The PD-Utility Function for Prospect Behavior and Related Researches

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***Abstract*—Based on Partial Distribution [11],[12], we put forward a PD-utility function of prospect behavior for the first time, the profiting utility function and losing utility function. The PD-utility function can reflect sufficiently the human's risk preferences properties to profiting or losing, describe and bring to light availably the important relations between profiting utility and losing utility, and interpret many conclusions in Daniel Kahneman's prospect theory in analytic way. Also we present the concepts and analytic expressions of essential indexes of realized level for prospect behavior, the limit value, the balanced value, and focus value, especially the method of calculating them. The limit level is beneficial to judge the reversal position of reality movement trend, and the latter is beneficial to judge that the focus of current reality is reasonableness or not. And we give out the calculating formula for the optimal value of realized level for prospect with its appearing probability.**

***Index terms*—Partial distribution, PD-utility function, prospect behavior, essential indexes, optimal value**

Ⅰ. Introduction

In recent years and in the international academic field , it becomes an important research trend that behavioral science is crossed and blended consciously to finance , economy and management science. In this respect, the behavior economics is most representative. Daniel Kahneman, the gainer of 2002 Nobel's prize of economic science, is a representation of economists those who study behavior economics. Using for reference of psychology researches, the researchers of behavior economics dedicate to discuss the psychology mechanism behind human's economic behavior. The main point of researches is that economic behavior is controlled not only by the rational profiting, but also by non-rational psychology.

Taking as a scale index, the utility is applied primarily to describe the approving degree of human's subjective attitude, preference, and value preference to something in reality. The utility theory is widely applied in many fields, like management, economics, risk decision, investment analysis etc. The importance of utility theory leaves no room for doubt. The traditional economics think that rational Economic person can estimate every kind of possibility of the future different consequents, and then maximize their expectation utility, but Daniel Kahneman and some other people do a lot of experiments with investigate, and the results enunciate: rational assumption should be doubted. So they establish the "prospect theory" [1]. Contrasting the experimental results, they find not all of individuals are rational and risk-avoiding [6]-[8]. Also D. Kahneman and some other people discover that human's decision lie on the difference between outcome and conceivability, instead of the result itself under the uncertain conditions. In other words, when people make a decision, there is usually a consult standard in his mind, and then looks into difference between the outcome and the consult standard. Comparing with the describing type of "prospect theory" is better than the axiom type of "expectation utility theory" in explaining human's attitude of detesting losing. Prospect theory can explain the obvious human's behavior of risk preference that the expectation utility theory could not explain, and make the decision theory of human behavior more perfect under the uncertain conditions.

This paper will establish a stochastic model to describe human's prospect behavior based on the partial distribution (PD) [11],[12], and give out a new kind of utility function. We call it the PD-utility function. By PD-utility function, we can interpret many of human's decision behavior.

Ⅱ. Basic Assumptions

Definition 1 We call the conceiving state level, that people think something should be in actuality, the prospect level (*PL*) of the thing , and the actual state level of the thing is called the realized level (*RL*) of the prospect.

The *RL* is the result of human behavior based on a *PL*. According to basic characters of human' prospect behavior, we give the basic assumptions as follow:

Assumption 1 Non-negative. The lowest limit of PL is no prospect, i.e. *PL* equal to zero. The lowest limit of *RL* is no realization, i.e. RL equal to zero. So the *PL* and *RL* are all non-negative.

Assumption 2 Rationality. When keep off the prospect level gradually, the possibility that prospect is realized steps down gradually, i.e. the more far from *PL* the RL is, the less the possibility that the *RL* appears is. The lower the *PL*, the larger the possibility that the RL equal to zero is.

Assumption 3 Fluctuation. There is fluctuation in *PL*, and the fluctuation spread (i.e. the variance) of *PL* is non-negative.

Assumption 4 Continuity. *RL* is stochastic, and varies continuously.

If a prospect level is an average of prospects from most of people, we call it an average prospect level (*APL*). The **Assumption 2** could be more expressed as: when the *RL* is lower than *APL*, the *RL* will become higher to tend to *APL*, and when the *RL* is higher than *APL*, the *RL* will become lower to tend to *APL*.

Ⅲ. PARTIAL DISTRIBUTION AND THE METHOD OF MEASURING *RL*

A. Partial Distribution and related results

Definition 1 (The Partial Distribution). Let X be a non-negative stochastic variable, and it follows the distribution of density

$$f(x)=\begin{cases} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx & x\geq 0 \\ 0 & x<0 \end{cases} \tag{1}$$

then X is said to have a Partial Distribution, and denotes $X\in P(\mu,\ \sigma^2)$. The partial distribution is a kind of truncated normal distribution.

Definition 2 (The Partial Process) If stochastic variable X is related to time, i.e. $\forall\ t\in[0,\infty)$, we have $X(t)\in P(\mu(t),\sigma^2(t))$, then the $\{X(t),\ t\in[0,\infty)\}$ is called a partial process.

