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Innovation, Rent Extraction, and Integration in Systems Markets

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Abstract:
We consider innovation incentives in markets where final goods comprise two strictly complementary components, one of which is monopolized. We focus on the case in which the complementary component is competitively supplied, and in which innovation is important. We explore ways in which the monopoly may have incentives to confiscate efficiency rents in the competitive sector, thus weakening or destroying incentives for independent innovation. We discuss how these problems are affected if the monopolist integrates into the competitive sector.

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INTEGRATION AND INNOVATION IN SYSTEMS MARKETS

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I. INTRODUCTION

In many high-technology markets, including the computer and communications industries, (a) a number of strongly complementary components are used together in a system to provide consumer benefits, and (b) some or all components are subject to significant technological progress as the result of suppliers' investments in R&D. In this paper, we examine how integration across components affects competition, with particular emphasis on R&D competition.

We study this problem in the following setting. Components *A* and *B* are valuable only when used together. There is a single producer of *A* labeled firm *M*, which may or may not also produce B. There is also at least one independent supplier of component *B*, which we label firm *N*.

A familiar intuition asserts that when *M* and *N* are the only producers of their respective components, integration through a merger of *M* and *N* would efficiently increase the incentives for each firm to innovate by internalizing what are otherwise

[1] We thank Richard Gilbert, Paul Milgrom, John Sutton, Jean Tirole, and participants in the Competition and Innovation in the Personal Computer Industry conference for helpful comments and suggestions. We thank the conference sponsors for financial support.

positive external effects on one another.[2] We show by example, however, that there are cases in which this intuition fails; integration can inefficiently reduce the incentives to innovate. Our example depends on the innovation's being valued differently by different consumers.

More importantly, the intuition does not carry over to market structures in which there are multiple suppliers of component *B* so that, if *M* enters the *B* market, it would be competing with independent B firms. Consideration of such markets is an important extension because there are many markets exhibiting this structure. For example, Microsoft supplies operating systems, and Microsoft and independent software vendors supply applications software that works with Microsoft's OS. Similarly, Intel supplies microprocessors, and Intel and other firms make motherboards designed around these microprocessors. Lastly, SBC supplies "access" that lets telephone subscribers make long-distance calls, and the non-access portions of those calls are supplied by a variety of long-distance phone companies, which in the future may include SBC.[3]

In these examples and many more, concern has been expressed about firm *M*'s being a supplier of component *B*.[4] Independent suppliers note that firm *M* may be able to

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2 We ignore the difficult question of why a merger would affect internalization.

3 In this example, *M*'s supply of *A* and its integration or non-integration into *B* are largely controlled by regulation, so that some important issues are rather different from those explored below; the analysis of the unregulated case is important for fully understanding regulated markets, however.

4 While we assume that *M* is a monopolist, our analysis applies more broadly. For example, retailers (dealers) may have to make specific investments in selling certain brands of cars. Independent service organizations may have to make specific investments in parts, in customer lists, and in learning how to repair certain brands of hardware. This can be true even if firm *M* faces competition in the overall systems market.

control the ability of independently supplied variants of component *B* to work with component *A*.[5] Independent suppliers fear that this power to exclude might be abused, especially if firm *M* is integrated into the supply of component *B* and attempts to limit the competition it faces as a supplier of that component. On the other side, many economists note that *M* may internalize through its sales of component *A* at least some of the gains from innovation in *B* that might otherwise be dissipated in double marginalization or escape to consumers. These commentators argue that firm *M* will therefore have particularly efficient incentives to innovate in *B* and to encourage others to do so in order to enhance the demand for component *A*.

In this paper, we formally examine firm *M*'s ex ante (*i.e.*, prior to innovation) and ex post (*i.e.*, after innovation has taken place) incentives to treat independent suppliers of component *B*. In particular, we explore how those incentives change if firm *M* enters the market for *B*. It is correct that an integrated firm *M* often has incentives to welcome and even encourage the competitive efforts of independent suppliers of component *B* because these efforts enhance the demand for firm *M*'s component *A*; and in our model those incentives dominate. However, firm *M* also has an ex post incentive, and in some cases even an ex ante incentive, to force the supplier of a superior *B* component to price lower than it would otherwise choose. Integration into development and production of *B* can strengthen firm *M*'s ability to engage in such a "squeeze."[6]

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5 This may be particularly likely in high-technology contexts, where the interface that joins components *A* and *B* may be subject to intellectual property protection, or may be changed frequently, or both.

6 We use the term "squeeze" without suggesting that the behavior is predatory or exclusionary. In our model, firm *M* does not at all wish to drive a superior B producer from the market, but only to expropriate some of its ex post rents.

Several strategies may enable *M* to force an independent producer of component *B* to charge a lower price than it otherwise might. In a *price squeeze,* an integrated firm *M* prices its variant of *B* lower than would a stand-alone supplier of component *B*, not in order to take sales from rivals but to push their prices lower. In an *investment squeeze,* an integrated firm *M* invests in improving its variant of component *B*, not with the goal of becoming the best or of making sales, but with the goal of driving the winning independent supplier of *B* to price its (still better) product lower than it otherwise would. Lastly, in an *exclusionary squeeze*, firm *M* demands a low price for an independently supplied component *B* as a *quid pro quo* for granting access.

In none of these potentially tempting strategies does firm *M* exclude rivals in order to earn greater profits from the sale of its own variant of *B*. (For instance, under an exclusionary squeeze in our model, firm *M* would rather not actually carry out the threat.) Although exclusion of rivals could indeed increase the profits of *M*'s operations in the *B* market, any such profits come (in our simple model) at the expense of profits in the *A* market.[7] In this sense, our model incorporates the "one monopoly rent theorem." Despite this, we find that integration by *M* can create efficiency problems.

We show that, while often efficiency-enhancing ex post, squeezes can also inefficiently reduce independent firms' incentives to invest in improving component *B*. Moreover, firm *M* may undertake activities in support of a squeeze that themselves are socially wasteful, such as excessive R&D. We show that integration can make these problems worse. We also argue that, even if the overall ex ante effects of a squeeze are negative for firm *M*, it may be difficult for firm *M* to commit not to engage in one ex post, and that difficulty is plausibly greater if firm *M* is integrated into the supply of component *B*.

The paper is organized as follows. In the next section we lay out a simple model. In Section III, we examine the effects of integration when there is just one provider of component *B*, either an independent firm or firm *M* itself. In Sections IV and V, we address our core topic: innovation in markets in which there are multiple suppliers of component *B*. Section IV analyzes squeezing through product-market and investment strategies. Section V then examines threats of exclusion as a means of squeezing independent suppliers. We also examine how firm *M* might use its power to exclude to implement certain private intellectual property policies. The paper closes with a few observations on the implications for competition and business policy.

