{cases} \delta_e \in (-\infty, -1) & \beta \in (0, 0.5) \\ \delta_e = -1 & \beta = 0.5 \\ \delta_e \in (-1, 0) & \beta \in (0.5, 1) \end{cases}$$

Therefore, $\forall \delta_e \in (-1,0) \subset (-1,1)$, $\exists \overline{\beta} \in (0.5,1) \subset \mathbb{R}_+$, such that $\overline{c} = 0$. Note that for $\delta_e \in (0,1)$, $\nexists \overline{\beta} \in (0,1) \subset \mathbb{R}_+$, such that $\overline{c} = 0$. □

Proposition 3 *To ensure that* $\overline{c} \neq 0$, *impose the stronger assumption that* δ_e *should be nonnegative.*

Proof. *This is a corollary from Lemma 7. To get the proof just take the negation of Lemma 7.* □

5.2.4 Proof of Proposition 4

Proposition 4 *If* $\beta < 1$, $\phi > 0$, $\forall \delta_e \in (0,1)$ *and* $z \neq \frac{\phi(1-\beta)}{1-\beta+2\beta\delta_e}$ *then* $\frac{\partial \overline{c}}{\partial z} < 0$.

Proof. *First, note that the denominator of equation (22) goes to zero at* $z = \frac{\phi(1-\beta)}{1-\beta+2\beta\delta_e}$, *which implies that* $\overline{c} \rightarrow \infty$ *(from equation (15)) and* $\frac{\partial \overline{c}}{\partial z}\mid_{z=\frac{\phi(1-\beta)}{1-\beta+2\beta\delta_e}} \rightarrow \infty$. *From equation (22),* $(1-\beta+\beta\delta_e) = (1-\beta(1-\delta_e)) > 0$, *and* $\beta \in (0,1)$ *implies that* $(-1+\beta) < 0$, *and the resources constraint is satisfied at the steady state, i.e.,* $\overline{c} + \overline{m} + \overline{i} \geq \overline{y}$, *therefore* $\frac{\partial \overline{c}}{\partial z} < 0$ □

5.2.5 Proof of Proposition 5

Proposition 5 *If* $\beta < 1$, *and* $\overline{y} > \overline{i} \implies \frac{\partial \overline{c}}{\partial \delta_e} < 0$

Proof. *Deriving equation (15) with respect to* δ_e *yields*

$$\frac{\partial \overline{c}}{\partial \delta_e} = \frac{\overline{z}\beta\left(A\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{\alpha}{\alpha-1}} - \delta_k\left(\frac{1-\beta+\delta_k\beta}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}}\right)(-\overline{z}-\phi+\beta\overline{z}+\beta\phi)}{(z-\phi-\beta\overline{z}+\beta\phi+2\beta\overline{z}\delta_e)^2} \tag{31}$$

The denominator is always positive. The numerator is made of $\overline{z}\beta\left(\overline{y}-\overline{i}\right)(-\overline{z}-\phi+\beta\overline{z}+\beta\phi)$ *which is equal to* $\overline{z}\beta\left(\overline{y}-\overline{i}\right)(-1+\beta)(1+\phi)$. *This last expression is always negative given that* $\beta < 1$, *and* $\overline{y} > \overline{i}$ □

6 Table

Table 1: Calibrated Values

Parameter	Value(s)	Definition
β	0.99	Discount factor
$1-\psi$	0.50	Weight of environmental quality in the utility
δ_k	0.06	Depreciation rate for capital
α	0.35	Capital's share in income
ρ	0.90	Degree of the shock persistence
δ_e	$\in \{0.2, 0.5, 0.8\}$	Depreciation rate for environment
ϕ	$\in \{0.2, 0.5, 0.8\}$	Percentage degradation due to consumption