If X is a non-negative stochastic variable, and $X\in P(\mu,\ \sigma^2)$, we have the following research results from [11], [12]:

1) The expected value $E(X)$ of X, is as follows

$$E(X)=\int_0^\infty tf(t)dt=$$

$$=\mu+\sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{\sigma^2}}dx \tag{2}$$

where, $R(X)=\sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$ is the margin of $E(X)$ and μ.

2) The variance $D(X)$, is as follows

$$D(X)=\int_0^\infty (t-E(X))^2dt$$

$$=\sigma^2+E(X)[\mu-E(X)] \tag{3}$$

3) $$\int_0^x e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx=\sqrt{\frac{\pi}{2}}\sigma\times(\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+\operatorname{sgn}(x-\mu)\sqrt{1-e^{-\frac{2}{\pi}(\frac{x-\mu}{\sigma})^2}}) \tag{4}$$

where, $\operatorname{sgn}(x)=\begin{cases}1 & x>0\\ 0 & x=0\\ -1 & x<0\end{cases}$,

4) $$\int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx=\sqrt{\frac{\pi}{2}}\sigma(\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+1) \tag{5}$$

B. The Method of Measuring RL

If we regard μ as *APL*, and σ as the fluctuation spread (standard variance) of *APL*, and assumption 1—assumption 3 are all tenable, the variable X of *RL* satisfies the partial distribution according to [11], [14], i.e. $X\in P(\mu,\ \sigma^2)$. So we have the following deductions:

1) The average of *RL* is

$$E(X)=\int_0^\infty xf(x)dx$$

$$=\mu+\sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$$

where, $R(X)=\sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$ the average margin of *RL* and μ.

2) The variance of *RL* is

$$D(X)=\int_0^\infty [x-E(\mu,\sigma)]^2 f(x)dx$$

$$=\sigma^2+E(\mu,\sigma)[\mu-E(\mu,\sigma)]$$

We could think $D(X)$ as the actual risk of *RL*.

Ⅳ. THE PROSPECT UTILITY FUNCTION AND ANALYSIS OF RISK PREFERENCE

Here, the rule that we obey is: an *APL* and a *RL* constitute a decision. If *RL* is lower than *APL*, the result of decision making is of loss; if *RL* is higher than *APL*, the result of decision making is of profit. This is applicable to circumstance that *RL* is hoped to run high.

A. The Prospect Utility Function

Definition 3 If *RL* follows the partial distribution, i.e. the $X\in P(\mu,\ \sigma^2)$, X is the variable of *RL*, for any $x\in[0,\infty)$, we call

$$U(x)=\int_0^x e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx \tag{6}$$

the profiting utility function of a prospect, where, x means a sample value of *RL*. And call

$$\overline{U}(x)=1-U(x)$$

$$=\int_x^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx \tag{7}$$

the losing utility function of a prospect.

The Profiting utility (6) can be comprehended as satisfying degree at the same time, and losing utility can also be comprehended as desponding degree. The $U(x)$, profiting utility function, is also called PD-utility function.

From **Definition 3**, we will see as follows:
1) A lower *APL* is easy to be satisfied, and a higher *APL* is easy to cause a disappointment.
2) The higher the x (*RL*) is, the higher the satisfying utility is; and the lower the x (*RL*) is, the higher the desponding utility is.
3) If μ and σ are separately different, the same x will cause the different satisfying utility $U(x)$ and the different desponding utility $\overline{U}(x)$.
4) The probability of an prospect being realized, i.e. the probability of getting profiting, is

$$P(\mu) = \int_{\mu}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \Big/ \int_{0}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \qquad (8)$$

5) The probability of an prospect not being realized, i.e. the losing probability, is

$$\overline{P}(\mu) = \int_{0}^{\mu} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \Big/ \int_{0}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \qquad (9)$$

B. Analysis of Risk Preference

According to the theory of risk preference and (6), we have

$$v(x) = -\frac{U''(x)}{U'(x)} = \frac{x-\mu}{\sigma^2},$$

so,

$$v(x)\begin{cases} > 0(x > \mu), \text{aversion of risk} \\ = 0(x = \mu), \text{neutrality of risk} \\ < 0(x < \mu), \text{pursuing of risk} \end{cases} \qquad (10)$$

The equation (10) can be explained for: in general, when *RL* is higher than *APL*, person is aversed by the risk; when the *RL* is equal to *APL*, person is neutral to the risk; when *RL* is lower than *APL*, person pursues the risk. This kind of condition is marked in the stock market. For example, one is tend to sell the stock when his stock is in an accrual; One is tend to do nothing when his stock price is equal to the current price in the market; one is tend to hold the stocks or buy stocks more at the lower prices when his stock is in a losing.

V. Analysis of PD-Utility Function Characteristics

A. Characteristic Analysis of Profiting Utility Curve

Let μ=10 (*APL*), and σ=3.8(standard variance of *APL*), corresponding curve of satisfying utility is drown in Fig.1.