II. THE MODEL

We analyze the following three-stage game.

Entry Stage: In the first stage, firm *M* and independent suppliers decide whether to enter the market for component *B*. We discuss alternative orders of decision making by firm *M* and the independent suppliers below.

R&D Stage: After the entry stage has been completed, and the firms' entry decisions are common knowledge, firms that are active in the market for *B* simultaneously invest in improving their quality levels (or lowering their costs). Let I_i denote firm *i*'s investment level, measured in dollars. This investment gives rise to a

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[7] Indeed, in our model firm *M* chooses never to take any profits in the *B* market.

distribution function for firm *i*'s product quality, q_i.[8] We assume the quality improvement enjoyed by one firm is independent of the R&D investments of other firms. Hence, we are ruling out both patent races and the possibility of spillovers across R&D programs while they are under way. Below, we consider the possibilities of licensing and ex post imitation.

At times, the analysis is simplified by assuming that there are only two possible outcomes of an R&D project, *success* and *failure*, and that all firms have the same R&D production function. We normalize failure as a quality of 0 and label success q_0. In this case, the distribution function takes the following form:

$$prob\ (\ q\ \text{£}\ q') = \begin{cases} 1 - \rho(I_i) \text{ for all } 0 \leq q' < q_0 \\ 1 \text{ for all } q' \geq q_0, \end{cases}$$

where the probability that firm *i* succeeds, $\rho(I_i)$, is assumed to be an increasing function of its investment.

Pricing Stage: At the start of this final stage, the R&D outcomes q_i may or may not be common knowledge; we consider both cases. We assume R&D investment levels remain private information. Given the information structure, suppliers of *B* simultaneously and non-cooperatively set prices for their components. Let p_i denote the price chosen by firm *i*. These prices—and the quality levels if not previously observed—are then commonly observed and firm *M* sets the price of component *A*. We assume for simplicity that the marginal costs of production are zero for component *A* and *c* for all

[8] As we discuss below, the model can also be interpreted as one of process, or cost-reducing, innovation.

variants of *B*. Throughout most of the exposition, we will subsume *c* in the demand curve and take $c = 0$ as a normalization.

Once all prices have been set, consumers decide whether to buy a system and, if there are multiple suppliers of component *B*, which one to patronize. We assume that components *A* and *B* must be used in fixed proportions (normalized as 1-to-1) to generate benefits. We also assume there is a unit mass of consumers, each of whom buys either 0 or 1 system. A type-θ consumer has a reservation price of $\theta + q$ for a system that combines one unit of *A* with one unit of good *B* having quality *q*.[9] Let $D(p)$ denote the number of consumers for whom $\theta \geq p$. We assume there exits a finite price, p_c, such that $D(p) = 0$ for all $p \geq p_c$.[10] At times, we restrict attention to the case of *inelastic demand*:

$$D(p) = \begin{cases} 0 \text{ if } p > p_0 \\ 1 \text{ if } p \leq p_0 \end{cases}$$

Before analyzing this model, we note a few limitations. First, this baseline model assumes away two mechanisms through which firm *M* might engage in a price squeeze: we do not allow *M* to act as a price leader by setting either the price of component *A* or the price of its component *B* before the independent firms set their prices. We return to this issue in subsection IV.C below.

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9 Given the assumption that all consumers value quality equally, an increase in quality is equivalent to a decrease in cost. In Section III, we briefly relax the assumption that all consumers value quality equally.

10 In addition to being realistic, the assumption of a finite choke price rules out certain mixed strategy equilibria that might otherwise exist in which a firm that has the lowest quality (or highest cost) makes equilibrium sales at a price strictly above cost.

Second, we assume away two reasons why firm *M* might wish to exclude an independently developed component *B*. When components *A* and *B* can be used in variable proportions to generate consumer benefits, firm *M* might be able to extract more surplus from buyers by excluding other component suppliers in order to create greater flexibility in its relative pricing of components *A* and *B*.[11] Clearly, this motive does not apply when goods A and *B* are used in fixed proportions, as in our model. We also assume that additional entry into the production of component *A* is impossible. If such entry were possible, firm *M* might wish to exclude based on fears that independent production of *B* could serve as a stepping stone into the *A*-market—so-called two-stage entry.

III. THE EFFECTS OF INTEGRATION WHEN THERE IS A SINGLE SUPPLIER OF EACH COMPONENT

As a benchmark, we first assume there is room for only one component *B* supplier in the entry stage. Either firm *M* (if it integrates) or firm *N* (otherwise) supplies component *B*. We examine the hypothesis that innovation incentives, like pricing incentives, are better when the control of strict complements is in a single hand.

First, suppose there is no integration. Suppose that, as result of its investments in the R&D stage, firm *N* has innovated and raised its product quality by Δq. By the following standard argument, the increase in q is equivalent to a cost reduction that increases the profits of both suppliers and raises consumer surplus.[12] Given our

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11 This issue is addressed in Katz (1989). If consumers have imperfect foresight about service or spare parts pricing, this can also potentially create a motive to exclude suppliers of those complements.

12 A similar logic applies if firm *M* sets the price of *A* first.

assumption that consumers value quality changes uniformly, the market demand for systems can be expressed as $D(h)$, where h is the hedonic price of systems and is equal to the sum of the two component prices minus q. Suppose firm N has set the price of B at p. Firm M can be thought of as choosing h to max $\{h - (p-q)\}D(h)$. The resulting quantity is $x^*(p-q)$, where $x^*(w)$ is the quantity that would be set by a monopolist with marginal cost ω facing demand curve D. This in turn defines a "derived" demand curve for firm N's pricing decision. Defining $n = p - q$, firm N chooses n to maximize $\{n + q\}x^*(n)$. Its choice of n is thus the profit-maximizing price for a monopolist with marginal cost $-q$ facing the demand curve $x^*(\cdot)$.

By the standard comparative statics of monopoly pricing, the $-\Delta q$ fall in $-q$ both makes firm N better off and also leads to a fall in n. The fall in n decreases firm M's effective marginal costs in its pricing problem. Hence, by the same comparative statics result, firm M's profits rise and its choice of h falls. The fall in h makes consumers better off. Thus, when firm N innovates, both firm M and consumers are made better off.

If the firms integrate, the incentive to make such an innovation is greater, because the previously external positive effect on M's profits is now internalized.[13] Moreover, the remaining externality from innovation—the external effect on consumers—is also positive. Hence, when innovation results in a uniform increase in systems reservation prices, the effects of a merger of firms M and N are as follows: (a) the private incentives for innovation increase; and (b) the increased innovation that results is socially desirable.