7 Figures

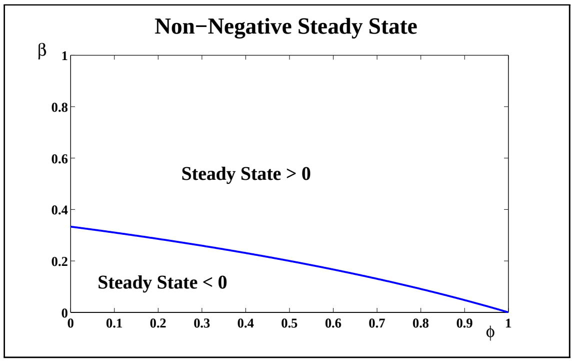

Figure 1: StSt Conditions on β and ϕ

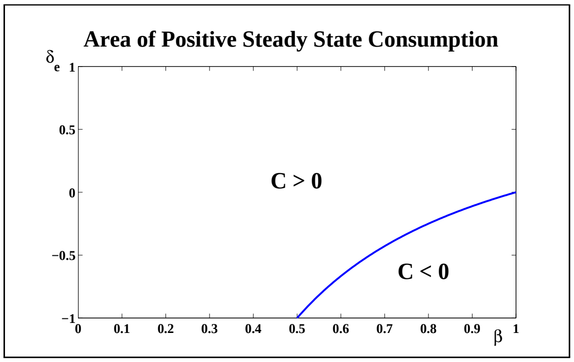

Figure 2: Area of positive consumption

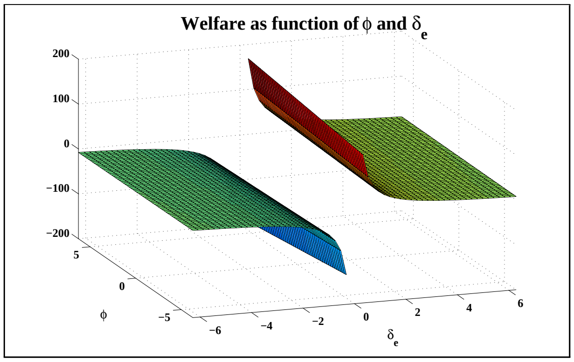

Figure 3: Welfare as function of ϕ and δ_e

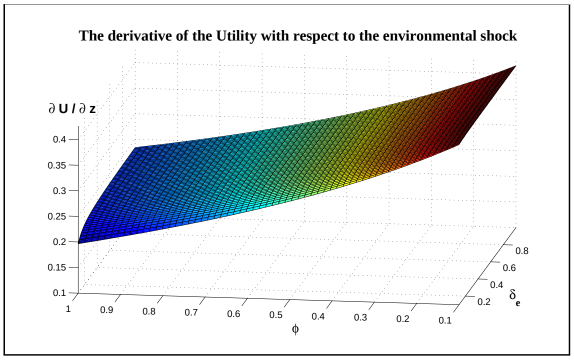

Figure 4: $\partial U/\partial z$ function of ϕ and δ_e

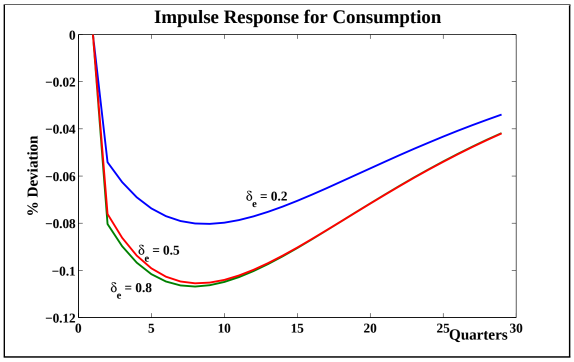

Figure 5: IR for different δ_e ($\phi = 0.5$)

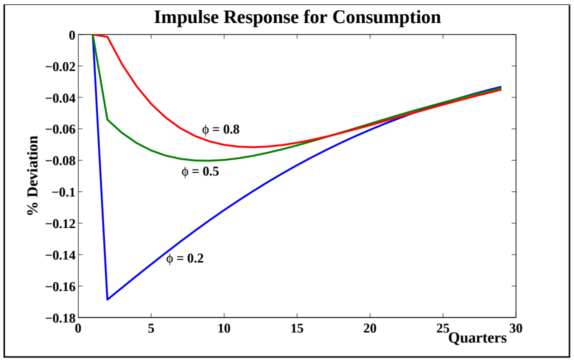

Figure 6: IR for different ϕ ($\delta_e = 0.2$)

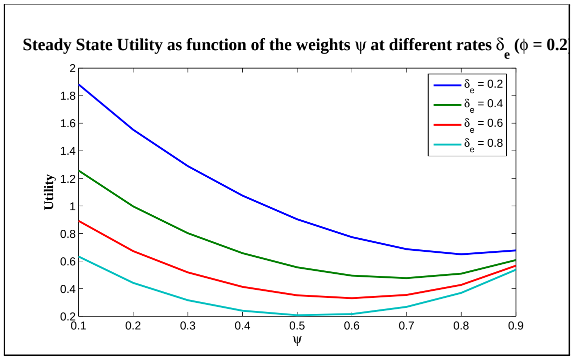

Figure 7: Utility, Consumption Weight and δ_e

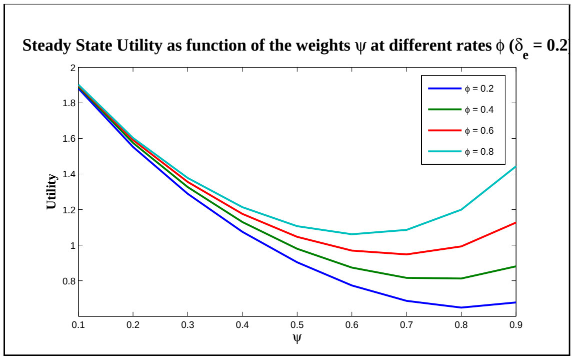

Figure 8: Utility, Consumption Weight and ϕ