In Fig.1, *RL* is in a losing when $x\in[0, 10)$, and *RL* is in an accrual when $x\in[10, 20]$. We see, the utility curve is a curve of the letter "S" form, and there is an inflexion on the curve. The curve is concave in the profiting field, and is protruding in the losing field. And the curve's gradient is steeper in the losing field than in the profiting field. This result is consistent with Kahneman's "unreasonable investor" theory [2].

B. Characteristic Analysis of relations between APL and Profiting Utility

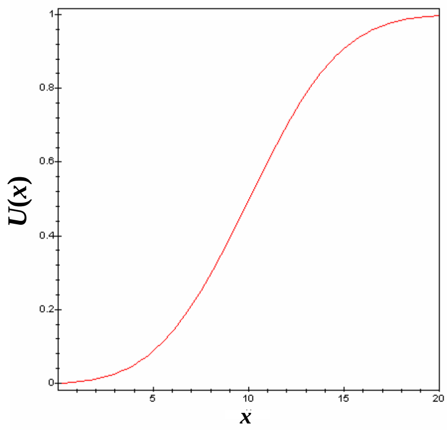

Fig.1. Profiting utility curve for fixing *APL*

In practice, the higher *APL* μ is, the less the profiting utility is, i.e. the more the desponding utility is. This Standpoint can be validated by the following analysis.
1) If *RL* x=10, σ=5.2, according to (6), the profiting utility will varies from high to low along with *APL* from small to large, this varying process is shown in Fig.2. And in Fig.2, the profiting utility is gradually down when the *APL* becomes larger and larger.

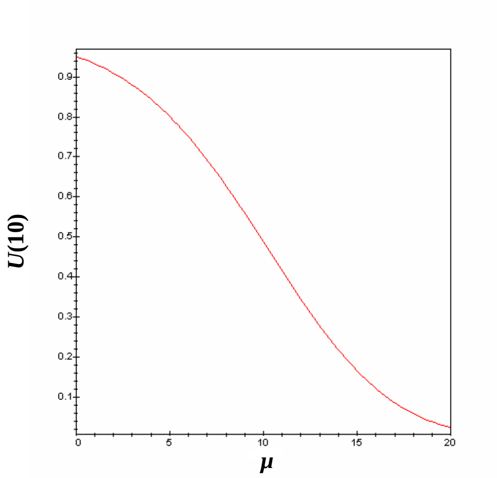

Fig.2. Profiting utility curve for varying *APL* and fixing *RP*

2) Again if σ=5.2, μ(*APL*) and x (*RL*) each vary independently from 0 to 20, then the profiting utility will vary correspondingly, this process is shown in Fig.3. Therefore, the relation between prospect behavior and profiting utility reflected by PD-utility function match the

actual circumstance in the basic characteristics. The related analysis about losing utility is similar to the profiting utility, and no longer describe here.

VI. COMPARISON ANALYSIS BETWEEN LOSING AND PROFITING UTILITY

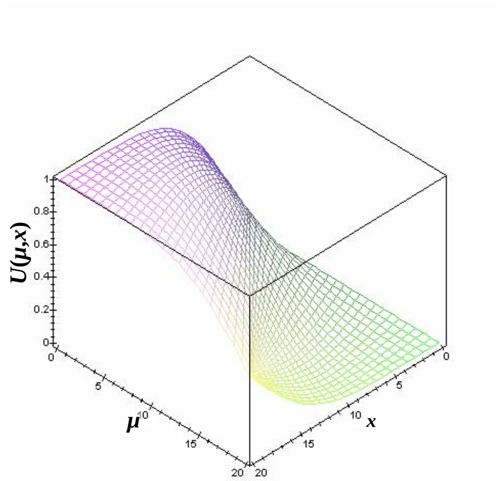

Fig.3. Profiting utility curve for both *APL* and *RL* varying

A. Comparison Analysis

For any $y \geq 0$, if losing utility is $\overline{U}(\mu - y)$, and profiting utility is $U(\mu + y)$, we have the following expression by using of (6), (7), (4) and (5),

$$\frac{\overline{U}(\mu - y)}{U(\mu + y)} = \frac{1 + \sqrt{1 - e^{-\frac{2}{\pi}\left(\frac{y}{\sigma}\right)^2}}}{\sqrt{1 - e^{-\frac{2}{\pi}\left(\frac{\mu}{\sigma}\right)}} + \sqrt{1 - e^{-\frac{2}{\pi}\left(\frac{y}{\sigma}\right)}}} > 1$$

That is $\overline{U}(\mu - y) > U(\mu + y)$, and also means the losing utility is larger than the profiting utility when the losing sum is equal to profiting sum.

In virtue of the concept "losing aversion" in perception psychology, D. Kahneman depict the liability people is more impressionable to decrease of his boon himself than increase of the same boon [8], [10], and make use of the gradient of loss function divided by gradient of profit function at original point to measure the degree of "losing aversion", and give, 2.0, an experience's estimate value. This means that the utility of giving up something is as two times as that of getting it [5].

However, according to the profiting utility given by expression (6) and losing utility given by (7), we get the following conclusion:

The losing utility is always larger than the profiting utility when the losing sum is equal to profiting sum, but the contrast value of loss utility to profit utility is determined by *APL*, the standard variance of *APL* and *RL*. We will see that in the following analysis.