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13 Formally, it is straightforward to establish that the equilibrium quantity for any given q is lower when the firms are not integrated due to double marginalization. Therefore, the value of a cost reduction (equivalently, a uniform increase in quality) is greater when the firms have merged.

These results are of course well known (at least informally) and at least in spirit very old (usually attributed to Cournot).

Perhaps surprisingly, integration need not increase the incentives to invest in socially valuable R&D for general patterns of quality improvements. Consider the following example in which we relax the assumption that all consumers equally value increased quality. One third of the consumers have a reservation price equal to 2, while two thirds have a reservation price of 1. If firm N innovates, the reservation price of the consumers with relatively high absolute willingness to pay rises to $2+\Delta q$, where $0 < \Delta q < 1$. The demands of consumers with lower willingness to pay are not affected by the innovation. The cost of innovation is I_0, where $0 < I_0 < (2/3)\Delta q$, and there are no production costs.

It is straightforward to show that, absent integration (and absent any other similar form of cooperation between the two suppliers), sequential double marginalization leads to an equilibrium price at which only the high-value consumers buy. In this case, it is profitable for firm N to innovate, because $I_0 < (2/3)\Delta q$. With integration, the equilibrium systems price is 1, with or without the innovation. Hence, the marginal consumer does not value increased quality, and there is no incentive to innovate. This outcome is inefficient since the gross consumption benefits would rise by more than the cost of the innovation: the inframarginal consumers are still buying and they value the increased quality. A feature of this example is of course that inframarginal customers value quality differently from marginal customers, as discussed by Spence (1975). Our argument above (and probably previous work by others) shows that absent this feature, the standard belief that integration improves integration incentives must hold.

Although this example qualifies one's confidence in the standard belief for general patterns of quality valuation, nevertheless we do not mean to take very vigorous issue here with the standard presumption that integration between a single *A*-firm and a single *B*-firm "normally" will improve incentives for innovation in each component. Rather, in the remainder of our paper, we explore how things may be very different when there are multiple *B*-firms, so that even this qualified intuition may be quite misleading. In order to isolate the effects of having multiple suppliers of *B*, we maintain our initial assumption that all consumers equally value quality improvements.

IV. INVESTMENT SQUEEZES AND PRICE SQUEEZES

We now turn to settings in which multiple firms are active in the *B* market, at least at the innovation stage. To find subgame-perfect (or perfect Bayesian) equilibrium, we solve the game by working backwards in time.

A. Analysis of the Pricing Stage

Our analysis of the pricing stage applies to both: (a) complete-information situations, in which the R&D outcomes, q_i, become common knowledge before any prices are set, and (b) incomplete-information situations, in which R&D outcomes become common knowledge after the prices of component *B* are set, but before the price of component *A* is set. Recall that R&D expenditures remain private information.

In order to determine their pricing strategies, suppliers of component *B* must form beliefs about how firm *M* will price component *A*. Working backwards, suppose *B*-firm i has quality q_i and sets price p_i. Define i's quasi-surplus as $s_i = q_i - p_i$. The demand for *A* depends only on its price and on the maximum of the quasi-surpluses; as in the previous section, the system surplus offered to a given consumer, and firm *M*'s profits in *A*, are both increasing functions of that maximum quasi-surplus.

Now consider equilibrium pricing strategies for the *B*-firms given their qualities and their predictions of how component *A* will be priced. Our interest is in how the prices of the *B* components are affected by firm *M*'s integration. Suppose that firm *M* is integrated into *B*.

Lemma 1. *It is a weakly dominant strategy for an integrated firm M to price its variant of B at cost.*

Proof: Let $Q(\cdot)$ denote the cumulative distribution function, as perceived by *M* in equilibrium, for the *highest* quasi-surplus offered by independent suppliers of *B*. (Of course, in the complete-information case $Q(.)$ is degenerate.) Thus, if firm *M* offers price *p* and quality *q*, its probability of making *B* sales is $Q(q- p)$.[15] Let $R^*(s)$ denote the maximal profits that can be earned from the sale of *A* given that consumers purchase units of *B* yielding quasi-surplus *s*. Note that R^* is an increasing function. Firm *M*'s expected profits from components *A* and *B* together are

$$\pi(p,q) = Q(q\text{-}p)\ \{R^*(q\text{-}p) + x^*(q\text{-}p)[p\text{-}c]\} + \int_{q\text{-}p}^{\infty} R^*(s)\ dQ(s)\ \ .$$

If it sells both components (i.e., if it sells B), firm *M* will earn total profits $R^*(q\text{-}c)$ for any *p* less than or equal to the *systems* price associated with $R^*(q\text{-}c)$ when it has quality *q* and unit cost *c*. This is immediate if *M* prices its *B* component at *c*, and follows for other prices because *M* will adjust the price of component *A* *pari passu* to offset any change in

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[14] Of course, it follows immediately that firm M has no incentive to hinder or exclude when it is not integrated.

[15] This assumes the tie-breaking rule that *M* wins sales if it offers the same surplus as an independent.

its price of component *B*. Hence, firm *M*'s expected profits from components *A* and *B*, if it charges such a price, can be expressed as:

$$\pi(p,q) \;=\; Q(q-p)\, R^*(q-c) \;+\; \int_{q-p}^{\infty} R^*(s)\, dQ(s) \;.$$

It follows that

$$\pi(p,q) - \pi(c,q) = \{Q(q-p) - Q(q-c)\}R^*(q-c) + \int_{q-p}^{q-c} R^*(s)\, dQ(s)$$

$$= \int_{q-p}^{q-c} \{ R^*(s) - R^*(q-c) \} dQ(s) \;.$$

Hence, sign{ $\pi(p,q) - \pi(c,q)$ } = $-$sign (p-c). Therefore, firm *M*'s expected profits are maximized by setting $p = c$. ***QED***

Now, consider the independent suppliers of *B*. We say that *firm i hopes to be best at quality q* if, according to its subjective beliefs about others' quality levels, there is strictly positive probability that $q_j < q$ for all $j \neq i$.[16] Of course, in the complete-information case, firm *i* hopes to be best if and only if it actually has strictly the highest quality level.

Lemma 2. *An independent supplier of component B: (i) never makes equilibrium sales at a price less than cost, and (ii) sets price strictly above cost for all quality realizations at which it hopes to be best.*

Proof: The first claim holds because making no sales dominates making sales below cost. To see why the second claim holds, it is useful to introduce additional notation. Let $F_i(\cdot)$ denote firm *i*'s subjective distribution function for $\max_{j\neq i} q_j$. Independent firm *i* hopes to be best at quality q^b if and only if there exists $q^a < q^b$ such that $F_i(q^a) > 0$.

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[16] These beliefs are derived from the R&D production functions of the rival suppliers as well as firm i's beliefs about its rivals' investment levels.

Let $s_k*(q)$ denote the equilibrium quasi-surplus offered by firm k when it has quality q. Firm rationality implies that $s_k*(q) \leq q - c$ for all k and q.[17] Hence, for $\varepsilon = q^b - q^a$, setting $s_i*(q^b) = q^b - c - \varepsilon$ yields expected profits of $\varepsilon F_i(q^a) > 0$. Of course, if firm i prices at cost when $q_i = q^b$, it makes zero expected profits. Therefore, $s_i*(q^b) < q^b - c$.

QED

Intuitively, each independent prices strictly above cost so as to make profits should it be lucky enough to have the highest quality. It trades off this desire against the fact that it sacrifices profits if it misses selling because of its markup. *M* suffers no such tradeoff: it can lower its price and makes up its sacrificed *B*-profits one-for-one in *A*.

Now consider the effects of firm *M*'s integrating on ex post efficiency (that is, taking the set of achieved quality levels as given). With complete information, the best quality wins (integration or not), and with the added assumption of inelastic demand this is the only ex post efficiency concern. Departing from that simplest model, integration may have two kinds of ex post efficiency effects: it may affect whether the best quality wins, and it may affect prices.

First, consider the possibility that the best quality fails to win under incomplete information. Suppose that firm *M* does not integrate. In the symmetric Bayesian equilibrium among the independent firms, the highest-quality firm makes all the sales. Now, suppose that *M* integrates. Because it prices at cost while its rivals (still) price above cost, *M* always sells *B* when it is efficient for it to do so, and sometimes when it is inefficient. This effect of integration lowers efficiency (conditional on the set of realized

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[17] There exist Nash equilibria in which firms that do not make sales set prices below cost. We rule these out as unreasonable. They would not, for example, survive

quality levels). As we noted, there is no ex post efficiency effect under complete information with inelastic demand; thus, integration weakly lowers ex post efficiency with inelastic demand.

Second, consider the efficiency effects of the changed pricing (holding the winning quality fixed). The change in pricing generates efficiency effects only with elastic demand. There is a direct effect because firm *M* prices its *B* variant at cost: if consumers buy that variant, efficiency is greater than if it were priced higher (as it typically would be if offered by an independent).[18] There may also be an indirect pricing effect of integration, mediated through the change in the independents' pricing strategies, and we discuss that next.

In this game, an independent firm cannot literally respond to the price of component *B* set by an integrated firm *M*: they are simultaneous choices. It is also simple to see that each independent's pricing strategy under complete information is unaffected by whether or not one of its rivals is *M*: the independent prices at cost unless it has the uniquely best quality, in which case all its rivals will price at cost whoever they are. But, under incomplete information, the independent firms' Bayesian pricing strategies may well respond to the knowledge that firm *M*'s strategy is to price at cost. That is, each independent makes its price a different function of its quality if it knows that one of its rivals is pricing at cost than it would in a symmetric Bayesian equilibrium.

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the introduction of trembles.

18 Although there remains a monopoly stage in which firm *M* sets the price of *A*, with elastic demand consumers still will be better off if the quasi-surplus offered in good *B* is higher.

It would seem natural for this response to take the form of lower prices (holding quality fixed). In this case, there is an improvement in efficiency due to integration when demand is not perfectly inelastic. Note that this effect also gives firm *M* a private incentive to integrate, which might be seen as an indirect price squeeze.

Interestingly, in some cases independent suppliers will respond to firm *M*'s pricing at cost by charging *higher* prices than they would do if competing against another independent firm rather than an integrated supplier.[19] In these situations, the indirect pricing effects are negative for both welfare and firm *M*'s profits.

The following proposition summarizes our analysis of ex post efficiency effects of integration:

Proposition 1. *Under complete information or a symmetric Bayesian equilibrium, the highest quality among B-suppliers efficiently makes all of the sales when M does not*

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[19] To see why, consider the following example of a success-failure model in which there are two suppliers of *B* and demand is inelastic. Recall that a firm's R&D succeeds with probability ρ and yields quality q_0 for its component *B.* With probability 1-ρ the R&D fails and yields a quality of 0. For simplicity, let $c = 0$.

If the two firms active in the market for *B* both are independents, each sets its price equal to 0 (cost) if its R&D fails and mixes over the interval $((1-\rho)q_0, q_0)$ if its project succeeds. Letting $G(\cdot)$ denote the distribution function for this mixed strategy, straightforward calculation shows that on this interval

$$G(p) = 1 - (1-\rho)(q_0-p)/(\rho p) .$$

Now suppose instead that firm *M* is one of the suppliers of component *B*. Firm *M* sets its price equal to zero no matter what the outcome of its R&D. Taking this fact into account, the independent firm competing against *M* sets $p = 0$ if its project fails and $p = q_0$ if it succeeds. Thus, an independent firm's response to firm *M*'s pricing strategy is to shift its pricing strategy *upward*.

Of course, one can understand this in terms of elasticity of the expected firm-specific demand curve. Firm *M*'s more aggressive pricing *lowers* that demand curve facing each independent, but (as we see here) need not make it *more elastic*.

integrate.[20] *When it integrates, firm M always makes equilibrium sales of component B if it has the highest quality and—under incomplete information—may make equilibrium sales of B when it is not the highest quality variant. With inelastic demand, ex post efficiency (i.e., conditional on R&D outcomes) is lowered by integration. With elastic demand, the ex post efficiency effects of integration are ambiguous.*

B. Analysis of the R&D Stage

Now, consider the incentives to conduct R&D. Begin by assuming that demand is inelastic and firms have complete information about product quality in the pricing stage. In this simplest form of our model, the supplier with the highest-quality *B* earns a per-unit margin equal to the difference between its quality and the second-highest quality.[21] This margin is also the social contribution of that highest-quality *B*-firm. Similarly, other *B*-firms earn no revenues, and their ex post social contributions are zero. For each independent *B*-firm, therefore, the incentive to improve its variant of *B* is efficient. This is of course a familiar result in the context of an isolated market.

If *M* is integrated into *B*, its total profits from the sale of *A* and *B* are equal to the maximum of its own quality and the *second*-highest quality level among the independents. As a result, *M*'s profits increase if it improves its *B*-product to a level between the two highest levels of the independent suppliers' variants of *B*. Firm *M* benefits even though there is no increase in total surplus. Consequently, an integrated

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20 Although we have discussed its properties conditional on existing, we have not proved that Bayesian equilibrium of this game exists. Rather generally, it does (see [We are still looking for an existence proof]). Note that when $F_i(\cdot)$ contains atoms, the only pricing equilibria may entail mixing.