B. The Examples

Let losing utility is $\overline{U}(\mu - y)$, and profiting utility is $U(\mu + y)$.

1) If μ=8(*APL*), and σ=12 (standard variance of *APL*),

When y=0.01, $\overline{U}(\mu - y)/U(\mu + y)$=2.013050943;

When y=7.99, $\overline{U}(\mu - y)/U(\mu + y)$=1.507478397.

The curve of other comparison results of losing utility to profiting utility is shown in Fig. 4.

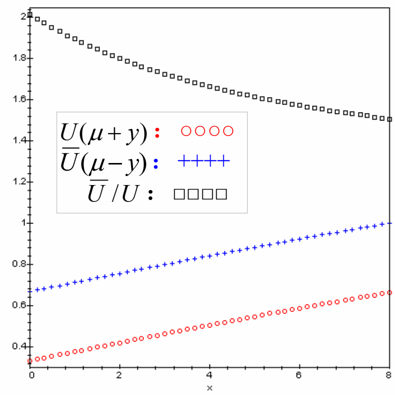

Fig. 4 The comparison of utility of loss and profit when μ=8,σ=12

2) If μ=8(*APL*), and σ=5 (standard variance of *APL*),

When y=0.01, $\overline{U}(\mu - y)/U(\mu + y)$=1.115030219;

When y=7.99, $\overline{U}(\mu - y)/U(\mu + y)$=1.0576318.

The curve of other comparison results of losing utility to profiting utility is shown in Fig. 5.

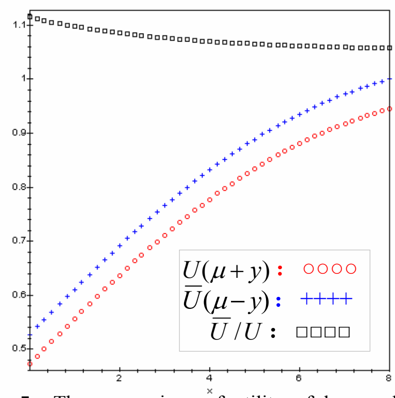

Fig. 5 The comparison of utility of loss and profit when μ=8,σ=5

3) If μ=8(*APL*), and σ =3 (standard variance of *APL*),

When y=0.01, $\overline{U}(\mu - y)/U(\mu + y)$=1.005435836;

When y=7.99, $\overline{U}(\mu-y)/U(\mu+y)$=1.002725270.

The curve of other comparison results of losing utility to profiting utility is shown in Fig. 6.

From the above examples, we could get the conclusion: despite losing utility is anyway larger than the profiting utility when the losing sum is equal to profiting sum. when human's prospect are very stable, the standard variance of fluctuation of *PL* is very small, people will accept loss or profit of same quantity as loss with the near utility; when human's prospect are not stable, the standard variance of fluctuation of *PL* is very large, people will accept loss or profit of same quantity as loss with the big margin of utility.

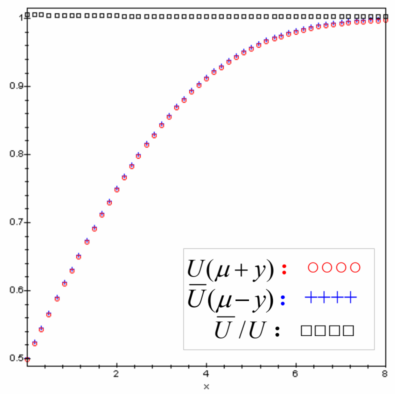

Fig. 6 The comparison of utility of loss and profit when μ=8,σ=3

Ⅶ. ANALYTIC ESSENTIAL INDEXES ABOUT *RP*

If the *RL* ascend to a certain degree, the higher foam prospect could be caused. When this kind of higher foam prospect is far from the reasonable *APL*, the inside force to drive *RL* to ascend will be exhausted gradually. Also, if *RL* descend to a certain degree, the lower prospect could be caused. When this kind of lower prospect is far from the reasonable *APL*, the inside force to drive *RL* to decline will be exhausted gradually. This exhaustion of force will make the intrinsic trend to move appear reversion. In order to evaluate efficiently the force to drive *RL* is exhausted or not, and know the terminal of *RL* going in intrinsic trend, we give out first two assistant functions as follows.

A. Two Assistant Functions

1) $H^{+}(x)=U(x)+xf(x)$

$$=\int_0^x f(t)dt+xf(x) \tag{11}$$

Expression (11) is called the assistant function of ascend behavior of *RL*.$H^{+}(x)$ means the addition of profiting utility (force of ascending behavior of *RL*) and the value of *RL* multiplying the probability of the *RL* appearing.

2) $H^{-}(x)=\overline{U}(x)-xf(x)$

$$=\int_x^{\infty} f(t)dt-xf(x) \tag{12}$$

Expression (11) is called the assistant function of descend behavior of *RL*. $H^{+}(x)$ means the addition of profiting utility (force of descending behavior of *RL*) and the value of *RL* multiplying the probability of the *RL* appearing.

B. The Analytic Indexes

X (*RL* variable) is assumed to follow Partial distribution, i.e. $X\in P(\mu,\ \sigma^2)$.

Definition 4 Let real number l>0 and any x>μ, when $H^{+}(x)$-$U(x)$ <l, then we call the ascending force of *RL* is obviously exhausted with l, where l is the fiducial level.

Definition 5 Let real number l>0 and any x, 0<x<μ, when $\overline{U}(x)$ -$H^{-}(x)$ <l, then we call the descending force of *RL* is obviously exhausted about l, where l is the fiducial level.

In the **Definition 4** and **Definition 5**, the less the l (obvious level) is, the more the possibility that the inside force to drive *RL* in intrinsic trend is exhausted is.

Because $\lim_{x\to\infty} H^{+}(x)=1$ and $\lim_{x\to\infty} U(x)=1$, the **Definition 4** could be explained as: when x (*RL*) is sufficiently large and the $xf(x)$ (the value of *RL* multiplying the probability of the *RL* occurring) is sufficiently small, the high *RL* is difficult to maintained with long time if probability of the *RL* appearing is very small. That *RL* may become the reversing point of ascending trend very much, and at the same time, the ascending force of *RL* is sufficiently exhausted with l (the significance level); Also, when x (*RL*) is sufficiently small and the $xf(x)$ is sufficiently small, the low *RL* is difficult to maintained with long time if probability of the *RL* appearing is very small. That *RL* may become the reversing point of descending trend very much, and at the same time, the descending force of *RL* is sufficiently exhausted with l.

Therefore, according to (11), if want to know whether the ascending force of *RL* is exhausted or not, we only need to estimate whether x is so large to make $xf(x)<l$ come into existence or not. Or according to (12), we only need to estimate whether x is so small to make $xf(x)<l$ come into existence or not.

Since $f(x)=\dfrac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx}$ (x≥0) and

$$e^{\frac{(x-\mu)^2}{2\sigma^2}}>1+\frac{(x-\mu)^2}{2\sigma^2} \tag{13}$$

we have $xf(x) < \dfrac{x}{c\left(1+\dfrac{(x-\mu)^2}{2\sigma^2}\right)}$.

And when $$\frac{x}{1+\dfrac{(x-\mu)^2}{2\sigma^2}} = cl \qquad (14)$$

the $xf(x) < l$ come into existence,

where, $c = \int_0^{\infty} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$, l is significance level. Expression (14) can be written as

$$x^2 - 2(\mu + \frac{\sigma^2}{cl})x + \mu^2 + 2\sigma^2 = 0 .$$

According to (4) and (5), we get

$$x = \mu + \frac{\sigma^2}{cl} \pm \sigma\sqrt{2\left(\frac{\mu}{cl}-1\right)+\left(\frac{\sigma}{cl}\right)^2} \qquad (15)$$

where, $c = \sqrt{\frac{\pi}{2}}\sigma\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu}{\sigma}\right)^2}}\right)$.

Therefore, on the ascending trend and with the significance level *l*, the possible highest value of *RL* reversing is

$$X^{+}(l) = \mu + \frac{\sigma^2}{cl} + \sigma\sqrt{2\left(\frac{\mu}{cl}-1\right)+\left(\frac{\sigma}{cl}\right)^2} \qquad (16)$$

and that, on the descending trend and with the significance level *l*, the possible lowest value of *RL* reversing is

$$X^{-}(l) = \mu + \frac{\sigma^2}{cl} - \sigma\sqrt{2\left(\frac{\mu}{cl}-1\right)+\left(\frac{\sigma}{cl}\right)^2} \qquad (17)$$

Here we need to see, when $|x-\mu|$ is larger, i.e. *x* away from μ, the left of inequation (13) is obviously larger than the right of inequation (13), for all that, and it will be made clear in the following examples that calculating formula (16) and (17) can't be influenced to apply.

By using (16) and (17), we can calculate the equilibrium value of *RL* ascending or descending.

Denoting:

$$X^0(l) = \frac{X^{+}(l) + X^{-}(l)}{2} = \mu + \frac{\sigma^2}{cl} \qquad (18)$$

We call $X^0(l)$ the equilibrium value of *RL*, and *EV* for short.

From (15), we have

$$2\left(\frac{\mu}{cl}-1\right)+\left(\frac{\sigma}{cl}\right)^2 \geq 0, \text{ i.e. } 2(\mu - cl) + \frac{\sigma^2}{cl} \geq 0 .$$

When $2(\mu - cl) + \frac{\sigma^2}{cl} = 0$, we get the conclusion as:

$$l^0 = \frac{\mu + \sqrt{\mu^2 + 2\sigma^2}}{2c} = \frac{\mu + \sqrt{\mu^2 + 2\sigma^2}}{\sqrt{2\pi}\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu}{\sigma}\right)^2}}\right)} \qquad (19)$$

If $l = l^0$, i.e. $2\left(\frac{\mu}{cl}-1\right)+\left(\frac{\sigma}{cl}\right)^2 = 0$, according to (16), (17), (18) and (19), we have

$$X(l^0) = X^0(l) = X^{+}(l) = X^{-}(l)$$
$$= \mu + \frac{\sigma^2}{cl} = \mu + \frac{2\sigma^2}{\mu + \sqrt{\mu^2 + 2\sigma^2}}$$
$$= \sqrt{\mu^2 + 2\sigma^2} ,$$

and call $X(l^0) = \sqrt{\mu^2 + 2\sigma^2}$ the focus value of *RL*, *FV* for short. *FV* reflects basic status of *RL* in the kernel meanings. So, *FV* can be used for the judgment that *RL* is of consistence to proper value or not in some period and on the basic meaning.