21 This is the standard Bertrand outcome.

firm *M* has strictly excessive incentives to improve its *B* product as long as there is positive probability that it will end up alone in second place overall. The reason is that an improvement in the *second-best B* causes the *best B* to price lower, thus transferring rents from the best *B* to firm *M*'s operations in *A*. Thus, by investing in R&D, firm *M* can squeeze ex post quasi-rents from the *B*-winner. Firm *M* does this by improving its variant of component *B* even if that improvement has no efficiency benefits.

Summarizing this discussion,

Proposition 2. *With complete information and inelastic demand: (i) conditional on the R&D level of firm M and the number of independent suppliers, an independent supplier undertakes the socially efficient amount of R&D whether or not M has integrated; and (ii) if M integrates into B, it has excessive incentives to innovate (conditional on the R&D levels of the independent firms).*

Now return to our more general model of demand. The welfare economics turn out to be very complex in general, but we can say something for the success-failure model. We say that a firm "is first" if it has the only successful R&D project, and that it "is second" if it and precisely one other firm succeed in their projects.

Proposition 3. *Suppose the conditions of the success-failure model hold. Then, conditional on the R&D levels of other suppliers, an independent supplier's level of R&D investment: (i) is less than is socially optimal when information is complete and demand is elastic, and (ii) is socially optimal when demand is inelastic. If it integrates, firm M: (iii) invests more in R&D than do its independent rivals, and (iv) may have socially insufficient or excessive incentives to innovate conditional on the R&D levels of other suppliers.*

Proof:

(i) An independent supplier invests too little in R&D because it ignores the social value of being second (second place profits are zero, but there is a social benefit from the reduction in the winner's markup), and it earns no more than the social value when it is first.

(ii) The result is familiar for the case of complete information. Under incomplete information, consider the following strategy *S* for a firm: Price at cost if R&D is unsuccessful, and at $q_0 + c$ if R&D is successful. It is not an equilibrium for all firms to adopt strategy *S*, because any one could profitably deviate by charging slightly less if successful (this increases expected profits when more than one firm is successful). It turns out that equilibrium instead involves successful firms mixing over prices up to $q_0 + c$, with no atom at that point. Because that price is in the closure of the support of the equilibrium pricing strategy for successful firms, any given firm could get its equilibrium payoff by playing strategy *S*. Because there is no atom at $q_0 + c$ and the equilibrium mixing involves prices no greater than that, a firm with successful R&D that played S would make sales with probability zero if in fact another firm had succeeded. It follows that a successful firm's expected payoff is equal to q_0 times the probability that no other firm also succeeded. This is the same expected payoff to success as under complete information, and coincides with the firm's social contribution. Of course, payoff to failure is zero. The result follows.

(iii) Firm *M* invests more because it internalizes benefits from being first or second, whereas independents internalize only (fewer of the) benefits from being first.

(iv) As we saw above, the case of inelastic demand shows strictly excessive incentives. The following is an example (recalling section III) in which firm *M*'s incentives are too low. There is room only for one *B* firm: under integration, it is *M*. Demand is elastic, so there is a positive externality on consumers from successful innovation: thus the integrated *M* has insufficient incentives to innovate. **QED**

C. Analysis of the Entry Stage

We now consider the entry stage, again for the case of complete information and inelastic demand. First, firm *M* has weakly excessive incentives to integrate when the total number of *B* suppliers is unaffected by M's integration decision.[22] This follows from the fact that the equilibrium in the absence of integration maximizes expected total surplus conditional on the number of firms—thus integration never can lead to a welfare improvement and (ignoring technical efficiencies of integration) can only make things worse. We show by example below that profitable integration can in fact lead to a strict fall in total surplus, so that firm *M*'s integration incentives are strictly excessive.

Complicating the picture, we note that independent suppliers also have excessive incentives to enter the market for component *B* (taking as given others' entry choices).[23] The argument is as follows. Let $W(I_1, I_2, \ldots, I_{n+1})$ denote the resulting level of expected total surplus when firm 1 through n+1 undertake R&D investments $(I_1, I_2, \ldots, I_{n+1})$. Let $(I_1^*, I_2^*, \ldots, I_n^*, 0)$ denote the vector of R&D levels that maximizes welfare subject to the

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22 In this case, firm *M*'s decision to integrate into *B* could be thought of as an acquisition of one of the fixed set of *B* suppliers.

23 This may seem confusing because of the result that each firm captures its social contribution in equilibrium. But that result takes all other firms' R&D efforts (or outcomes) as given, whereas our assumption here is that others' R&D choices respond to incremental entry by others.

constraint that $I_{n+1} = 0$. As noted above, $(I_1^*, I_2^*, \ldots, I_n^*, 0)$ can be supported as a Nash equilibrium in the R&D stage given Bertrand product-market competition and inelastic demand, and given that firm n+1 is "out." Now, suppose that firm n+1—an independent supplier—chooses to enter the market for B in the entry stage. Let $(I_1^{**}, I_2^{**}, \ldots, I_{n+1}^{**})$ denote the equilibrium R&D levels: note that other firms react to the new entry. Firm n+1's expected profits are $W(I_1^{**}, I_2^{**}, \ldots, I_{n+1}^{**}) - W(I_1^{**}, I_2^{**}, \ldots, I_n^{**}, 0)$, its social contribution taking as given all others' actions. But the true social contribution of its entry (taking account of others' reactions to that entry) is $W(I_1^{**}, I_2^{**}, \ldots, I_{n+1}^{**}) - W(I_1^*, I_2^*, \ldots, I_n^*, 0)$. By definition, $W(I_1^{**}, I_2^{**}, \ldots, I_n^{**}, 0) \leq W(I_1^*, I_2^*, \ldots, I_n^*, 0)$. Therefore, firm n+1 has weakly excessive incentives to enter. These incentives will (generically) be strictly excessive when entry induces rival suppliers to change their equilibrium R&D levels.

Proposition 4. *With complete information and inelastic demand: (i) M has weakly excessive incentives to integrate when the total number of B suppliers is unaffected by M's integration decision; (ii) moreover, profitable integration can strictly reduce total surplus; and (iii) independent suppliers have excessive incentives to enter the market for component B.*

To illustrate our argument above, consider the following example of a success-failure model. With or without integration, there are two producers of B—that is, either there are two independents or there is M plus one independent. The R&D cost function is $I(\rho) = \rho^2/(2k)$, where $0 < k < 1$, and $q_0 = 1$.

If both suppliers of B are independent, each will choose to invest $\rho = k/(1+k)$ and earn expected profit (net of R&D costs) of $k/[2(1+k)^2]$. If unintegrated, M earns expected profit of $\rho^2 = [k/(1+k)]^2$. Expected welfare is equal to $k/(1 + k)$, which is the maximum possible given the technology and the number of innovators.

If *M* integrates with (or replaces) one of the *B*-firms, the integrated firm will set $\rho = k$ (higher than each *B*-firm chooses absent integration, because of the quality squeeze) and will earn expected (integrated) profit of $k/2$. The remaining independent *B*-firm will set $\rho = k(1-k)$, which is lower than absent integration (because the integrated firm sets a higher ρ). Expected welfare is $k(1 - k)^2 + k^2/2$.

It follows (from these and simple further calculations) that integration is privately profitable, even if *M* has to pay full price for its partner (as opposed to threatening to cut it off, or merely to integrate with the other *B*-firm instead), and yet inefficient. Integration leads the integrated firm to do inefficiently more R&D (higher ρ), and this causes a welfare loss and a transfer of profit (on average) from a successful independent firm to the integrated firm, despite the (socially efficient) downward adjustment of the independent's R&D spending in response.

More generally, it is not clear in what circumstances firm *M* has excessive integration incentives when independents' entry decisions and R&D efforts (which *M* broadly values) are affected by *M*'s integration decision. There is, however, one case in which *M* fully internalizes the effects of its integration decision. Suppose demand is inelastic and there is free entry into *B* after *M* has committed either to integration or non-integration. Assuming away integer problems, the independent suppliers of *B* make zero expected profits given *M*'s integration decision. Given that consumers earn zero surplus in equilibrium, firm *M* internalizes all efficiency effects of its integration decision and thus has efficient incentives to integrate.

Proposition 5. *Suppose demand is inelastic and there is free entry into B after M has committed to integrating or not. Assuming away integer problems, firm M has efficient incentives to integrate under complete or incomplete information.*

This is a strong form of the internalization claim. It is important to note that even in this special case the logic does not carry over to decisions, such as the level of R&D investment, made by firm *M* after independent suppliers have made their entry decisions.

D. Analysis of Alternative Pricing Subgames

In this part, we briefly examine what would happen under some alternative simple pricing subgames (structures of the product-market stage). In particular, we examine the outcomes when: (1) firm *M* sets the price of component *A* before the independent firms set their prices; and (2) the independent firms set their prices after firm *M* has set the price of its component *B* but before it sets the price of component *A*. The goal is not to proliferate cases but to illuminate some arguably robust effects that are missing from our main model. To keep the illustrations relatively simple, we examine them in the simple case of complete information and inelastic demand.

Firm *M* sets the price of A first. In this case, *M* sets the price of A to be equal to (just below) the highest of the *B*-qualities. The highest-quality *B* then makes all the sales of *B* and makes infinitesimal quasi-profits: *M* has extracted all the quasi-profits through pricing A so high ex post. Naturally, the prospect of this destroys independent innovation in *B*. In a sense, *M* is behaving like a short-sighted regulator.

Firm *M*'s integrating may then be the only way to have innovation. If demand is inelastic, firm *M* has efficient incentives to innovate given that it is the sole innovator. However, it could nevertheless be socially and privately optimal to have multiple innovators. Firm *M* itself might want to combine the advantages of integration with the advantages of excellence through diversity, by keeping the independents flourishing while it enters. Under this ex post pricing game, it cannot do so. Firm *M* would like to commit to a better policy, but it is not so clear how to commit to ignoring good news about the supply of complement *B* in pricing component *A*.

Firm M prices its component B first. Firm *M* will in due course price *A* at the highest offered quasi-surplus, as in our main pricing game. But now there is first a price squeeze through the pricing of *B*. Suppose that *M*'s *B*-affiliate does not create the highest-quality *B*. Now, because the pricing of others' *B* products can react to the pricing of *M*'s, *M* has an incentive to price its inferior product *below* cost so as to affect the pricing of the superior product. Specifically, suppose that the highest-quality *B* is of quality q, and *M*'s is of quality $q_M < q$. Then if *M* sets the price of its *B* at infinitesimally above cost minus ($q - q_M$), the highest-quality *B* will have to respond by pricing its product just above cost. *M*'s inferior *B*-affiliate makes no sales in this price squeeze, but transfers all of the superior *B*-firm's quasi-rents to M's complementary operations in *A*.

Again, this pricing game (whose outcome is ex post optimal for *M*) destroys all ex ante incentives for independent innovation by *B*. Note that, taking firm *M*'s R&D level as given, independent suppliers of *B* conduct less than the socially optimal amount of R&D. This situation contrasts with the equilibrium in the main pricing game, where the independent suppliers conduct the socially efficient amounts of R&D *given M*'s (excessive) R&D level.

While this pricing game as described is particularly artificial, we think that its implication is not. Presumably in the (much more complex) pricing game played in reality, independent *B*-firms would react to the prices charged by M's affiliate. Qualitatively, then, the squeeze would work essentially as described.

Again, firm *M* itself may well have an incentive to try to avoid this problem, but it may be quite difficult. While the prospects for a commitment "not to price below cost" may be somewhat brighter than the prospects for a commitment to ignore innovations in *B* when pricing *A*, we are skeptical that it would be a manageable commitment. It seems quite possible that the most manageable kind of commitment would be to refrain from integration. That would preserve incentives for independent R&D (in some versions of the model), but loses internalization benefits and possible technical efficiencies of integration.

V. COOPERATION AND EXCLUSION

Whether or not it is an integrated supplier of component *B*, firm *M* may be able to take actions that improve or worsen an independent supplier's ability to offer value to consumers. Most dramatically, *M* might deny the independent supplier access to component *A*, but there are many other, less dramatic (or blatant) ways. For example, firm *M* might adjust how promptly and openly it gives an independent supplier of component *B* information about component *A*'s specifications and the design of the interface between the two components. In other cases, firm *M* may affect an independent's costs or quality through firm *M*'s choice of design for component *A* and the interface. In this section, we examine both firm *M*'s incentives to threaten or engage in exclusion and its incentives to help independent suppliers.

A. Cooperation and Integration

We first consider firm *M*'s incentives to cooperate with independent firms' attempts to improve their product qualities. Firm *M*'s incentives depend, in part, on what forces drive the pricing of the winning variant of *B*. When the leading innovation is *non-drastic*, the leader offers the same quasi-surplus to consumers as does the second-best variant when the latter is priced at cost. When an innovation is *drastic*, the winning supplier prices *B* as if it had no competition from other suppliers of *B*.