C. Determining the Significance Level

If the value of *l*, the significance level, is too small, it will cause reversing values calculated by (16) and (17) to be too high in ascending trend or too low in descending trend; and If the value of *l* is too large, it will cause reversing values calculated by (16) and (17) to be too low in ascending trend or too high in descending trend. The two kinds of cases are all difficult to use for practice. Therefore, we can usually determine the value field of *l* by using of following inequation (19).

$$\frac{1}{4}l^0 \leq l \leq \frac{1}{2}l^0 \qquad (20)$$

where, l^0 is determined by expression (19).

In usually, the value of *l* could be larger when the σ, standard variance of *APL*, is smaller; and the value of *l* should be smaller when the σ is larger.

D. The examples

In all of the following examples, the parameters, such as μ and σ, is estimated by the method in [14],[15], and the estimated parameters all pass the statistic test of significance.

1) ***DJX*** (***1/100DJ INDU***). We take the close points of ***DJX*** as sample data. Time: Jun. 19, 2002—Dec. 24, 2002. The estimated values of parameters in partial distribution

$P(\mu,\sigma^2)$ are as follows:

The estimated value of μ is $\hat{\mu}$=84.84577713, and the estimated value of σ is $\hat{\sigma}^2$=28.65615031.

So the ***DJX*** index follows the partial distribution *P*(84.84577713, 28.65615031) in the period of time mentioned above. By using of (19), we calculate the l^0=6.335679489, and the *FL*, focus level of *RL*, is: $X(l^0)$ =85.01448140.

When 0<l≤l^0, the varying process of $X^+(l), X^-(l)$ and $X^0(l)$ are shown in Fig.7. Let $l=\frac{l^0}{4}$=1.583919872 and by (16) and (17), we get $X^+(l)$= 99.35843320, $X^-(l)$= 73.02971640. And in the period of time: Jun. 19, 2002 -Dec. 24, 2002, the actual maximum value of ***DJX*** is X_{max}=95.62, and the minimum value of ***DJX*** is X_{min}=72.86.

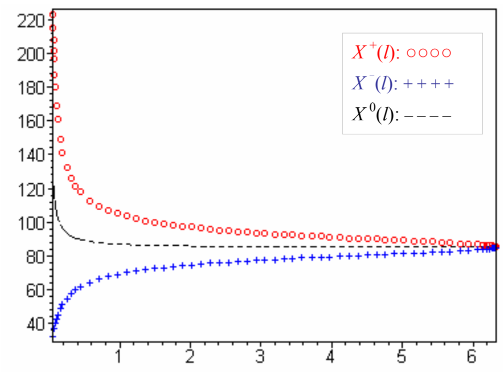

Fig. 7 ***DJX***: the varying process of $X^+(l)$,$X^-(l)$ and $X^0(l)$

2) ***CSMI***(China Shengzhen stock market index). We take the close points of ***CSMI*** as sample data. Time: Apr. 2, 2002—Oct. 8, 2002. The estimated values of parameters in partial distribution $P(\mu,\sigma^2)$ are as follows:

The estimated value of μ is $\hat{\mu}$=478.0363182, and the estimated value of σ is $\hat{\sigma}^2$=650.3925613.

So the ***CSMI*** index follows the partial distribution *P*(478.0363182, 650.3925613) in the period of time mentioned above. By using of (19), we calculate the l^0=7.488587154, and the *FV*, focus value of *RL*, is: $X(l^0)$= 478.7161101.

When 0<l≤l^0, the varying process of $X^+(l), X^-(l)$ and $X^0(l)$ are shown in Fig.8. Let $l=\frac{l^0}{3}$=2.496195718 and by (16) and (17), we get $X^+(l)$= 533.2262202, $X^-(l)$= 430.9981354. And in the period of time: Apr. 2, 2002—Oct. 8, 2002, the actual maximum value of ***CSMI*** is X_{max}= 512.38, and the minimum value of ***CSMI*** is X_{min}=429.47.

3) ***ZYIC***(Zongyi Incorporated Company). We take the close points of ***ZYIC*** as sample data. Time: Jan. 23, 2002—Aug. 8, 2002. The estimated values of parameters in partial distribution $P(\mu,\sigma^2)$ are as follows:

The estimated value of μ is $\hat{\mu}$=12.84800010, and the estimated value of σ is $\hat{\sigma}^2$=1.915181026.