With complete information and inelastic demand, innovations always are non-drastic. Moreover, efficiency depends solely on the quality of the best variant. Firm *M*'s profits, however, can depend on the quality of the second-best variant. Indeed, when it is not integrated into the supply of *B*, firm *M*'s profit is equal to the second-highest *B*-quality level. And an integrated *M*'s profit is equal to the maximum of its own quality and the second-best independent quality. The dependence on the second-best indpendent quality gives *M* incentives to cooperate with the general process of quality improvement by *B*-firms. The incentive is not exactly aligned with efficiency, however, because *M* is concerned with the *second*-best variant of *B*, while welfare depends on the *best* variant.

This suggests two kinds of imperfections in *M*'s incentives to help independents innovate. First, *M* may take actions that help the second-best independent at the expense of helping the best.[24] Second, firm *M* may be unwilling to take actions that would improve the best of the independent variants. As an extreme case, when firm *M* is integrated and there is only one independent supplier of component *B*, firm *M*'s profit is

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[24] We consider one type of action—creating ex post spillovers—in the next part of this section.

equal to its quality level in *B*. Hence, in this case, firm *M* has no incentive to help the independent supplier.[25]

Under complete information, the lack of incentive to help the sole independent supplier holds for any R&D function and demand structure such that innovation is non-drastic. For sufficiently large differences in quality, the profit-maximizing price of the independent *B* will be driven by the overall demand for systems when demand is elastic. In other words, the innovation will be drastic.[26] In this case, a higher quality level by the leading supplier of *B* can result in increased profits for firm *M*, creating incentives for cooperation.

When information is incomplete, firm *M* can have incentives to help an independent supplier even when firm *M* is integrated, there is only one independent firm, and the innovation is non-drastic. The reason is that independent firm *N* cannot observe integrated firm *M*'s quality level and thus *N* must price based on its subjective beliefs about *M*'s actions. A higher quality of its variant generally will induce firm *N* to offer a higher level of quasi-surplus to consumers. This higher level of quasi-surplus will increase firm *M*'s profits from the sale of component *A*.

Summarizing this discussion, we have:

Proposition 6. *Under complete information, an integrated firm M facing a single independent supplier of component B has no incentive to cooperate with that supplier*

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25 Firm *M* also has no incentive to harm the supplier in this case.

26 Interpreting $-q$ as a cost, the standard conditions for an innovation to be non-drastic (and for the proposition to hold) can be applied when demand takes any of the following forms: $D(p) = \alpha + \beta p$, $D(p) = \alpha e^{\beta p}$, or $D(p) = \alpha p^{\beta}$, where α and β are constants. For this class of demand functions, a monopolist facing demand *D*

when innovations are non-drastic. With drastic innovations, multiple independent component suppliers, or incomplete information, firm M has incentives to cooperate.

B. Exclusion and Integration

We now return to the concern that firm *M* will use its control of the interface with component *A* to exclude competitors in *B*. In our model, there are three ways in which *threatening* exclusion could be profitable for firm *M*. First, firm *M* could demand side payments, or *access charges*, in return for granting interoperability. Second, firm *M* could insist that a supplier of component *B* commit to charging a low price for its output, which would increase the profits firm *M* would enjoy from the sale of component *A* (a strategy we labeled an *exclusionary squeeze* in the introduction). Third, if the threats and commitments can be made prior to the conduct of R&D, firm *M* might use the threat of exclusion (or the promise to exclude others) as a means of increasing an independent supplier's R&D investment.

The next Proposition (not formally proved) records our claim that in our model,[27] there is no reason to exclude independent suppliers if the above options are not available.

Proposition 7. *Suppose side payments are infeasible, firm M cannot condition access to A on B's price, and there is no possibility of binding access negotiations prior*

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would choose the same price as it would if it faced demand $x^*(p)$, where x^* is derived from *D* in the manner discussed in the text above. See the Appendix.

[27] Recall that our model rules out two other reasons firm *M* might work to drive independent suppliers of *B* out of the market. First, firm *M* faces no threat of two-stage entry. Second, because components *A* and *B* are perfect complements, there is no gain from excluding suppliers of component *B* in order to raise the price of component *B* paid by consumers.

to B's conducting its R&D. Then there is no incentive for firm M to threaten or engage in exclusion, whether or not it has integrated.

In closing, we note that actual and threatened exclusion are in principle available to *M* as strategies whether or not it is integrated into *B* ex ante. However, if it were illegal for *M* to engage in exclusion, integration could arguably make it harder to verify whether it had done so. If *M* is not integrated into *B*, verifying whether a threat to exclude has been carried out might be relatively easy. The reason is that, in carrying out the threat, *M* would have to find some other independent *B* partner who would not be cut off (or else no *B* would be available). By contrast, if *M* is integrated into B, then it could (with colorable arguments about confidentiality, appropriability of innovations, intellectual property rights, and so on) "close the interface" to all independent suppliers.

C. Spillovers and Grantbacks

We now consider another use of firm *M*'s ability to exclude independent suppliers. Firm *M* may be able to use its control over access to component *A* to extract commitments from independent suppliers of *B* to share their intellectual property with one another or with firm *M*. That is, firm *M* may be able to mandate licensing. We also address how firm *M* may use its ability to exclude as a response to the presence of significant ex post technological spillovers.[28] Ex post technological spillovers arise when, if one firm is the most successful innovator, some of the result leaks to other firms whether or not that firm grants a license to its intellectual property. In other words, other firms can imitate the innovator.

First, suppose that firm *M* is not integrated. It turns out that it generally has incorrect incentives even ex ante:

Proposition 8. *Suppose the number of suppliers of component B is fixed. With complete information and inelastic demand, firm M can have socially excessive incentives to generate ex post technological spillovers or mandate licensing.*

Proof: With complete information and inelastic demand, there is no social value to spillovers—the zero-spillover equilibrium yields the second-best levels of R&D investment, and improvements in the quality of the second-best variant of *B* have no value.

Spillovers can have a private value for firm *M*, however. When its variant of *B* is not the best one, firm *M* benefits from the increased competition due to an improvement in the second-best variant. When its variant of *B* is the best, firm *M* is indifferent with respect to the second-best quality.