So the ***ZYIC*** index follows the partial distribution *P*(12.84800010, 1.915181026) in the period of time mentioned above. By using of (19), we calculate the l^0=3.725104101, and the *FV*, focus value of *RL*, is: $X(l^0)$= 12.92231742.

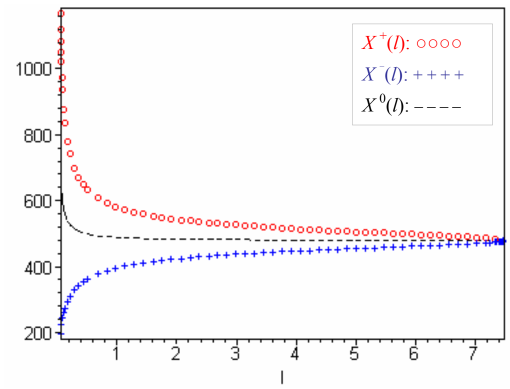

Fig. 8 ***CSMI***: the varying process of $X^+(l)$,$X^-(l)$ and $X^0(l)$

When 0<l≤l^0, the varying process of $X^+(l), X^-(l)$ and $X^0(l)$ are shown in Fig.9. Let $l=\frac{l^0}{3.2}$=1.164095032 and by (16) and (17), we get $X^+(l)$= 16.25168021, $X^-(l)$= 10.39286193. And in the period of time: Jan. 23, 2002—Aug. 8, 2002, the actual maximum value of ***ZYIC*** is X_{max}= 14.68, and the minimum value of ***ZYIC*** is X_{min}=10.41.

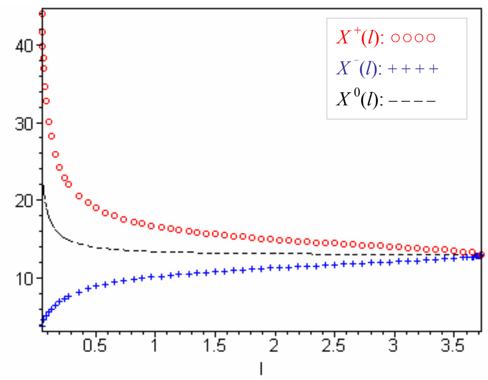

Fig. 8 ***ZYIC***: the varying process of $X^+(l)$,$X^-(l)$ and $X^0(l)$

In the example 2) and example 3), the actual maximum values of ***CSMI*** and ***ZYIC*** (533.2262202 and 16.25168021) are all separately lower than the theoretic maximum values. The reason is that China stock market is in the slumping trend at this time.

Ⅷ. THE CALCULATING FORMULA FOR THE OPTIMAL VALUE OF *RL*

A. The Calculating Formula

Theorem If *RL* variable, *X*, follows Partial Distribution $P(\mu,\sigma^2)$, then the optimal value of *RL* with its appearing probability is

$$X^* = \frac{\mu + \sqrt{\mu^2 + 4\sigma^2}}{2} \qquad (21)$$

where, μ is *APL*, and σ is the standard. The optimal value of *RL* with its appearing probability is optimal value of *RL* for short.

Proof. According to expression (1), we denote

$$G(x) = xf(x) = cxe^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where, $c = 1\Big/\int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$.

Let $\frac{dG(x)}{dx} = \frac{c}{\sigma^2}e^{-\frac{(x-\mu)^2}{2\sigma^2}}(\sigma^2 - x(x-\mu)) = 0$,

i.e. $x = \frac{\mu \pm \sqrt{\mu^2 + 4\sigma^2}}{2}$

since $x > 0$, we have

$$X^* = \frac{\mu + \sqrt{\mu^2 + 4\sigma^2}}{2}.$$

$$\frac{d^2G(x)}{dx^2}\Big|_{x=X^*} = -\frac{c}{\sigma^2}e^{-\frac{(X^*-\mu)^2}{2\sigma^2}}(2X^*-\mu)$$

$= -\frac{c}{\sigma^2}e^{-\frac{(X^*-\mu)^2}{2\sigma^2}}\sqrt{\mu^2 + 4\sigma^2} < 0$, the result as follows.

The expression (21) is the calculating formula for the optimal value of *PL*.

Corollary 1 If *RL* variable, *X*, follows Partial Distribution $P(\mu,\sigma^2)$, and the optimal value of *RL* is given by (21), then the optimal margin between *RL* and μ is

$$R^* = \frac{2\sigma^2}{\mu + \sqrt{\mu^2 + 4\sigma^2}} \qquad (22)$$

Proof. According to formula (21), $R^* = X^* - \mu$, the result as follows.

Corollary 2 If *RL* variable, *X*, follows Partial Distribution $P(\mu,\sigma^2)$, and the optimal value of *RL* is given by (21), then the corresponding variance of *RL* , the actual risk, is

$$D^* = D(X) + [E(X) - X^*]^2 \qquad (23)$$

Proof. According to expression (1) and (3),

$$D^* = \int_0^{\infty}(x - X^*)^2 f(x)dx$$

$= D(X) + [E(X) - X^*]^2$, the result as follows.