Firm *M* balances the gain from improvements in the second-best variant against possible reductions in the levels of R&D investments made by independent suppliers. For some parameter values, the ex post private benefits of spillovers will exceed the private losses from the reduction in R&D investment. **QED**

The excessive incentive to impose spillovers emerges starkly when firm *M* is integrated and faces only one rival. Suppose there are two *B*-firms: an integrated *M*, and one independent, *N*. Suppose that firm *M* can force *N* to provide intellectual property to firm *M* in return for access to component *A* (a "grantback"). With complete information,

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[28] Ex ante spillovers would mean investment by one firm helps prospects of another, even if the former firm fails to be the leader. The analysis of how firm *M* would

firm *M*'s overall profit is always equal to its own quality level. The independent firm's R&D is socially desirable but of no value to *M* absent spillovers. Thus even if only a small amount of independent R&D will be forthcoming with a grantback, *M* would rather impose the grantback than encourage more independent innovation.[29]

Sometimes, however, there can be spillovers even in the absent of mandatory sharing or licensing. For example, one supplier may be able to reverse engineer the products of others. If these baseline spillovers are high enough, they may destroy the incentives to innovate: any firm that makes successful R&D investments may find its product quickly imitated and the resulting product-market rivalry may be so intense that recovery of the R&D outlays is impossible. In this case, firm *M* may restore intellectual property rights—and, thus, the incentives to innovate—by using its control of access to component *A* to keep imitators out of the market for *B*.

In this setting, firm *M* can have incentives to commit to exclusion if there are such high technological spillovers that incentives to innovate in component *B* are very low relative to the gross consumption benefits that innovation would create. In this case, firm *M* may use its control of access as a backdoor means of restoring intellectual property rights. It could either innovate itself or appoint an exclusive innovator.[30] Notice that this incentive can arise in either the complete or incomplete information pricing games.

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respond to them could be different.

29 Even if the spillover must be symmetric, *M* doesn't care, because if it wins the R&D competition in *B* it can optimally price *B* at cost and take the extra profits in A: it has no need to take its profit margin in B.

30 Ironically, with perfect spillovers there might not be a problem. At most one firm would enter anyway since multiple firms never can earn profits or cover any fixed costs of entry.

Proposition 9. *In the presence of high baseline technological spillovers, firm M may use its control of access as a backdoor means of restoring intellectual property rights.*

Of course, firm *M* can restore incentives only if it can credibly commit to blocking imitators from access. It might appear that exclusive contracts could be used, but notice that the contract would have to be signed prior to the R&D investment. But then the value of having multiple innovators is lost.

VI. CONCLUSION: PUBLIC POLICY PROBLEM, OR MANAGEMENT PROBLEM?

We have seen how there are complex—and often ex ante inefficient—effects of some strategies that firm *M* may pursue, and how some of these strategies may be made possible or easier if *M* integrates into the *B* market. These lessons apply even in the simplest case of inelastic demand and complete information, where the "one monopoly rent" or internalization theorem seems at its strongest. In this section we raise the question whether, or to what extent, these findings suggest the existence of a public policy problem, rather than just a management problem for *M*.

Much of our analysis concerned incentives to extract quasi-rents from independent *B*-innovators. The ex post incentive is of course clear, but following the internalization argument just rehearsed, one might ask whether an intelligently rapacious *M* would efficiently avoid such problems. Our analysis demonstrates that firm *M* may internalize ex ante a great deal of the effects. But the analysis also showed that in some cases *M* has even an ex ante incentive to squeeze independent firms.

An intriguing way to think of some of this is that firm *M* plays a role like that of a "public-interest" regulator: in many ways it has broadly efficient incentives (since it captures through *A* at least a part of what in a stand-alone *B* market would be consumer benefits), but problems arise because it has a great deal of power and commitments are difficult. Amidst concern lest excessive antitrust zeal bring regulation-style problems to the computer industry, we should not forget that an industry with a single gatekeeper would also be in some sense a regulated industry.

VII. APPENDIX

In this appendix we provide some results on the effects of double marginalization. Let $p^*(w)$ denote the profit-maximizing price for a monopolist facing demand curve *D* with unit cost ω. In other words, $p^*(w) = \text{argmax}_p D(p)\{p - w\}$. We examine how the price chosen by an "upstream" monopolist (such as a uniquely and drastically successful *B* firm in our model), knowing that its price will in turn be marked up by the downstream monopolist, compares to a unified single-monopolist choice. In particular, if these prices are the same, then we can say whether or not an efficiency difference is drastic without conditioning on the vertical structure.

Lemma A1. *If there exist constants* a *and* b *such that* $p^*(w) = a + bw$*, then* $argmax_p\ D[p^*(p)]\{p - c\} = argmax_p\ D(p)\{p - c\}$.

Proof: Let $p_0 = argmax_p\ D[p^*(p)]\{p - c\}$. Then p_0 satisfies the following first-order condition:

$$D(a + bp_0) + bD'(a + bp_0)\{p_0 - c\} = 0.$$

By the definition of p^*, the following first-order condition must also hold:

$$D(p^*(p_0)) + D'(p^*(p_0))\{p^*(p_0) - p_0\} = 0,$$

or

$$D(a + bp_0) + D'(a + bp_0)\{a + (b-1)p_0\} = 0.$$

Substitution then yields,

$$-D'(a + bp_0)\{a + (b\text{-}1)p_0\} + bD'(a + bp_0)\{p_0 - c\} = 0,$$

which is satisfied if and only if

$$\{a + (b\text{-}1)p_0\} + b\{p_0 - c\} = 0.$$

This last equation implies that $p_0 = a + bc$. However, by hypothesis, $a + bc =$ $\text{argmax}_p\ D(p)\{p - c\}$. **QED**

In words, Lemma A1 states conditions under which double marginalization by firm *M* has no effect on the price chosen by an independent supplier of *B.*

Lemma A2. *There exist constants* a *and* b *such that* $h^*(p) = a + bp$ *if and only if there exists a constant* g *such that* $gD'^2 = DD''$.

Proof: $p^*(w) = \text{argmax}_p\ D(p)\{p - w\}$ if and only if the following first-order condition is satisfied:

$$D(p^*(\omega)) + D'(p^*(\omega))\{p^*(\omega) - \omega\} = 0,$$

Total differentiation yields

$$\{2D'(p^*(\omega)) + D''(p^*(\omega))\{p^*(\omega) - \omega\}\}\text{d}p^* - D'(p^*(\omega))\text{d}\omega = 0,$$

or,

$$\text{d}p^*/\text{d}\omega = D'/\{2D' + D''(p^* - \omega)\ .$$

By the first-order condition for the optimality of p^*, $(p^* - \omega) = -D/D'$. Using this fact, one obtains

$$\text{d}p^*/\text{d}\omega = D'/\{2D' - D''D/D'\}\ .$$

Hence, $\text{d}p^*/\text{d}\omega \equiv \beta$ if and only if $D'^2 = \beta\{2D'^2 - D''D\}$. **QED**

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