B. The examples

1) The optimal value of ***DJX*** and corresponding comparison analysis.

From the first example in Ⅶ, $\hat{\mu}$ =84.84577713 and $\hat{\sigma}^2$ =28.65615031.according to (2), (3), (21), (22) and (23), we have the calculating results shown in table Ⅰ.

2) The optimal pricing of ***MSFT*** and corresponding comparison analysis.

We take the close prices of ***MICROSOFT CP*** (***MSFT***) as sample data. Time: Jan. 29, 2002 -Dec. 24, 2002.

By use of the method in [14],[15], the estimated values of parameters in partial distribution $P(\mu,\sigma^2)$ are as follows:

$\hat{\mu}$ =53.58500013; $\hat{\sigma}^2$ = 24.62632700.

According to (2), (3), (21), (22) and (23), we have the calculating results shown in table Ⅱ.

We see, from table Ⅰ and table Ⅱ, that the average margin between X^* and μ are higher than the average margin between $\overline{X}$ (E(X)) and μ, the risks on one unit of

TABLE Ⅰ
COMPARISON OF CALCULATING RESULTS ABOUT ***DJX***

	Index price *X*	Average margin between *X* and μ *R*	Actual risk *D*	Risk on one unit of index price *D/X*	Index price with its appearing probability X_P
Optimal value X^*	85.18218725	0.33641012	28.76932208	0.3377387105	42.50707408
Average value $\overline{X}$	84.84577713	0.60151352×10^{-54}	28.65615031	0.3377439784	42.42288856
A higher value X^h	90	5.154222870	55.22216370	0.6135795968	28.30764034
Comparing	$X^h > X^* > \overline{X}$	$X^h > X^* > \overline{X}$	$\overline{X} > X^* > X^h$	$X^* > \overline{X} > X^h$	$X^* > \overline{X} > X^h$
Note	Where, *A*>*B* means the *A* surpasses the *B*; A higher value is arbitrarily given				

optimal value, X^* , are lower than that of the average value of *RL*, and the optimal value X^* with its appearing probability are higher than the average value $\overline{X}$ with its appearing probability. The later is important for director of an industrial enterprise or the fund manager to do their works. If we hope to have a higher value of *RL*, the risks on one unit of *RL* must be obviously higher, and the values with its appearing probability must be obviously lower.

TABLE Ⅱ
COMPARISON OF CALCULATING RESULTS ABOUT *MSFT*

	Stock price X	Average margin between X and μ R	Actual risk D	Risk on one unit of stock price D/X	stock price with its appearing probability X_P
Optimal price X^*	54.04069976	0.4556996300	24.83398915	0.4595423316	26.90666480
Average price $\overline{X}$	53.58500013	0.95044035×10^{-25}	24.62632700	0.4595750104	26.79250006
A higher price X^h	60	6.414999870	65.77855032	1.096309172	13.00929717
Comparing	$X^h > X^* > \overline{X}$	$X^h > X^* > \overline{X}$	$\overline{X} > X^* > X^h$	$X^* > \overline{X} > X^h$	$X^* > \overline{X} > X^h$
Note	Where, *A*>*B* means the *A* surpasses the *B*; A higher value is arbitrarily given				

Ⅸ CONCLUSIONS AND REMARKS

According to the basic characteristics of human's prospect behavior, this paper bring forward the PD-utility function based on partial distribution, and get the following conclusions:

1) Many results in "prospect theory" from Daniel Kahneman, such as "the utility curve is concave in the profiting field, and is protruding in the losing field", can be analytically described by the PD-utility function.

2) That the comparison value between losing utility and profiting utility is higher or lower is decided by to prospect level, standard variance of prospect level and realized level. And "losing utility is anyway larger than the profiting utility when the losing sum is equal to profiting sum" can be analytically proved.

3) The actual phenomena of "the higher the expectation is, the more the desponding utility is" can be analytically evaluated.

4) We give out the analytic standard to estimate that the intrinsic moving trend of realized level is reversed or not, and the calculating method of reversing value under the different significance level.

5) We give out the concepts of equilibrium value and focus value of realized level, and the calculating method of them. They can be used for the estimating that the realized level is too higher or lower than the proper value in some period and on the basic meaning.

6) We give out the calculating formula for the optimal value of realized level. By the formula, the optimal value of realized level with its appearing probability could be obtained from average prospect level and the standard variance of it.

7) By use of (4), (16), (17), (18) and (21), we can get the utility results of reversal limit level, focus level and the optimal level of prospect behavior.

In this paper, the foundation that we discuss is that when prospect level is invariable, the higher the realized level is, the higher the profit is. This is applicable to circumstance that one has already hold some kind of assets such as stocks ,etc. but, when did not hold some kind of assets, we may hope the realized level lower, so that we can buy them by lower price. In this case, the lower the realized level is, the higher the latent profit is. For the latter, we can probably adopt with the contrary way discussed in this paper, and no longer discuss here.

The textual data source is in the websites:
http://finance.yahoo.com
http://www.stockstar.com.cn

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